Part 1

Question 1

1)

P(X,Y,X) = P(X) x P(Y|X) x P(Z|X,Y) product chain rule

= P(X) x P(Y|X) x P(Z|Y) conditional independent

X	Y	Z	Working	P(X,Y,Z)
0	0	0	= P(X=0) x P(Y=0|X=0) x P(Z=0|Y=0) = 0.35 x 0.1 x 0.7	=0.0245
0	0	1	= P(X=0) x P(Y=0|X=0) x P(Z=1|Y=0) = 0.35x0.1x0.3	= 0.0105
0	1	0	= P(X=0) x P(Y=1|X=0) x P(Z=0|Y=1) =0.35x0.9x0.2	= 0.063
0	1	1	= P(X=0) x P(Y=1|X=0) x P(Z=1|Y=1) =0.35x0.9x0.8	= 0.252
1	0	0	= P(X=1) x P(Y=0|X=1) x P(Z=0|Y=0) =0.65x0.6x0.7	= 0.273
1	0	1	= P(X=1) x P(Y=0|X=1) x P(Z=1|Y=0) =0.65x0.6x0.3	= 0.117
1	1	0	= P(X=1) x P(Y=1|X=1) x P(Z=0|Y=1) =0.65x0.4x0.2	= 0.052
1	1	1	= P(X=1) x P(Y=1|X=1) x P(Z=1|Y=1) =0.65x0.4x0.8	= 0.208
				=1

2)

P(X,Y) = P(Y|X) x P(X) Product Rule

P (X = 0, Y = 0) = P(Y=0|X=0) x P(X=0) = 0.1 x 0.35 = 0.035

P (X = 0, Y = 1) = P(Y=1|X=0) x P(X=0) = 0.9 x 0.35 = 0.315

P (X = 1, Y = 0) = P(Y=0|X=1) x P(X=1) = 0.6 x 0.65 = 0.39

P (X = 1, Y = 1) = P(Y=1|X=1) x P(X=1) = 0.4 x 0.65 = 0.26

	Y=0	Y=1
X=0	0.035	0.315
X=1	0.39	0.26
		=1

3)

(a) P (Z = 0),

P(Z = 0) = 0.0245 + 0.063+0.273+0.052 = 0.4125

(b) P (X = 0, Z = 0),

P(X = 0, Z = 0) = 0.0245 + 0.273 = 0.2975

(c) P (X = 1, Y = 0|Z = 1),

P(X=1,Y=0,Z=1) = 0.117

P(Z=1) = 1-P(Z=0) = 1-0.4125 = 0.5875

P (X = 1, Y = 0|Z = 1) = 0.117/0.5875 = 0.1991

(d) P (X = 0|Y = 0, Z = 0).

P(Y=0,Z=0) = 0.2975

P(X=0,Y=0,Z=0) = 0.0245

P (X = 0|Y = 0, Z = 0) = 0.0245/0.2975 = 0.08235

Question 2

(i) P (B = t, C = t) = P(C=t) x P(B=t|C=t) = 0.4 x 0.2 = 0.08 product rule

(ii) P (A = f |B = t) = 1 – P(A=t|B=t) = 1 – 0.3 = 0.7 Normalisation rule

(iii) P (A = t, B = t|C = t) = P(A=t|C=t) x P(B=t|C=t) = 0.5 x 0.2 = 0.1 Conditional independence

(iv) P (A = t|B = t, C = t) = P(A=t|C=t) = 0.5 conditional independence

(v) P (A = t, B = t, C = t) = P(C=t) x P(A=t|C=t) x P(B=t|A=t,C=t) product chain rule

= P(C=t) x P(A=t|C=t) x P(B=t|C=t) = 0.4 x 0.5 x 0.2 = 0.04 conditional independence.

Part 2

1.

