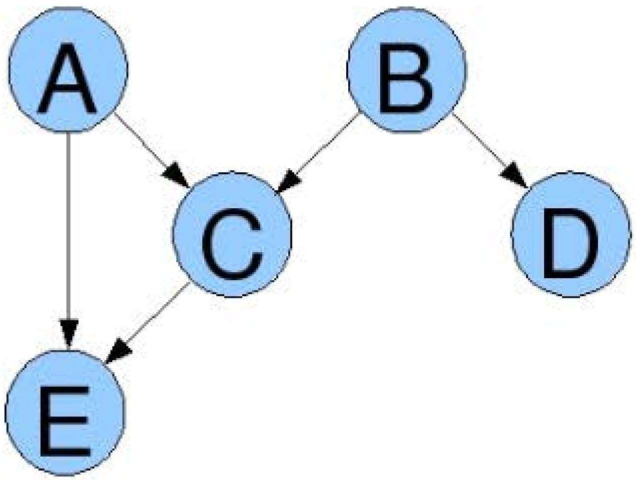
Bayesian Network Fundamentals

Latest Submission Grade 100%

1. Factorization.

1 / 1 point

Given the same model as above, which of these is an appropriate decomposition of the joint distribution $P(A, B, C, D)$?



- ○ $P(A, B, C, D) = P(A)P(B)P(C|A)P(C|B)P(D|B)$
- ◉ $P(A, B, C, D) = P(A)P(B)P(C|A, B)P(D|B)$
- ○ $P(A, B, C, D) = P(A)P(B)P(A, B|C)P(B|D)$
- ○ $P(A, B, C, D) = P(A)P(B)P(C)P(D)$

Correct

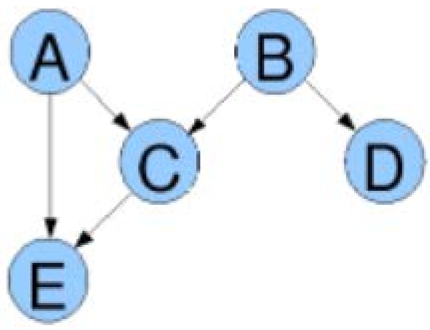
We can read off the appropriate factorization from the graph by examining the parents of each variable in the graph: A and B have no parents, while C is a child of A, B and D is a child of B.

This gives us $P(A, B, C, D) = P(A)P(B)P(C|A, B)P(D|B)$.

2. Independent parameters.

1 / 1 point

How many independent parameters are required to uniquely define the CPD of C (the conditional probability distribution associated with the variable C) in the same graphical model as above, if A, B, and D are binary, and C and E have three values each?



If you haven't come across the term before, here's a brief explanation: A multinomial distribution over m possibilities $x_1, \ldots, x_m$ has m parameters, but $m-1$ independent parameters, because we have the constraint that all parameters must sum to 1, so that if you specify $m-1$ of the parameters, the final one is fixed. In a CPD $P(X|Y)$, if X has m values and Y has k values, then we have k distinct multinomial distributions, one for each value of Y, and we have $m-1$ independent parameters in each of them, for a total of $k(m-1)$. More generally, in a CPD $P(X|Y_1, \ldots, Y_r)$, if each Y_i has k_i values, we have a total of $k_1 \times \ldots \times k_r \times (m-1)$ independent parameters.

Example: Let's say we have a graphical model that just had $X \rightarrow Y$, where both variables are binary. In this scenario, we need 1 parameter to define the CPD of X. The CPD of X contains two entries $P(X=0)$ and $P(X=1)$. Since the sum of these two entries has to be equal to 1, we only need one parameter to define the CPD.

Now we look at Y. The CPD for Y contains 4 entries which correspond to: $P(Y=0|X=0)$, $P(Y=1|X=0)$, $P(Y=0|X=1)$, $P(Y=1|X=1)$. Note that $P(Y=0|X=0)$ and $P(Y=1|X=0)$ should sum to one, so we need 1 independent parameter to describe those two entries; likewise, $P(Y=0|X=1)$ and $P(Y=1|X=1)$ should also sum to 1, so we need 1 independent parameter for those two entries.

Therefore, we need 1 independent parameter to define the CPD of X and 2 independent parameters to define the CPD of Y.

