

Copyright Notice

These slides are distributed under the Creative Commons License.

[DeepLearning.AI](#) makes these slides available for educational purposes. You may not use or distribute these slides for commercial purposes. You may make copies of these slides and use or distribute them for educational purposes as long as you cite [DeepLearning.AI](#) as the source of the slides.

For the rest of the details of the license, see <https://creativecommons.org/licenses/by-sa/2.0/legalcode>



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



DeepLearning.AI

Vectors and Linear Transformations

Machine Learning motivation

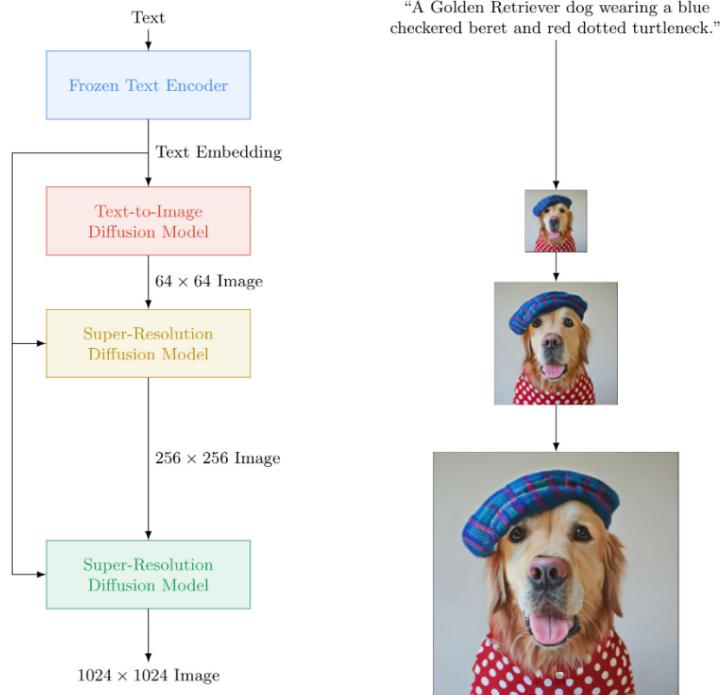
Neural Networks - AI generated images



AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."



The screenshot shows an AI interface for generating text from images. On the left, there's a 'Model' section containing a large clock icon. On the right, there's an 'Output' section with a text box containing the text "wall clock - wall clock." Above the text box is a timestamp "3.0s". In the top right corner of the interface, there are edit icons (pen and delete).

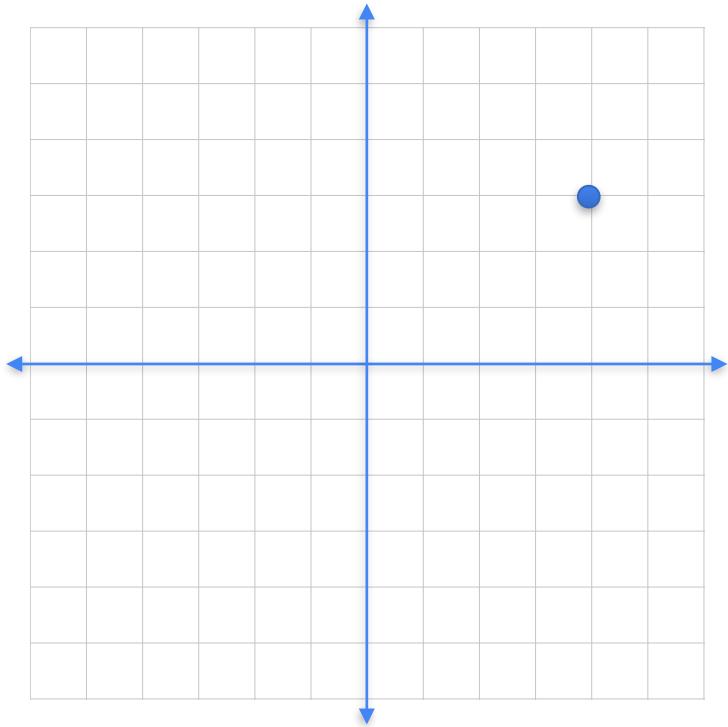


DeepLearning.AI

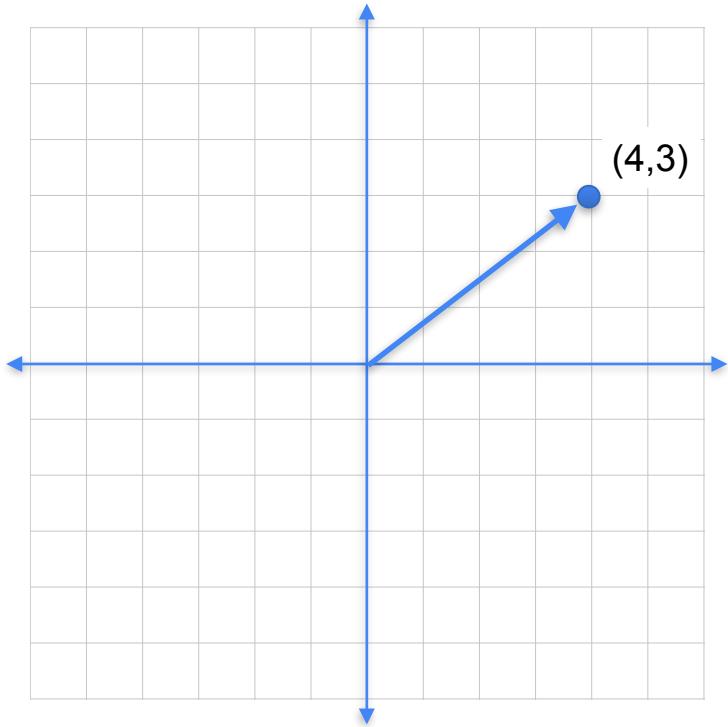
Vectors and Linear Transformations

Vectors and their properties

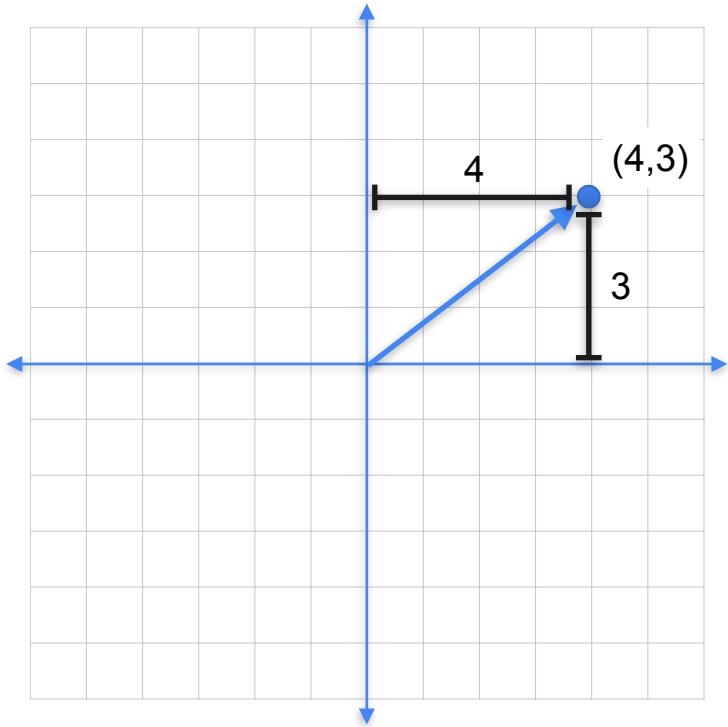
Vectors



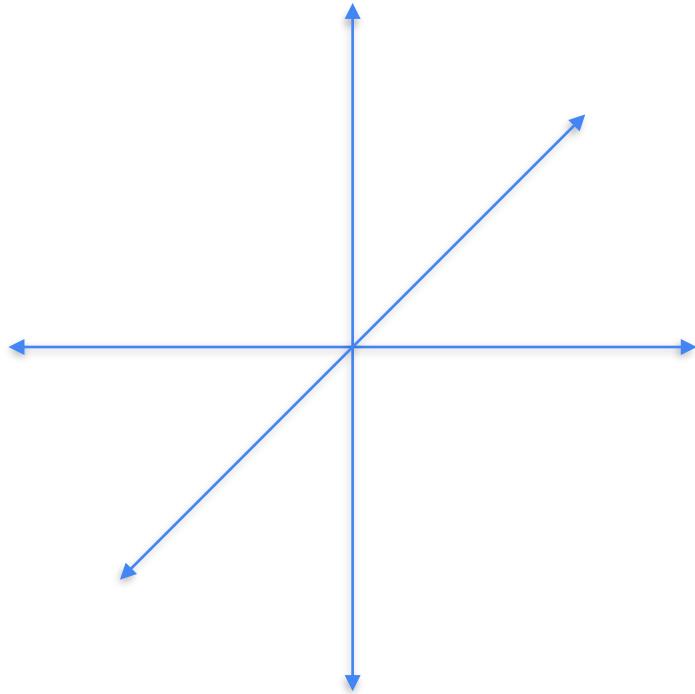
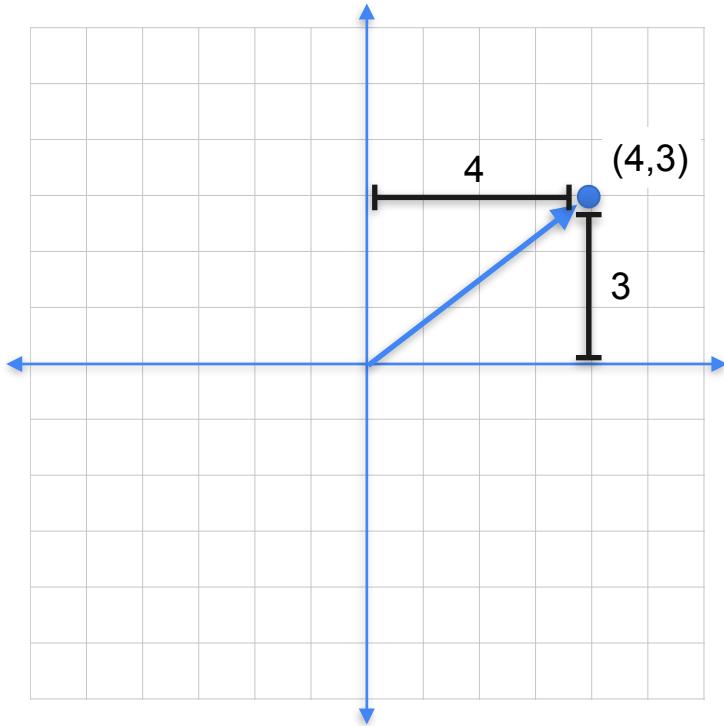
Vectors



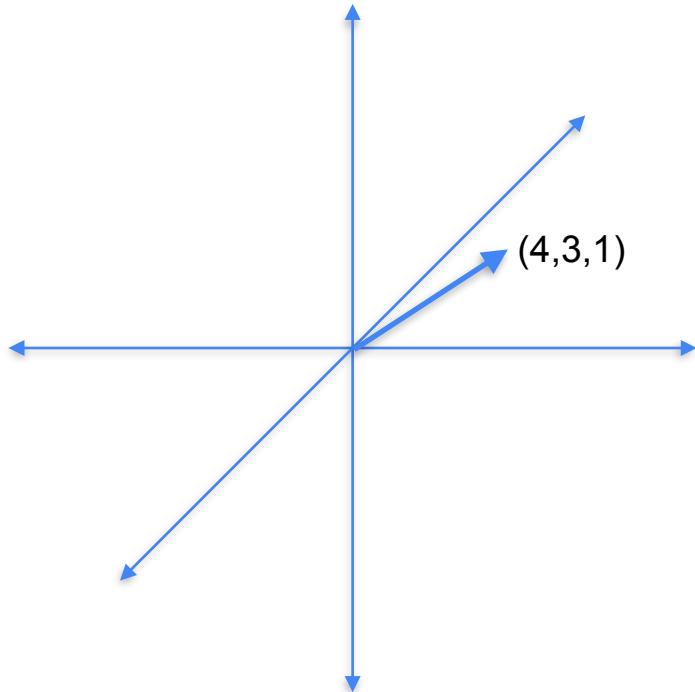
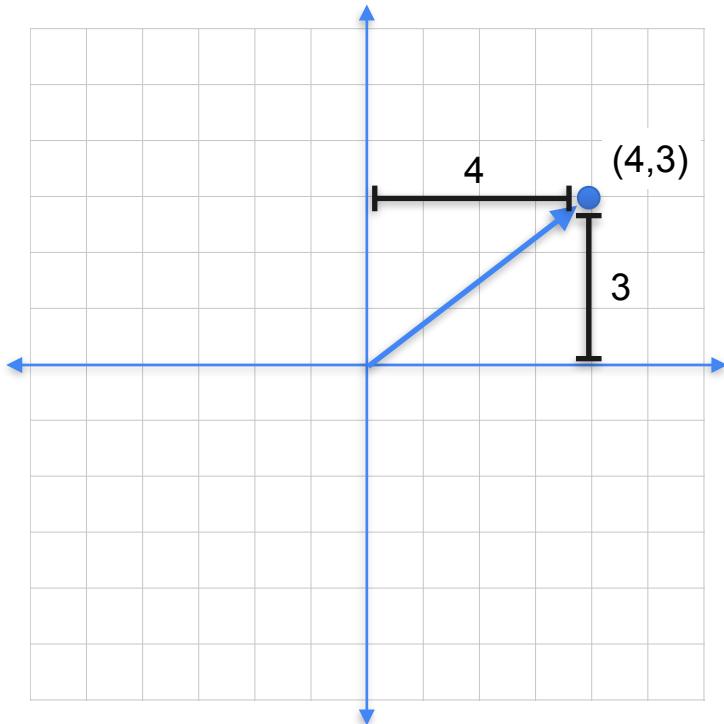
Vectors



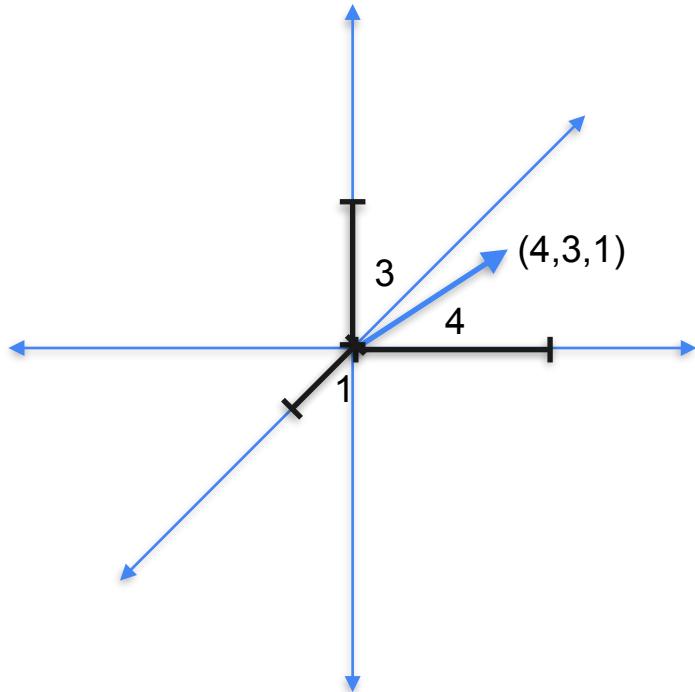
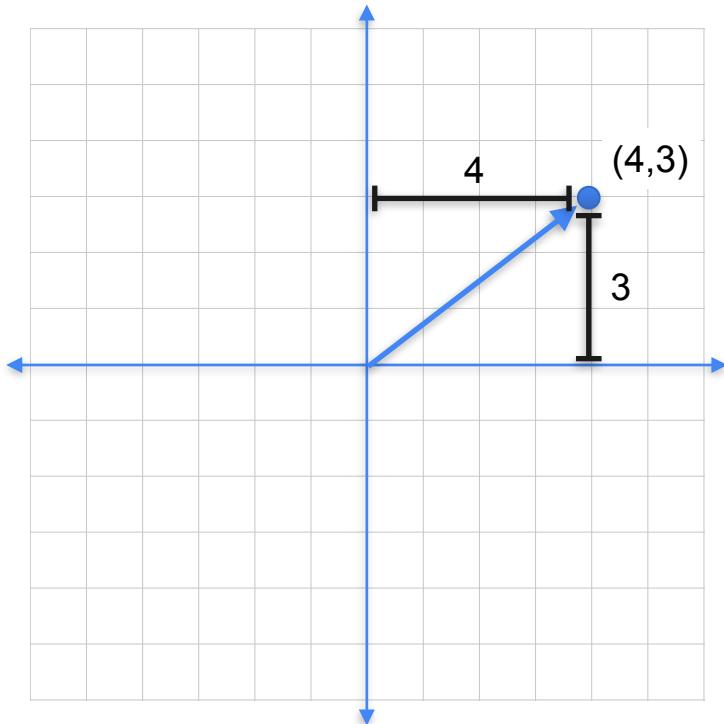
Vectors



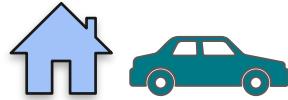
Vectors



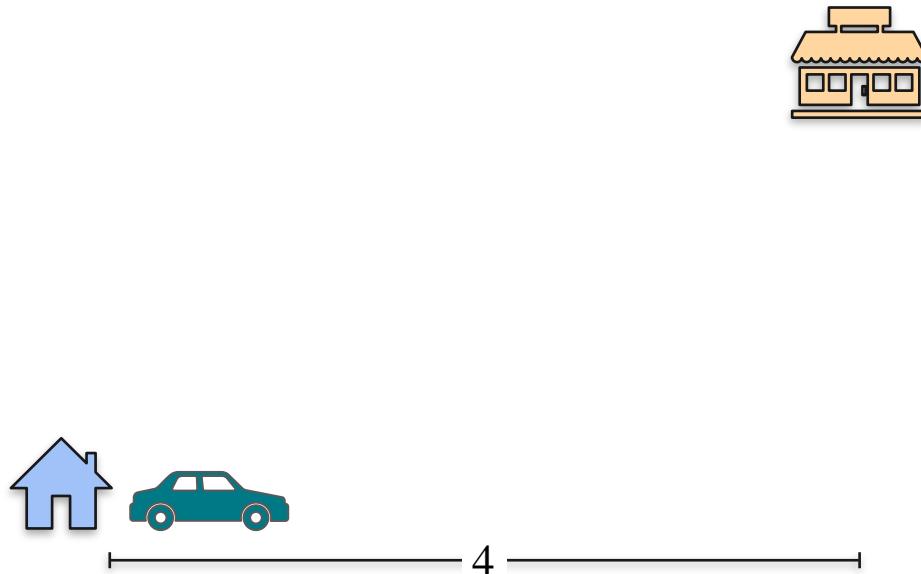
Vectors



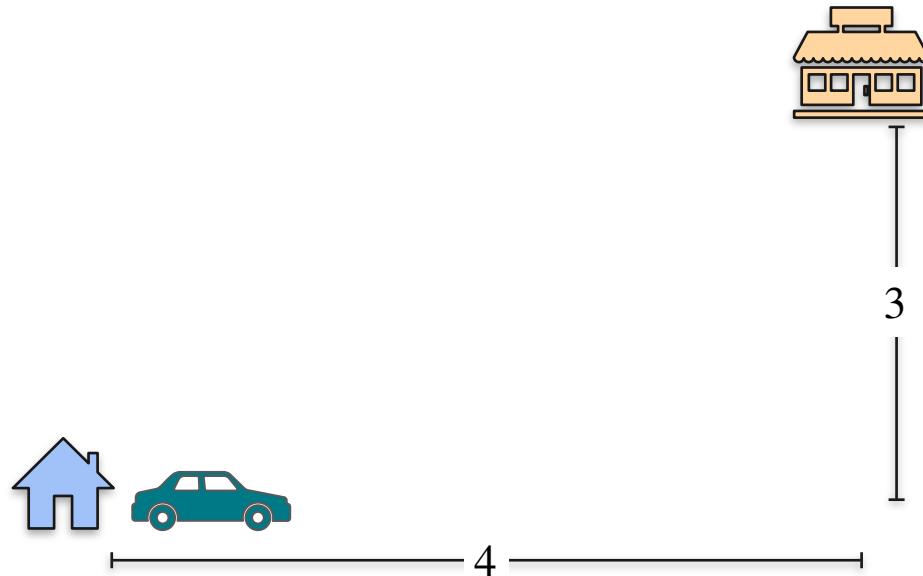
How to get from point A to point B?



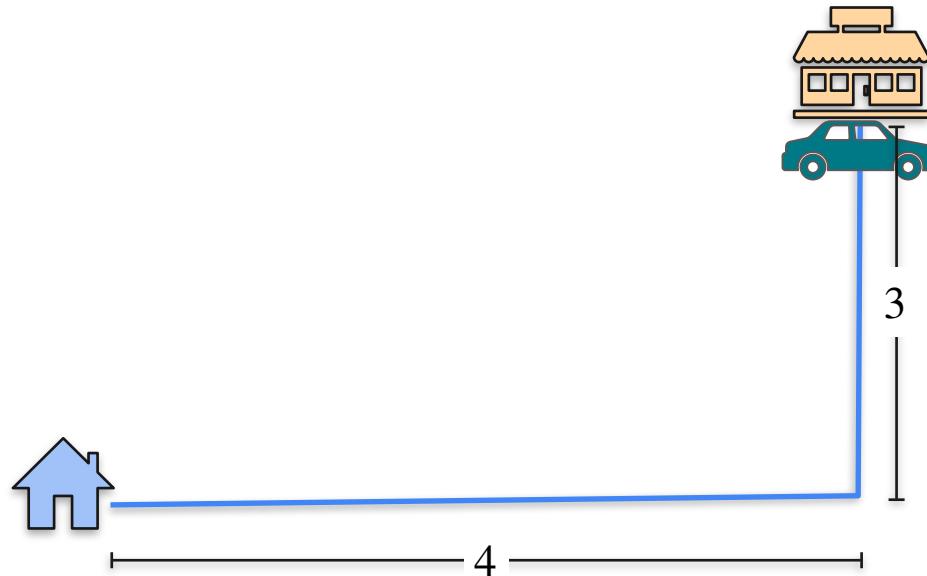
How to get from point A to point B?



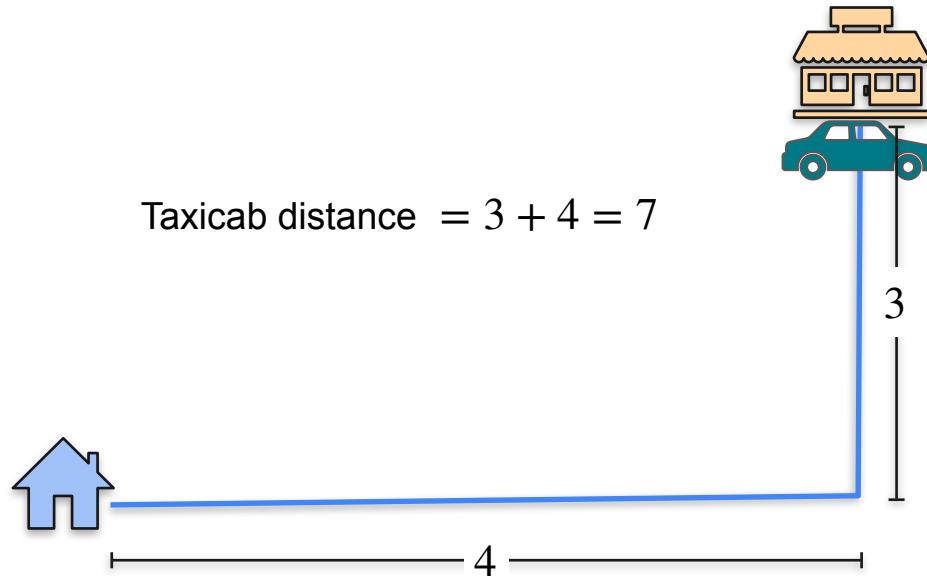
How to get from point A to point B?



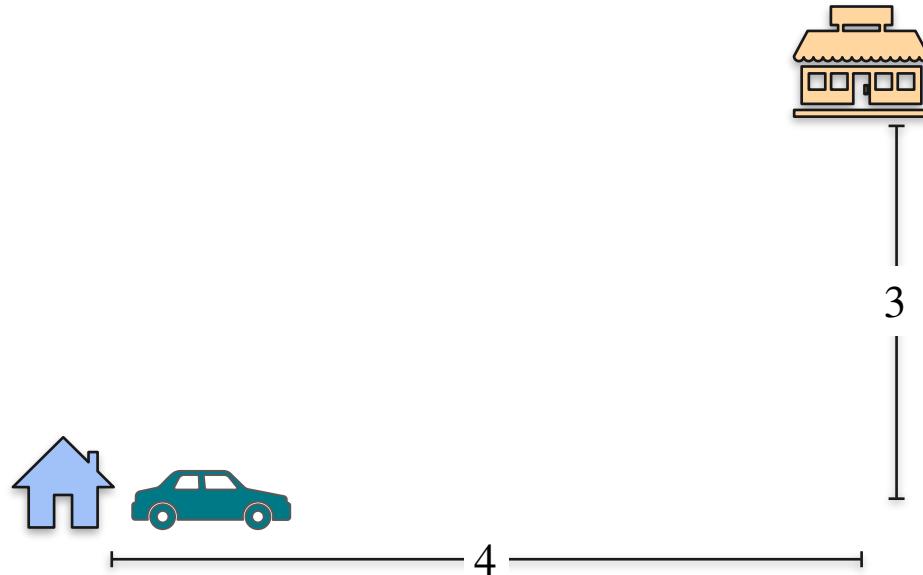
How to get from point A to point B?



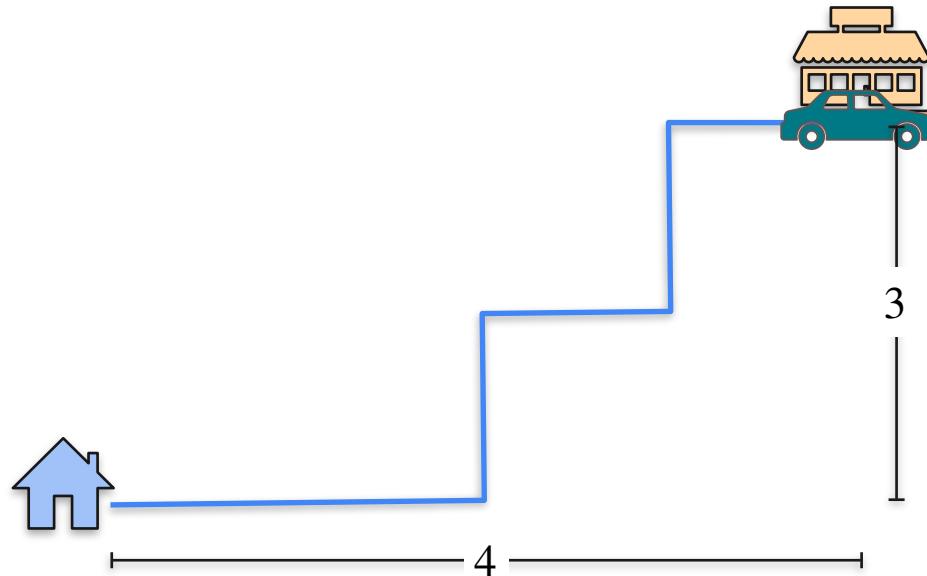
How to get from point A to point B?



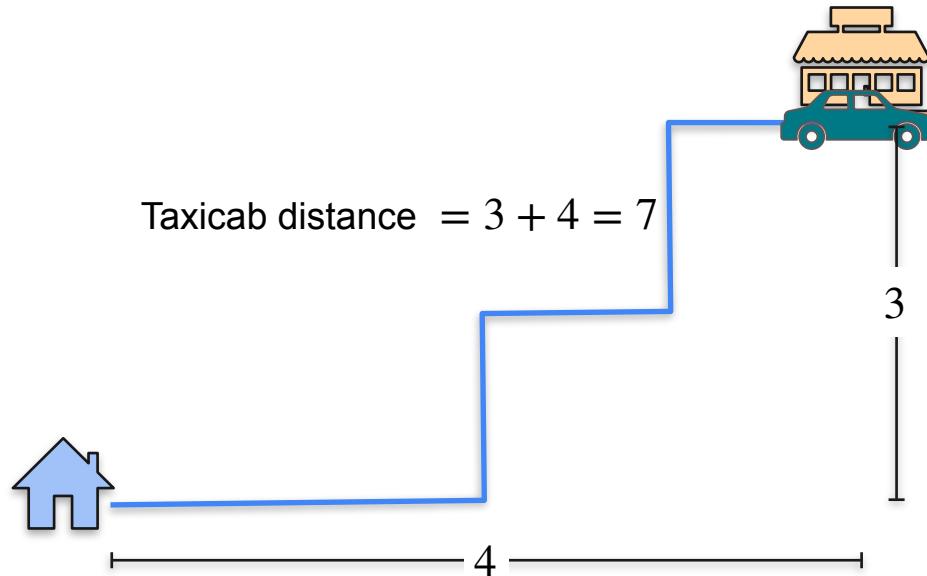
How to get from point A to point B?



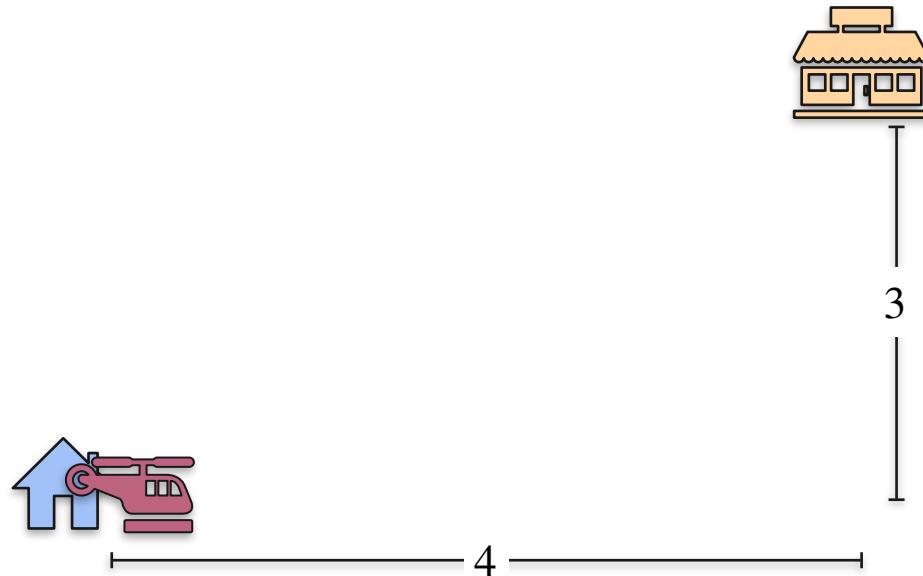
How to get from point A to point B?



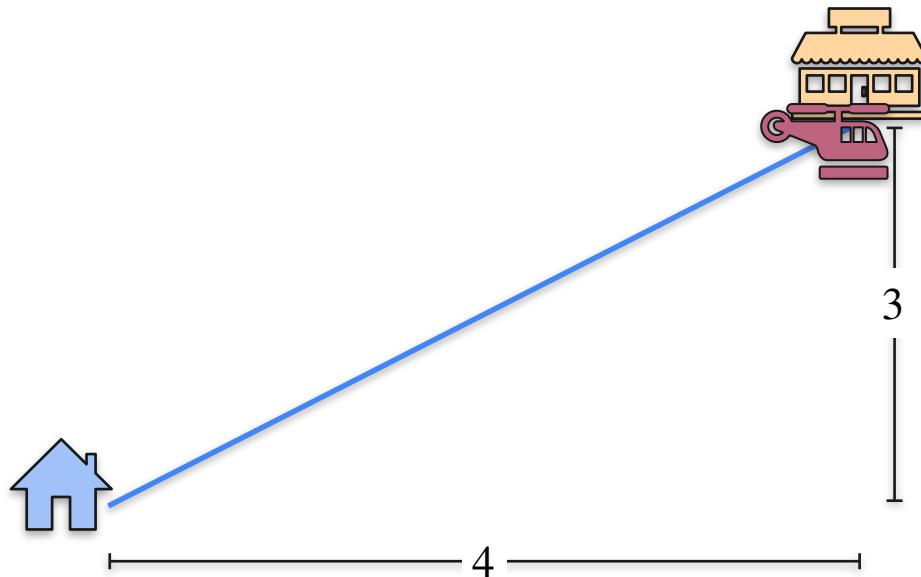
How to get from point A to point B?



