Constructive MA Generation for 2D Models

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Abstract

Medial axis (MA) is widely used in many fields nowadays. However, the efficiency of generating MA of complicated models with current methods is still not satisfactory. A novel approach to constructively generate MA is proposed in this paper. With this method, the MA of the resultant model set up with two primitives and a Boolean operation upon them is generated by combining the MAs of the two primitives in a certain way instead of regenerating from scratch. First, the properties of MA are investigated. During Boolean operation, the boundaries that will vanish are found and the region of the model on the basis of which some new MA segments(MASs) need to be generated is determined, and the new MASs are generated based on the region using tracing method. The MA of the resultant model is constructed by combining the new generated MASs with the reserved MASs of the two primitives at last.

1. Introduction

Since medial axis(MA) and medial axis transformation(MAT) were proposed in 1967 [1], they are widely used in many fields. Currently, a lot of researches have been conducted on MA generation. Most of them are about generating MA of 3D mesh models or 2D discrete polygons. Of almost all the existing approaches, a common characteristic is that they are all devised to generate the MA of a model in an integral way rather than in a constructive way. One problem with current MA generation algorithms is that their efficiency is still not satisfactory for complex models. Especially, once a complicated model whose MA has been generated is changed, its MA has to be regenerated from scratch no matter how little the modification is. To solve the problem, a constructive MA generation method is proposed for 2D models in this paper. The feature of the proposed approach is that the MA of a model is procedurally generated along with the modeling process of the model based on the MAs of the primitives used to construct the model. In this way, when the model is modified by performing Boolean operations, its MA can be just adapted locally rather than regenerated from scratch. Thus the computational efficiency can be improved dramatically.

2. Related work

The approaches to generating MA can be classified into three categories: thinning, tracing and Voronoi graph based methods.

Thinning method is to generate the MA of a model by generating the MA of another approximate model. Based on the diffusing process of combined wave, G.L.Scott *et al* propose a method to determine the symmetric axis of a model which is the superset of MA [2]. K.Siddiqi *et al* [3] put forward a vector field based thinning method. In their method, each voxel is given a vector pointing to the nearest boundary point and if the average flux for a voxel into its adjacent voxels is positive the voxel must be on MA. I. Ragnemalm [4] gives a shortest Euclidean distance to the boundary for each voxel and obtained MA by calculating the local gradient maxima..

Based on local continuity, tracing method is to generate the MA by tracing some special MA points. U.Montanari *et al* set forward a method to calculate the MA for the deformed multiply-connected region [5]. In their method, the key issue is to determine the crucial bifurcation points and offset the boundary inward. D.T.Lee proposes a method whose computation complexity is $O(n\log n)$ for convex polygons and $O(n^2)$ for concave polygons [6]. For the multiply-connected region with h holes, V. Srinivasan and L. R. Nackman put forward a method whose computational complexity is $O(nh+n\log n)$ [7]. Similarly, Gursoy and Patrikalakis propose a method for 2D models consisting of

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multiply-connected regions with the boundary being line and arc [8-9]. T. Culver *et al* [10] propose a space partition based method to reduce the computational complexity for tracing.

Based on the duality relation between Voronoi diagram and Delaunay triangulation, MA can also be calculated approximately from the Voronoi diagram. If the sampling density approaches infinity, the Voronoi vertices in this case converge to MA. J.M. Reddy and G.M. Turkiyyah [11] set forward a method of 3D skeleton of polyhedron based on the generalized Voronoi diagram. They generalize the conventional Delaunay triangulation with two isolated nodes being connected with a straight line. And the generalized triangulation is still dual with Voronoi diagram and is easier to be calculated. D. Lavender *et al* [12] generate the Voronoi diagram based on Octree (Quadtree on 2D) which can divide the space into Voronoi zones according to a given resolution. For 2D zone and 3D space, J.W. Brandt *et al* propose a continuous method to approximate the skeleton with boundary discretization [13-14].

3. Preliminaries

3.1. Basic concepts

Definition 1. **MA and MAT of a 2D model**

The following definition is always used for practical calculation of MA [10]:

A point set in the 2D model, the maximum inscribed circle centered at each point of which is tangent with the boundary edges at least at two points of the 2D model.