Probability of: 'no-recurrence-eventsage10-19' = 0.005050505050505051

Probability of: 'no-recurrence-eventsage20-29' = 0.010101010101010102

Probability of: 'no-recurrence-eventsage30-39' = 0.1111111111111111

Probability of: 'no-recurrence-eventsage40-49' = 0.31313131313131315

Probability of: 'no-recurrence-eventsage50-59' = 0.3282828282828283

Probability of: 'no-recurrence-eventsage60-69' = 0.1919191919191919

Probability of: 'no-recurrence-eventsage70-79' = 0.030303030303030304

Probability of: 'no-recurrence-eventsage80-89' = 0.005050505050505051

Probability of: 'no-recurrence-eventsage90-99' = 0.005050505050505051

Probability of: 'no-recurrence-eventsmenopauselt40' = 0.03125

Probability of: 'no-recurrence-eventsmenopausege40' = 0.4583333333333333

Probability of: 'no-recurrence-eventsmenopausepremeno' = 0.5104166666666666

Probability of: 'no-recurrence-eventstumor-size0-4' = 0.03980099502487562

Probability of: 'no-recurrence-eventstumor-size5-9' = 0.024875621890547265

Probability of: 'no-recurrence-eventstumor-size10-14' = 0.12935323383084577

Probability of: 'no-recurrence-eventstumor-size15-19' = 0.11442786069651742

Probability of: 'no-recurrence-eventstumor-size20-24' = 0.17412935323383086

Probability of: 'no-recurrence-eventstumor-size25-29' = 0.15920398009950248

Probability of: 'no-recurrence-eventstumor-size30-34' = 0.1691542288557214

Probability of: 'no-recurrence-eventstumor-size35-39' = 0.05970149253731343

Probability of: 'no-recurrence-eventstumor-size40-44' = 0.0845771144278607

Probability of: 'no-recurrence-eventstumor-size45-49' = 0.014925373134328358

Probability of: 'no-recurrence-eventstumor-size50-54' = 0.024875621890547265

Probability of: 'no-recurrence-eventstumor-size55-59' = 0.004975124378109453

Probability of: 'no-recurrence-eventsinv-nodes0-2' = 0.7970297029702971

Probability of: 'no-recurrence-eventsinv-nodes3-5' = 0.08415841584158416

Probability of: 'no-recurrence-eventsinv-nodes6-8' = 0.039603960396039604

Probability of: 'no-recurrence-eventsinv-nodes9-11' = 0.01485148514851485

Probability of: 'no-recurrence-eventsinv-nodes12-14' = 0.009900990099009901

Probability of: 'no-recurrence-eventsinv-nodes15-17' = 0.019801980198019802

Probability of: 'no-recurrence-eventsinv-nodes18-20' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsinv-nodes21-23' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsinv-nodes24-26' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsinv-nodes27-29' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsinv-nodes30-32' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsinv-nodes33-35' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsinv-nodes36-39' = 0.0049504950495049506