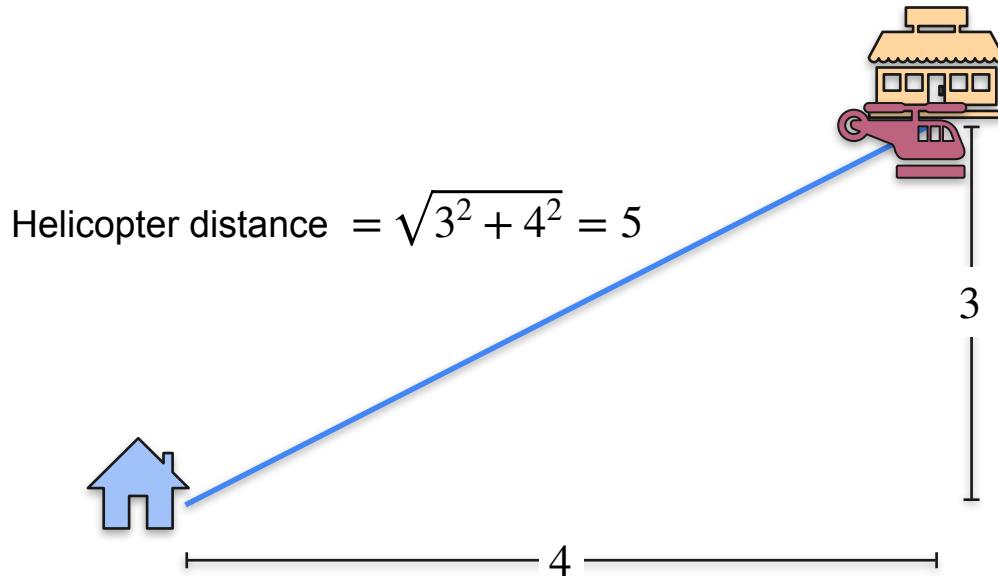
How to get from point A to point B?



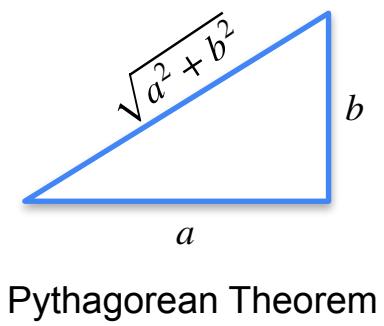
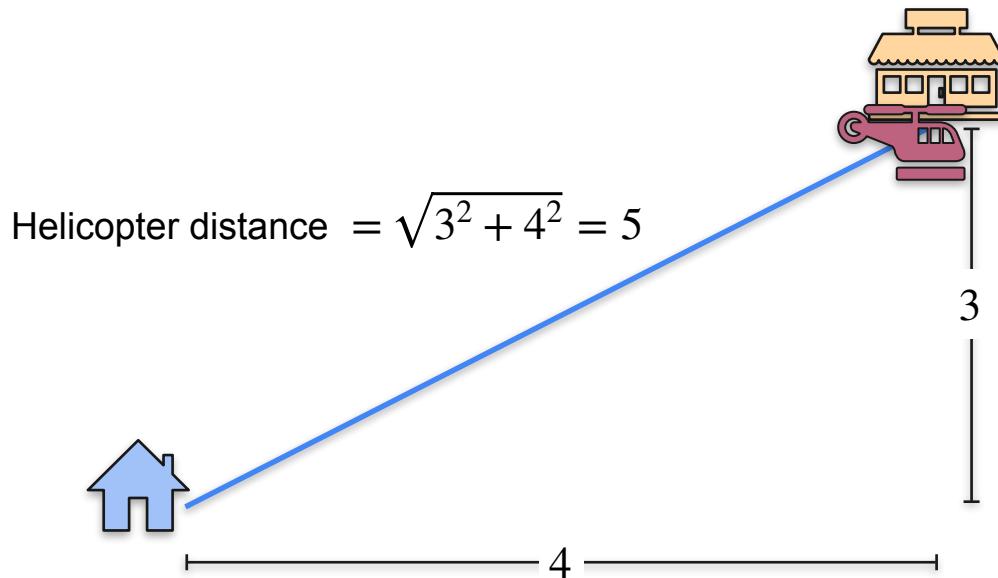
How to get from point A to point B?



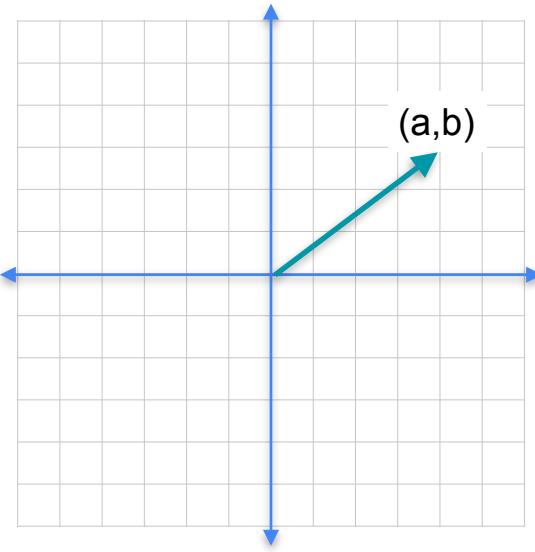
How to get from point A to point B?



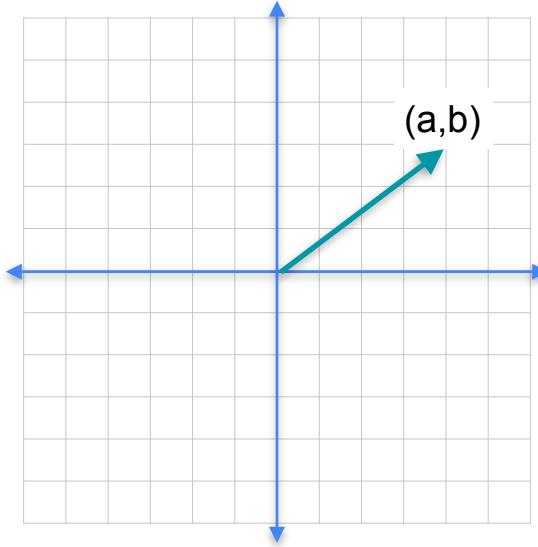
How to get from point A to point B?



Norms

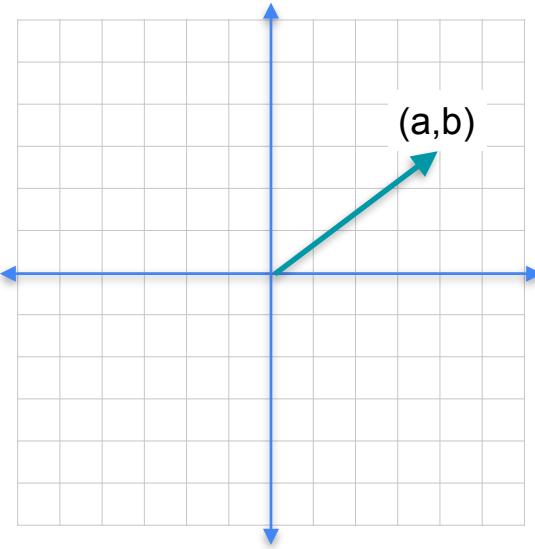


Norms

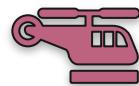


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

Norms

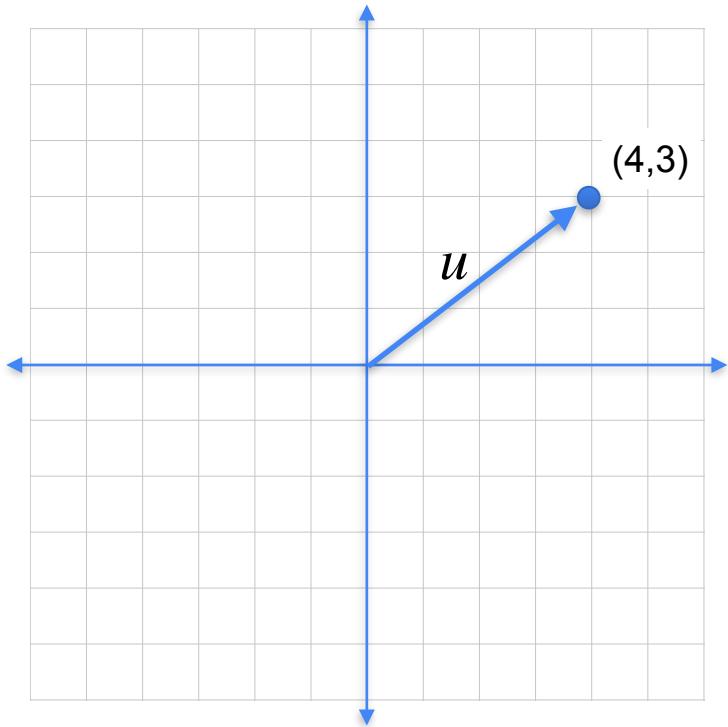


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

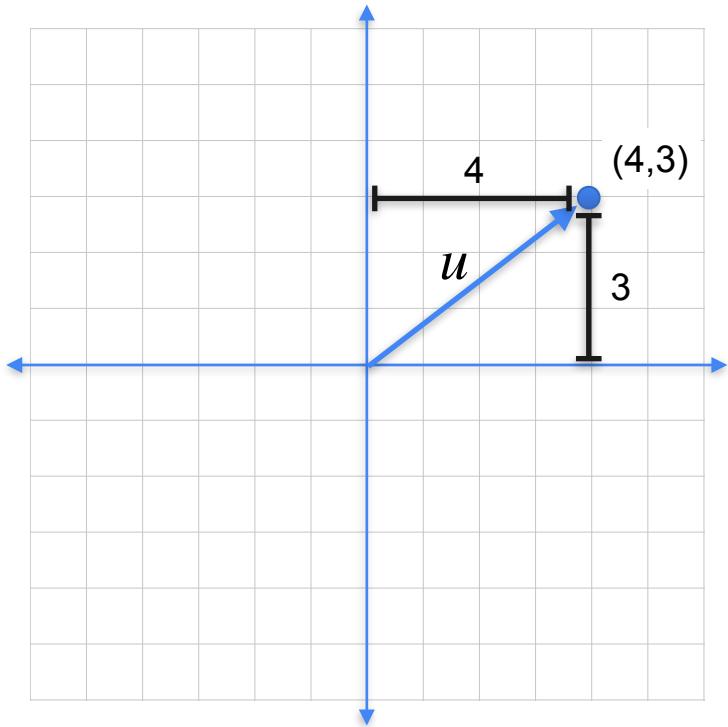


$$\text{L2-norm} = |(a, b)|_2 = \sqrt{a^2 + b^2}$$

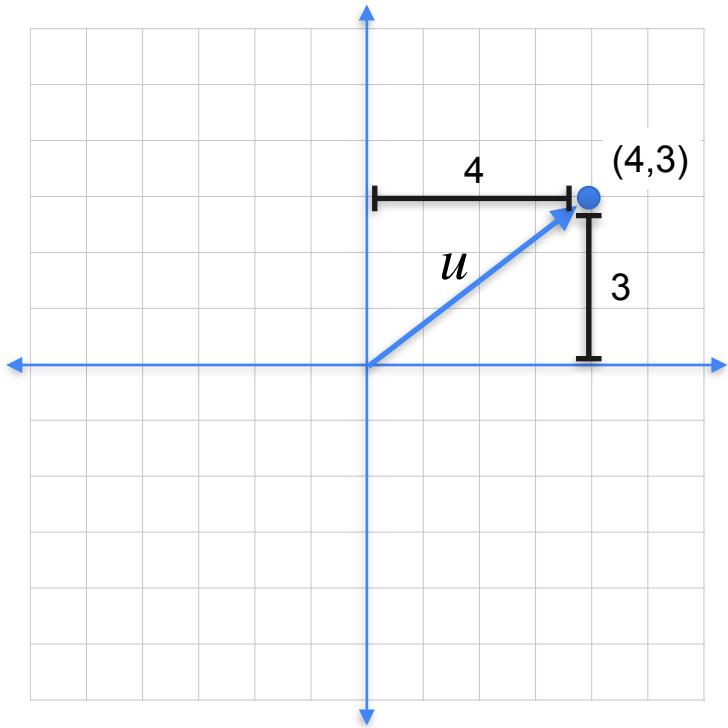
Norm of a vector



Norm of a vector

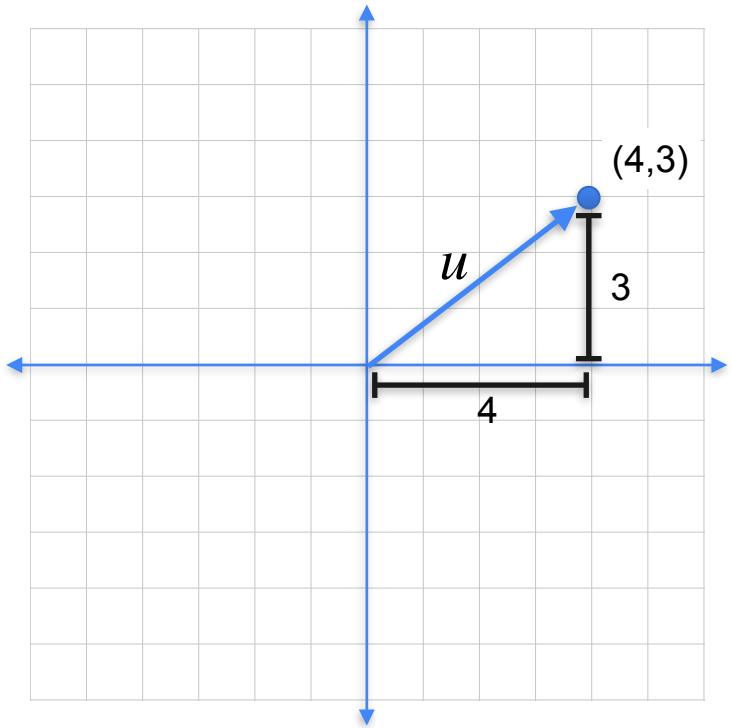


Norm of a vector

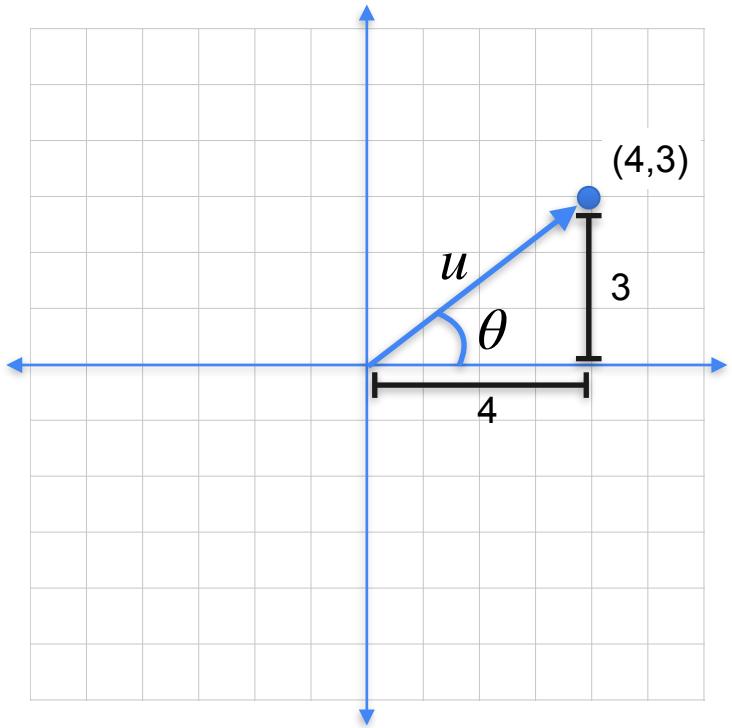


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

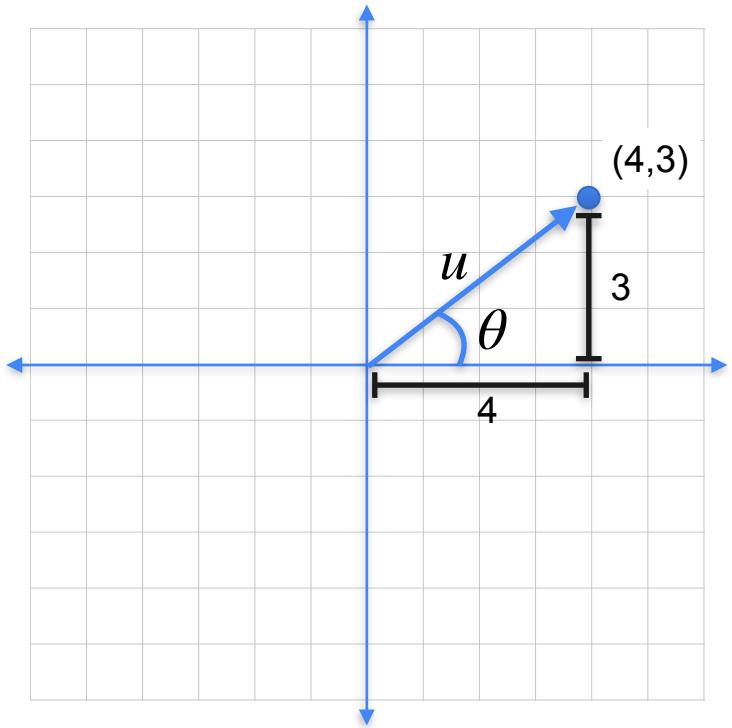
Direction of a vector



Direction of a vector

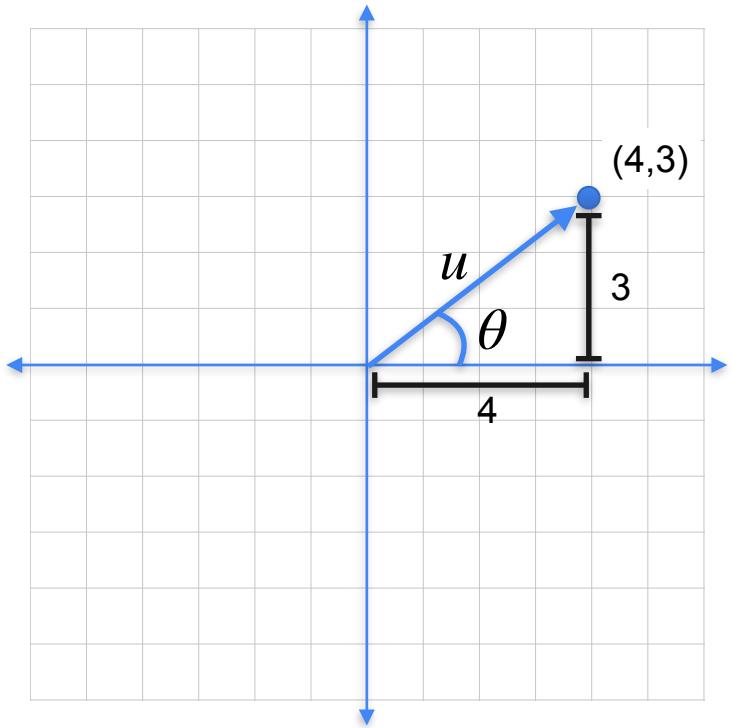


Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

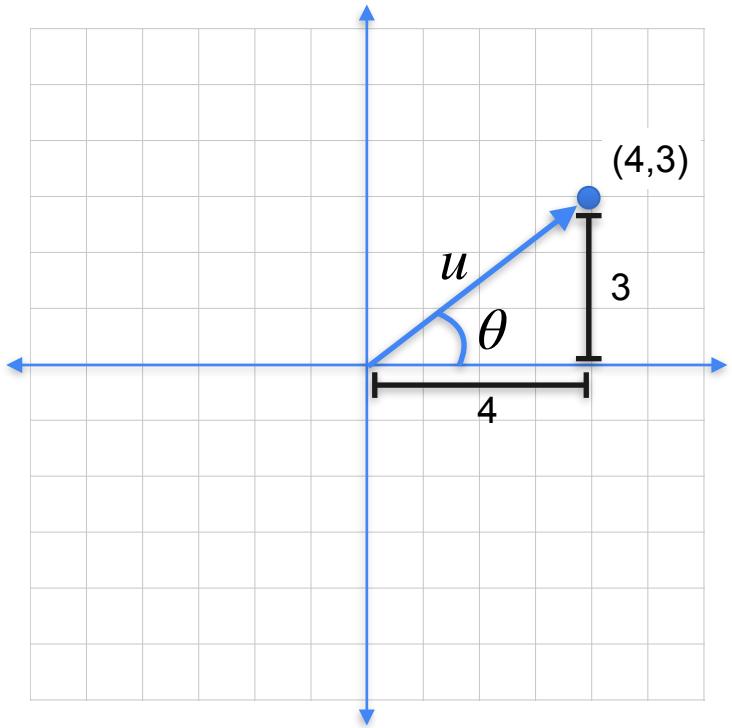
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

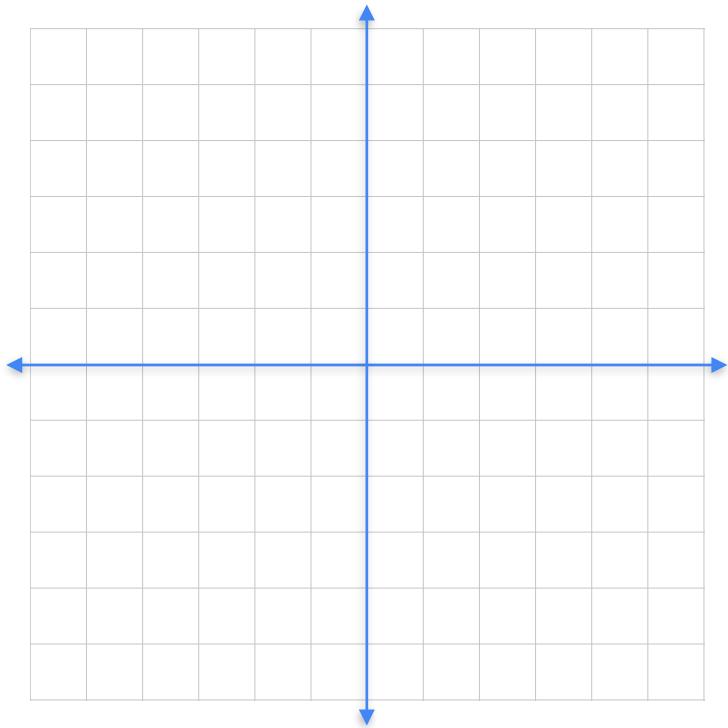


DeepLearning.AI

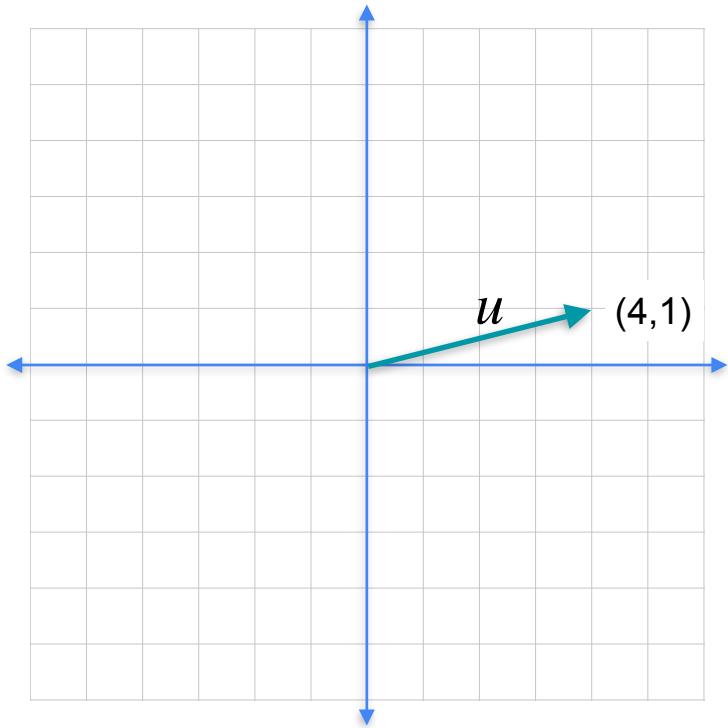
Vectors and Linear Transformations

**Sum and difference of
vectors**

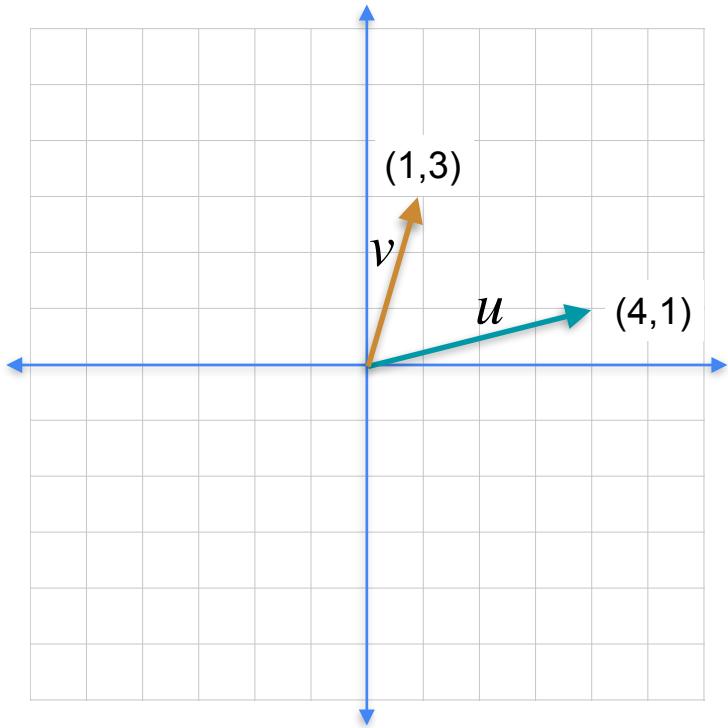
Sum of vectors



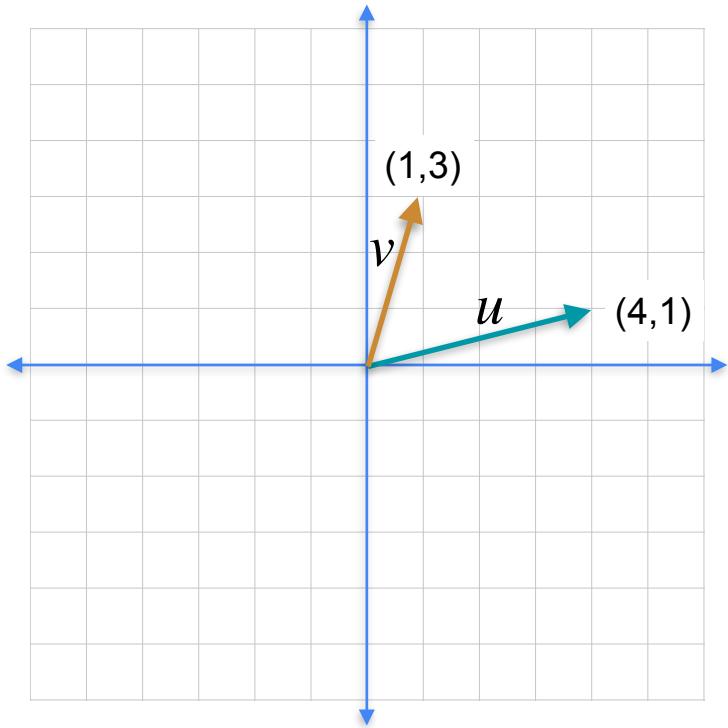
Sum of vectors



Sum of vectors

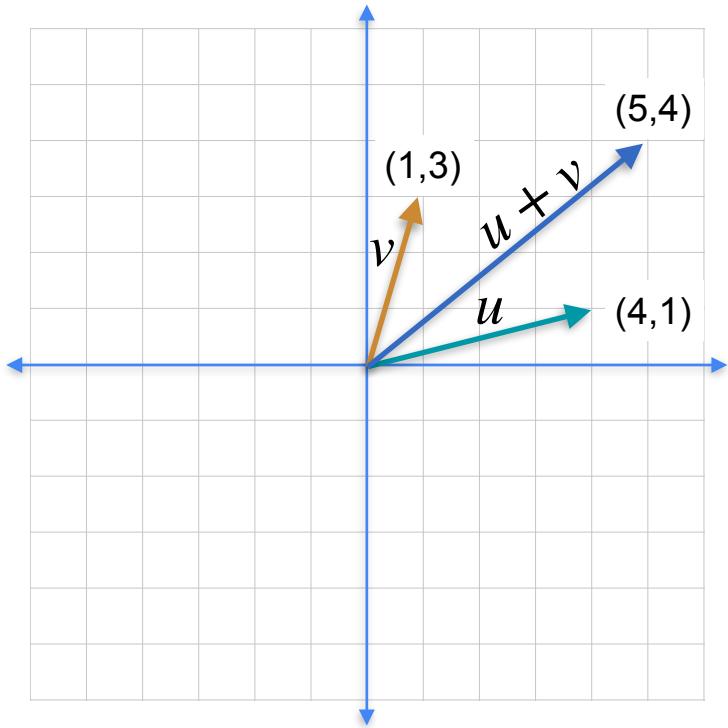


Sum of vectors



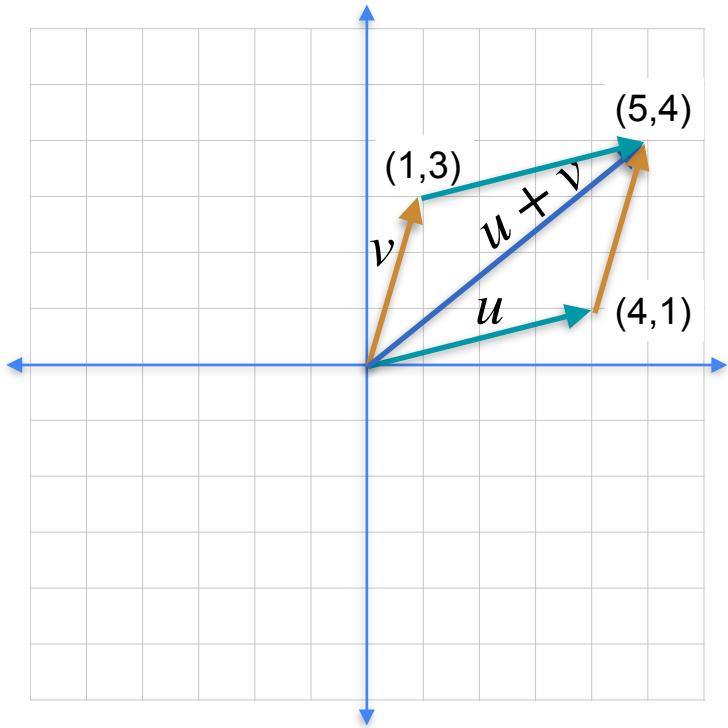
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors



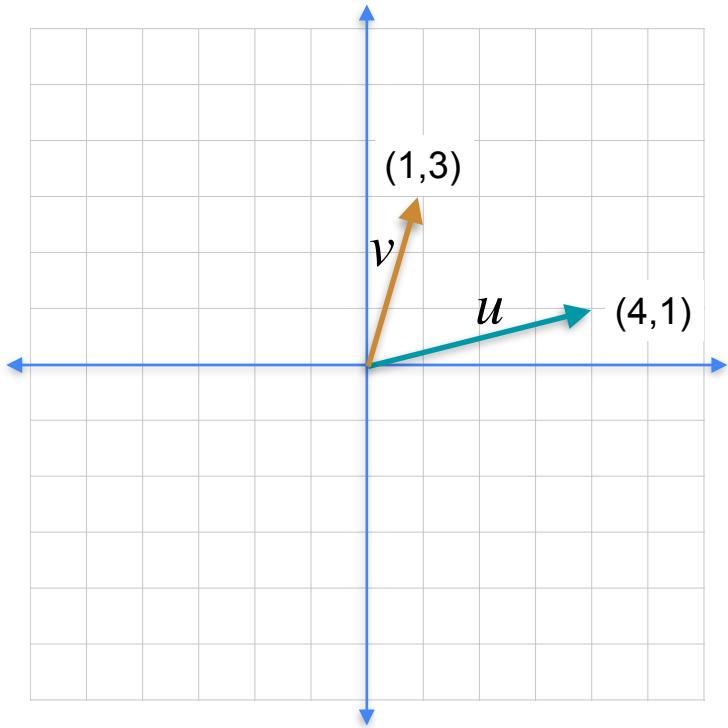
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors

