Basic Definitions

Latest Submission Grade 100%

1. Factor product.

1 / 1 point

Let X, Y and Z be binary variables.

If $\phi_1(X, Y)$ and $\phi_2(Y, Z)$ are the factors shown below, compute the selected entries (marked by a '?') in the factor $\psi(X, Y, Z) = \phi_1(X, Y) \cdot \phi_2(Y, Z)$, giving your answer according to the ordering of assignments to variables as shown below.

Separate each of the 3 entries of the factor with spaces, e.g., an answer of

0.1 0.2 0.3

means that $\psi(1, 1, 1) = 0.1$, $\psi(1, 2, 1) = 0.2$, and $\psi(2, 2, 2) = 0.3$. Give your answers as exact decimals without any trailing zeroes.

X	Y	$\phi_1(X,Y)$
1	1	0.8
1	2	0.5
2	1	0.5
2	2	0.6

×

Y	Z	$\phi_2(Y,Z)$
1	1	0.2
1	2	0.2
2	1	0.9
2	2	1.0

=

X	Y	Z	$\psi(X,Y,Z)$
1	1	1	?
1	1	2	
1	2	1	?
1	2	2	
2	1	1	
2	1	2	
2	2	1	
2	2	2	?

0.16 0.45 0.6

Correct

2. Factor reduction.

1 / 1 point

Let X, Z be binary variables, and let Y be a variable that takes on values 1, 2, or 3.

Now say we observe $Y = 1$. If $\phi(X, Y, Z)$ is the factor shown below, compute the missing entries of the reduced factor $\psi(X, Z)$ given that $Y = 1$, giving your answer according to the ordering of assignments to variables as shown below.

As before, separate the 4 entries of the factor by spaces.

X	Y	Z	$\phi(X,Y,Z)$
1	1	1	14
1	1	2	60
1	2	1	40
1	2	2	27
1	3	1	42
1	3	2	85
2	1	1	4
2	1	2	59
2	2	1	54
2	2	2	3
2	3	1	96
2	3	2	30

X	Z	$\psi(X,Z)$
1	1	?
1	2	?
2	1	?
2	2	?

14 60 4 59

Correct

3. **Properties of independent variables.**

1 / 1 point

Assume that A and B are independent random variables. Which of the following options are always true? You may select 1 or more options.

- [x] $P(B|A) = P(B)$

Correct

In intuitive terms, this means that the value of B is not dependent on the value of A. We can derive this from $P(A, B) = P(A) \times P(B)$ as follows:

$$\begin{aligned} P(A,B) &= P(A) \times P(B) && \text{(by definition of independence)} \\ &= P(B|A) \times P(A) && \text{(by chain rule of probabilities)} \\ \text{therefore} \quad P(B|A) &= P(B). \end{aligned}$$

- [x] $P(A|B) = P(A)$

Correct

In intuitive terms, this means that the value of A is not dependent on the value of B. We can derive this from $P(A, B) = P(A) \times P(B)$ as follows:

$$\begin{aligned} P(A,B) &= P(A) \times P(B) && \text{(by definition of independence)} \\ &= P(A|B) \times P(B) && \text{(by chain rule of probabilities)} \\ \text{therefore} \quad P(A|B) &= P(A). \end{aligned}$$

- [ ] $P(A) + P(B) = 1$
- [ ] $P(A) = P(B)$

4. **Factor marginalization.**

1 / 1 point

Let X, Z be binary variables, and let Y be a variable that takes on values 1, 2, or 3.

If $\phi(X, Y, Z)$ is the factor shown below, compute the entries of the factor

4. Factor marginalization.

1 / 1 point

Let X, Z be binary variables, and let Y be a variable that takes on values 1, 2, or 3.

If $\phi(X, Y, Z)$ is the factor shown below, compute the entries of the factor

$\psi(Y, Z) = \sum_X \phi(X, Y, Z)$,

giving your answer according to the ordering of assignments to variables as shown below.

Separate the 4 entries of the factor with spaces, and do not add any extra trailing or leading zeroes or decimal points.

X	Y	Z	$\phi(X,Y,Z)$
1	1	1	68
1	1	2	95
1	2	1	65
1	2	2	63
1	3	1	57
1	3	2	5
2	1	1	40
2	1	2	40
2	2	1	14
2	2	2	78
2	3	1	16
2	3	2	89

Y	Z	$\psi(Y,Z)$
1	1	?
1	2	?
2	1	?
2	2	?
3	1	
3	2	

108 135 79 141

Correct