Probability of: 'no-recurrence-eventsnode-capsyes' = 0.1256544502617801

Probability of: 'no-recurrence-eventsnode-capsno' = 0.8743455497382199

Probability of: 'no-recurrence-eventsdeg-malig1' = 0.2916666666666667

Probability of: 'no-recurrence-eventsdeg-malig2' = 0.5104166666666666

Probability of: 'no-recurrence-eventsdeg-malig3' = 0.19791666666666666

Probability of: 'no-recurrence-eventsbreastleft' = 0.5078534031413613

Probability of: 'no-recurrence-eventsbreastright' = 0.49214659685863876

Probability of: 'no-recurrence-eventsbreast-quadleft_up' = 0.34536082474226804

Probability of: 'no-recurrence-eventsbreast-quadleft_low' = 0.36597938144329895

Probability of: 'no-recurrence-eventsbreast-quadright_up' = 0.10824742268041238

Probability of: 'no-recurrence-eventsbreast-quadright_low' = 0.09278350515463918

Probability of: 'no-recurrence-eventsbreast-quadcentral' = 0.08762886597938144

Probability of: 'no-recurrence-eventsirradiatyes' = 0.15706806282722513

Probability of: 'no-recurrence-eventsirradiatno' = 0.8429319371727748

Probability of: 'recurrence-eventsage10-19' = 0.011494252873563218

Probability of: 'recurrence-eventsage20-29' = 0.011494252873563218

Probability of: 'recurrence-eventsage30-39' = 0.1839080459770115

Probability of: 'recurrence-eventsage40-49' = 0.3103448275862069

Probability of: 'recurrence-eventsage50-59' = 0.25287356321839083

Probability of: 'recurrence-eventsage60-69' = 0.19540229885057472

Probability of: 'recurrence-eventsage70-79' = 0.011494252873563218

Probability of: 'recurrence-eventsage80-89' = 0.011494252873563218

Probability of: 'recurrence-eventsage90-99' = 0.011494252873563218

Probability of: 'recurrence-eventsmenopauselt40' = 0.012345679012345678

Probability of: 'recurrence-eventsmenopausege40' = 0.38271604938271603

Probability of: 'recurrence-eventsmenopausepremeno' = 0.6049382716049383

Probability of: 'recurrence-eventstumor-size0-4' = 0.022222222222222223

Probability of: 'recurrence-eventstumor-size5-9' = 0.011111111111111112

Probability of: 'recurrence-eventstumor-size10-14' = 0.022222222222222223

Probability of: 'recurrence-eventstumor-size15-19' = 0.07777777777777778

Probability of: 'recurrence-eventstumor-size20-24' = 0.15555555555555556

Probability of: 'recurrence-eventstumor-size25-29' = 0.2111111111111111

Probability of: 'recurrence-eventstumor-size30-34' = 0.25555555555555554

Probability of: 'recurrence-eventstumor-size35-39' = 0.08888888888888889

Probability of: 'recurrence-eventstumor-size40-44' = 0.07777777777777778

Probability of: 'recurrence-eventstumor-size45-49' = 0.022222222222222223

Probability of: 'recurrence-eventstumor-size50-54' = 0.044444444444444446

Probability of: 'recurrence-eventstumor-size55-59' = 0.011111111111111112

Probability of: 'recurrence-eventsinv-nodes0-2' = 0.4725274725274725

Probability of: 'recurrence-eventsinv-nodes3-5' = 0.17582417582417584

Probability of: 'recurrence-eventsinv-nodes6-8' = 0.12087912087912088

Probability of: 'recurrence-eventsinv-nodes9-11' = 0.06593406593406594

Probability of: 'recurrence-eventsinv-nodes12-14' = 0.03296703296703297

Probability of: 'recurrence-eventsinv-nodes15-17' = 0.04395604395604396

Probability of: 'recurrence-eventsinv-nodes18-20' = 0.01098901098901099

Probability of: 'recurrence-eventsinv-nodes21-23' = 0.01098901098901099

Probability of: 'recurrence-eventsinv-nodes24-26' = 0.02197802197802198

Probability of: 'recurrence-eventsinv-nodes27-29' = 0.01098901098901099

Probability of: 'recurrence-eventsinv-nodes30-32' = 0.01098901098901099

Probability of: 'recurrence-eventsinv-nodes33-35' = 0.01098901098901099

Probability of: 'recurrence-eventsinv-nodes36-39' = 0.01098901098901099

Probability of: 'recurrence-eventsnode-capsyes' = 0.4

Probability of: 'recurrence-eventsnode-capsno' = 0.6

Probability of: 'recurrence-eventsdeg-malig1' = 0.1111111111111111

Probability of: 'recurrence-eventsdeg-malig2' = 0.35802469135802467

Probability of: 'recurrence-eventsdeg-malig3' = 0.5308641975308642

Probability of: 'recurrence-eventsbreastleft' = 0.55

Probability of: 'recurrence-eventsbreastright' = 0.45

Probability of: 'recurrence-eventsbreast-quadleft_up' = 0.30120481927710846

Probability of: 'recurrence-eventsbreast-quadleft_low' = 0.3855421686746988

Probability of: 'recurrence-eventsbreast-quadright_up' = 0.1686746987951807

Probability of: 'recurrence-eventsbreast-quadright_low' = 0.08433734939759036

Probability of: 'recurrence-eventsbreast-quadcentral' = 0.060240963855421686

Probability of: 'recurrence-eventsirradiatyes' = 0.3875

Probability of: 'recurrence-eventsirradiatno' = 0.6125

2.