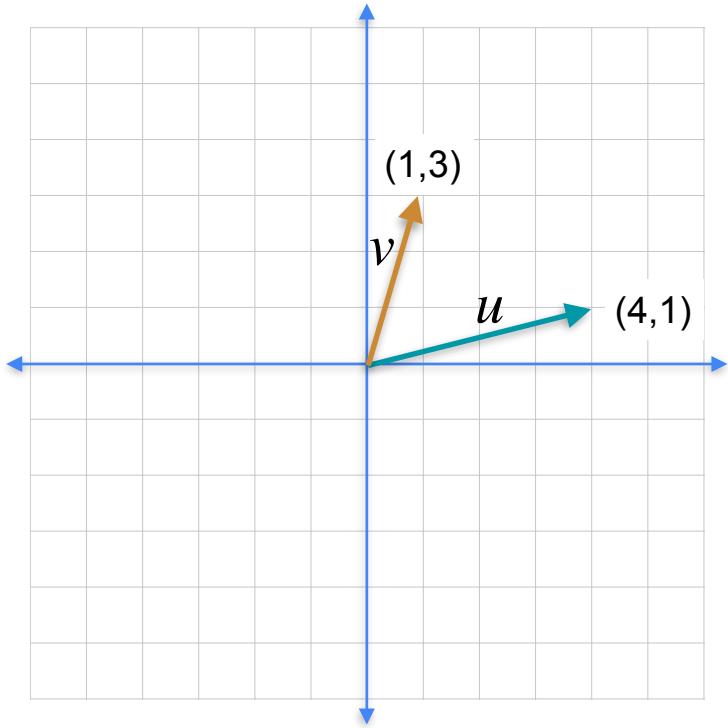


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Difference of vectors

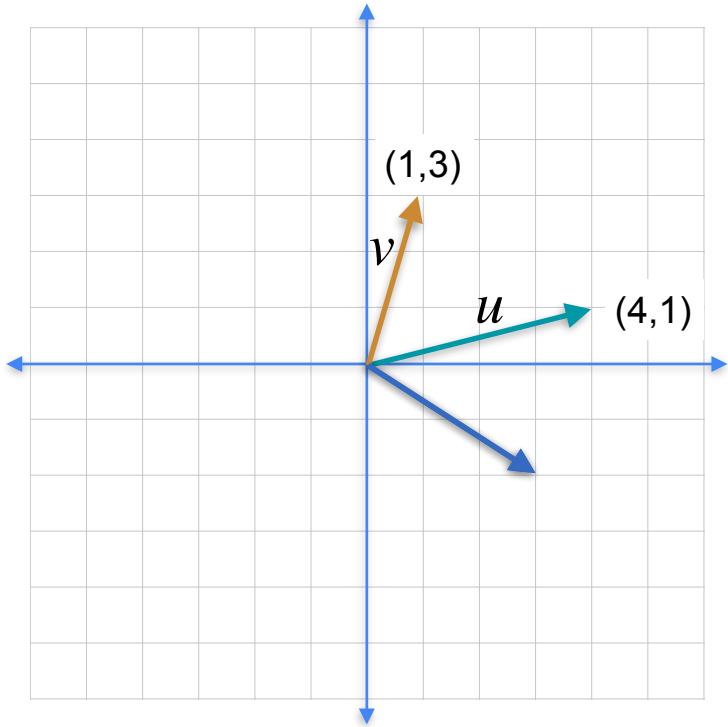


Difference of vectors



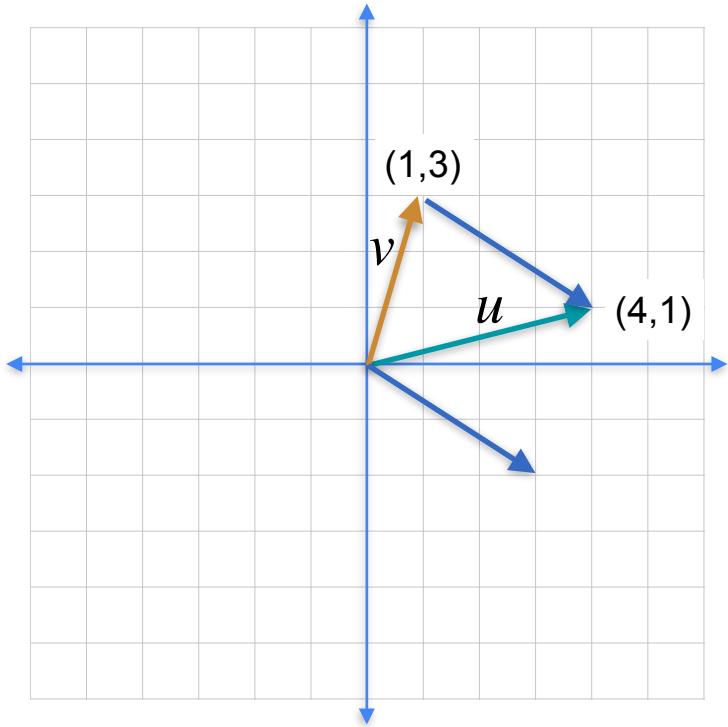
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



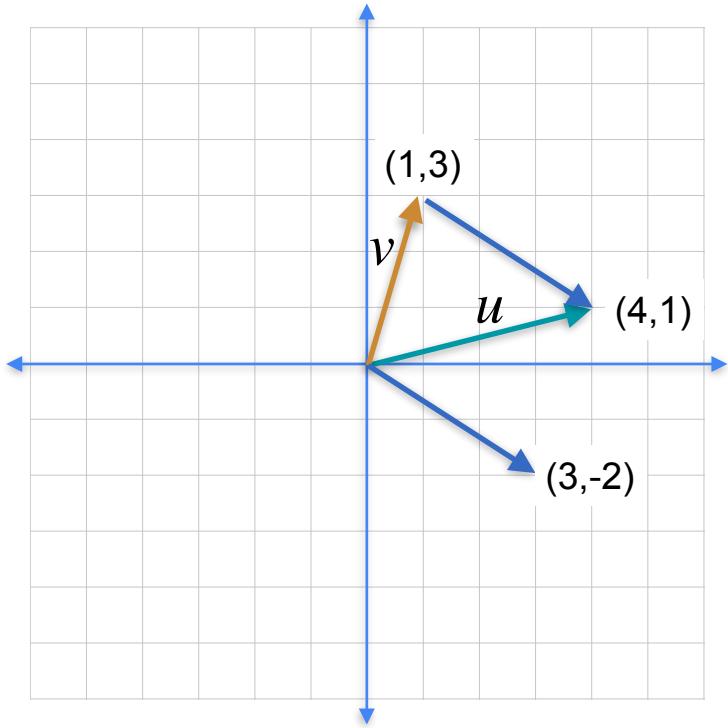
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