- (●) 8
- ( ) 6
- ( ) 4
- ( ) 11
- ( ) 3
- ( ) 7
- ( ) 12

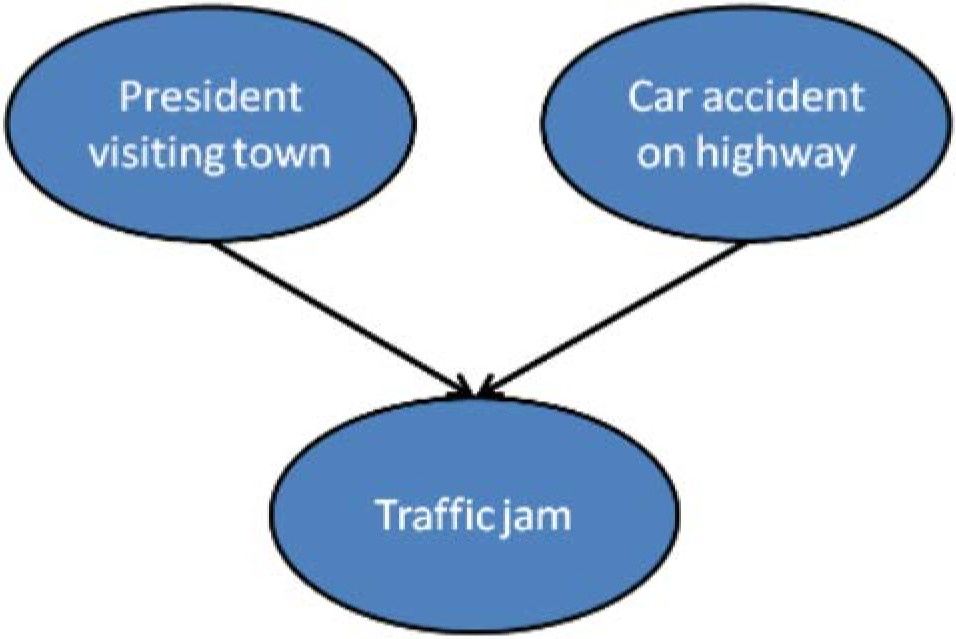
✓ **Correct**

In a Bayesian network, the conditional probability distribution associated with a variable is the conditional probability distribution of that variable given its parents. There are 4 possibilities for the values of C's parents (A and B, which are binary). For each of these possibilities, there are 3 possible values for C, which corresponds to 2 free parameters (since the 3 numbers have to sum to 1). So there are 4*2=8 total free parameters.

3. *Inter-causal reasoning. 1 / 1 point

Consider the following model for traffic jams in a small town, which we assume can be caused by a car accident, or by a visit from the president (and the accompanying security motorcade).

P(President = 1) = 0.01 P(Accident = 1) = 0.1



P(Traffic = 1 | President = 0, Accident = 0) = 0.1
P(Traffic = 1 | President = 0, Accident = 1) = 0.5
P(Traffic = 1 | President = 1, Accident = 0) = 0.6
P(Traffic = 1 | President = 1, Accident = 1) = 0.9

Calculate P(Accident = 1 | Traffic = 1) and P(Accident = 1 | Traffic = 1, President = 1). Separate your answers with a space, e.g., an answer of

0.15 0.25

means that P(Accident = 1 | Traffic = 1) = 0.15 and P(Accident = 1 | Traffic = 1, President = 1) = 0.25. Round your answers to two decimal places and write a leading zero, like in the example above.

0.35 0.14

Correct

To calculate the required values, we can apply Bayes' rule. For instance,

$$P(A=1|T=1,P=1) = \frac{P(A=1,T=1,P=1)}{P(T=1,P=1)}$$
$$= \frac{P(A=1,T=1,P=1)}{P(A=0,T=1,P=1)+P(A=1,T=1,P=1)}.$$

We can then use the chain rule of Bayesian networks to substitute the correct values in, e.g., Calculate P(Accident = 1 | Traffic = 1) and P(Accident = 1 | Traffic = 1, President = 1). Separate your answers with a space, e.g., an answer of

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We can then use the chain rule of Bayesian networks to substitute the correct values in, e.g.,

$$P(A=1,T=1,P=1) = P(P=1) \times P(A=1) \times P(T=1|P=1,A=1)$$

This example of inter-causal reasoning meshes well with common sense: if we see a traffic jam, the probability that there was a car accident is relatively high. However, if we also see that the president is visiting town, we can reason that the president's visit is the cause of the traffic jam; the probability that there was a car accident therefore drops correspondingly.