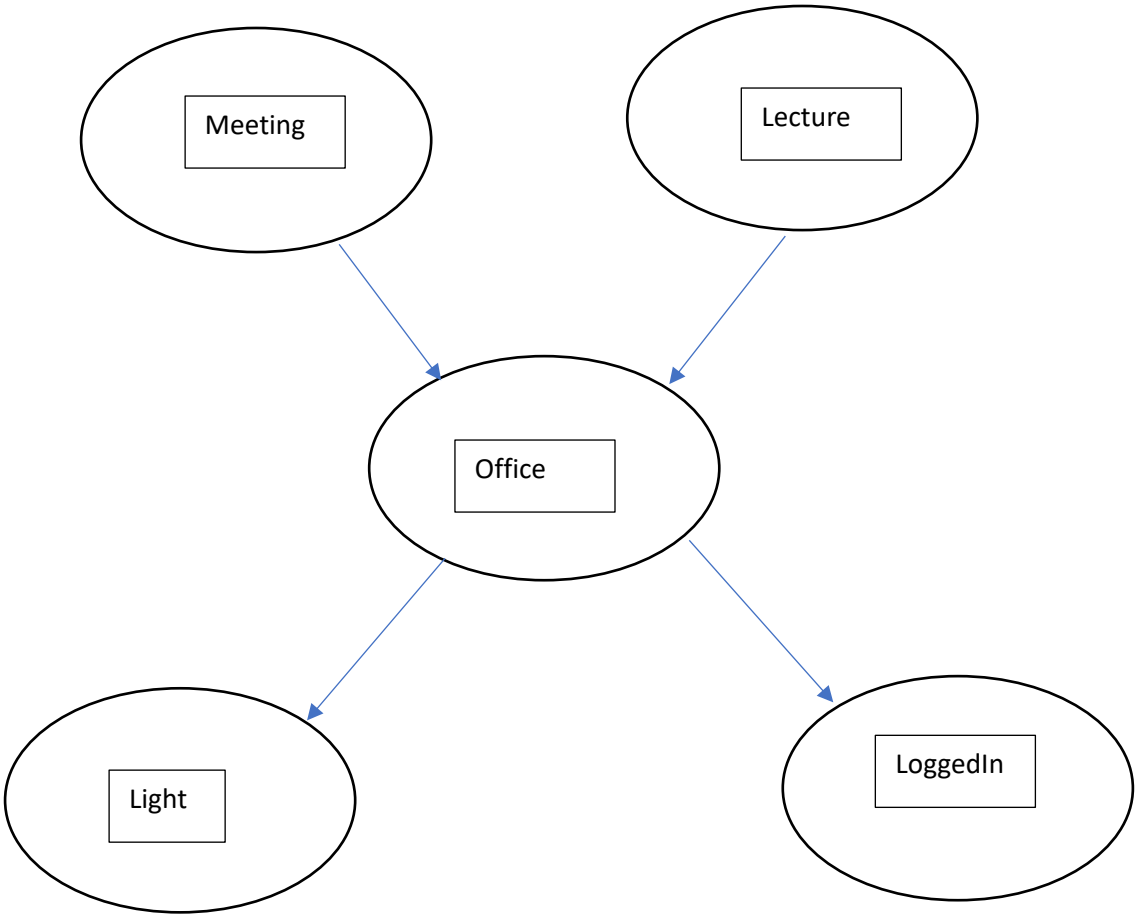
```
no-recurrence-events = 0.7063197026022305
recurrence-events = 0.2936802973977695
```

3.

```
no-recurrence-events score: 4.017731924138001e-06
recurrence-events score: 7.096642912782161e-06
Prediction: recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 0.00033365875538186835
recurrence-events score: 2.6021024013534593e-05
Prediction: no-recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 4.707378797592419e-05
recurrence-events score: 9.715041077797865e-07
Prediction: no-recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 0.00015163354504655905
recurrence-events score: 1.0219165794406317e-05
Prediction: no-recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 4.34523614378287e-06
recurrence-events score: 1.9205993914751727e-06
Prediction: no-recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 0.0006000123072810078
recurrence-events score: 3.880403296274196e-05
Prediction: no-recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 0.0002071279390811688
recurrence-events score: 7.415569159796725e-05
Prediction: no-recurrence-events Actual: no-recurrence-events

no-recurrence-events score: 0.00031504865826699285
recurrence-events score: 8.934762413310772e-06
Prediction: no-recurrence-events Actual: recurrence-events

no-recurrence-events score: 3.9123945169671666e-05
recurrence-events score: 6.483884504037111e-05
Prediction: recurrence-events Actual: recurrence-events

no-recurrence-events score: 4.1017039290784815e-05
recurrence-events score: 5.283165151437645e-05
Prediction: recurrence-events Actual: recurrence-events

accuracy is: 8/10
```

Part 3

1.