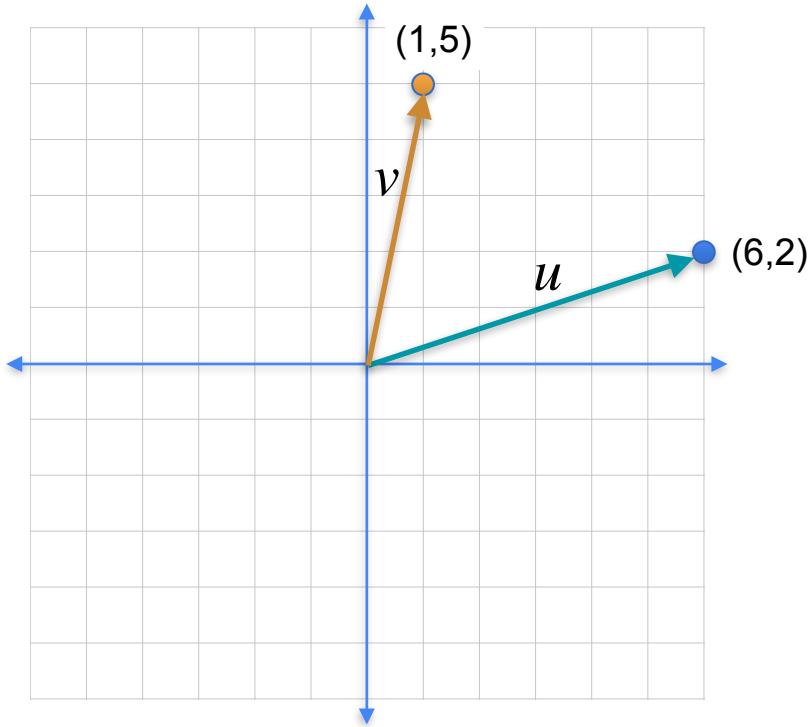


DeepLearning.AI

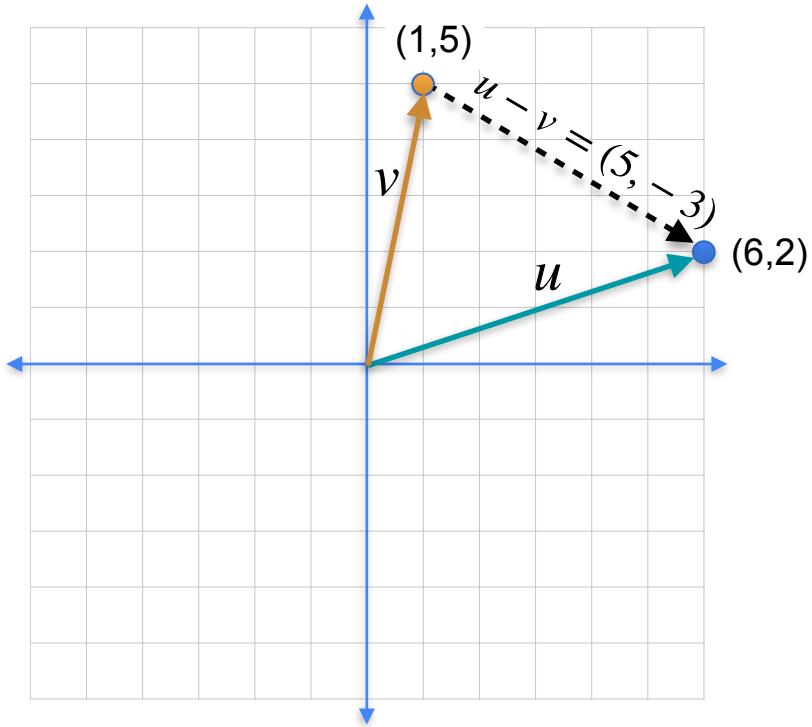
Vectors and Linear Transformations

Distance between vectors

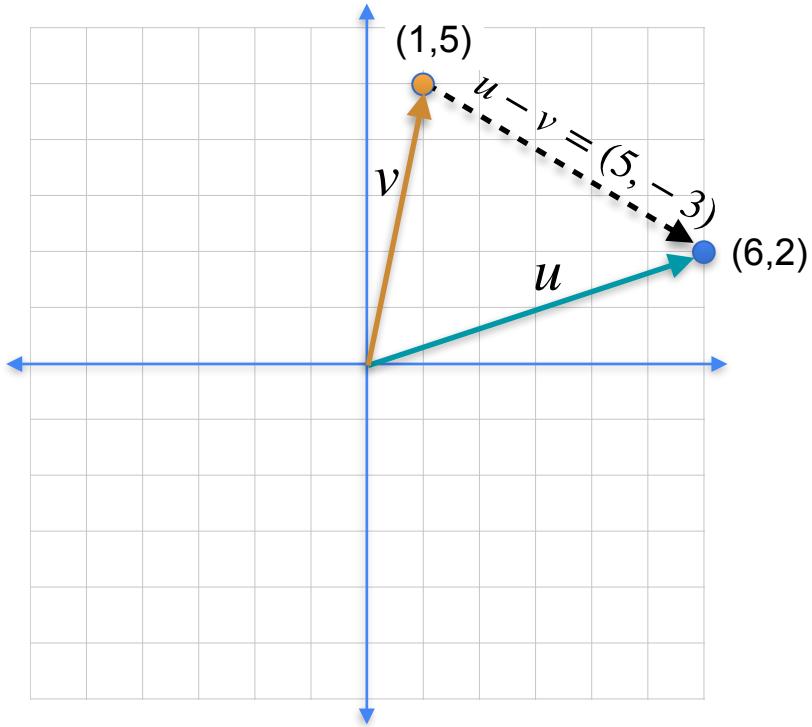
Distances



Distances

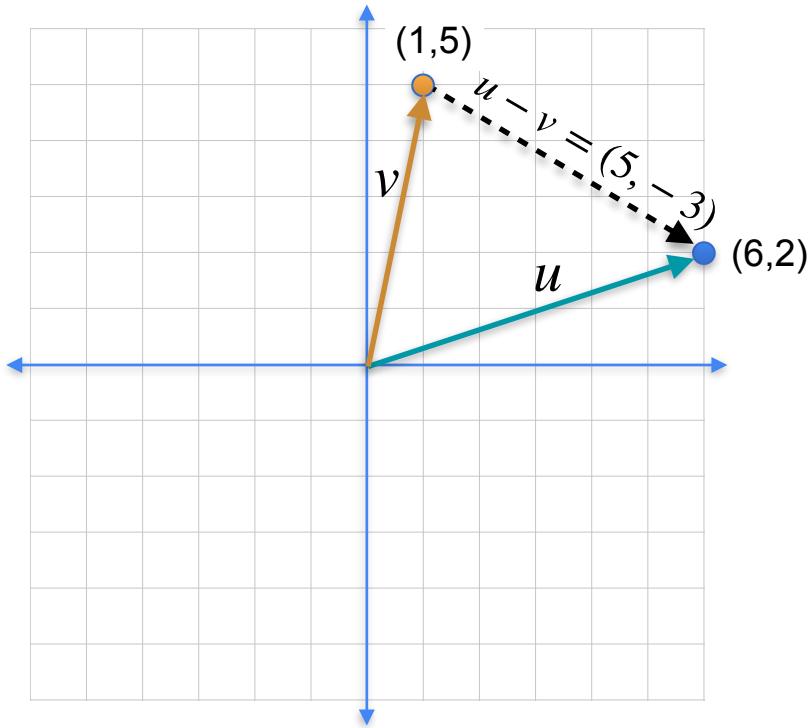


Distances



$$|u - v|_1 = |5| + |-3| = 8$$

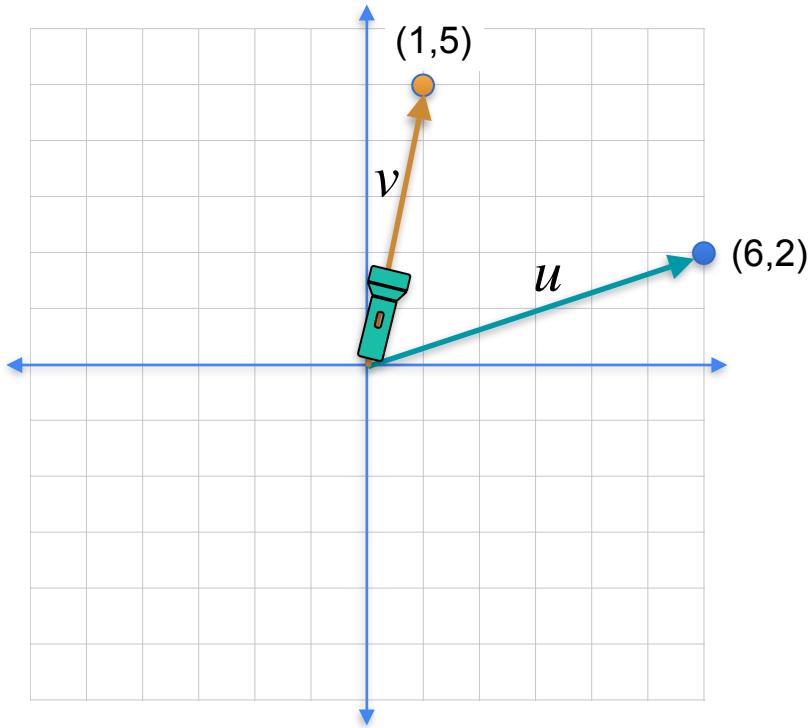
Distances



$$|u - v|_1 = |5| + |-3| = 8$$

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



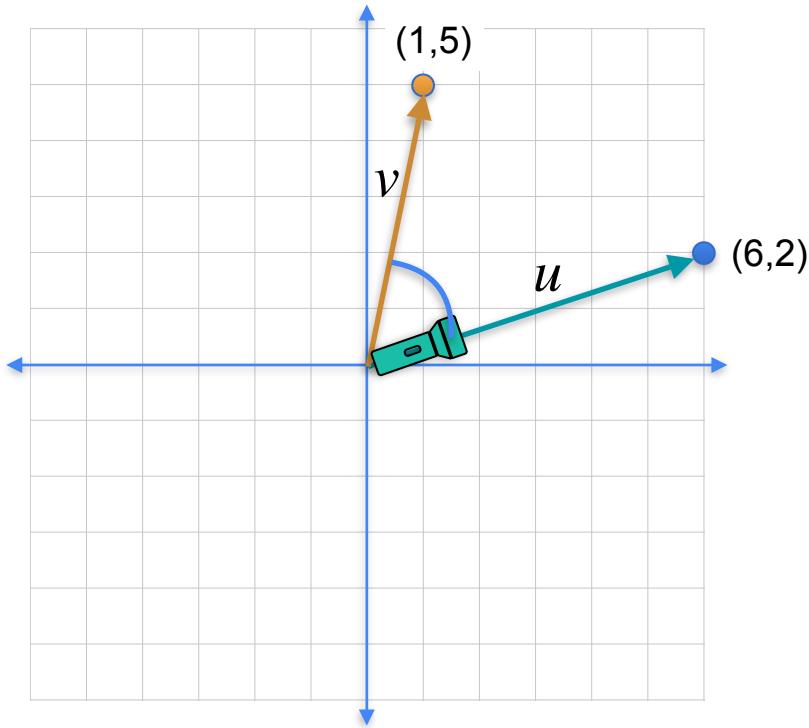


L1-distance $|u - v|_1 = |5| + |-3| = 8$



L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



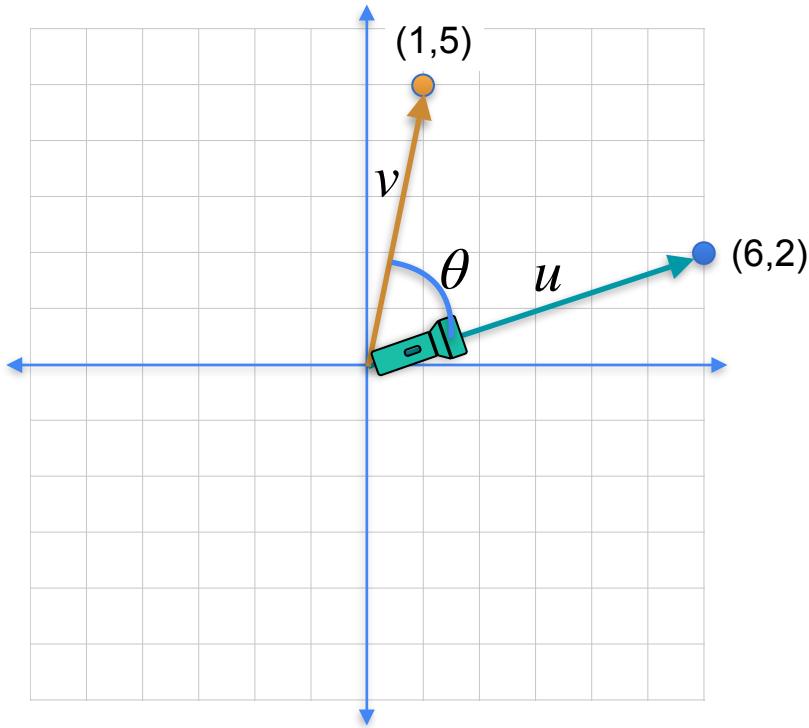


L1-distance $|u - v|_1 = |5| + |-3| = 8$



L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



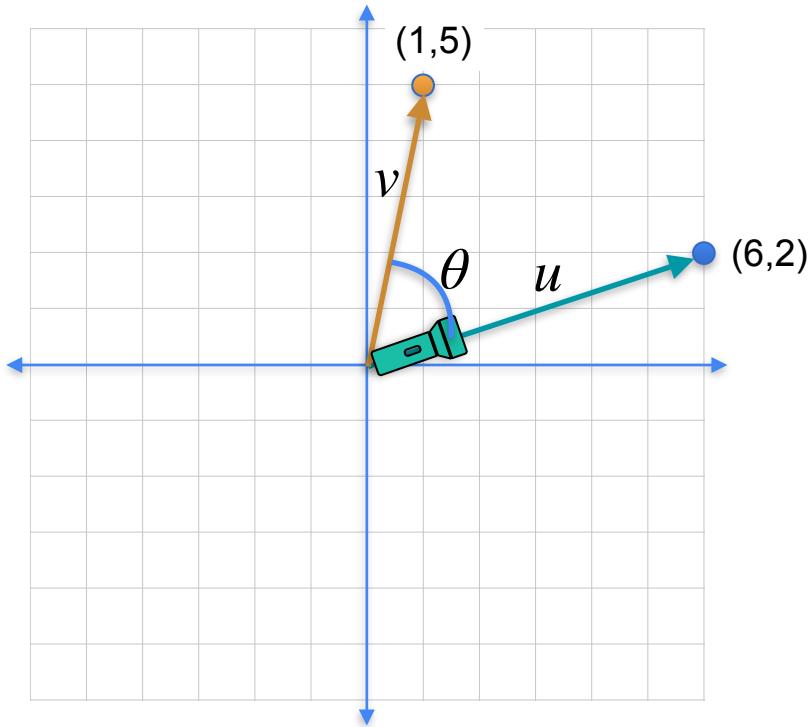


L1-distance $|u - v|_1 = |5| + |-3| = 8$



L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances





L1-distance $|u - v|_1 = |5| + |-3| = 8$



L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$



$\cos(\theta)$
Cosine distance

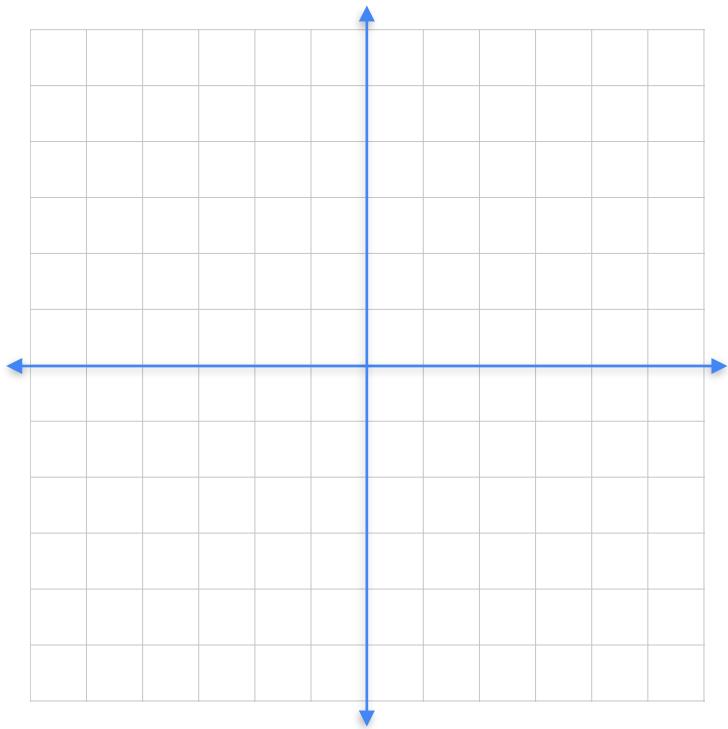


DeepLearning.AI

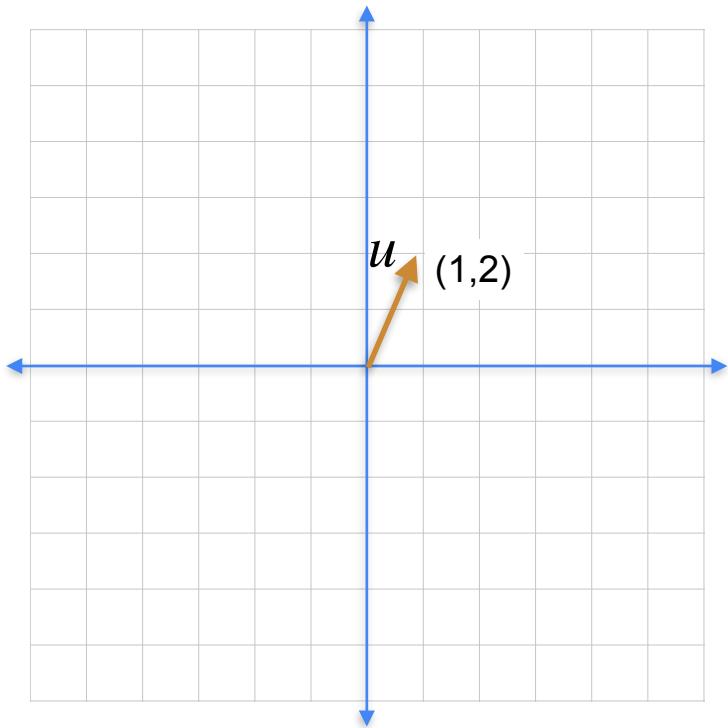
Vectors and Linear Transformations

Multiplying a vector by a scalar

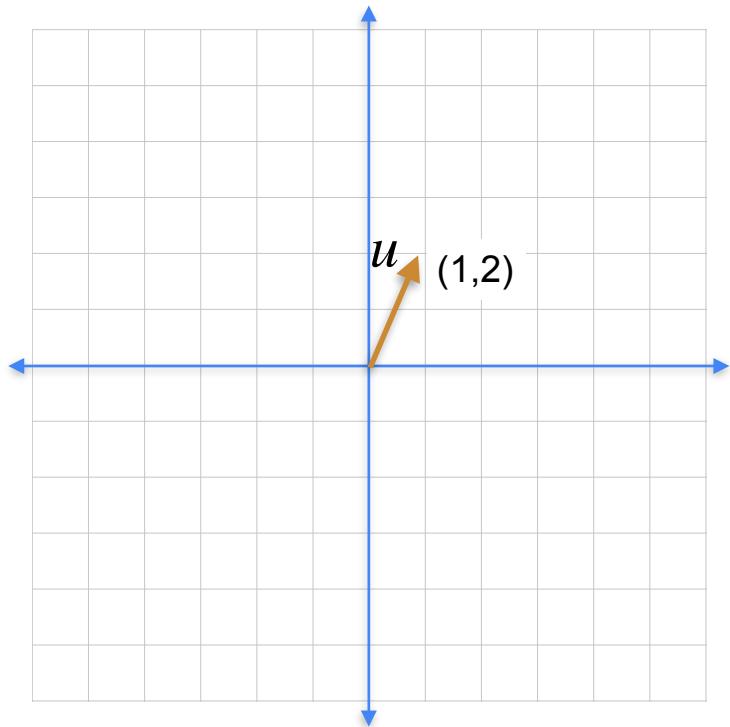
Multiplying a vector by a scalar



Multiplying a vector by a scalar

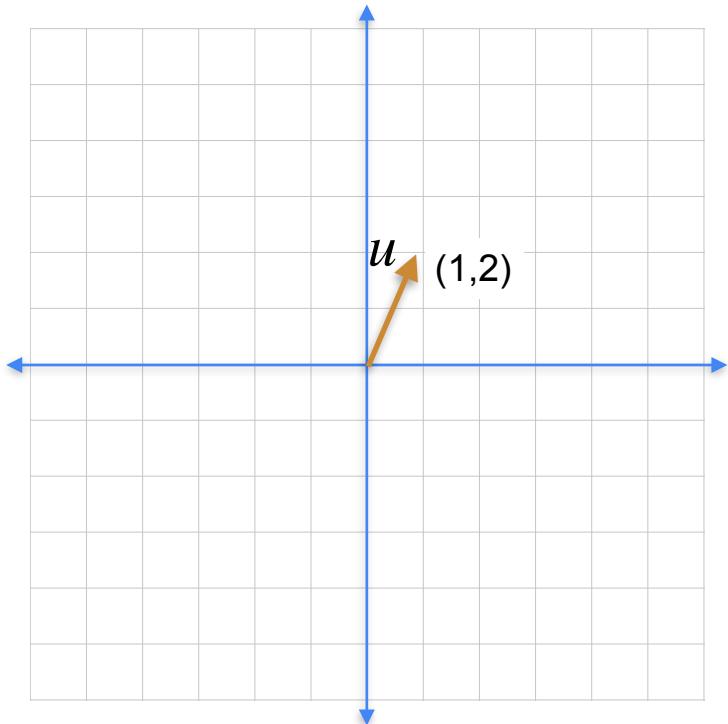


Multiplying a vector by a scalar



$$u = (1, 2)$$

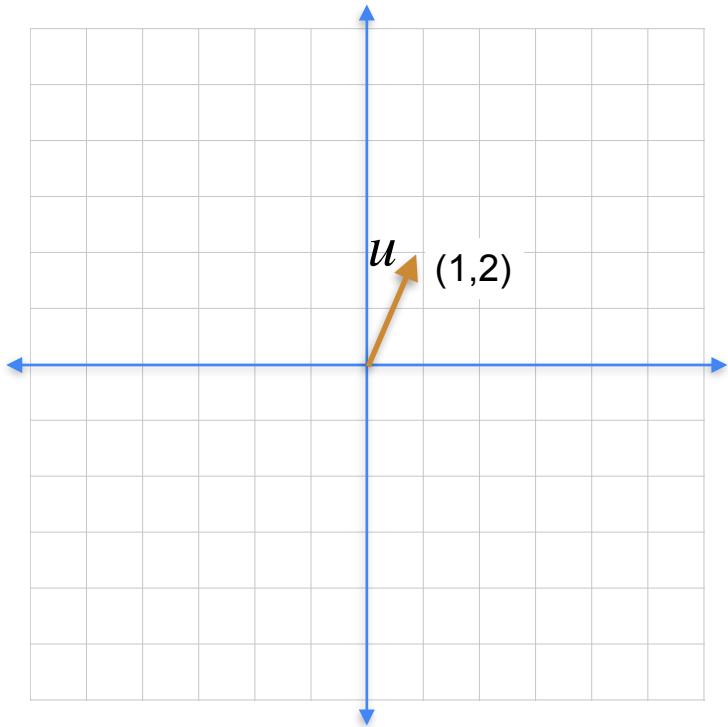
Multiplying a vector by a scalar



$$u = (1, 2)$$

$$\lambda = 3$$

Multiplying a vector by a scalar

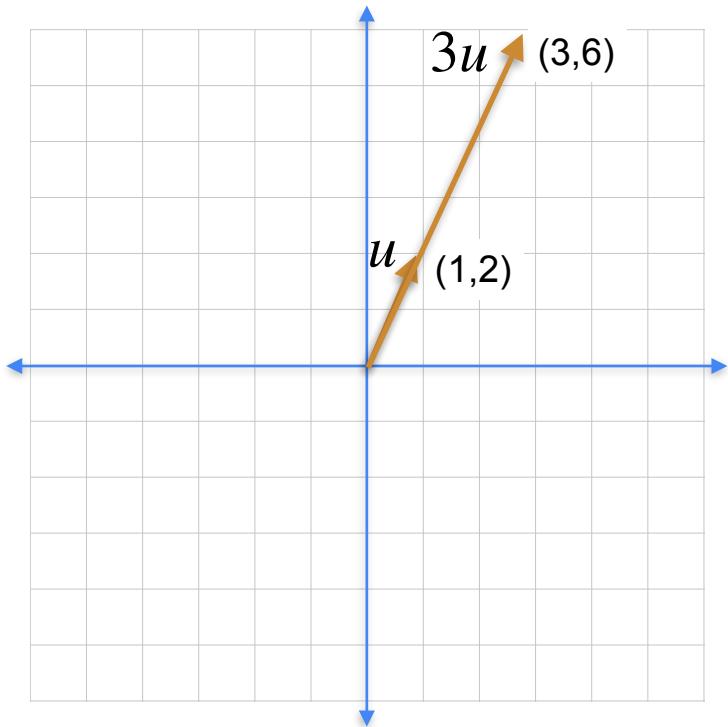


$$u = (1, 2)$$

$$\lambda = 3$$

$$\lambda u = (3, 6)$$

Multiplying a vector by a scalar

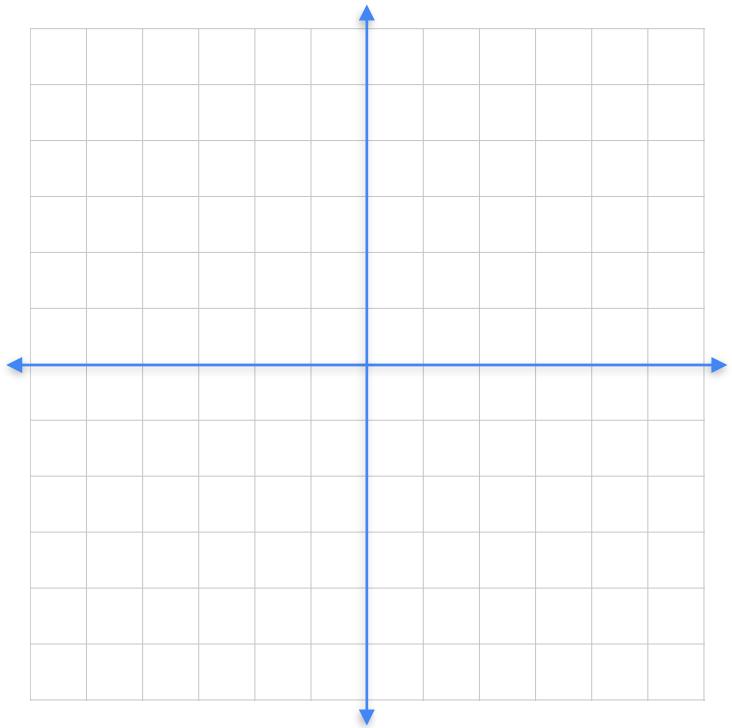


$$u = (1, 2)$$

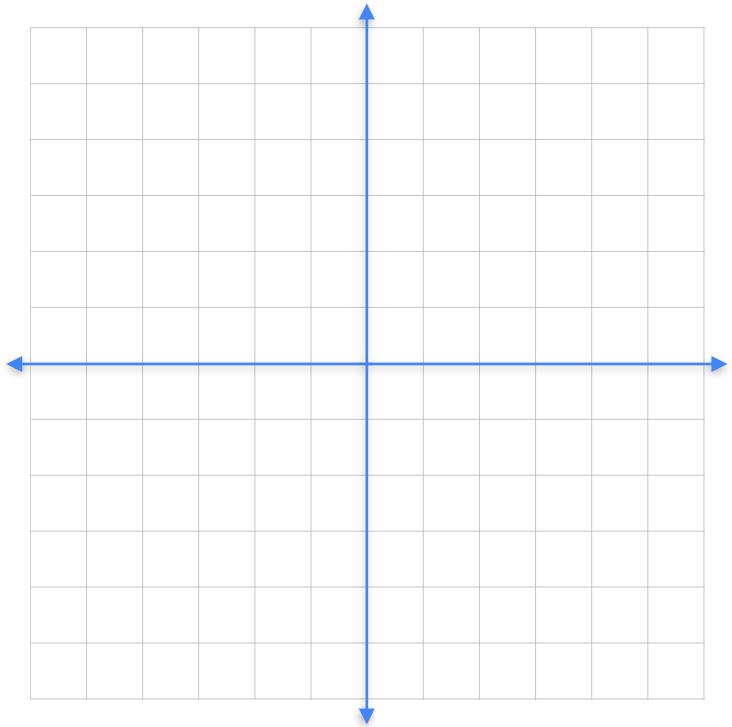
$$\lambda = 3$$

$$\lambda u = (3, 6)$$

If the scalar is negative

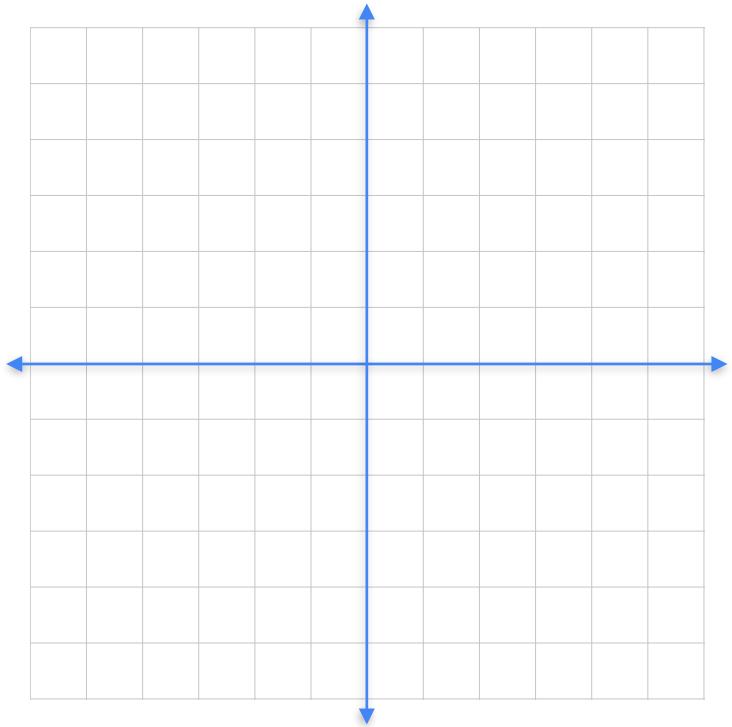


If the scalar is negative



$$u = (1, 2)$$

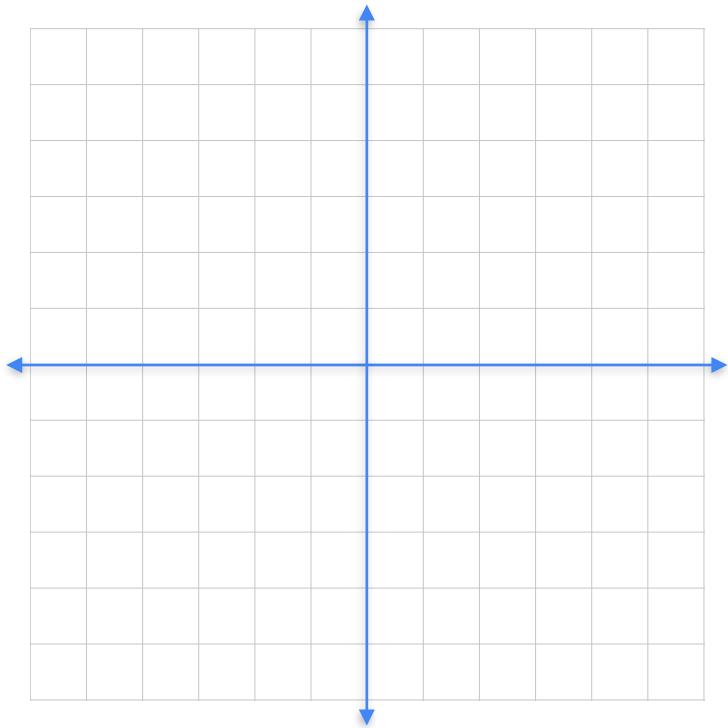
If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

If the scalar is negative

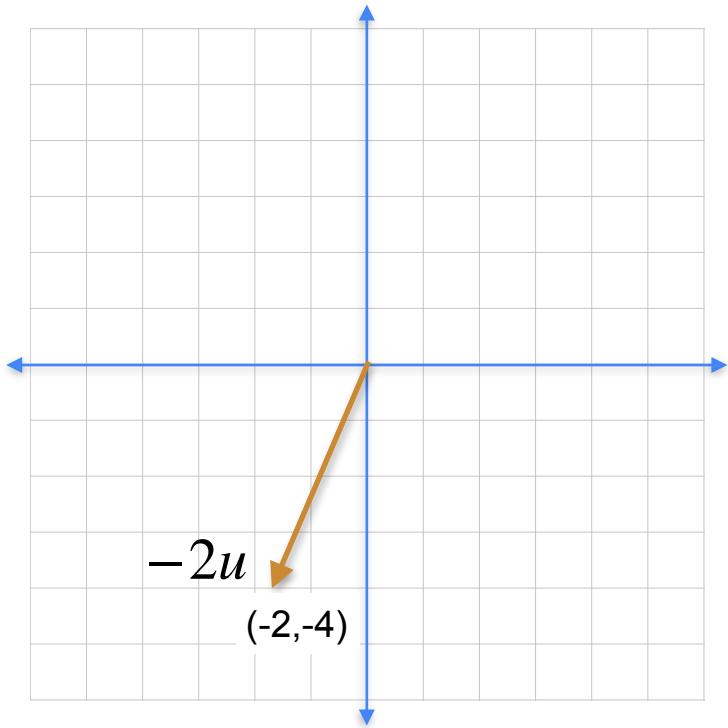


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative

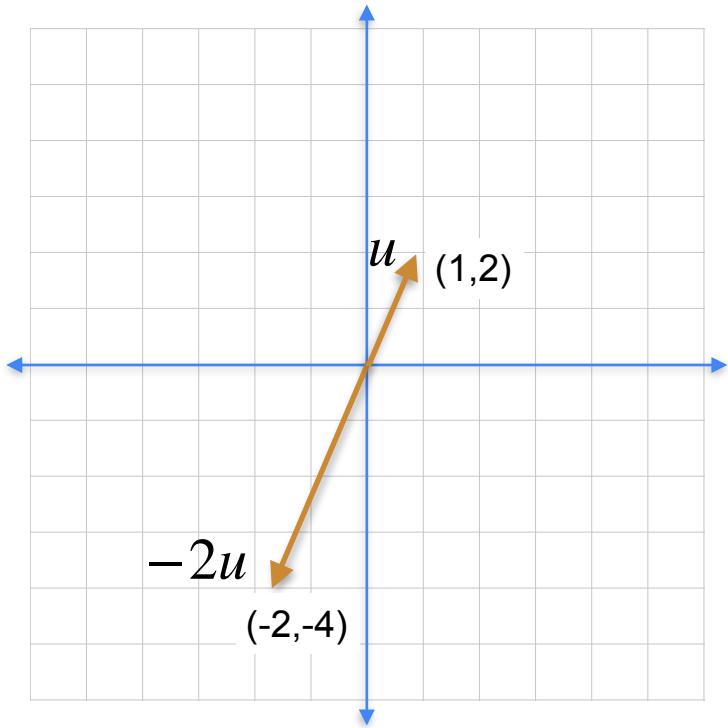


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



DeepLearning.AI

Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

| | |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

| | |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |

Prices

apples: \$3

bananas: \$5

cherries: \$2

| | |
|--|---|
| \$  | 3 |
| \$  | 5 |
| \$  | 2 |

Total price

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

| | |
|---|---|
|   | 2 |
|     | 4 |
|  | 1 |

Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$



| | |
|--|---|
| \$  | 3 |
| \$  | 5 |
| \$  | 2 |

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

| | |
|---|---|
|  | 2 |
|  | 4 |
|  | 1 |

Prices

apples: \$3

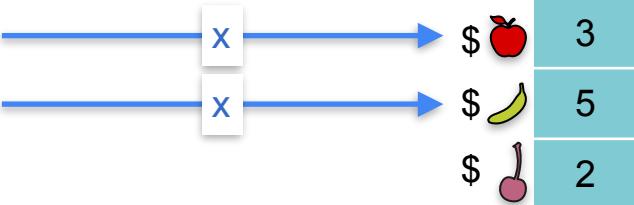
bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$



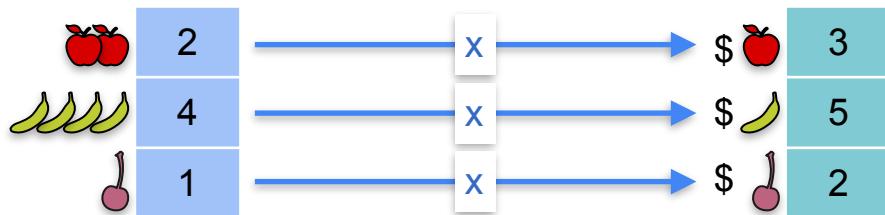
A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry



Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$

$$1 \times 2 = 2$$

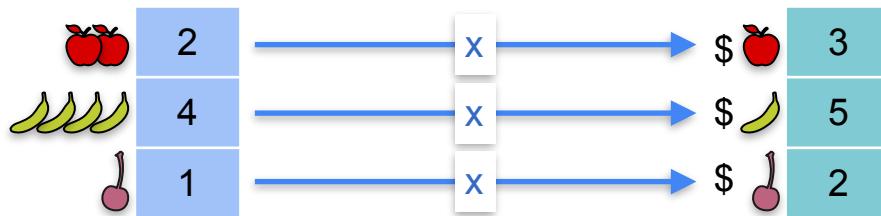
A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry



Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$

$$1 \times 2 = 2$$

$$6 + 20 + 2 = 28$$

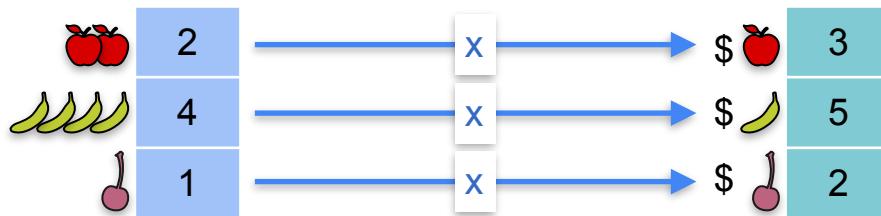
A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry



Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

\$28

The dot product

$$\begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} \begin{array}{c|c} & 2 \\ \hline 4 & \\ \hline 1 & \end{array} \cdot \begin{array}{c} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{array} \begin{array}{c|c} 3 \\ \hline 5 \\ \hline 2 \end{array} = \$28$$

The dot product

$$\begin{matrix} \text{apple} & \text{apple} \\ \text{banana} & \text{banana} \\ \text{cherry} & \end{matrix} \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{matrix} \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = \$28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

The diagram shows the dot product of two vectors. The first vector, on the left, represents the quantity of three types of fruit: 2 apples, 4 bananas, and 1 cherry. The second vector, on the right, represents the price per fruit: \$3 for an apple, \$5 for a banana, and \$2 for a cherry. The result of the dot product is \$28.

| | | |
|----|----|---|
| 2 | 4 | 1 |
| • | \$ | 3 |
| \$ | 5 | |
| \$ | 2 | |

= \$28

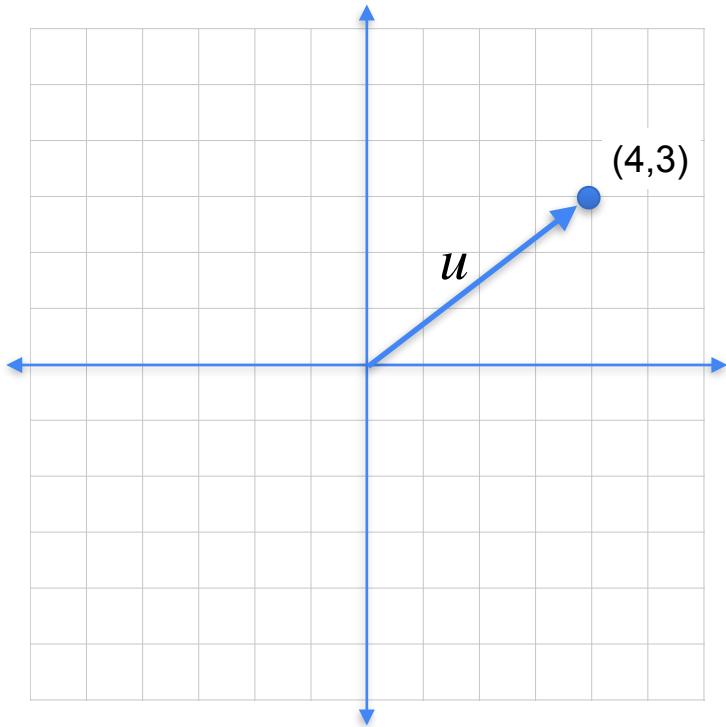
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

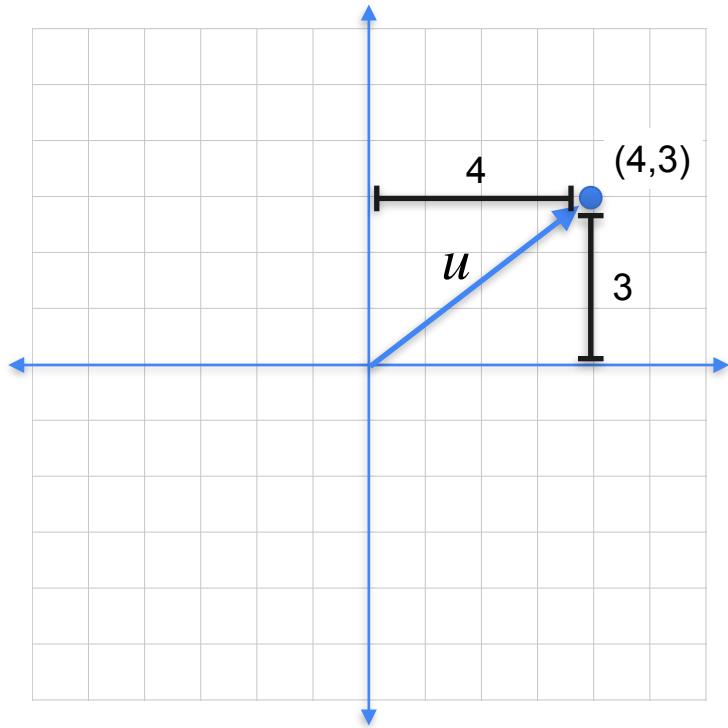
$$\begin{matrix} 2 & | & 4 & | & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

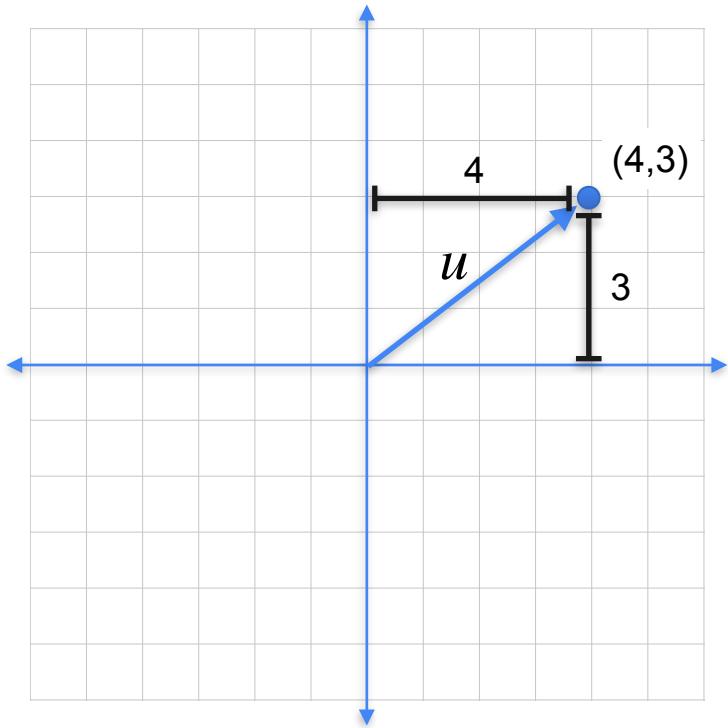
Norm of a vector using dot product



Norm of a vector using dot product

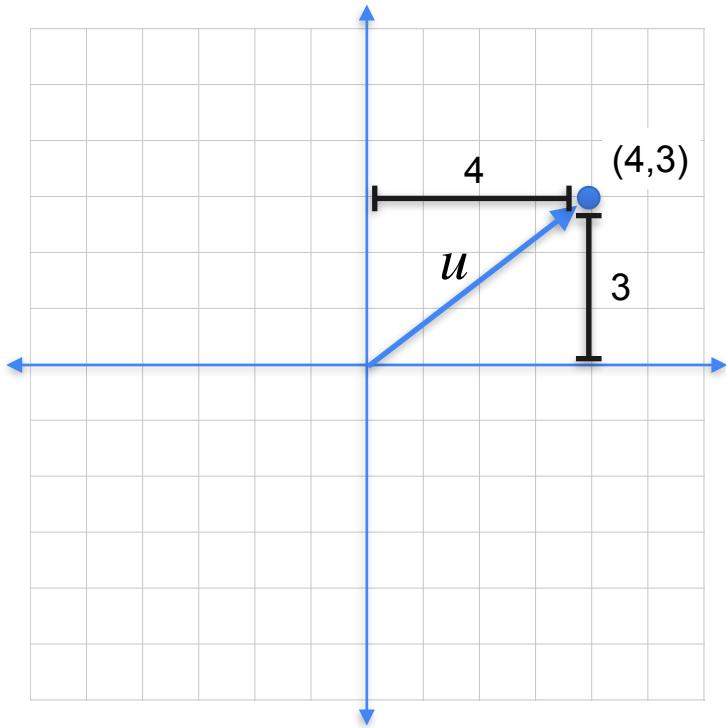


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

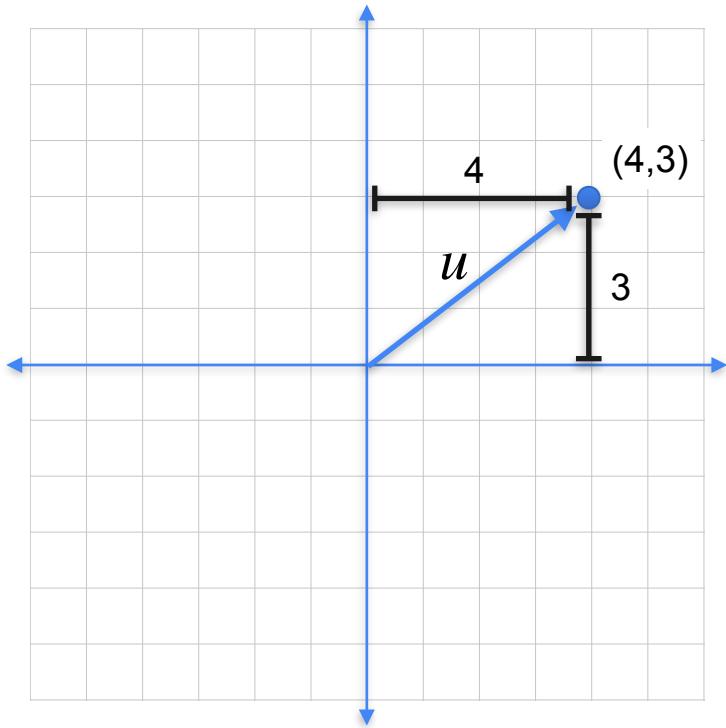
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 & \\ \hline 3 & \\ \hline \end{array} = 25$$

Norm of a vector using dot product

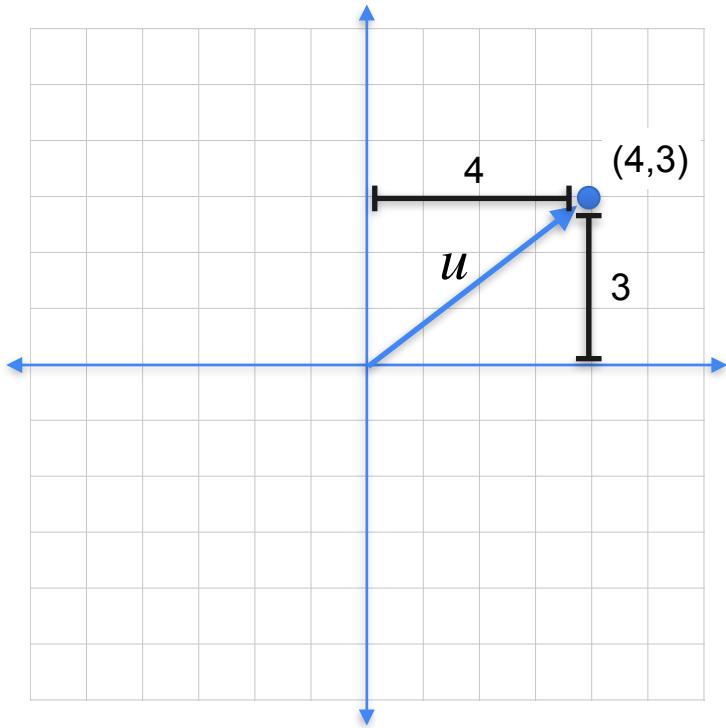


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{dot\ product(u, u)}$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{dot\ product(u, u)}$$

$$\| u \|_2 = \sqrt{\langle u, u \rangle}$$

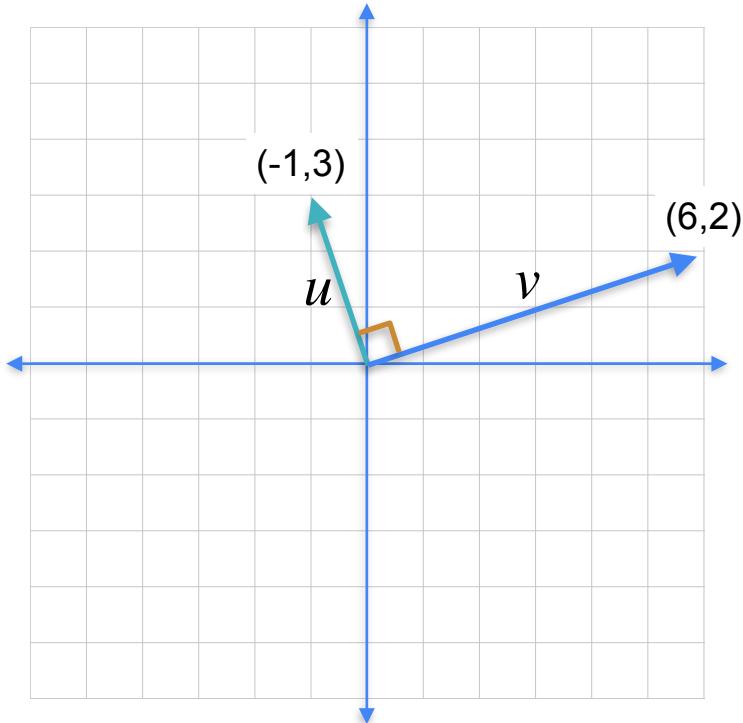


DeepLearning.AI

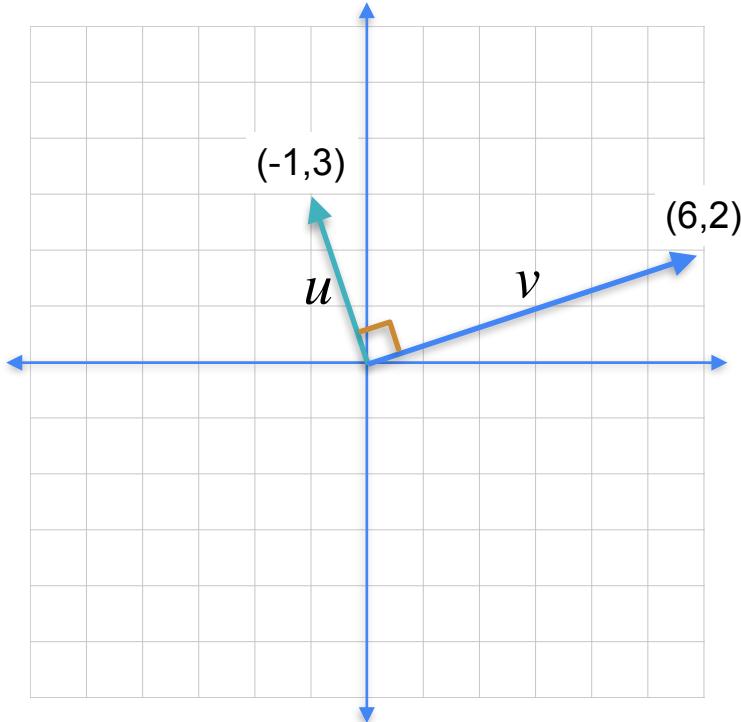
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0

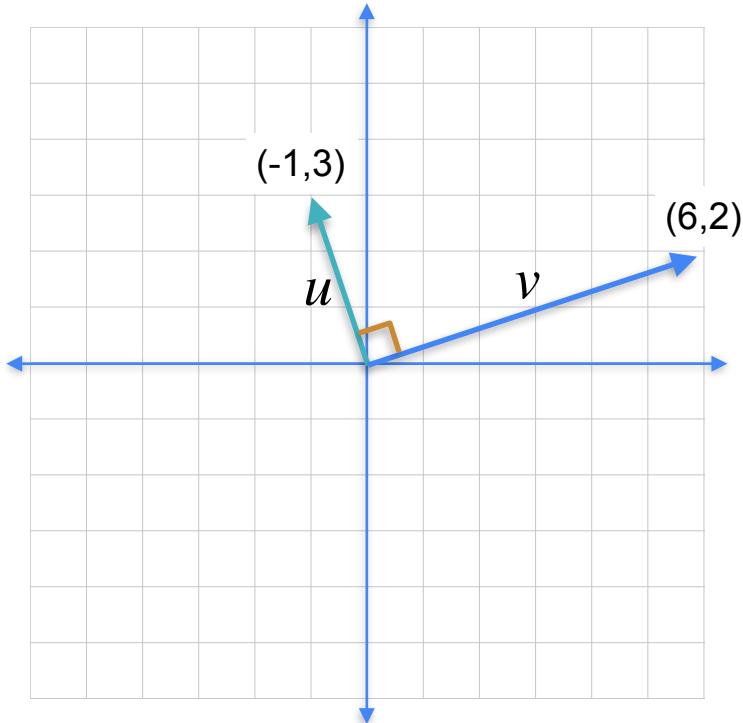


Orthogonal vectors have dot product 0



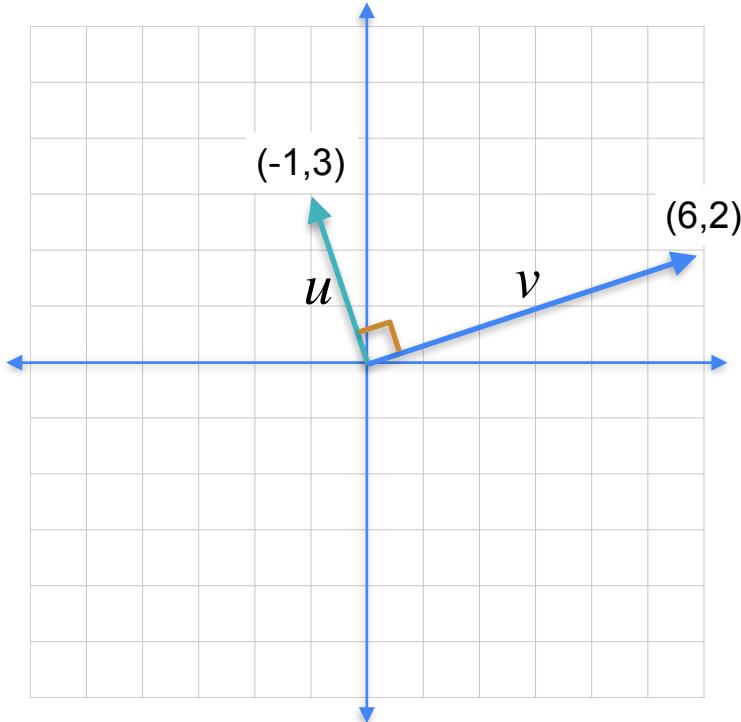
| | |
|---|---|
| 6 | 2 |
|---|---|

Orthogonal vectors have dot product 0



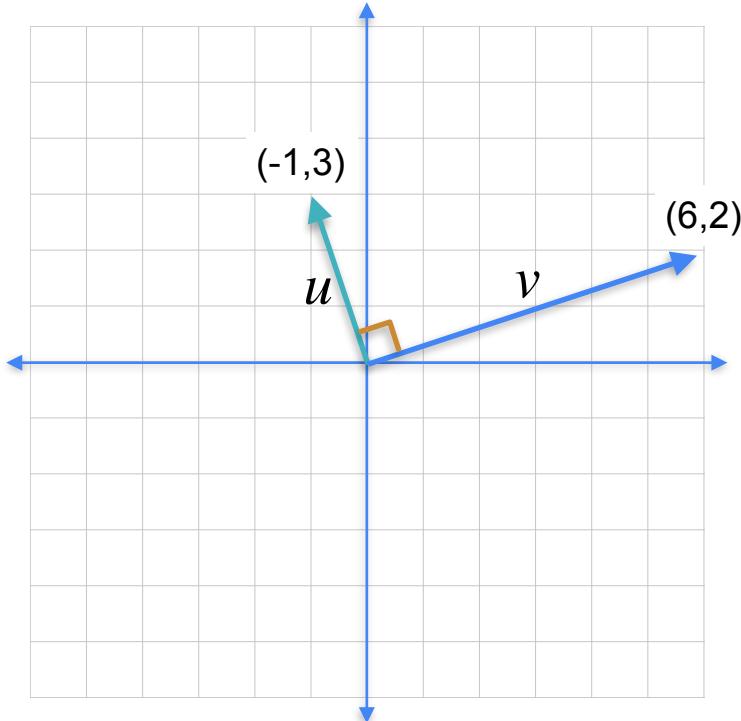
| | | |
|---|---|----|
| 6 | 2 | -1 |
| 3 | | |