Definition 2. **Generative elements of MA**: the boundary elements tangent with the maximum inscribed circle during the generation of MA. For a 2D model, all the boundary edges and the concave vertices are the generative elements of a certain MA. The generative elements of the same MAS are called conjugated boundaries each other.

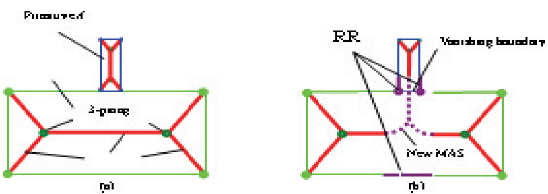


Figure 1. Illustration of some concepts

Definition 3. **N-prong MA point**: the center point of the maximum inscribed circle that is tangent with the model boundary at N different points.

Definition 4. **Constructive generation of MA**: The MA of a model is procedurally generated along with the modeling process of the model based on the MAs of the primitives used to construct the model. As shown in Figure1, given the MAs of Primitive *A* and Primitive *B*, the MA of the resultant model constructed by performing the Boolean union operation upon primitive *A* and *B*.

Definition 5. **Vanishing boundary**: The boundary of primitives that is to vanish in the resultant model after the Boolean operation.

Definition 6. **Regeneration region (RR)**: In order to correctly construct the MA of the resultant model after a Boolean operation is performed upon two primitives, the MASs of the two primitives need to be combined with certain MASs that can be generated based on a certain region of the resultant model. Such region is called Regeneration region.

3.2. Properties of MA

Property 1: MAT is semi-continuous.

As shown in Figure 1(b), the RR for each Boolean operation is definite. The MA of the resultant model is obtained just by subtracting some MASs from the MASs of the Primitive *A* and Primitive *B* and adding some new MASs.

Property 2: MAT is region-separable.

As shown in Figure 2, if the region 1, 2 or 3 in Figure 2(a) is modified, the MASs needed to be modified in the region 1', 2' or 3' are shown in Figure 2(b) respectively, and the MASs of other regions need not be changed.

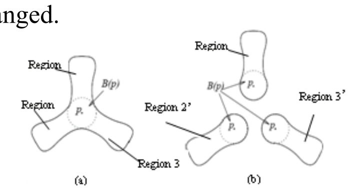


Figure 2. Region-separable of MA

3.3. The basic MA types

Usually, each MAS is generated from two generative elements. And the basic types of boundary elements are point, straight line, circle (arc) and freeform curve. In this paper, only the first three types of boundary elements are considered. All the possible

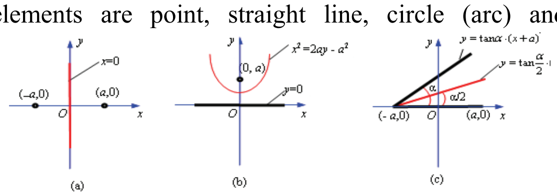


Figure 3. MA forms corresponding to three simple type combinations

type combinations of generative elements are listed in Table 1.

For the type combinations (1), (2) and (3) listed in Table 1, the forms of MA are line, parabola and line respectively as shown in Figure 3. For the other complicated combinations (4), (5), (6) in Table 1, the forms of MA should be deduced according to geometric relationships. The deduction process is given in [17] and the results are given in Table 2 and illustrated in Figure 4.

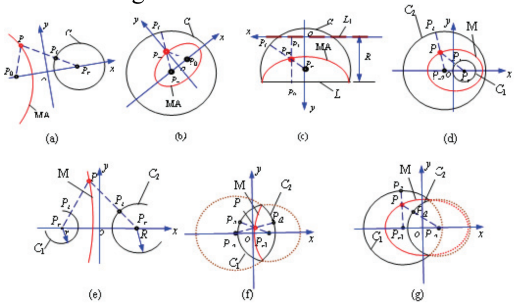


Figure 4. MA forms for complicated combination

Table 1. All possible types of combinations of generative elements

	Point	Straight line	Circle
Point	Point-point (1)	Point-line (2)	Point-circle (4)
Straight line		Line-line (3)	Line-circle (5)
Circle			Circle-circle(6)