P(Meeting)
0.7

P(Lecture)
0.6

Meeting	Lecture	P(Office|Meeting, Lecture)
T	T	0.95
T	F	0.75
F	T	0.8
F	F	0.06

Light	P(Light | Office)
T	0.5

F	0.02

LoggedIn	P(LoggedIn | Office)
T	0.8
F	0.2

2. Free Parameters = 1+1+4+2+2 = 12

3. P(Meeting=true) x P(Lecture=false)xP(office|Meeting=true,Lecture=false)xP(light=false|office)xP(LoggedIN=true|office)

=0.6 x 0.3 x 0.8 x 0.5 x 0.8 = 0.0576

4. P(Office=true) =

P(office=true | meeting=true , lecture=true) x P(meeting=true) x P(lecture=true) +

P(office= true | meeting=true, lecture= false) x P(meeting=true) x P(lecture= false) +

P(office=true | meeting= false, lecture=true) x P(meeting= false) x P(lecture=true) +

P(office=true |meeting= false, lecture= false) x P(meeting= false) x P(lecture= false) +

= (0.95 x 0.7 x 0.6) + (0.75 x 0.7 x 0.4) + (0.8 x 0.3 x 0.6) + (0.06 x 0.3 x 0.4)

= 0.399 + 0.21 + 0.144 + 0.0072

= 0.7602

5. P(loggedOn=true, light=false | Office=true)

= P(loggedOn=true | Office=true) x P(light=false | Office=true) conditional prob rule

= 0.8 x 0.5 = 0.4

Part 4

1. Evidence = Xray
   Hidden = Cancer, Smoker, Dyspnoea
   Query = Pollution
2. Order of hidden variables: D -> C -> S

1) Initial factors:

$f_1(P) = P(P)$:

P	P(P=t)
t	0.90
f	0.10

$f_2(S) = P(S)$:

S	P(S=t)
t	0.30
f	0.70

$f_3(C, P, S) = P(C|P,S)$:

C	P	S	P(C|P,S)
t	t	t	0.05
f	t	t	0.95
t	t	f	0.02
f	t	f	0.98
t	f	t	0.03
f	f	t	0.97
t	f	f	0.001
f	f	f	0.999

$f_4(C) = P(X=t|C)$:

C	P(X=t|C)
t	0.90
f	0.20

$f_5(D, C) = P(D|C)$:

D	C	P(D|C)
t	t	0.65
f	t	0.35
t	f	0.30
f	f	0.70

Iteration 1: Eliminate D from $f_5(D, C)$ to get $f_6(C)$ (+2)

C	$f_6(C)$
t	0.65+0.35 = 1
f	0.30+0.70 = 1

Iteration 2:

1) Join all factors containing C, $f_3(C, P, S)$, $f_4(C)$ and $f_6(C)$ (×16)

C	P	S	$f_7(C, P, S)$ to get $f_7(C, P, S)$
t	t	t	0.05 × 0.90 × 1 = 0.045
f	t	t	0.95 × 0.20 × 1 = 0.19
t	t	f	0.02 × 0.90 × 1 = 0.018
f	t	f	0.98 × 0.20 × 1 = 0.196
t	f	t	0.03 × 0.90 × 1 = 0.027
f	f	t	0.97 × 0.20 × 1 = 0.194
t	f	f	0.001 × 0.90 × 1 = 0.0009
f	f	f	0.999 × 0.20 × 1 = 0.1998

2) Eliminate C from $f_7(C, P, S)$ to get $f_8(P, S)$ (+4)

P	S	$f_8(P, S)$
t	t	0.045+0.19 = 0.235
f	t	0.018+0.196 = 0.214
t	f	0.027+0.194 = 0.221
f	f	0.0009+0.1998 = 0.2007

Iteration 3:

1) Join all factors containing S, $f_2(S)$ and $f_8(P, S)$ to obtain $f_9(P, S)$ (×4)

P	S	$f_9(P, S)$
t	t	0.3 × 0.235 = 0.0705
f	t	0.3 × 0.214 = 0.0642
t	f	0.7 × 0.221 = 0.1547
f	f	0.7 × 0.2007 = 0.14049

2) Eliminate S from $f_9(P, S)$ to obtain $f_{10}(P)$ (+2)

P	$f_{10}(P)$
t	0.0705+0.1547 = 0.2252
f	0.0642+0.14049 = 0.20469

Iteration 4 Join all factors containing P, $f_1(P)$ and $f_{10}(P)$ to create $f_{11}(P)$ (×2)

P	$P_{11}(P)$
t	$0.90 \times 0.2252 = 0.20268$
f	$0.10 \times 0.20469 = 0.020469$

Normalise probabilities in $f_{11}(P)$

P	norm $f_{11}(P)$
t	$0.20268 / (0.20268 + 0.020469) = 0.90827$
f	$0.020469 / (0.20268 + 0.020469) = 0.09173$

$P(P=t \mid X=t) = 0.90827$