Orthogonal vectors have dot product 0



$$\begin{matrix} 6 & 2 \end{matrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

Orthogonal vectors have dot product 0



$$\begin{matrix} 6 & 2 \end{matrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

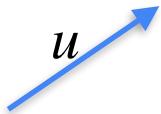
$$\langle u, v \rangle = 0$$

The dot product

The dot product

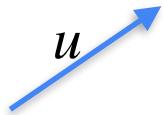


The dot product

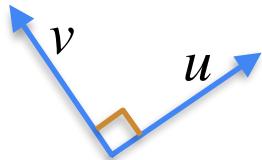


$$\langle u, u \rangle = |u|^2$$

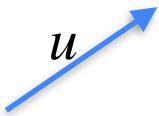
The dot product



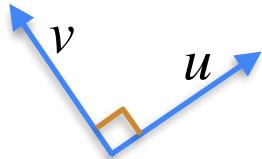
$$\langle u, u \rangle = |u|^2$$



The dot product

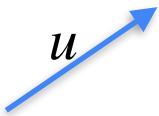


$$\langle u, u \rangle = |u|^2$$

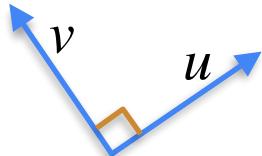


$$\langle u, v \rangle = 0$$

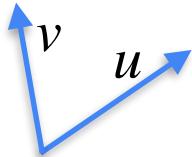
The dot product



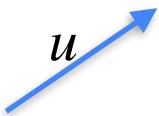
$$\langle u, u \rangle = |u|^2$$



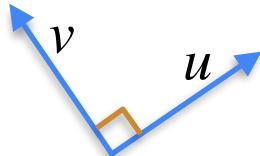
$$\langle u, v \rangle = 0$$



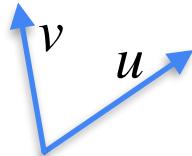
The dot product



$$\langle u, u \rangle = |u|^2$$



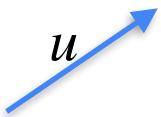
$$\langle u, v \rangle = 0$$



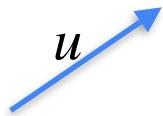
$$\langle u, v \rangle = ?$$

The dot product

The dot product

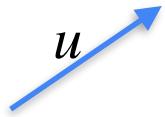
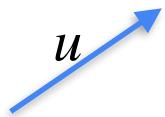


The dot product



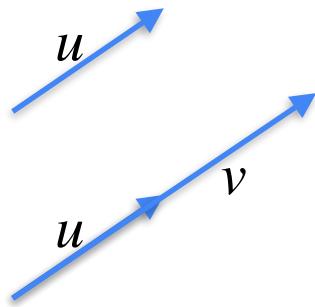
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



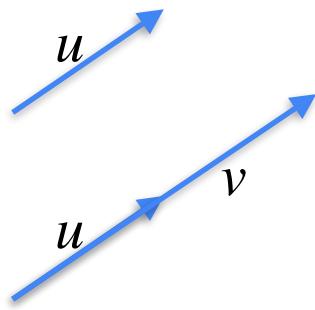
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

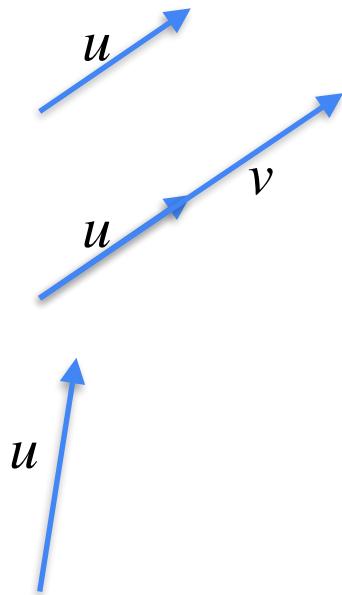
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

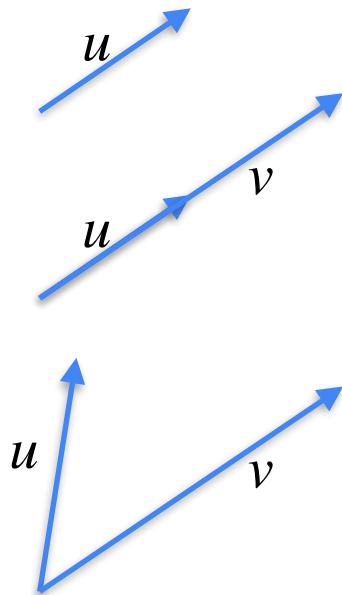
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

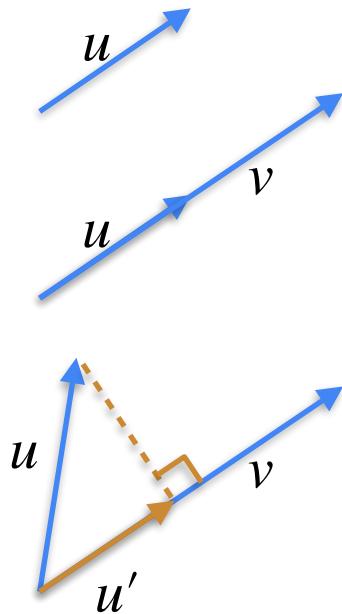
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

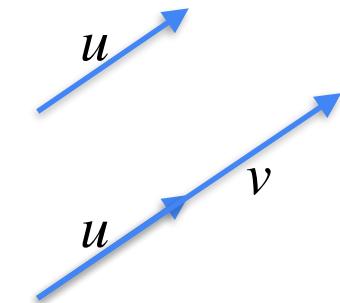
The dot product



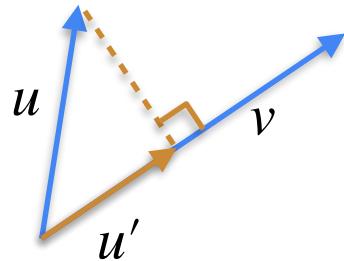
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product



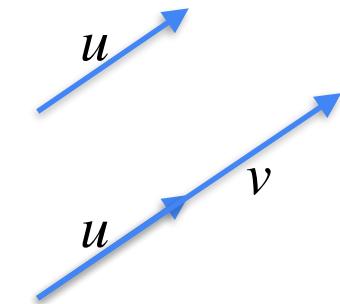
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



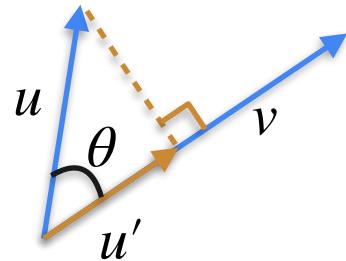
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



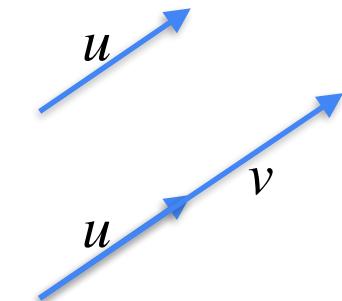
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



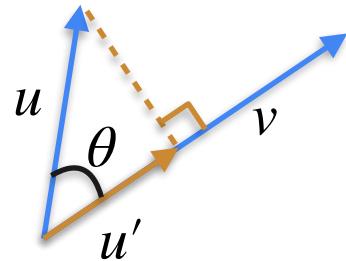
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



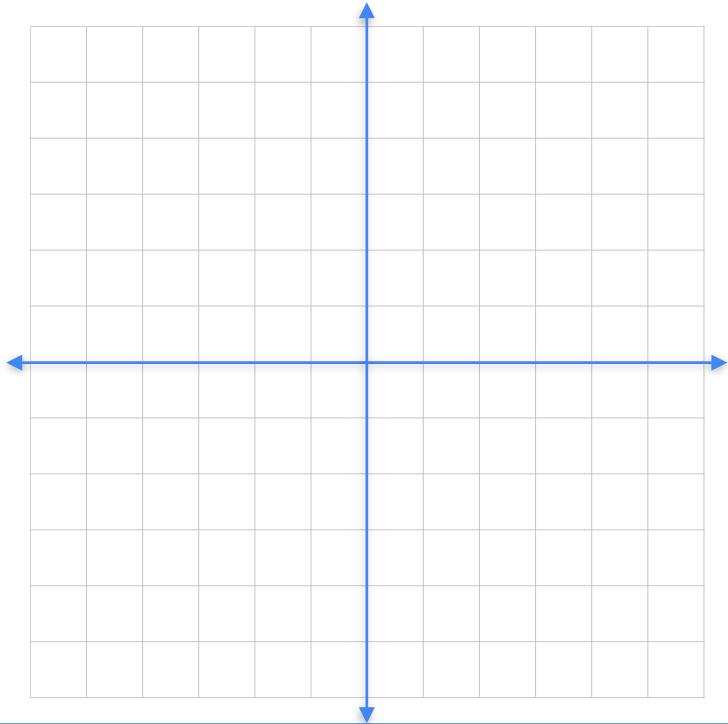
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



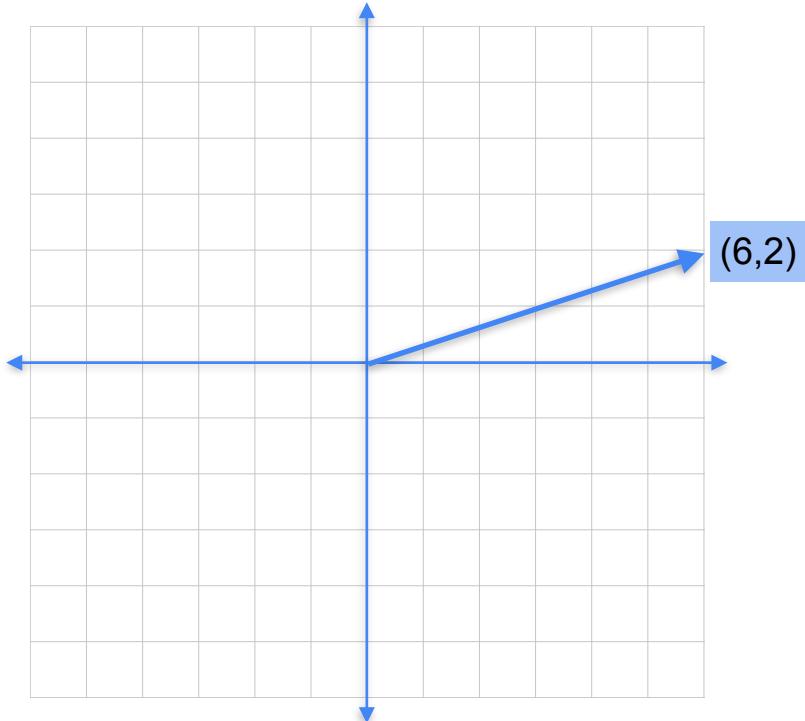
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

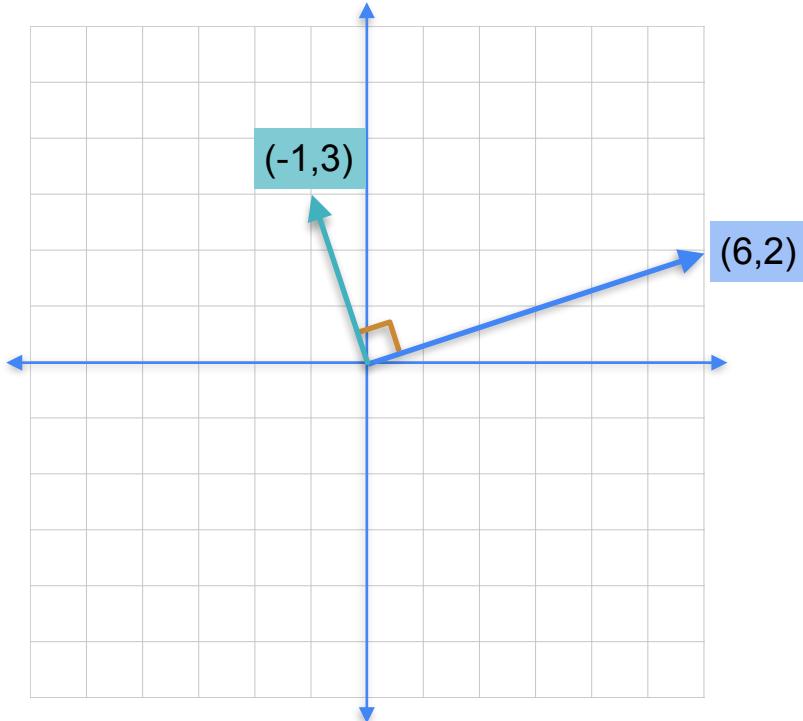
Geometric dot product



Geometric dot product

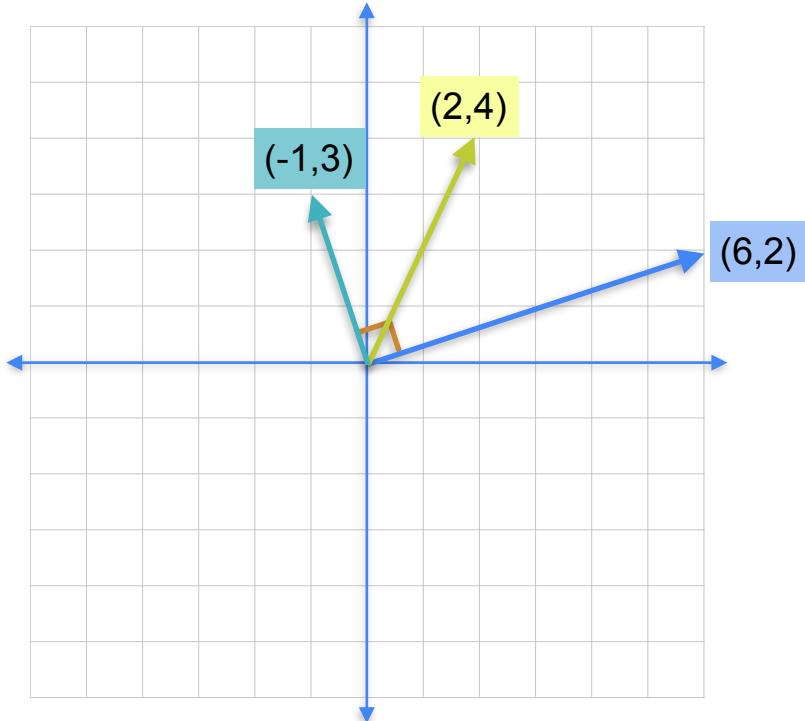


Geometric dot product



$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = 0$$

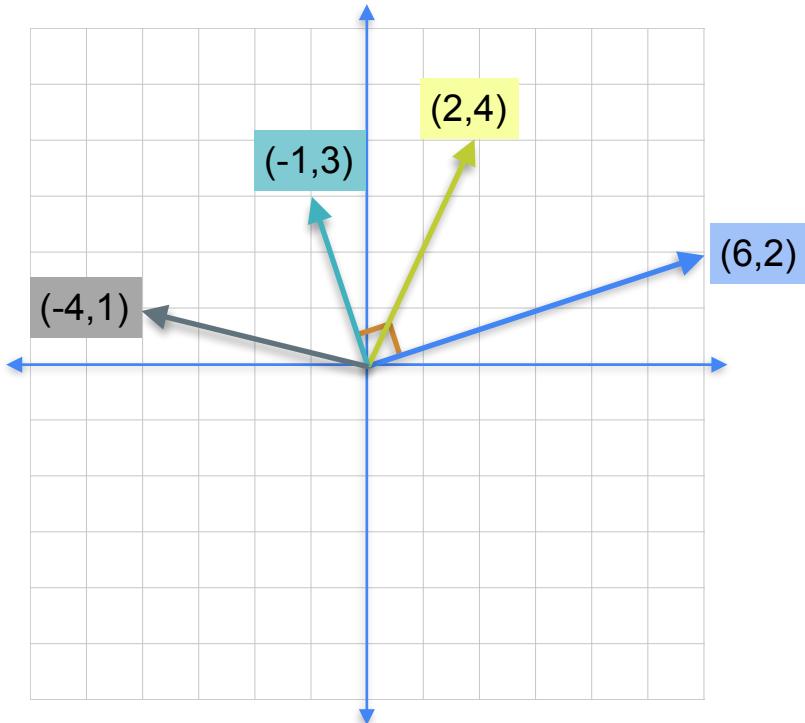
Geometric dot product



$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Geometric dot product

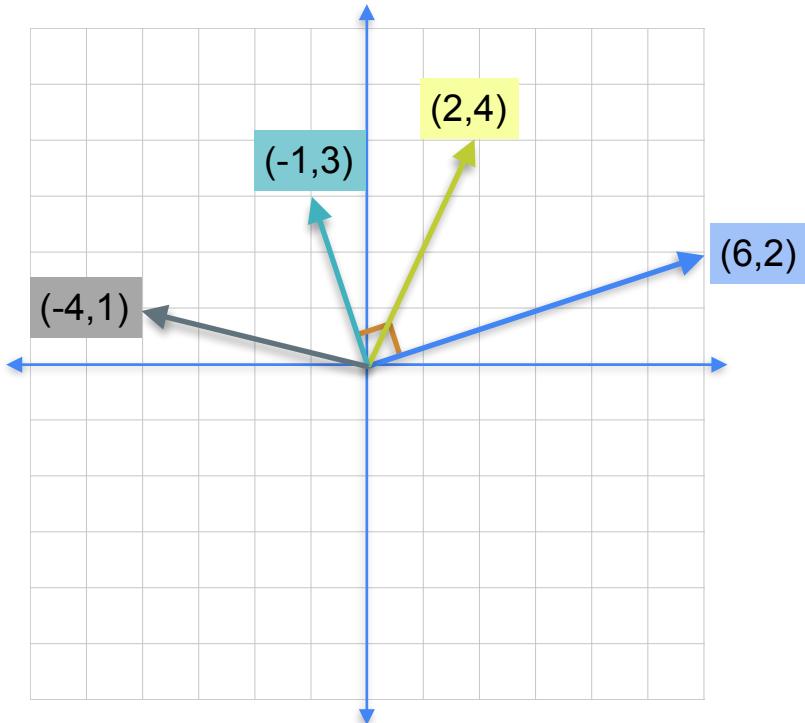


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

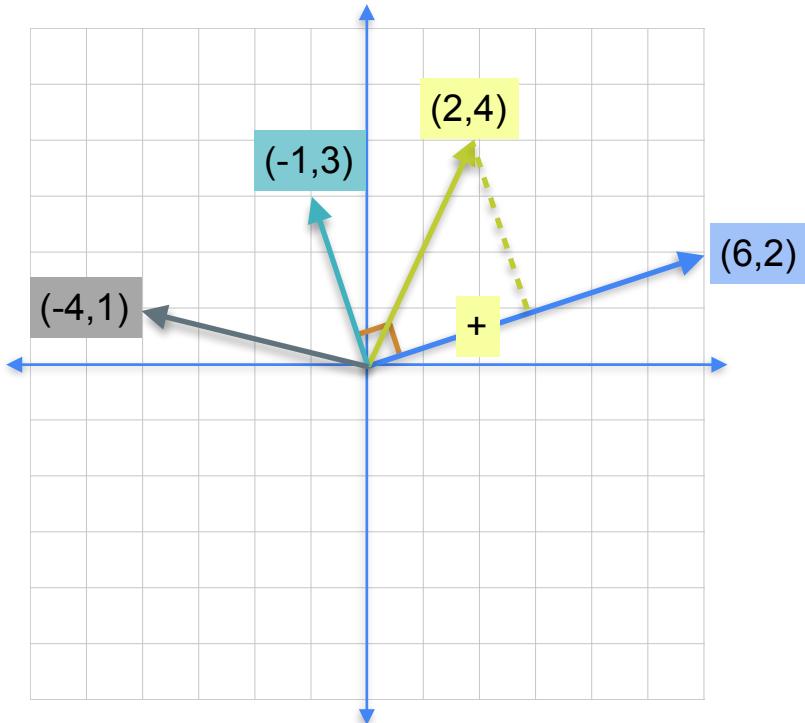


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

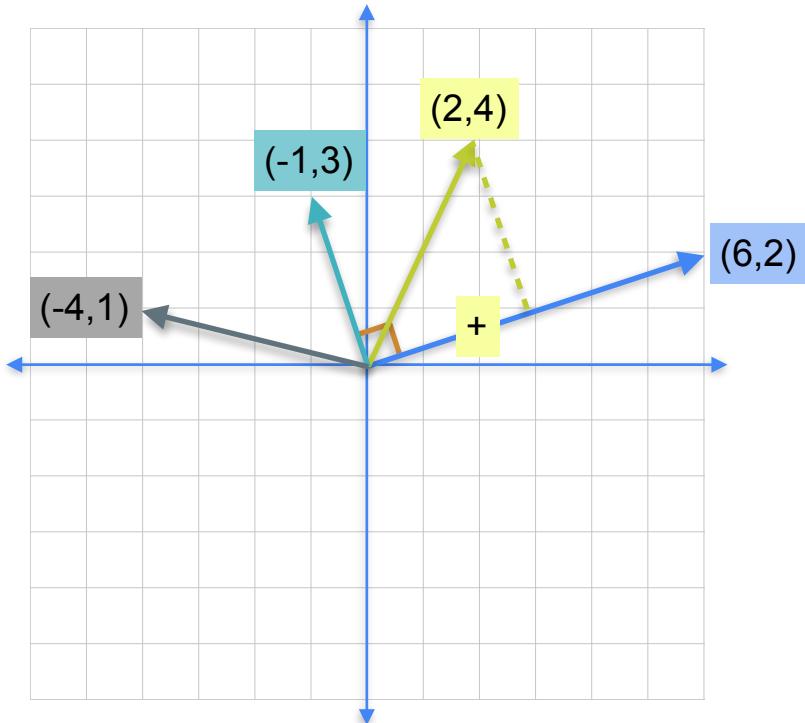


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

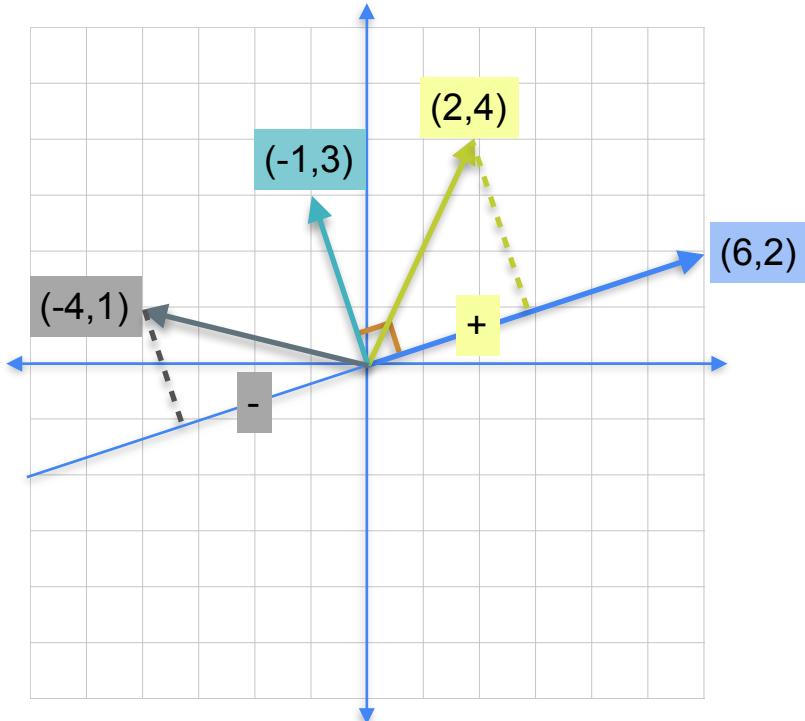


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

Geometric dot product

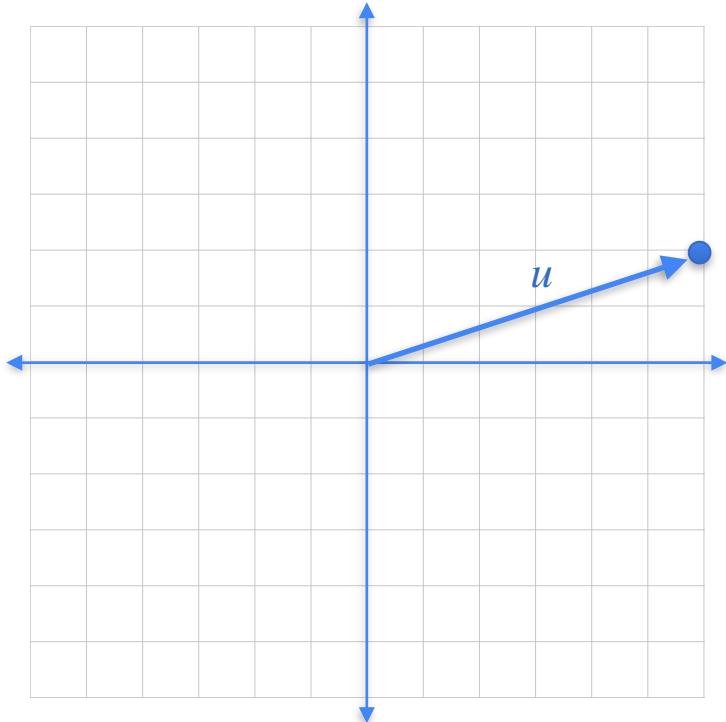


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

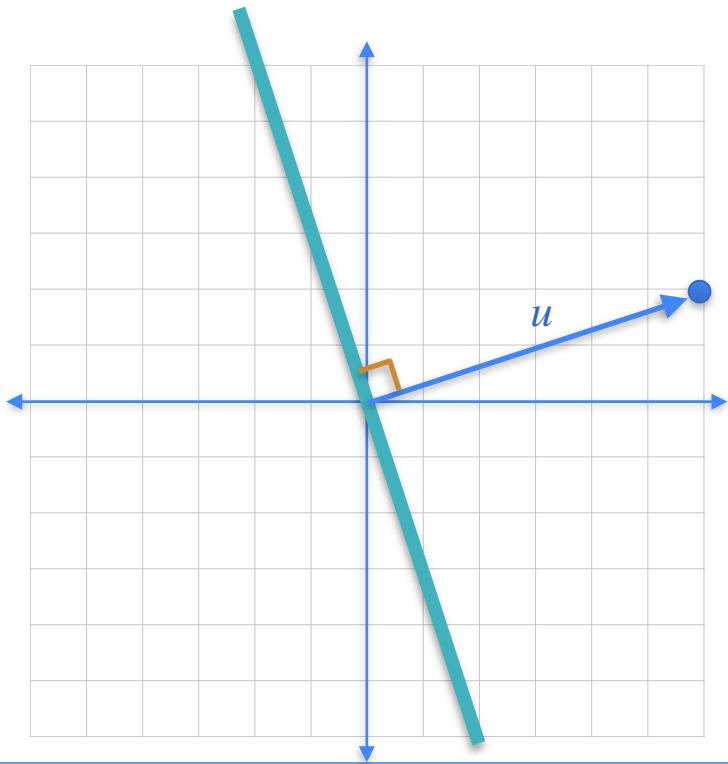
$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

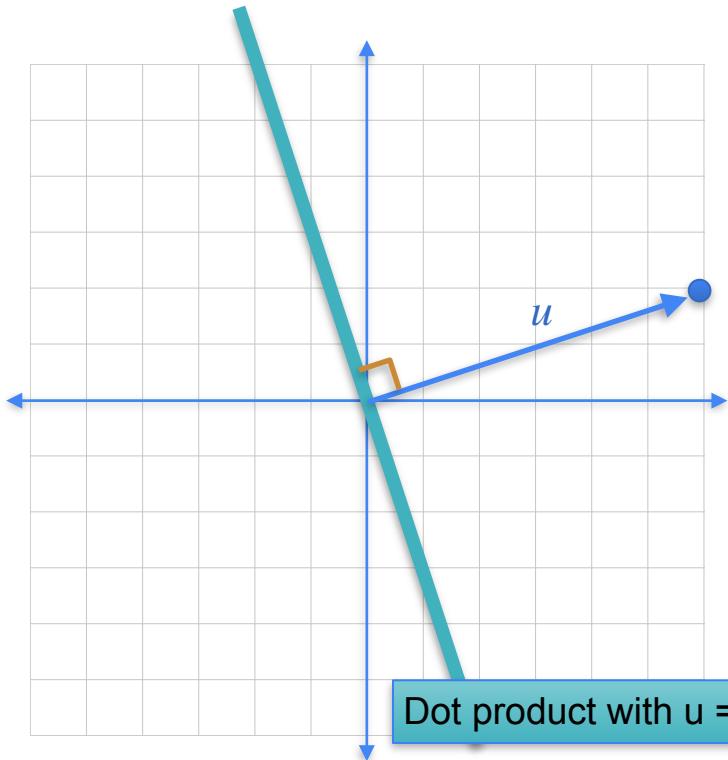
Geometric dot product



Geometric dot product

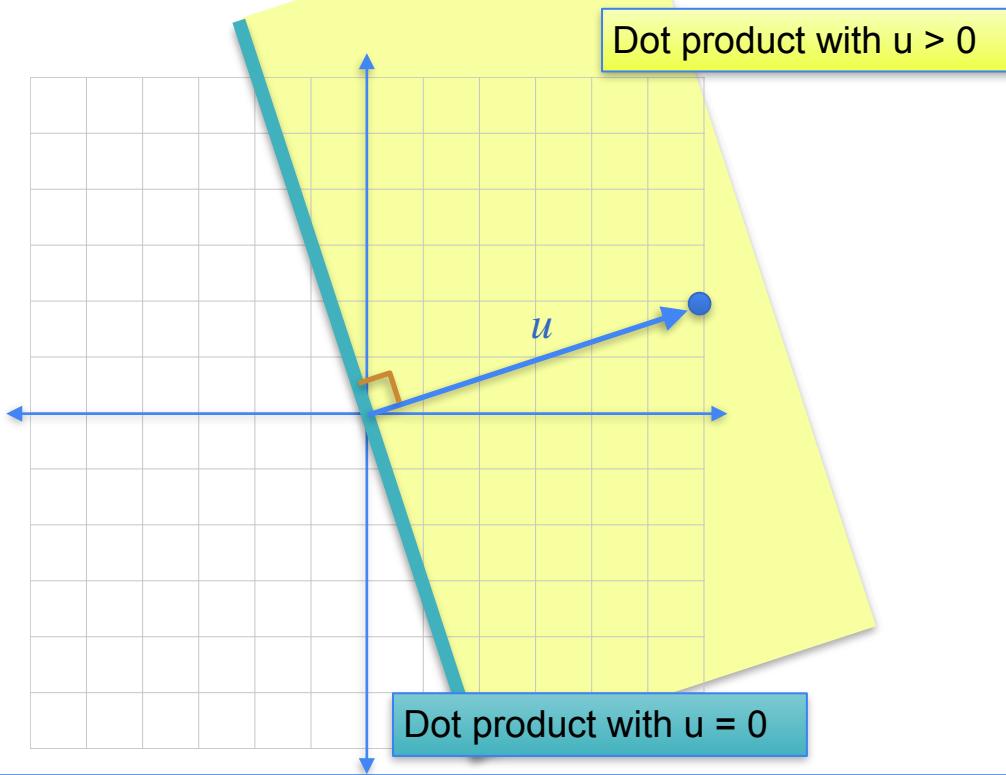


Geometric dot product



$$\langle u, v \rangle = 0$$

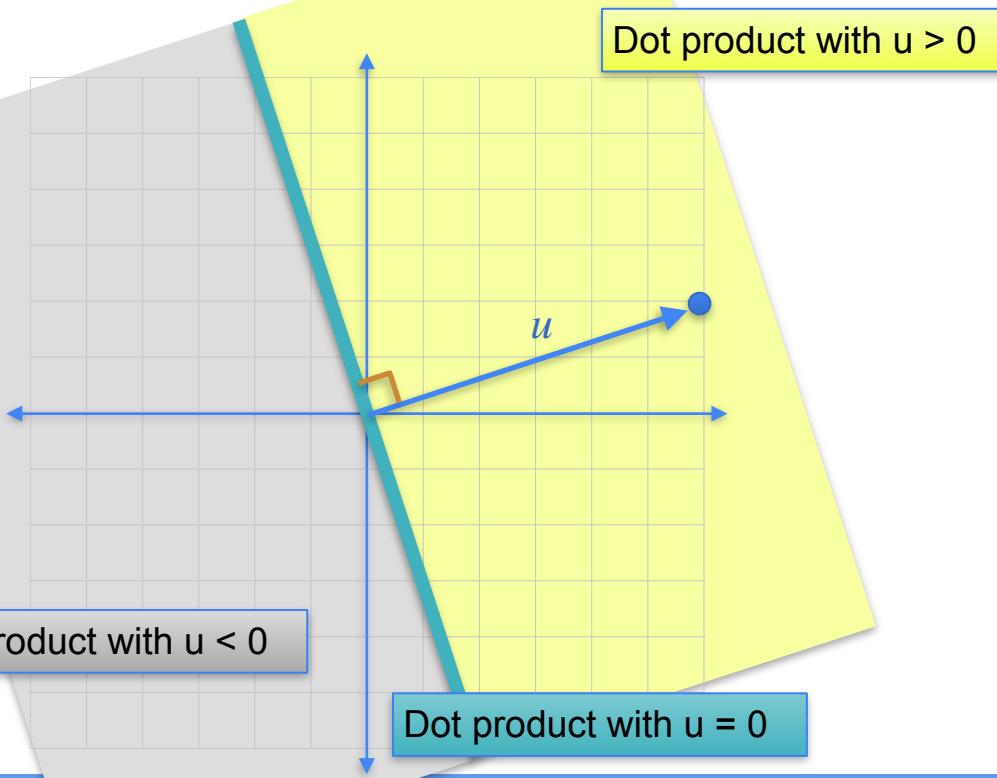
Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$



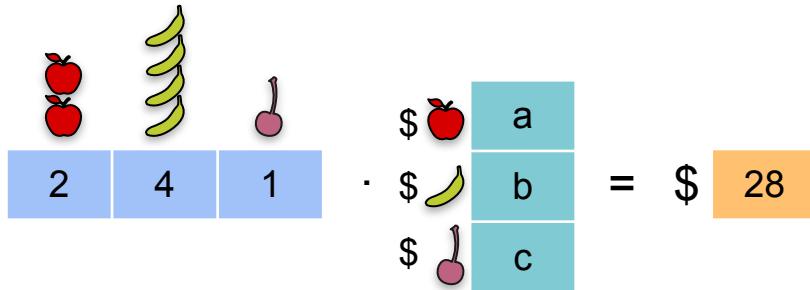
DeepLearning.AI

Vectors and Linear Transformations

**Multiplying a matrix by a
vector**

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

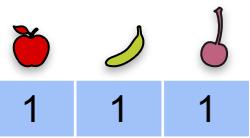
$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

| | | | | |
|---|---|---|------|---|
| | | | \$ | a |
| 1 | 1 | 1 | · \$ | b |
| | | | \$ | c |

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Diagram illustrating the first equation $a + b + c = 10$:

Three fruit icons (apple, banana, cherry) are shown above a row of three blue boxes, each containing the number 1.