Table 2. MA forms for complicated combination

	Figure	Type of MA
Point-circle(1)	5(a)	hyperbola
Point-circle(2)	5(b)	ellipse
Line-circle	5(c)	parabola
Circle-circle(1)	5(d)	ellipse
Circle-circle(2)	5(e)	hyperbola
Circle-circle(3)	5(f)	hyperbola
Circle-circle(4)	5(g)	ellipse

4. The process of constructive generation of MA

To shorten the computational time of constructively generating MA, the key issue is to efficiently determine the RR firstly. And then generate the new MASs for the RR and combine the new MASs with the reserved MASs of the two primitives involved in constructing the model to get the MA of the resultant model. The process can be divided into five steps as shown in Figure 5.

4.1. Perform Boolean operation and break the effected MA

For union operation, as shown in Figure 6(a), the new generated concave points V_1 and V_2 break the boundary edges of primitives A and B into several

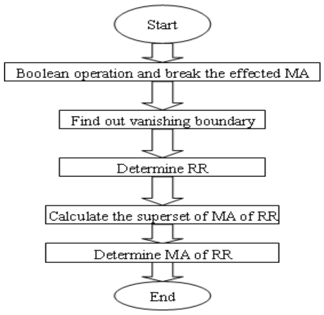
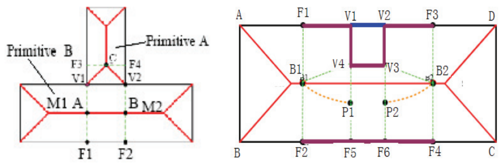


Figure 5. Overview of the method

segments, and they also break the corresponding MASs into several segments. To calculate the broken points on MAS when conducting union operation, it needs to generate a straight line that passes through the new concave point and perpendicular to the edge it depends on, and then the intersection point of the corresponding MAS and the generated straight line is the broken point. But for Boolean subtraction, this will be more complex. The way to get the broken points is to make a conic. Then the intersection point of the MAS and the conic is the real broken point. Which type of the conic curve to be used to make the intersection depends on the type of MA between the concave point and the conjugated boundary edge that generates the MAS as mentioned in 3.3. For example, in Figure 6(b), the conjugated boundary edge that generates M_1M_2 is BC, and it is a straight line segment, so we use parabola B_1P_1 (the focus is V_4 and the directrix is BC) and parabola B_2P_2 (the focus is V_3 and the directrix is BC) to do the intersection with M_1M_2.



(a) (b)

Figure 6. Break the effected MA

4.2. Determination of vanishing boundary and RR

Some boundary edges of the two primitives which will vanish after a Boolean operation is performed. This implies the useful heuristics for determining the RR of the resultant model. They should be found out. It is observed from Figure 7 that the boundary edges of the resultant model of conducting Boolean intersection operation between primitives A and B are just the vanishing boundary edges when conducting Boolean union operation between them. We use a set named as **ES** to contain all the vanishing boundary edges. And points V_1 and V_2 should be saved into RR. We use a set named as **BPS** to record the broken points mentioned above.

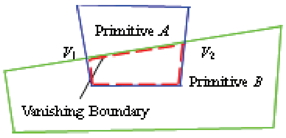


Figure 7. Determination of vanishing boundary

After determining the vanishing boundary edges, the RR can be determined. And the determination of RR can be conducted as follows:

(1) Clear the set of RR, add the newly generated concave points to the set. Clear **BPS.**

(2) Initiate an empty stack called **MS**. Get an edge VE_i from the set **ES**, push all MASs that depend on it into the stack **MS**.

(3) While the stack is not empty, pop up MAS from **MS**, get the conjugated generated element and add it into the set of RR. For Boolean subtraction operation, we should also add the other generated elements of the MAS which will not vanish in the resultant model into RR. Then remove the MA from the resultant model and add the endpoint of the MA in **BPS** if the endpoint is not on the boundary edges**.**

As shown in Figure 8(b), for Boolean subtraction operation, besides F_2F_4, F_1V_1 and F_3V_2 which also generate points on B_1B_2 should be added in the set of RR. But V_1V_2 will vanish, so it should not be added into RR.