The row of icons is multiplied by a column vector:

$$\begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ \text{banana} \\ \text{cherry} \end{matrix} \cdot \begin{matrix} a \\ b \\ c \end{matrix} = \$ 10$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates three linear equations using fruit icons (apple, banana, cherry) and price vectors (\$). The first equation, $a + b + c = 10$, is shown as a row vector of fruit icons (1 apple, 1 banana, 1 cherry) multiplied by a column vector of prices (\$1 for apple, \$1 for banana, \$1 for cherry) resulting in a total value of \$10. The second equation, $a + 2b + c = 15$, is shown as a row vector of fruit icons (1 apple, 2 bananas, 1 cherry) multiplied by a column vector of prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$15. The third equation, $a + b + 2c = 12$, is implied by the vertical alignment of the second equation's structure.

$$\begin{matrix} \text{apple} & \text{banana} & \text{cherry} \\ 1 & 1 & 1 \end{matrix} \cdot \begin{matrix} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{matrix} = \$\text{10}$$
$$\begin{matrix} \text{apple} & \text{banana} & \text{cherry} \\ 1 & 2 & 1 \end{matrix} \cdot \begin{matrix} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{matrix} = \$\text{15}$$

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: one apple, one banana, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where each component is labeled with a dollar sign (\$) and a fruit icon. The result is equal to a scalar value of 10.

| | | |
|----|--------|---|
| 1 | 1 | 1 |
| \$ | apple | a |
| \$ | banana | b |
| \$ | cherry | c |

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ & \text{apple} & a \\ \$ & \text{banana} & b \\ \$ & \text{cherry} & c \end{bmatrix} = \$ 10$$

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It shows a vector of fruits (one apple, two bananas, one cherry) multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$. The result is equal to a scalar value of 15.

| | | |
|----|--------|---|
| 1 | 2 | 1 |
| \$ | apple | a |
| \$ | banana | b |
| \$ | cherry | c |

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ & \text{apple} & a \\ \$ & \text{banana} & b \\ \$ & \text{cherry} & c \end{bmatrix} = \$ 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a blue horizontal bar divided into three equal segments, each labeled with the number 1. To the right of the fruit vector is a multiplication sign (\cdot). Next is a vector of prices: a red apple icon above a blue box labeled 'a', a yellow banana icon above a blue box labeled 'b', and a pink cherry icon above a blue box labeled 'c'. Below this price vector is a blue horizontal bar divided into three equal segments, each labeled with a dollar sign (\$) and the letter 'c'. After the multiplication sign, there is an equals sign (=) followed by a yellow box containing the number 10.

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a blue horizontal bar divided into three equal segments, each labeled with the number 1. To the right of the fruit vector is a multiplication sign (\cdot). Next is a vector of prices: a red apple icon above a blue box labeled 'a', a yellow banana icon above a blue box labeled 'b', and a pink cherry icon above a blue box labeled 'c'. Below this price vector is a blue horizontal bar divided into three equal segments, each labeled with a dollar sign (\$) and the letter 'c'. After the multiplication sign, there is an equals sign (=) followed by a yellow box containing the number 15.

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a blue horizontal bar divided into three equal segments, each labeled with the number 1. To the right of the fruit vector is a multiplication sign (\cdot). Next is a vertical stack of three boxes labeled 'a', 'b', and 'c' from top to bottom. Each box contains a dollar sign (\$) and a fruit icon: an apple for 'a', a banana for 'b', and a cherry for 'c'. To the right of this stack is another multiplication sign (\cdot). Then comes a blue box containing the number 10, which is highlighted in orange. The entire expression is preceded by an equals sign (=).

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It follows a similar structure to the first diagram. On the left, there is a vector of fruits: an apple, two bananas, and a cherry. Below this vector is a blue horizontal bar divided into three segments, with the middle segment labeled with the number 2 and the other two labeled with 1. To the right is a multiplication sign (\cdot). Next is a vertical stack of three boxes labeled 'a', 'b', and 'c'. Each box contains a dollar sign (\$) and a fruit icon: an apple for 'a', a banana for 'b', and a cherry for 'c'. To the right is another multiplication sign (\cdot). Then comes a blue box containing the number 15, which is highlighted in orange. The entire expression is preceded by an equals sign (=).

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + 2c = 12$ using vectors. It follows the same structure as the previous diagrams. On the left, there is a vector of fruits: an apple, a banana, and two cherries. Below this vector is a blue horizontal bar divided into three segments, with the middle segment labeled with the number 2 and the other two labeled with 1. To the right is a multiplication sign (\cdot). Next is a vertical stack of three boxes labeled 'a', 'b', and 'c'. Each box contains a dollar sign (\$) and a fruit icon: an apple for 'a', a banana for 'b', and a cherry for 'c'. To the right is another multiplication sign (\cdot). Then comes a blue box containing the number 12, which is highlighted in orange. The entire expression is preceded by an equals sign (=).

Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation $a + b + c = 10$. It shows a vector of fruit counts (1 apple, 1 banana, 1 cherry) multiplied by a vector of fruit prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$10.

| | | |
|----|--------|---|
| 1 | 1 | 1 |
| \$ | apple | a |
| 1 | banana | b |
| \$ | cherry | c |

$$1 \cdot \$1 + 1 \cdot \$2 + 1 \cdot \$1 = \$10$$

$$a + 2b + c = 15$$

A diagram illustrating the equation $a + 2b + c = 15$. It shows a vector of fruit counts (1 apple, 2 bananas, 1 cherry) multiplied by a vector of fruit prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$15.

| | | |
|----|--------|---|
| 1 | 2 | 1 |
| \$ | apple | a |
| 1 | banana | b |
| \$ | cherry | c |

$$1 \cdot \$1 + 2 \cdot \$2 + 1 \cdot \$1 = \$15$$

$$a + b + 2c = 12$$

A diagram illustrating the equation $a + b + 2c = 12$. It shows a vector of fruit counts (1 apple, 1 banana, 2 cherries) multiplied by a vector of fruit prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$12.

| | | |
|----|--------|---|
| 1 | 1 | 2 |
| \$ | apple | a |
| 1 | banana | b |
| \$ | cherry | c |

$$1 \cdot \$1 + 1 \cdot \$2 + 2 \cdot \$1 = \$12$$

Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation $a + b + c = 10$. It shows a vector of fruit counts (1 apple, 1 banana, 1 cherry) multiplied by a vector of fruit prices (\$1 for apple, \$1 for banana, \$1 for cherry) resulting in a total value of \$10.

| | | |
|----|--------|---|
| 1 | 1 | 1 |
| \$ | apple | a |
| \$ | banana | b |
| \$ | cherry | c |

$$\begin{matrix} \text{apple} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \text{banana} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \text{cherry} \\ 1 \\ \$ \end{matrix} = \$ \boxed{10}$$

$$a + 2b + c = 15$$

A diagram illustrating the equation $a + 2b + c = 15$. It shows a vector of fruit counts (1 apple, 2 bananas, 1 cherry) multiplied by a vector of fruit prices (\$1 for apple, \$1 for banana, \$1 for cherry) resulting in a total value of \$15.

| | | |
|----|--------|---|
| 1 | 2 | 1 |
| \$ | apple | a |
| \$ | banana | b |
| \$ | cherry | c |

$$\begin{matrix} \text{apple} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \text{banana} \\ 2 \\ \$ \end{matrix} \cdot \begin{matrix} \text{cherry} \\ 1 \\ \$ \end{matrix} = \$ \boxed{15}$$

$$a + b + 2c = 12$$

A diagram illustrating the equation $a + b + 2c = 12$. It shows a vector of fruit counts (1 apple, 1 banana, 2 cherries) multiplied by a vector of fruit prices (\$1 for apple, \$1 for banana, \$1 for cherry) resulting in a total value of \$12.

| | | |
|----|--------|---|
| 1 | 1 | 2 |
| \$ | apple | a |
| \$ | banana | b |
| \$ | cherry | c |

$$\begin{matrix} \text{apple} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \text{banana} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \text{cherry} \\ 2 \\ \$ \end{matrix} = \$ \boxed{12}$$

Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation $a + b + c = 10$. It shows three blue boxes representing the counts of apples, bananas, and cherries (1, 1, 1). To the right is a teal matrix with columns labeled a , b , and c , representing the price of each fruit (\$1, \$2, \$1). Below the matrix is the equation $\begin{matrix} \text{apple} \\ 1 \\ \text{banana} \\ \$ \\ \text{cherry} \end{matrix} \cdot \begin{matrix} a \\ \$1 \\ b \\ \$2 \\ c \\ \$1 \end{matrix} = \$10$.

$$a + 2b + c = 15$$

A diagram illustrating the equation $a + 2b + c = 15$. It shows three blue boxes representing the counts of apples, bananas, and cherries (1, 2, 1). To the right is a teal matrix with columns labeled a , b , and c , representing the price of each fruit (\$1, \$2, \$1). Below the matrix is the equation $\begin{matrix} \text{apple} \\ 1 \\ \text{banana} \\ \$ \\ \text{cherry} \end{matrix} \cdot \begin{matrix} a \\ \$1 \\ b \\ \$2 \\ c \\ \$1 \end{matrix} = \$15$.

$$a + b + 2c = 12$$

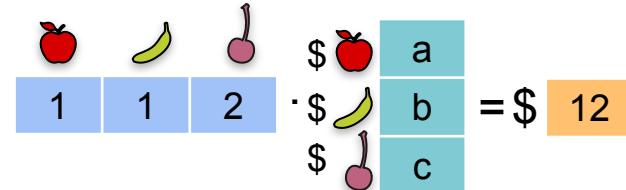
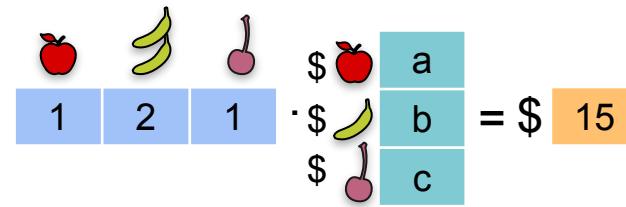
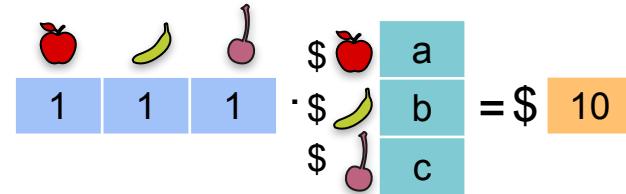
A diagram illustrating the equation $a + b + 2c = 12$. It shows three blue boxes representing the counts of apples, bananas, and cherries (1, 1, 2). To the right is a teal matrix with columns labeled a , b , and c , representing the price of each fruit (\$1, \$2, \$1). Below the matrix is the equation $\begin{matrix} \text{apple} \\ 1 \\ \text{banana} \\ \$ \\ \text{cherry} \end{matrix} \cdot \begin{matrix} a \\ \$1 \\ b \\ \$2 \\ c \\ \$1 \end{matrix} = \$12$.

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

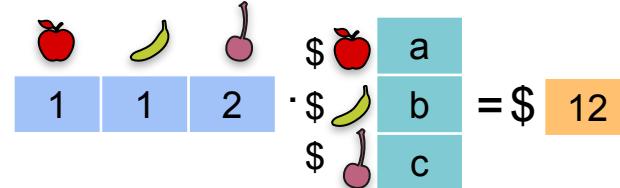
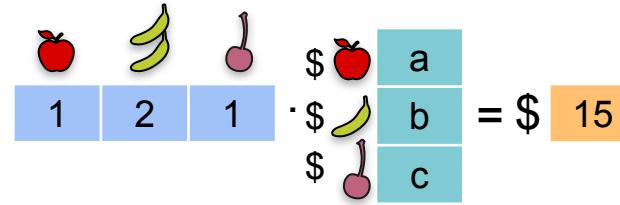
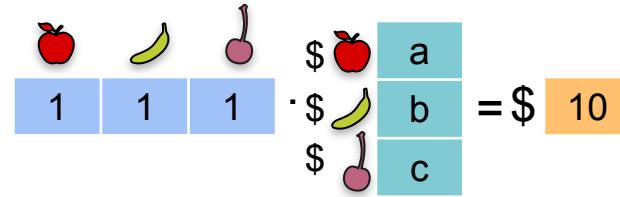


Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

The diagram illustrates the conversion of a system of linear equations into a matrix multiplication problem. It features three rows of fruit icons (apple, banana, cherry) above three corresponding columns of variables (a, b, c). To the left is a 3x3 matrix with values 1, 1, 1; 1, 2, 1; and 1, 1, 2. To the right is a vertical stack of dollar signs (\$) followed by the same 3x3 matrix. To the far right is another vertical stack of dollar signs (\$) followed by a 3x1 column vector with values 10, 15, and 12. This visualizes the system of equations as a matrix equation: $\begin{matrix} \text{apple} & \text{banana} & \text{cherry} \\ 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \cdot \begin{matrix} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{matrix} = \begin{matrix} \$10 \\ \$15 \\ \$12 \end{matrix}$.

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$



DeepLearning.AI

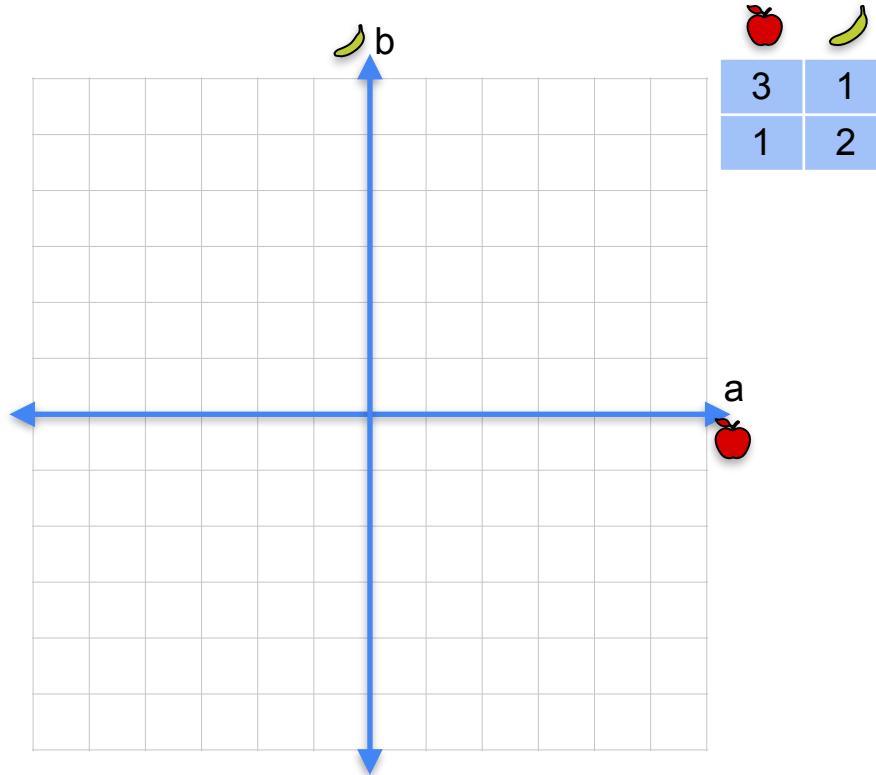
Vectors and Linear Transformations

**Matrices as linear
transformations**

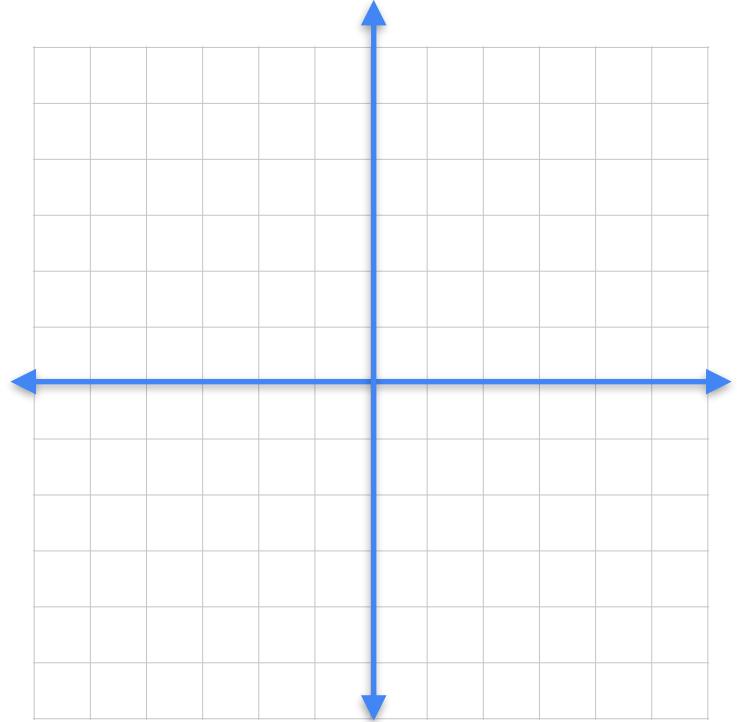
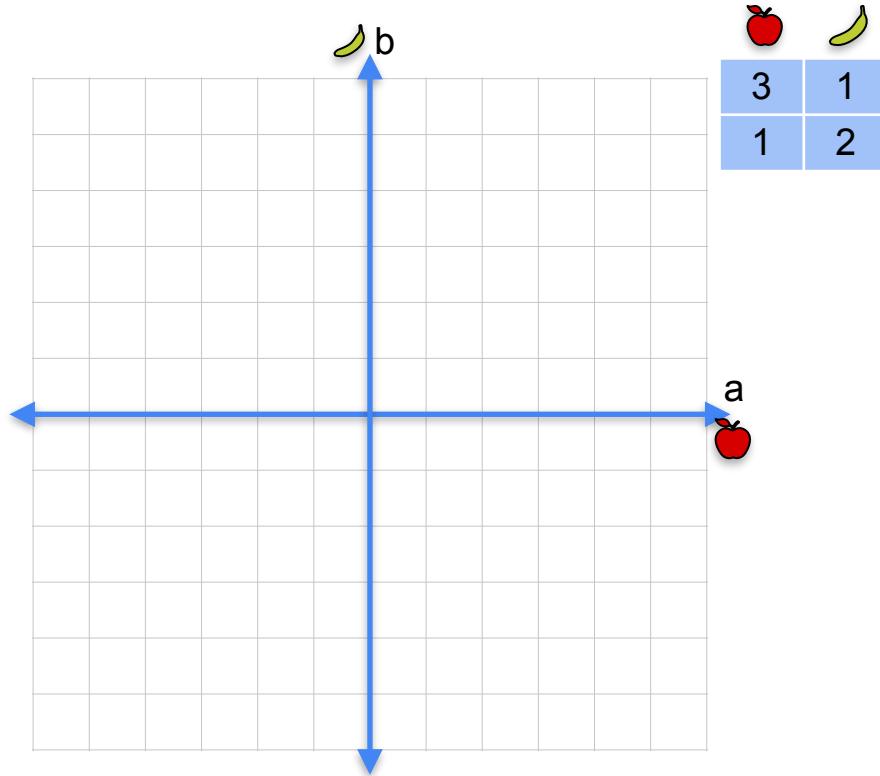
Matrices as linear transformations

| | |
|---|---|
|  |  |
| 3 | 1 |
| 1 | 2 |

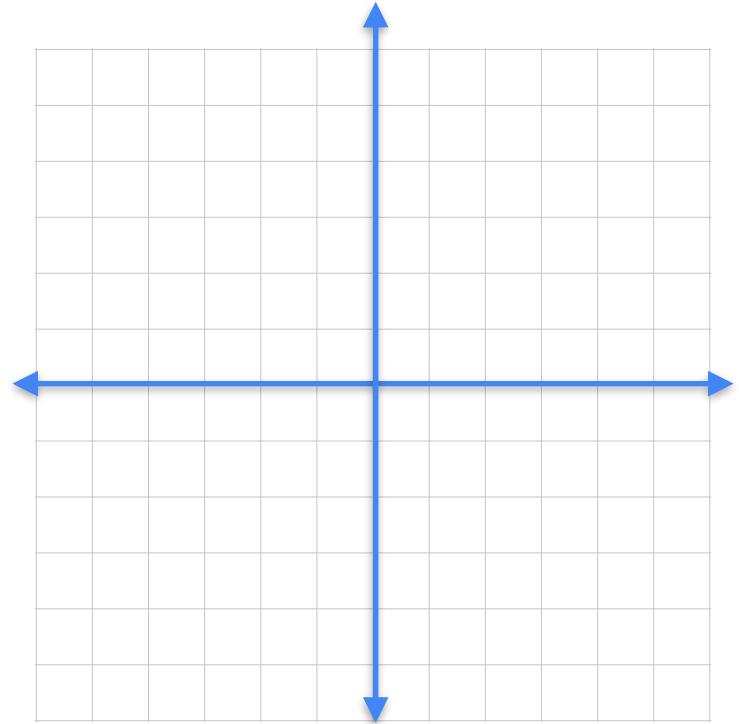
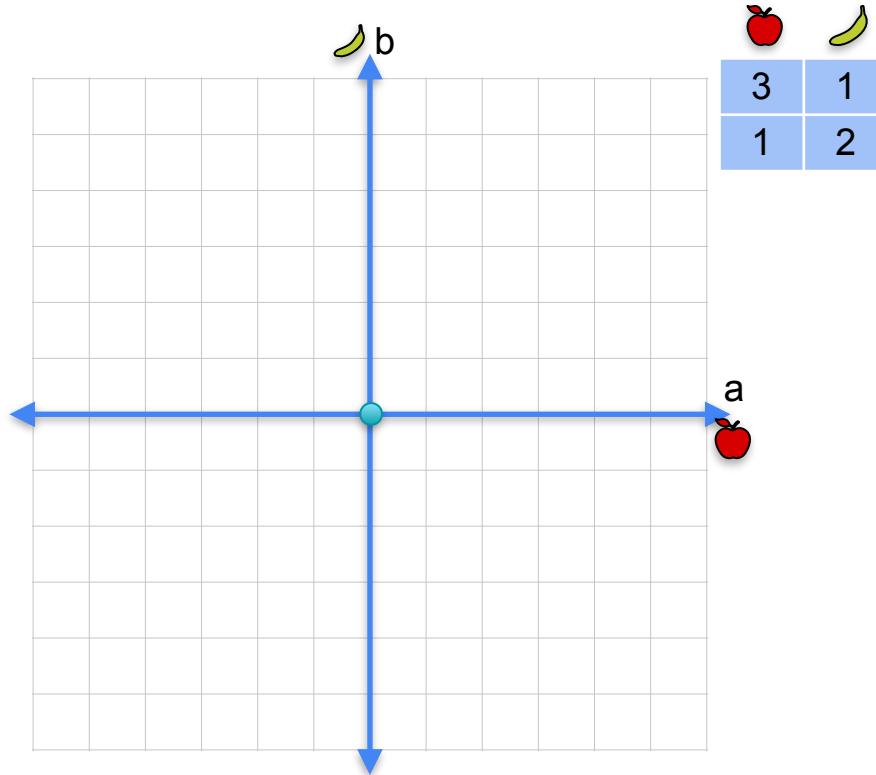
Matrices as linear transformations