(4) Remove VE_i from the set **ES.**

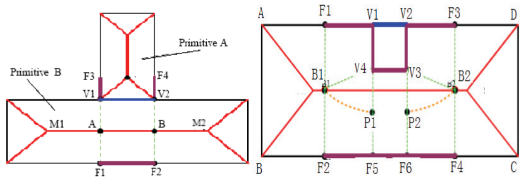
(5) Go to step (2), until the set **ES** is empty.

(6) For subtraction operation, some of the edges of the subtrahend primitive model which will not vanish in the resultant model should also be put into the set of RR.

In Figure 8, V_1V_4, V_3V_4 and V_2V_3 should be added into the set of RR.

(7) Refine the set of RR. There may be some overlapped parts between the boundary edges of RR and they should be refined.

After the seven steps above, in Figure 8(a), **RR** = { V_1, V_2, F_1F_2, V_1F_3, V_2F_4 }, **BPS** = { A, B, C }. And in Figure 8(b), **RR** = { V_3, V_4, F_2F_4, F_1V_1, F_3V_2, V_1V_4, V_4V_3, V_3V_2 }, **BPS** = { B_1, B_2 }.



(a) (b)

Figure 8. (a) Determined RR in union operation; (b) Determined RR in subtraction

4.3. Calculate the superset of MA in RR.

The superset of new MAS is the set consisting of all the MASs which can be generated based on every two elements in RR, and it includes all the new MASs as well as the unnecessary MASs. Obviously it is easy to get the superset of new MAS using the methods given in section 3.3 just by combine every two boundary elements in RR.

4.4. Determine the final set of new MASs.

In this paper, tracing method is used to effectively select the suitable MASs. Firstly, take a broken point as the start point to trace the new MAS.

According to the tracing method, after an effective MAS (MA_i) and its endpoints ($P_{st,i}$ and $P_{end,i}$) are determined, the next effective MASs MA_{i+1}^{j} that are adjacent to MA_i can be determined similarly. Here without consideration of the degeneration, the number of MASs connecting MA_i must be j=1 or j=2 since each generative element of the 2-prong MA MA_i or the adjacent boundary of each generative element must be the one of the next MAS according to the continuity of MA. Supposing that MAS MA_i and its start point $P_{st,i}$ are known, the process for determining { MA_{i+1}^{j} } (j=1, 2) and the end point of MA_i can be described as follows :

(1) Find out all MASs that intersect with MA_i and arrange them and the intersection points according to the distances between $P_{st,i}$ and the intersection points in increasing order.

(2) Make a circle disc with the given intersection point as its center and the distance between the given intersection point and each generative element of MA_i as its radius. Then check:

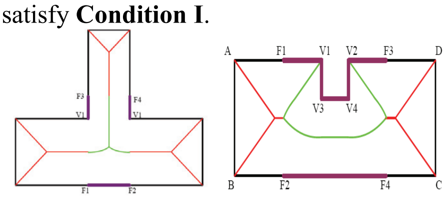
Condition I: The circle disc lies inside **RR**, and besides the tangent points with the generator of MA_i, there is another one and only one tangent point with the boundary edges of the model.

If so, the intersection point is the end point of MA_i.

(3) Divide each of the segment(s) { MA_{i+1}^{j} } (j=1, 2)

into two segments with $P_{st,i+1}$ being the start point of them. Furthermore, decide which segment is the effective MAS.

(4) Repeat above steps until there is no intersection point between the current MAS MA_i and any of the left MASs or all the intersection points do not satisfy **Condition I**.



(a) (b)

Figure 9. MA in resultant model

After the new MASs are determined for the RR, the final MA of the resultant model can be obtain by combining the new MASs with the reserved MASs of the two primitives. Then the algorithm ends.

Figure 9 shows the resultant MA based on RR. The MASs in green color are newly generated ones and those in red color are the reserved.

4.5. Discussion for Boolean intersection

Generally, we make model by union and subtraction operation. However, the proposed constructive MA generation method is not very suitable for Boolean intersection and the reasons are: (1). There are only few edges of the involved models that will be reserved during Boolean intersection. Therefore few MASs can be reserved. (2) Even if both generative elements are not deleted during Boolean intersection, it is guaranteed that the MAS will be reserved. Thus the time to determine which MAS will not be deleted is relatively considerable. As shown in Figure 10, $V_1V_2V_3V_4$ is a square after intersection operation, and no MAS can be reserved in the primitives. (3) Boolean intersection is not used frequently and the edges of the resultant models of Boolean intersection are always few. Therefore the computational efficiency will not be low if the MA is generated from scratch for the resultant model.