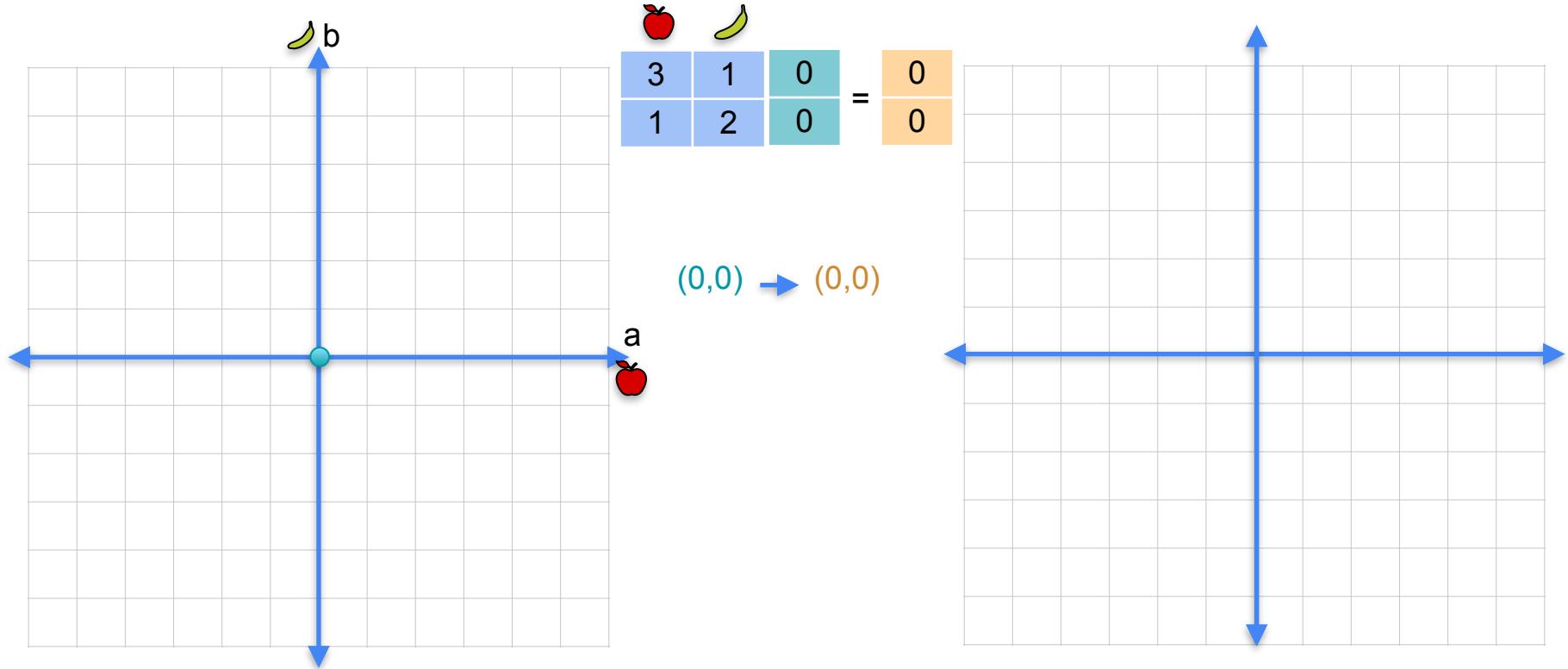
Matrices as linear transformations



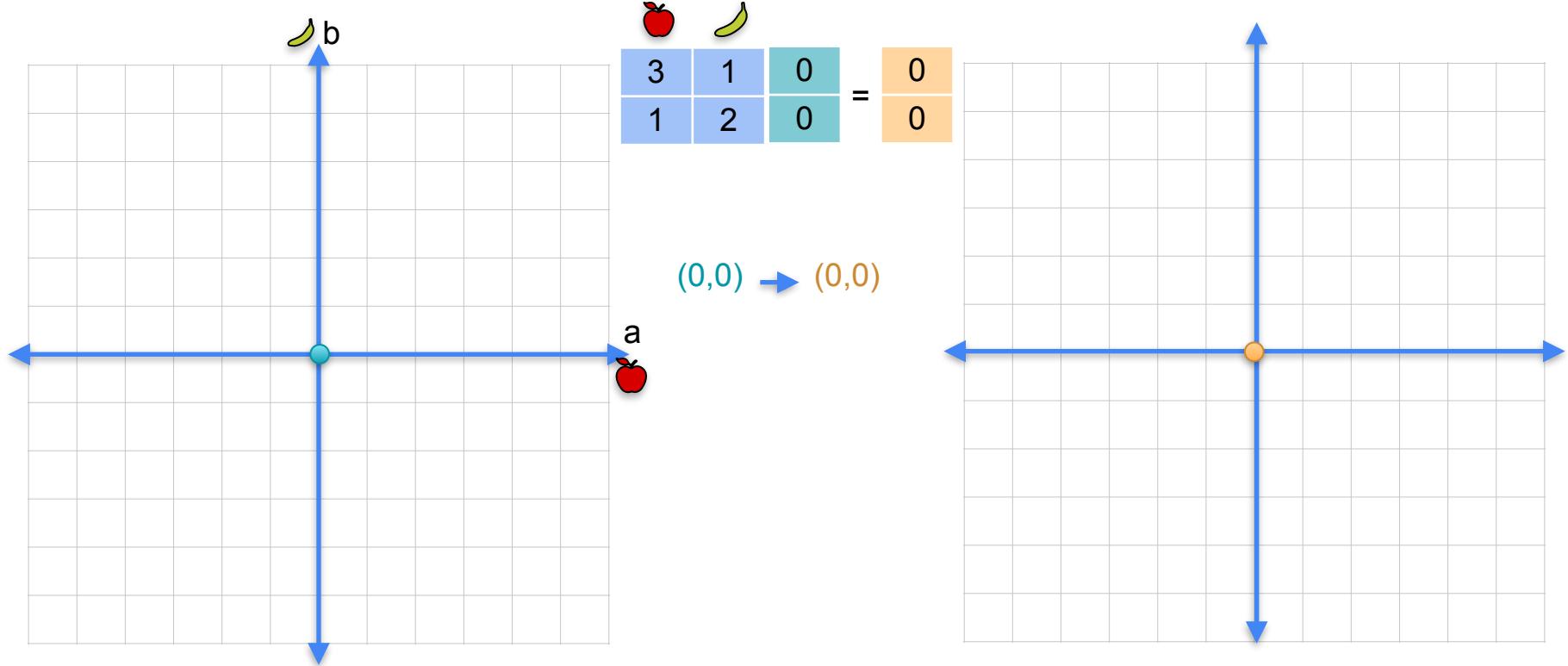
Matrices as linear transformations



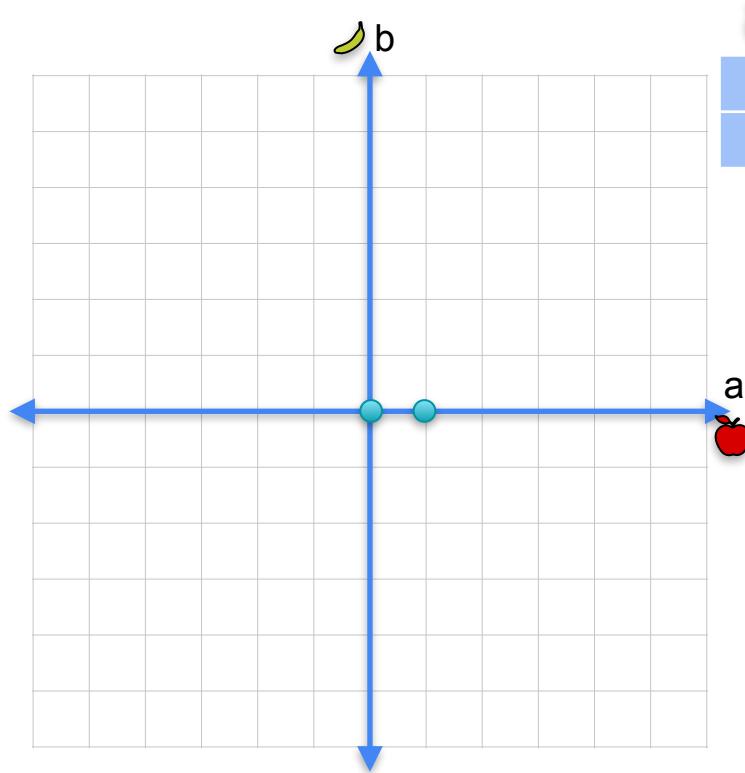
Matrices as linear transformations



Matrices as linear transformations

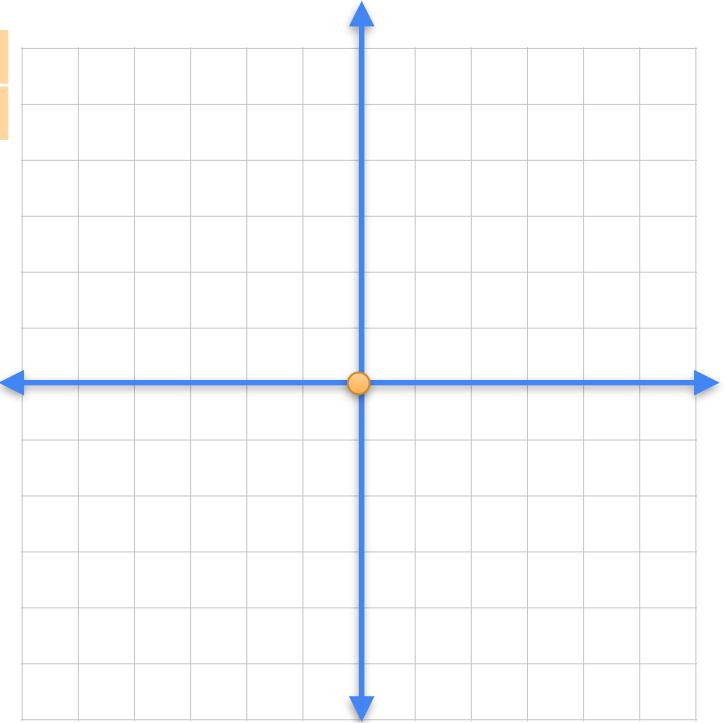


Matrices as linear transformations

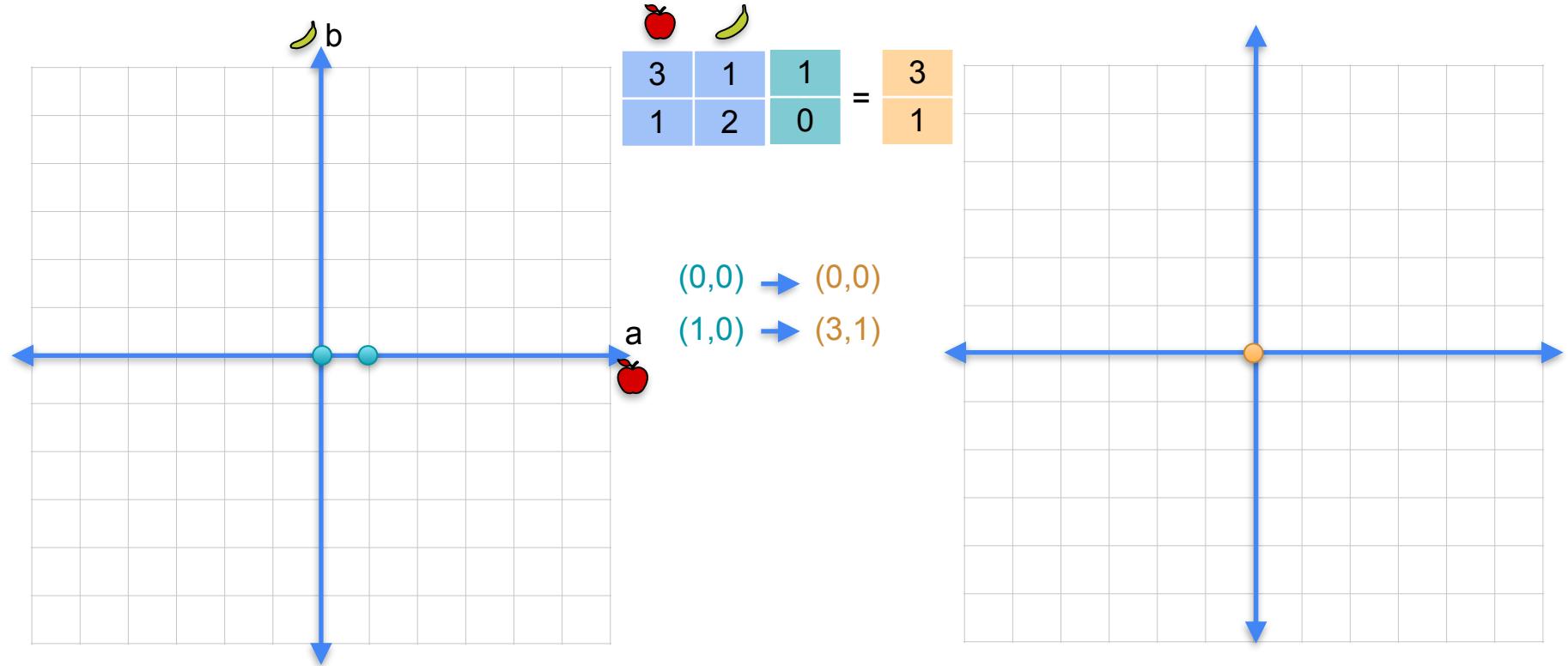


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ = & \begin{matrix} 0 \\ 0 \end{matrix} \end{array}$$

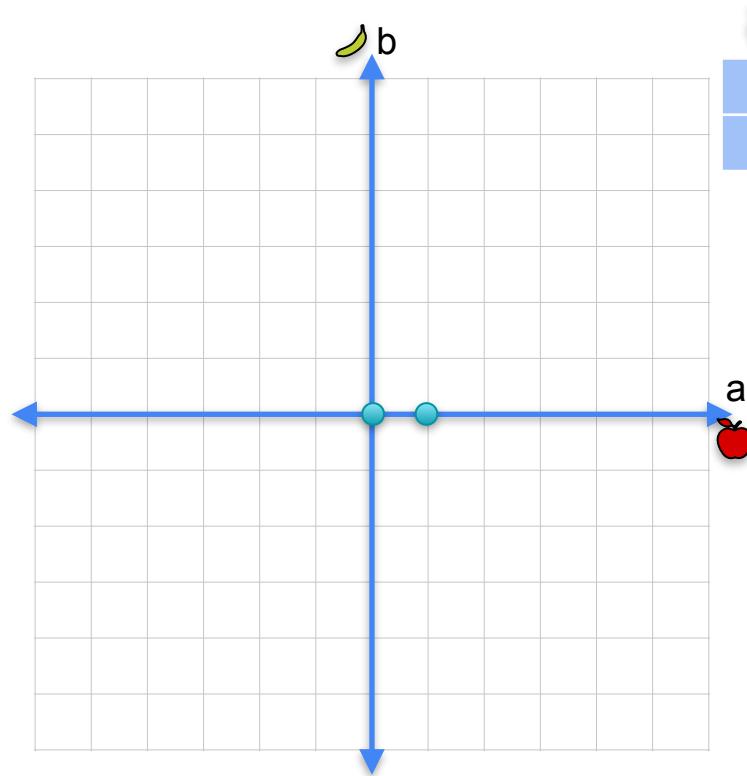
$(0,0) \rightarrow (0,0)$



Matrices as linear transformations

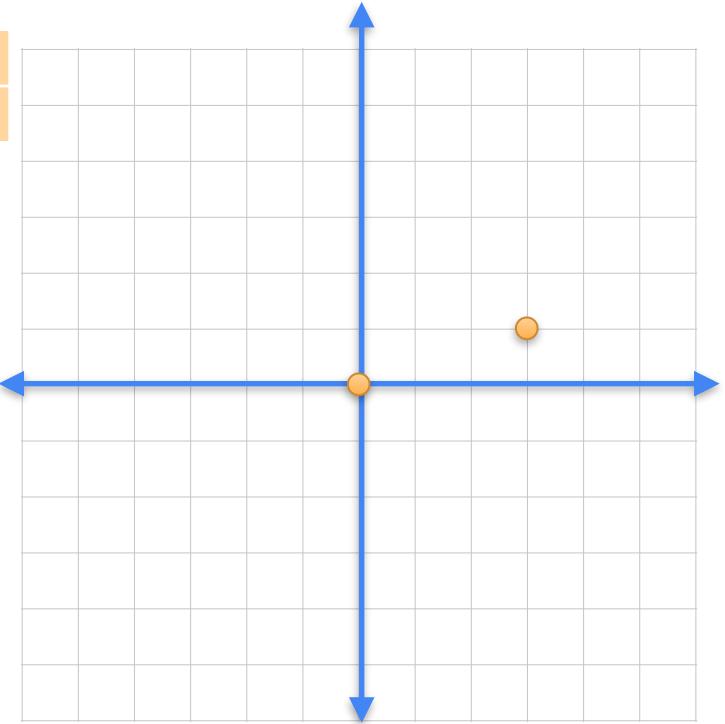


Matrices as linear transformations

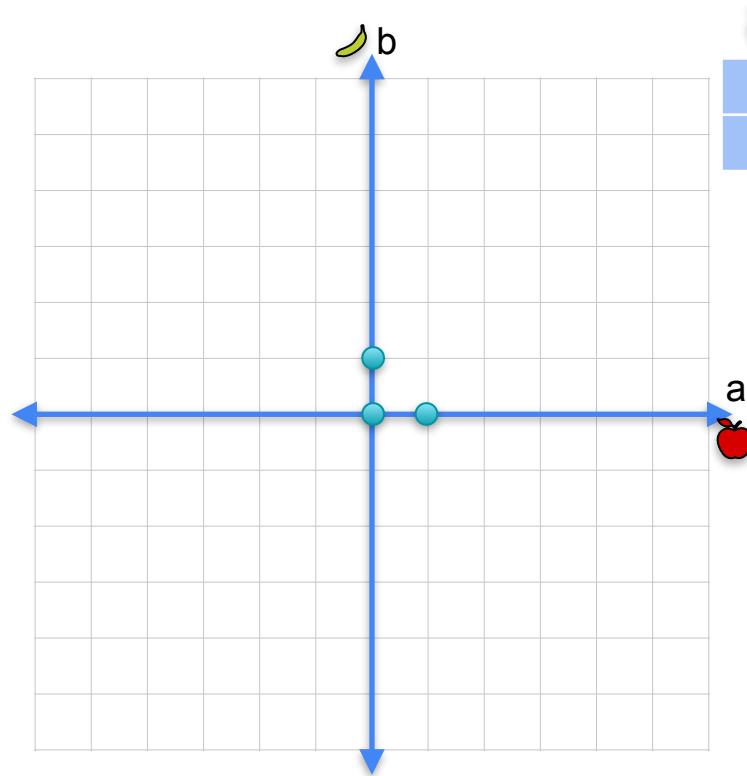


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

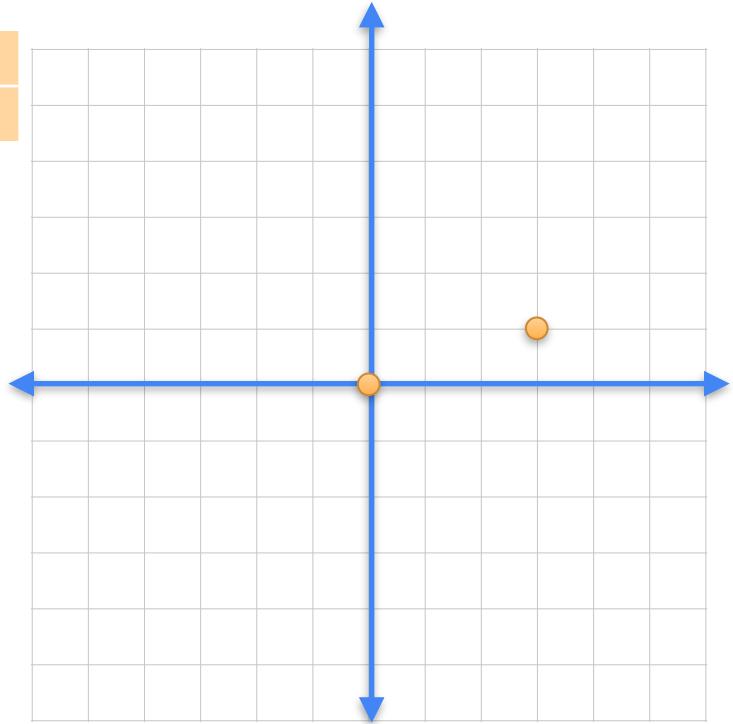


Matrices as linear transformations

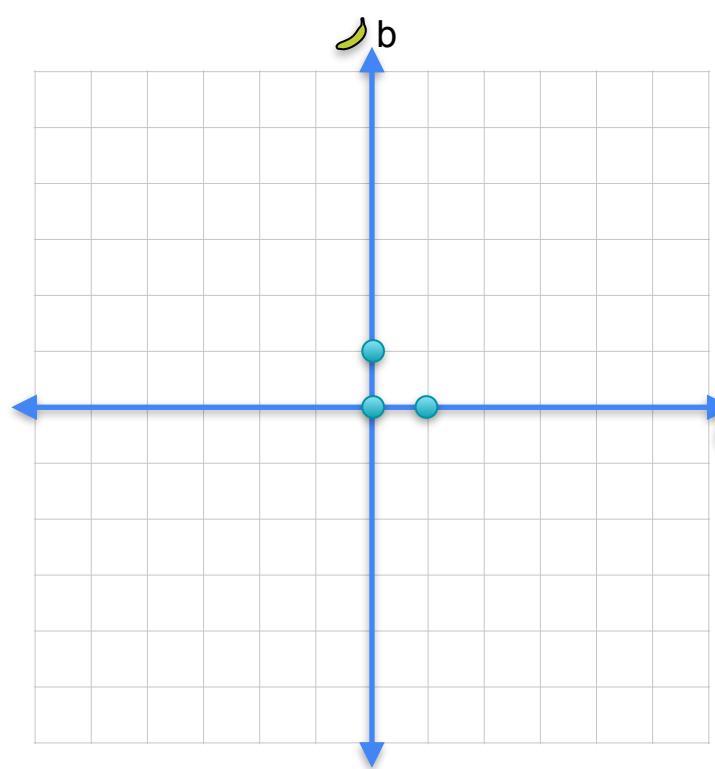


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

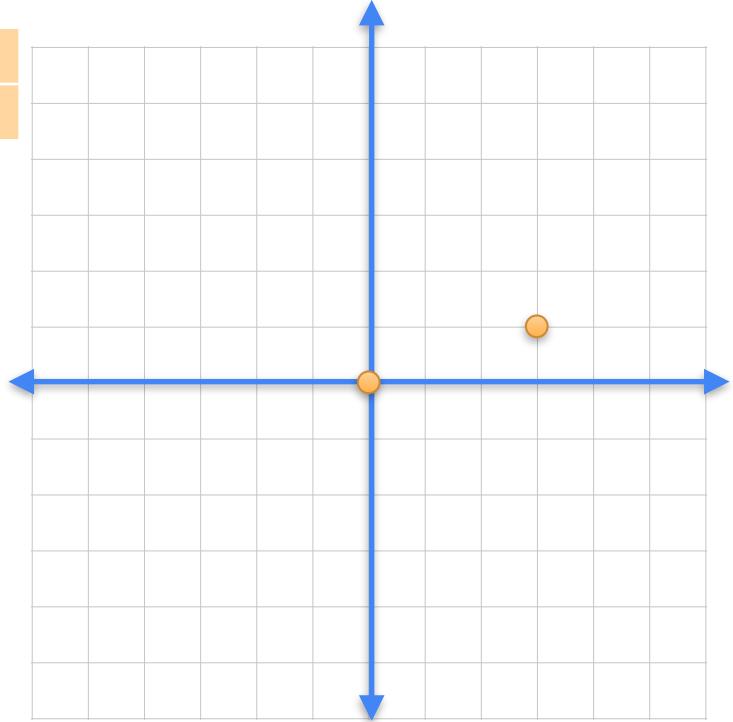


Matrices as linear transformations

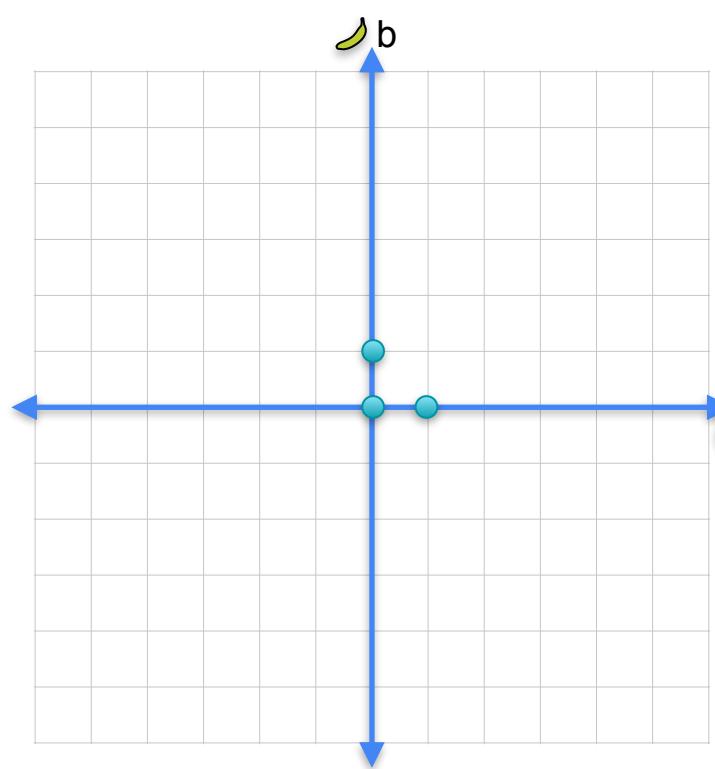


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} = \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

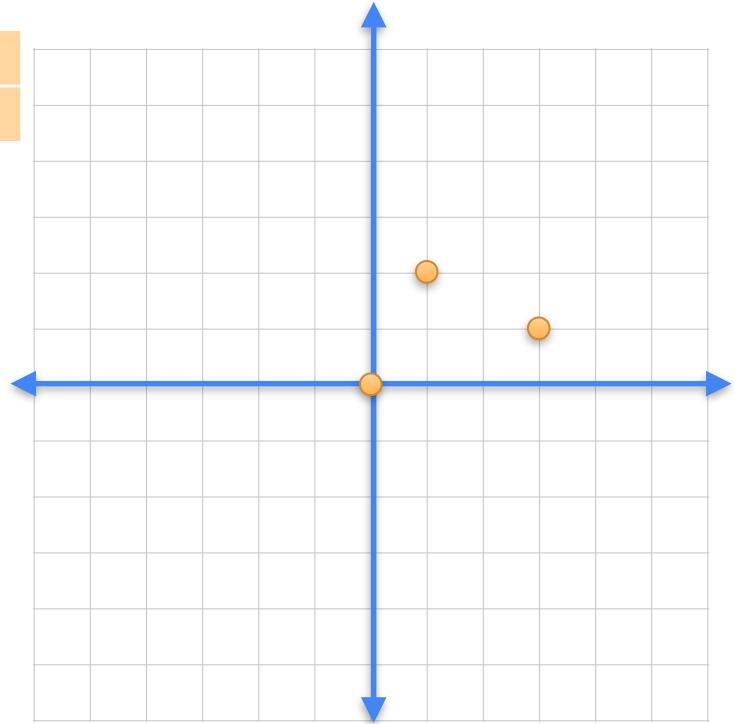


Matrices as linear transformations

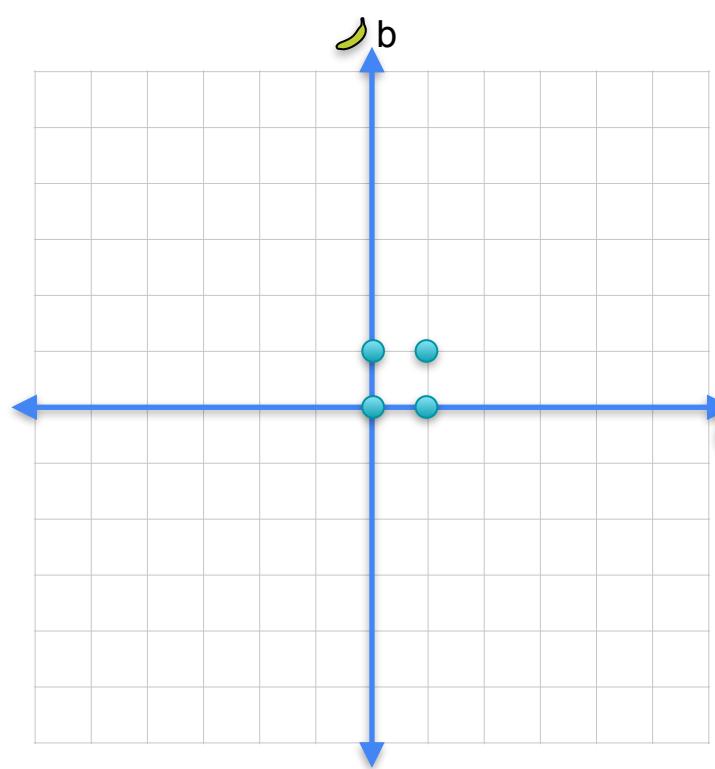


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 1 \\ 2 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

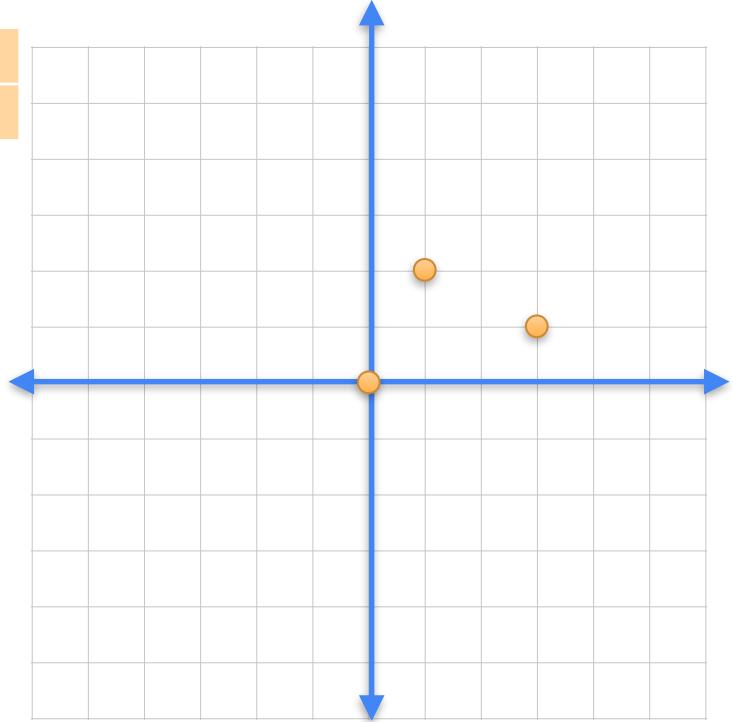


Matrices as linear transformations

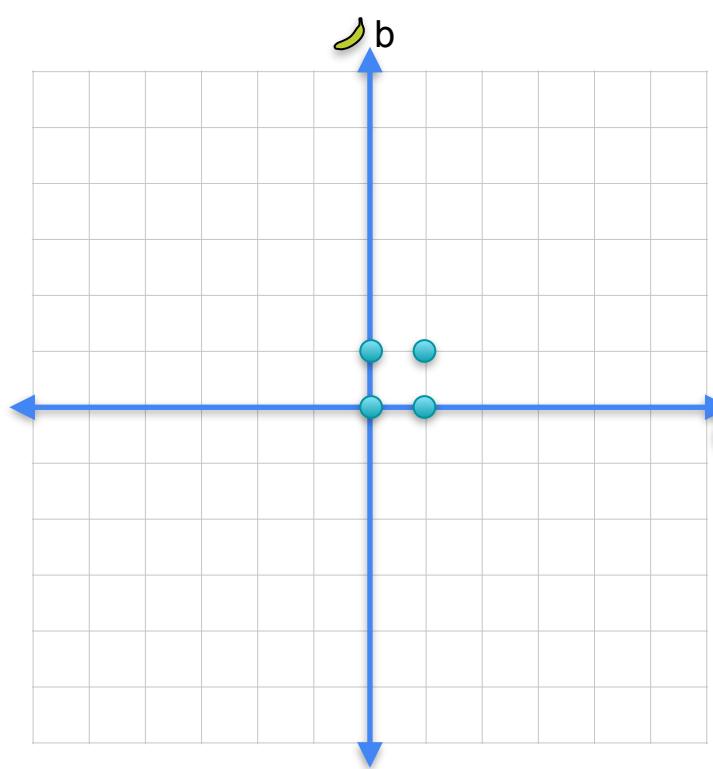


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} = \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$