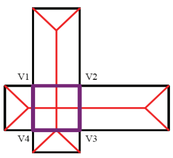
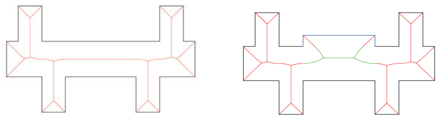


Figure 10. Intersection operation

5. Examples and discussion

The proposed method is implemented with visual C++.net 2005 based on the geometric modeling kernel ACIS 13.0 (test on PC with Pentium IV 2.8G CPU and 512M RAM).

In order to show the advantage of it, the proposed method is compared with the method in which MA is generated using tracing from scratch for the whole model [6].

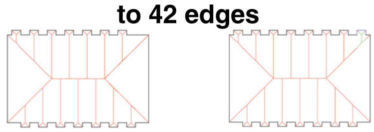


(a) (b)

Figure 10. A model modified from 20 edges to 24edges

(a) (b)

Figure 11. A model modified from 38 edges to 42 edges



(a) (b)

Figure 12. A model modified from 60 edges to 64 edges

Figure 10-12 show the local modification on 2D models ((a) to (b)). The new generated MASs based on RR are marked in green. Table 3 shows the calculation time of the two methods. T1 denotes the time using the tracing method that generate the MA from scratch for the whole model, and T2 denotes the time using the proposed method.

Table 3 Comparison of calculation time of two methods

Figure	Number of edges	T1(s)	T2 (s)	Ratio(T1/T2)
16	24	3.800	0.094	40.43
17	42	20.000	0.095	210.53
18	64	60.080	0.099	606.87

As shown in table 3, it can be seen that if MA is generated from scratch for the whole model, it is time-wasting. In contrast, the computational time is dramatically reduced if the proposed method is used.

A more elaborate test is conducted and the result is given in Figure 13. It can be seen from the figure that in the case that the model is very complicated and just locally modified by performing a Boolean operation, the computational time keeps in a certain level. Whereas if the MA is regenerated from scratch for the whole model, it will increase in a terrible way, and is approximate to $O(n^2)$ which is corresponding to the theory in [6]. Based on the analysis, it is obvious that although it is not easy to calculate the complexity for the proposed method in any case since it is difficult to determine the number of elements (including edges and concave vertexes) affected during the Boolean operation, the computational efficiency of the proposed method is improved dramatically in the case that the model is very complicated and just locally modified.

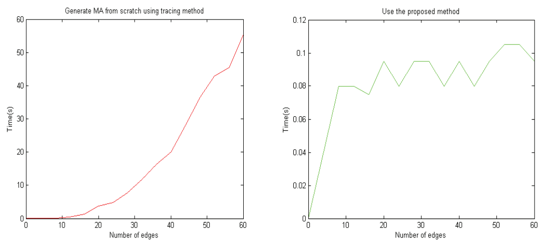
6. Conclusions

A novel approach to constructively generate MA of a 2D model is proposed in this paper. The major contributions of the work include:

(1) With the approach, the MA of a model which is procedurally generated based on the MAs of the primitives is used to construct the model. Similarly, when the model is modified locally by performing a Boolean operation, the MA of the resultant model is obtained by adapting the original MA locally rather than regenerating from scratch. Therefore, the computational efficiency can be improved dramatically, especially for complicated models with minor modification.

(2) The concept RR is set forward and the methods of automatically determining the RR of all Boolean operations are given, which guarantee that the original MASs can be reused as much as possible to generate the MA of the resultant model.

The work is still in its infancy and there is a lot of work to do, such as extending the method to 3D models and enabling the method to deal with parametric modification.



(a) (b)

Figure 13. Relation of the time and the number of edges using two methods

Acknowledgements

The authors are very grateful to the financial support from NSF of China (No.60403049) and the 863 High-Technology Project (No.2006AA01Z313, 2006AA01Z335).

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