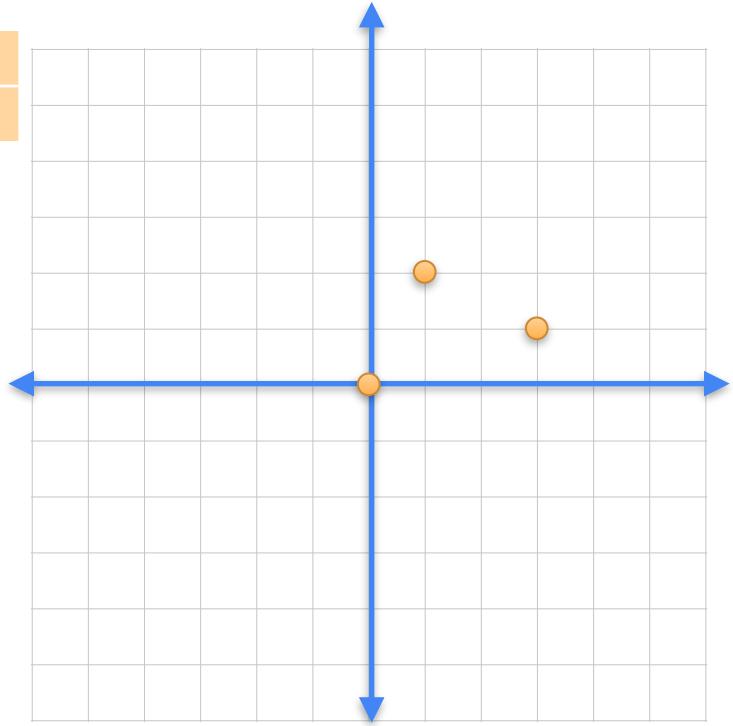


Matrices as linear transformations

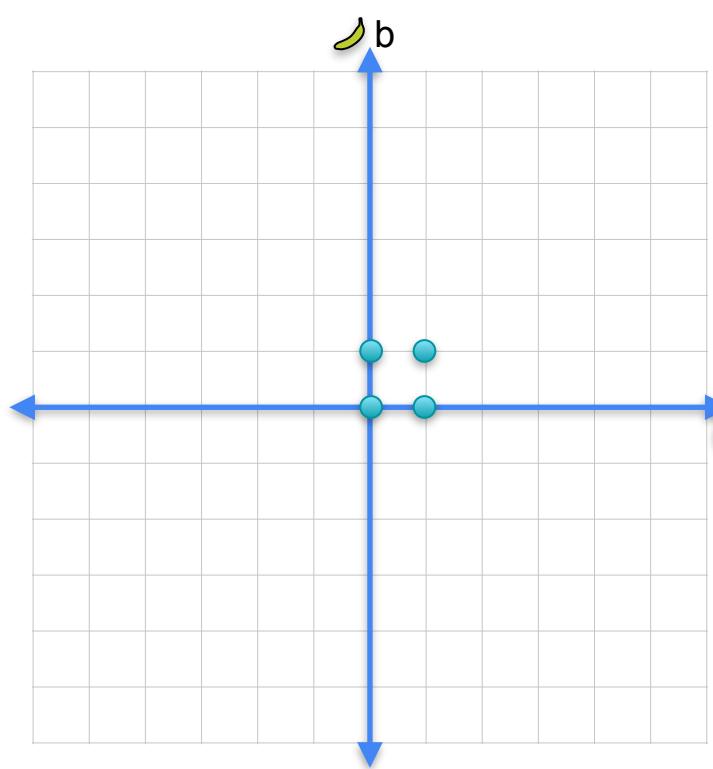


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

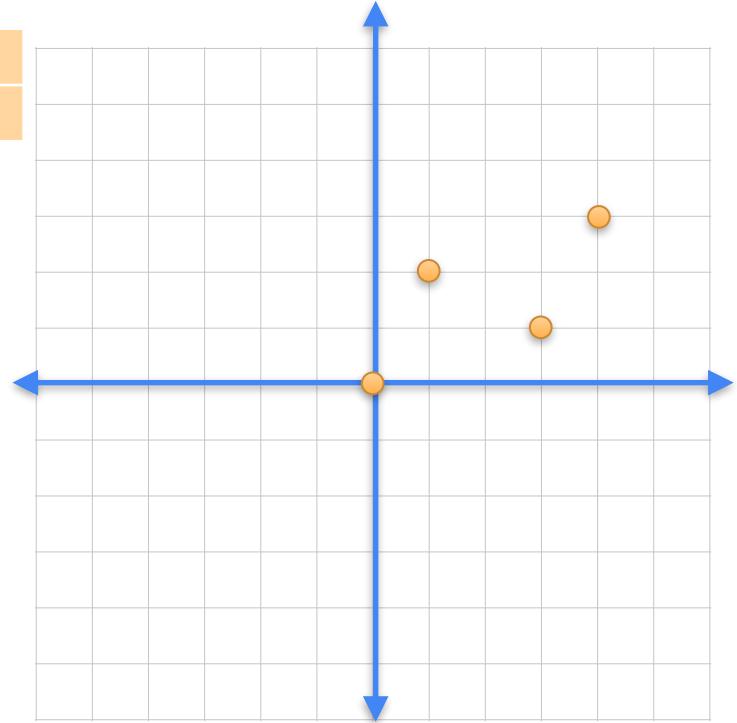


Matrices as linear transformations

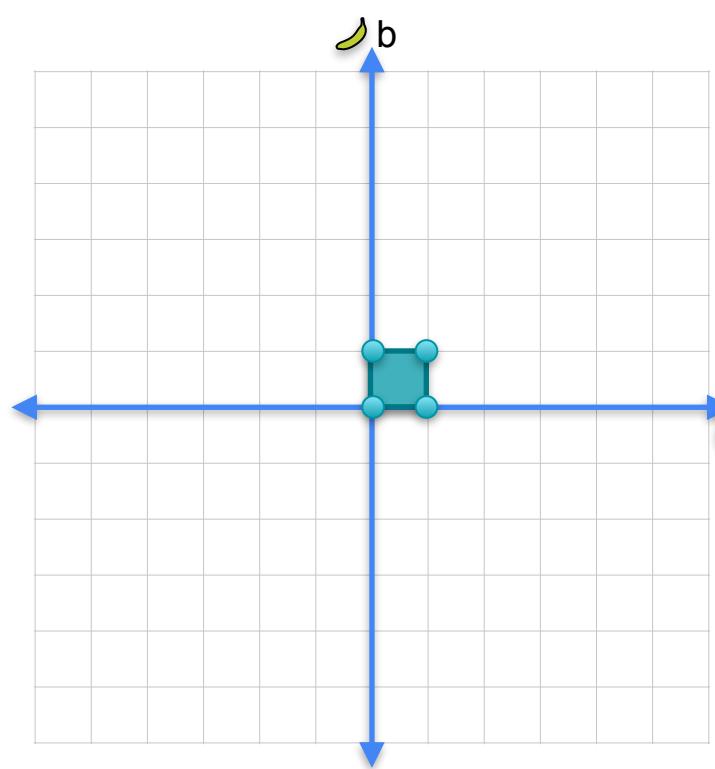


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

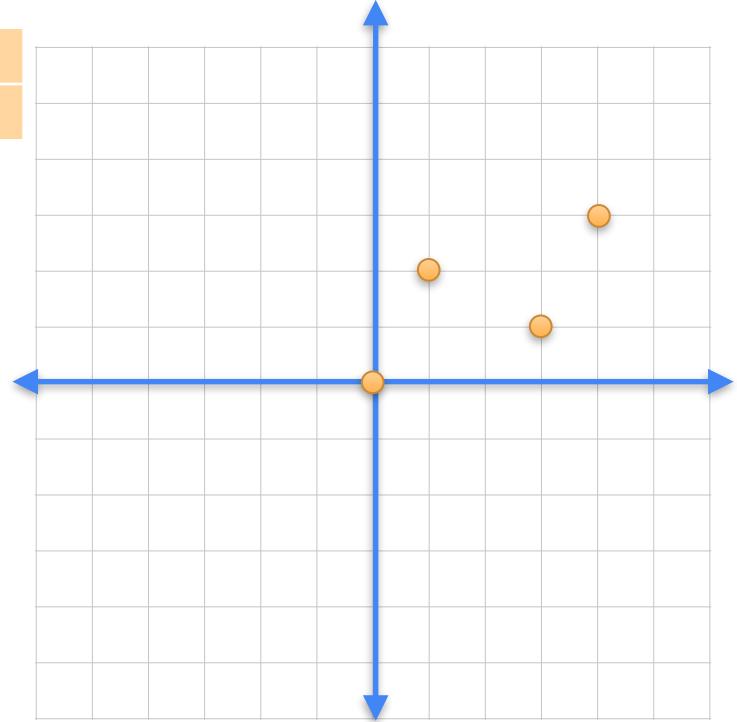


Matrices as linear transformations

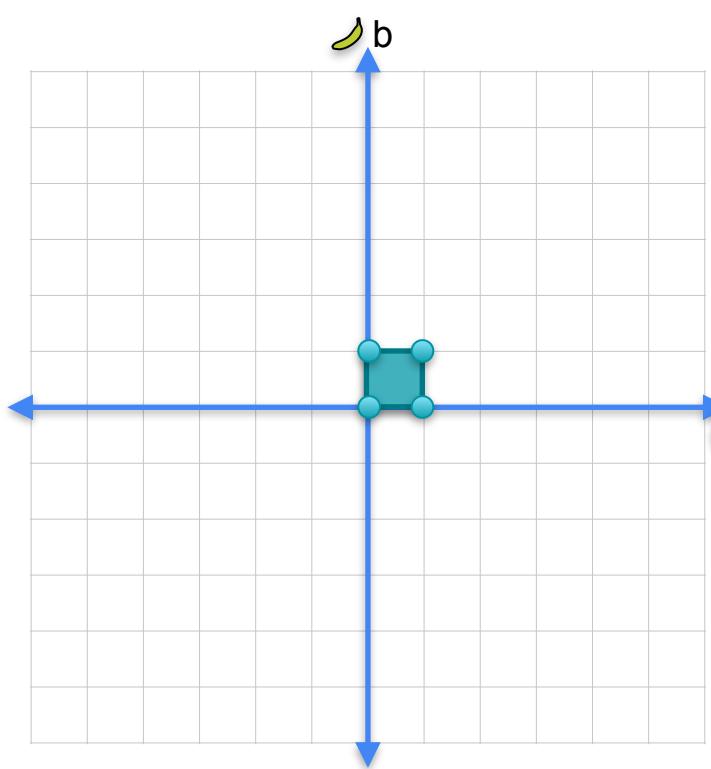


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

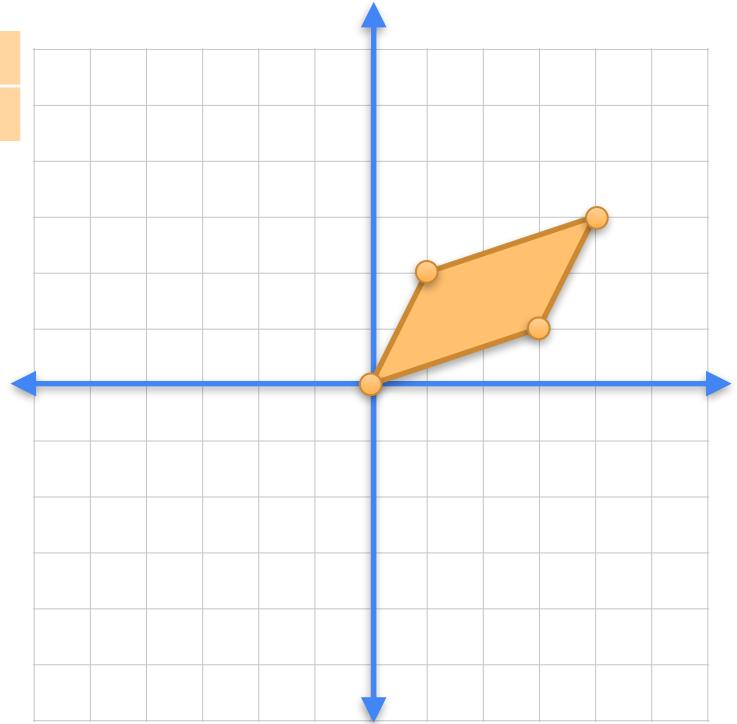


Matrices as linear transformations

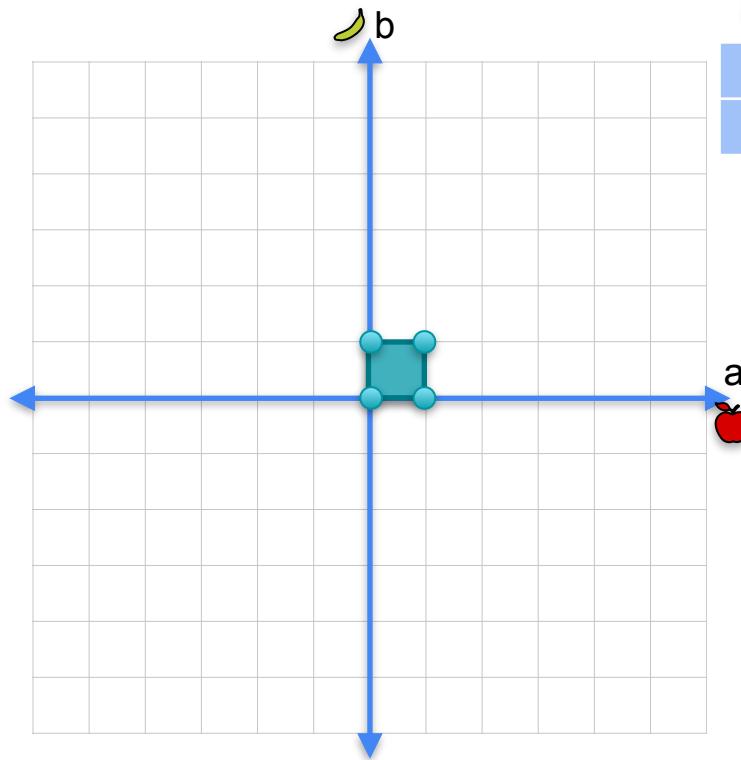


$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|c|} \hline 3 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 & \\ \hline 3 & \\ \hline \end{array} \end{array}$$

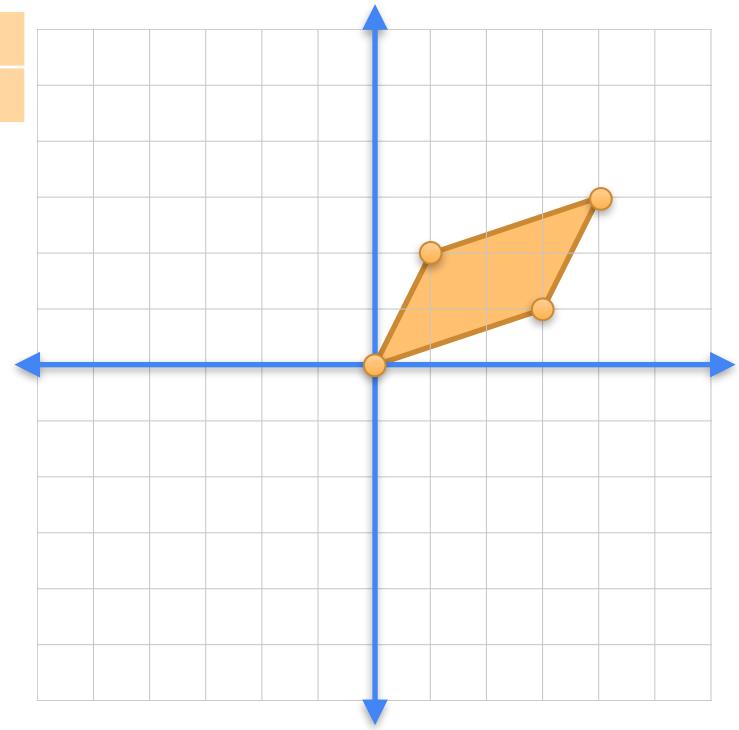
$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (4,3) \end{aligned}$$



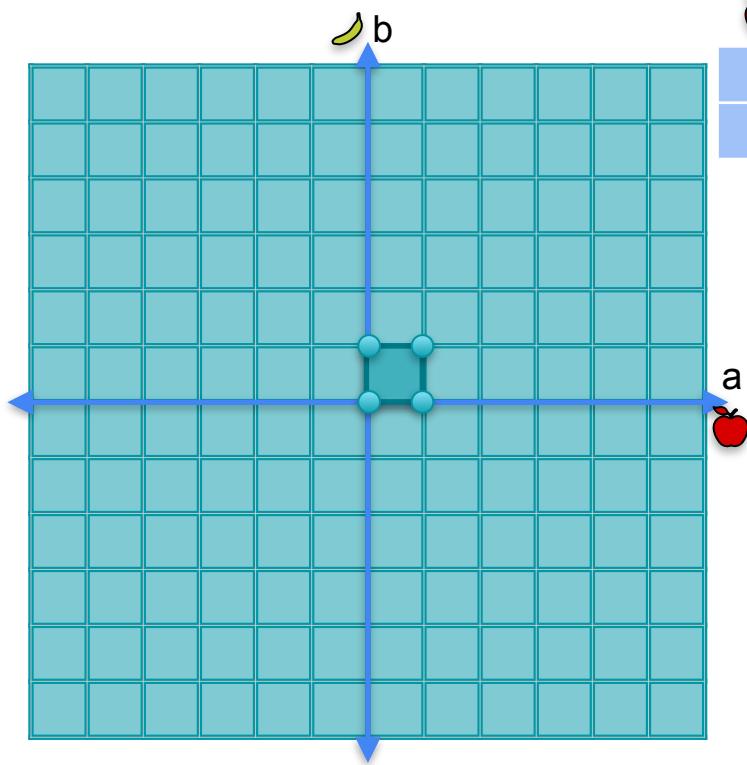
Matrices as linear transformations



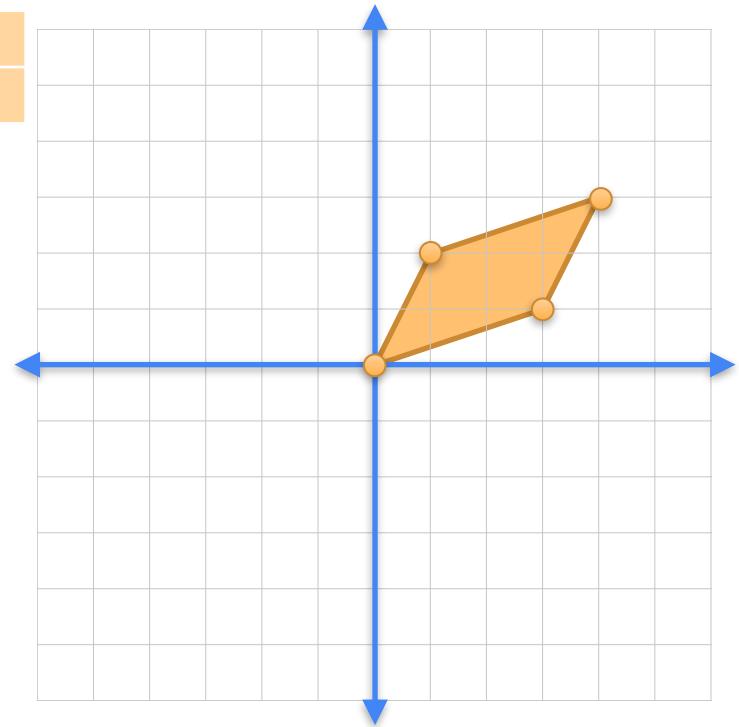
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} & = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$



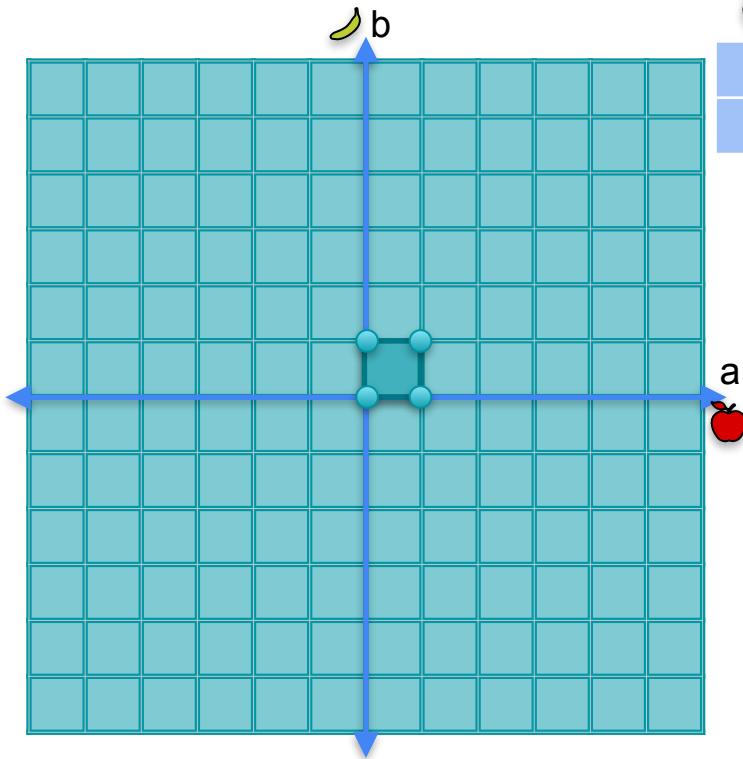
Matrices as linear transformations



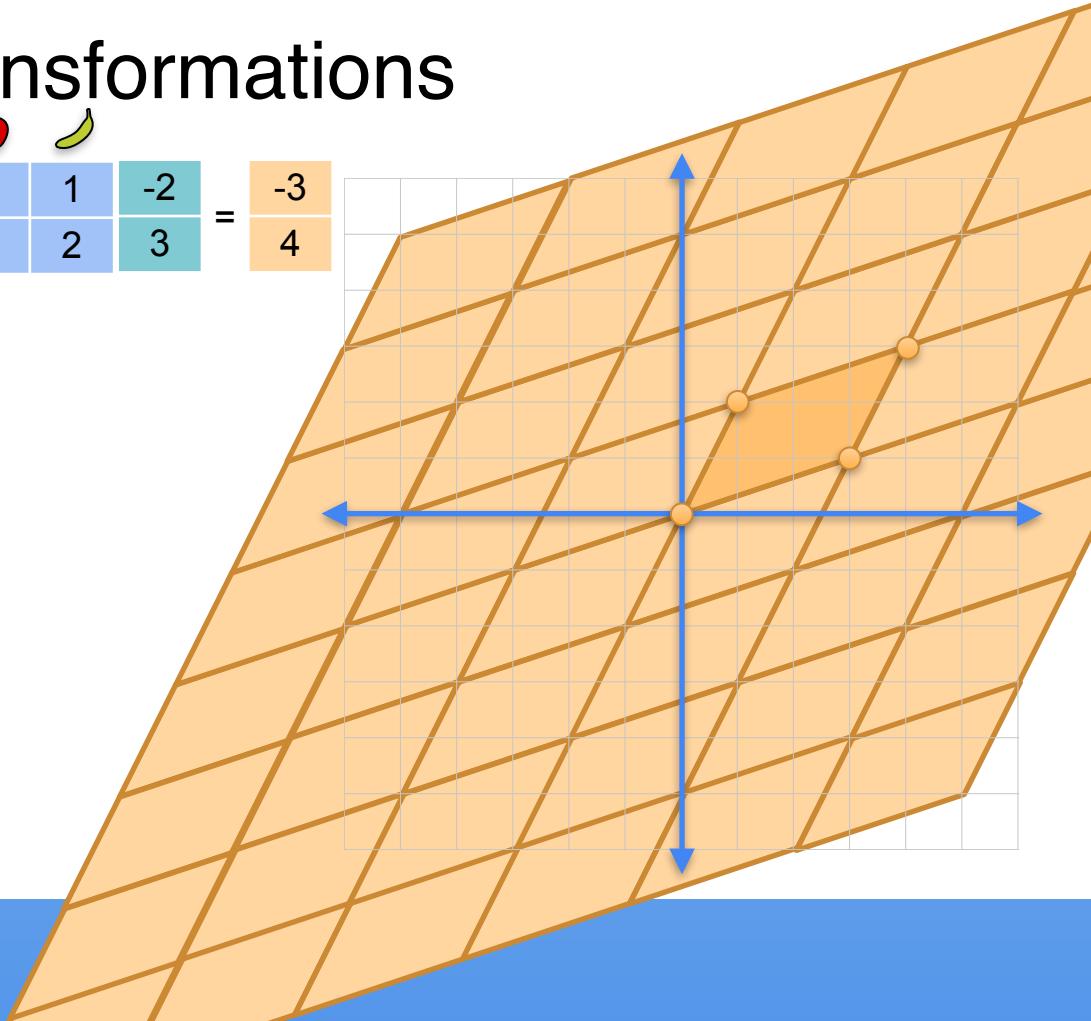
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} -2 \\ 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$



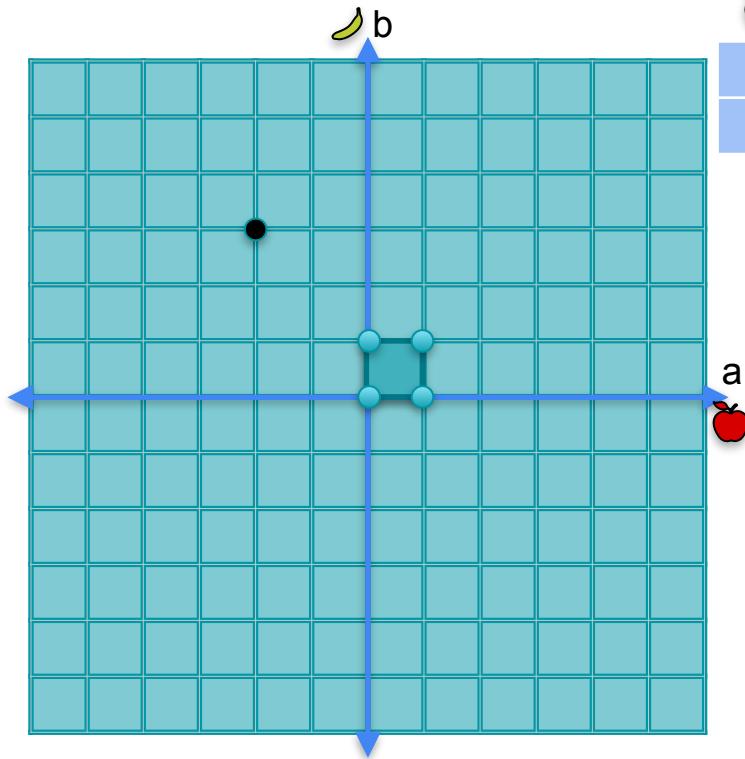
Matrices as linear transformations



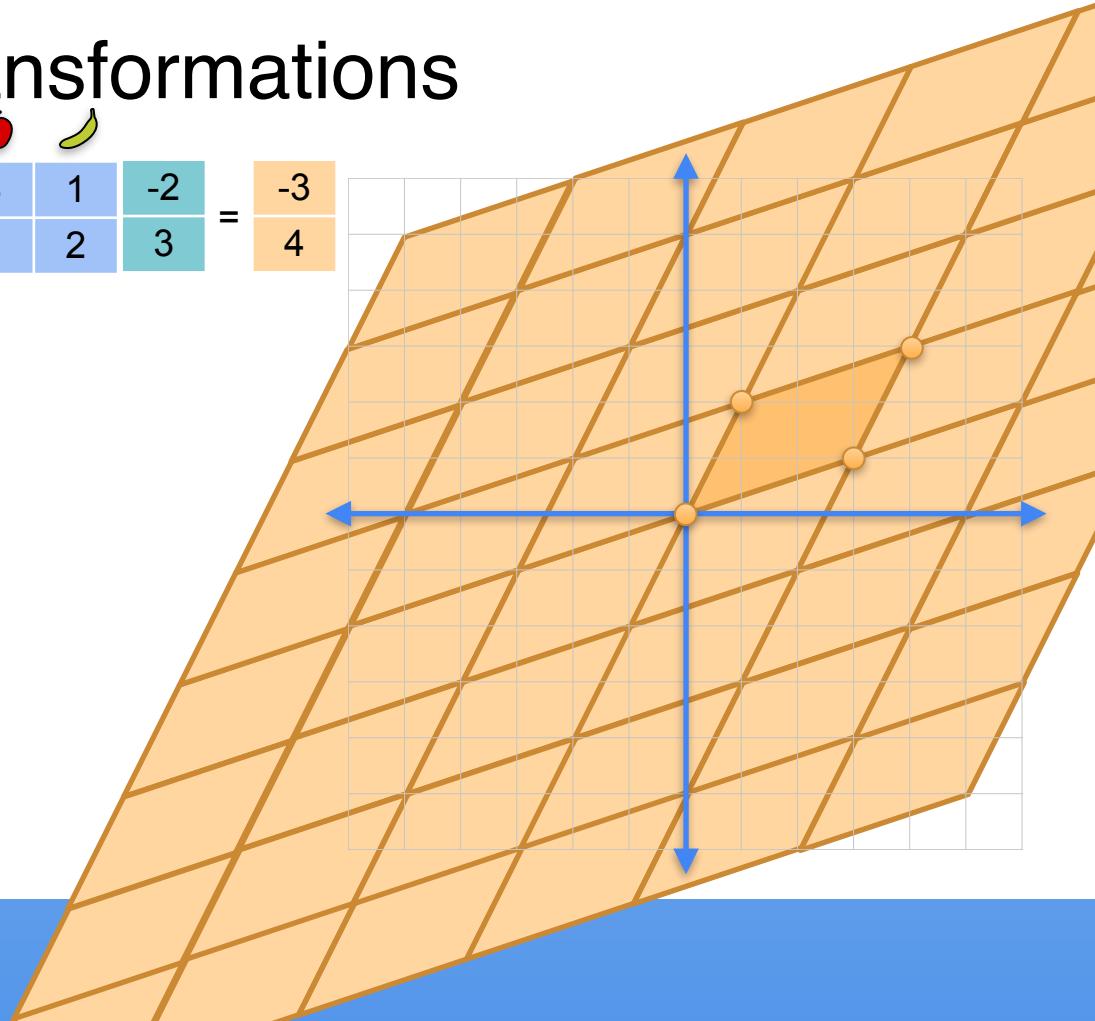
$$\begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} = \begin{matrix} -3 & 4 \\ -3 & 4 \end{matrix}$$



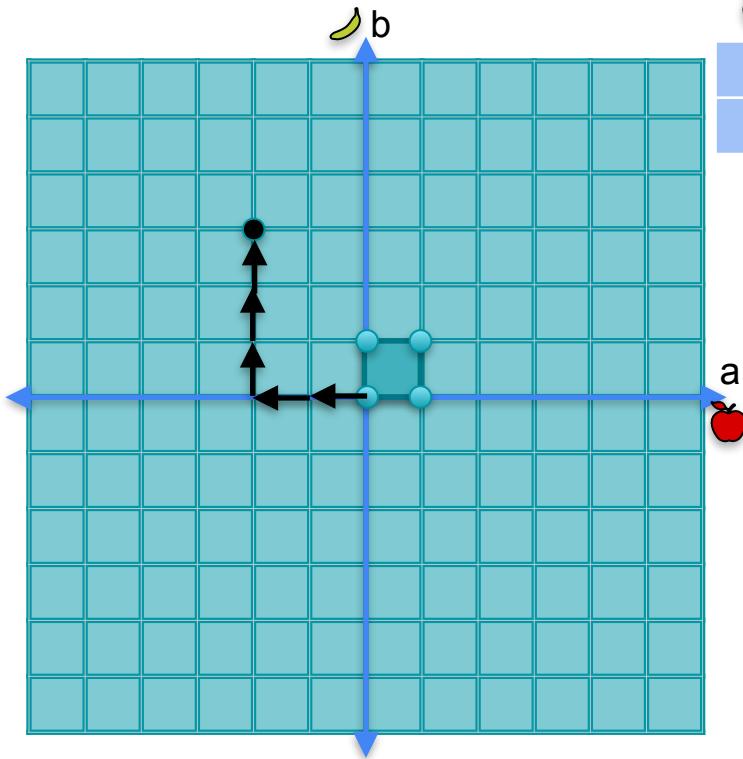
Matrices as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} & = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$

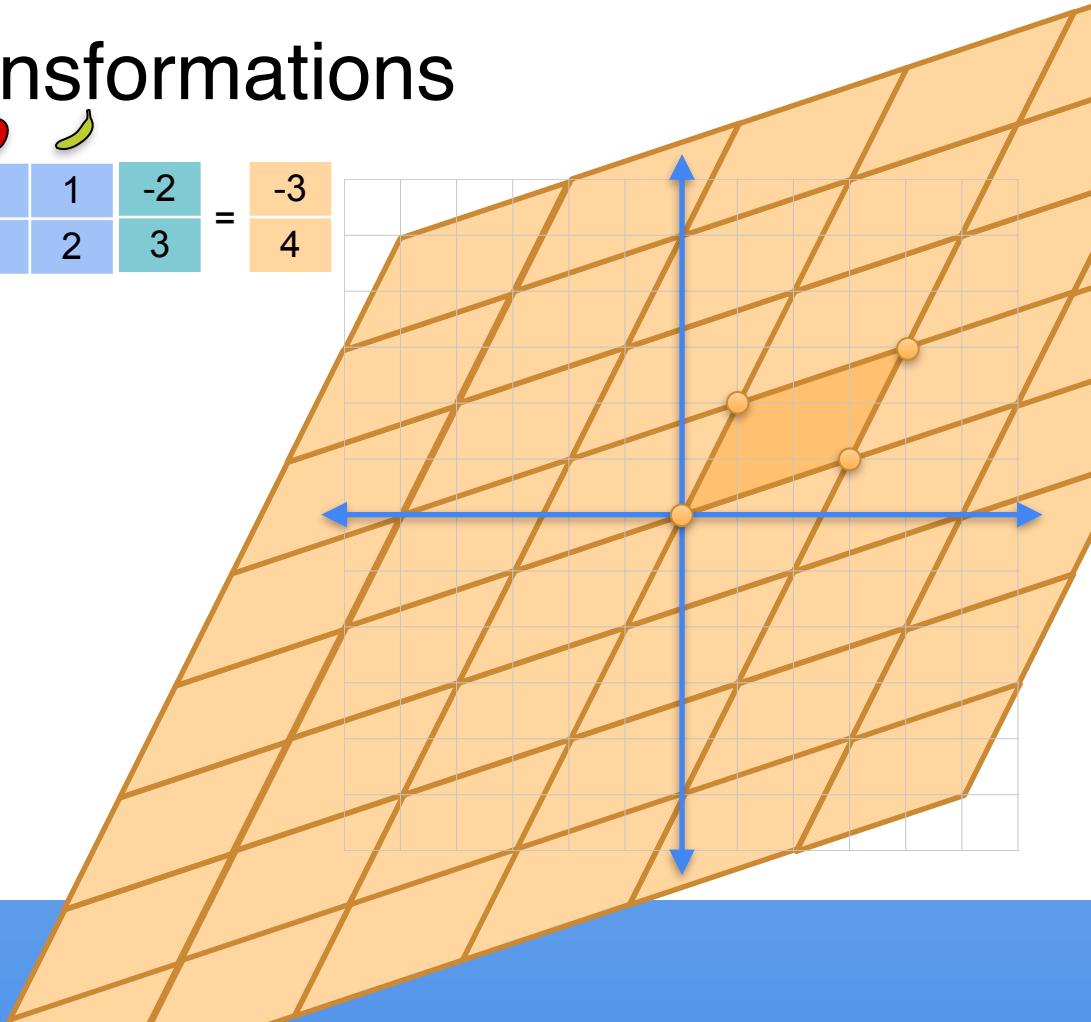


Matrices as linear transformations

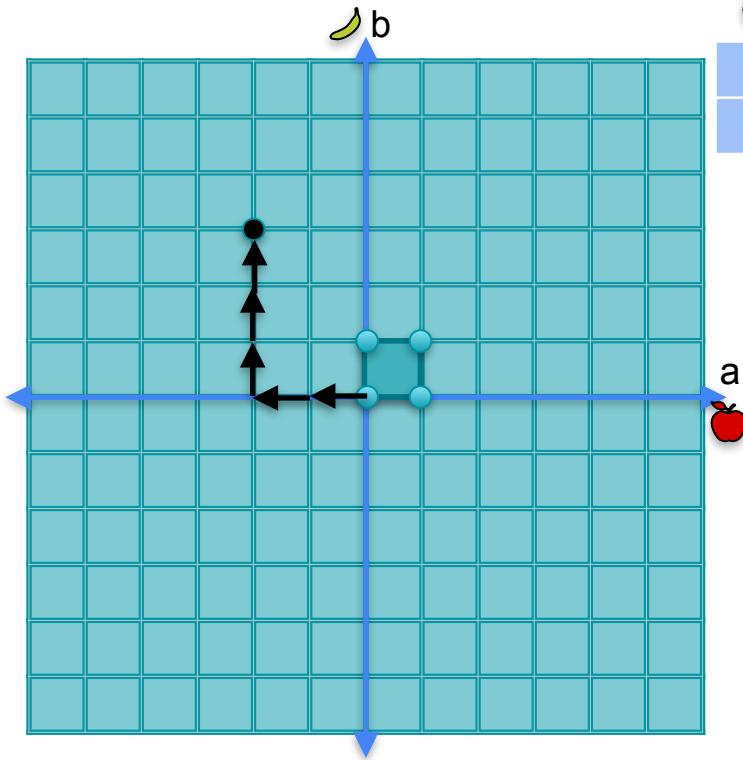


A diagram showing a red apple and a yellow banana above a matrix multiplication equation. The equation is $\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

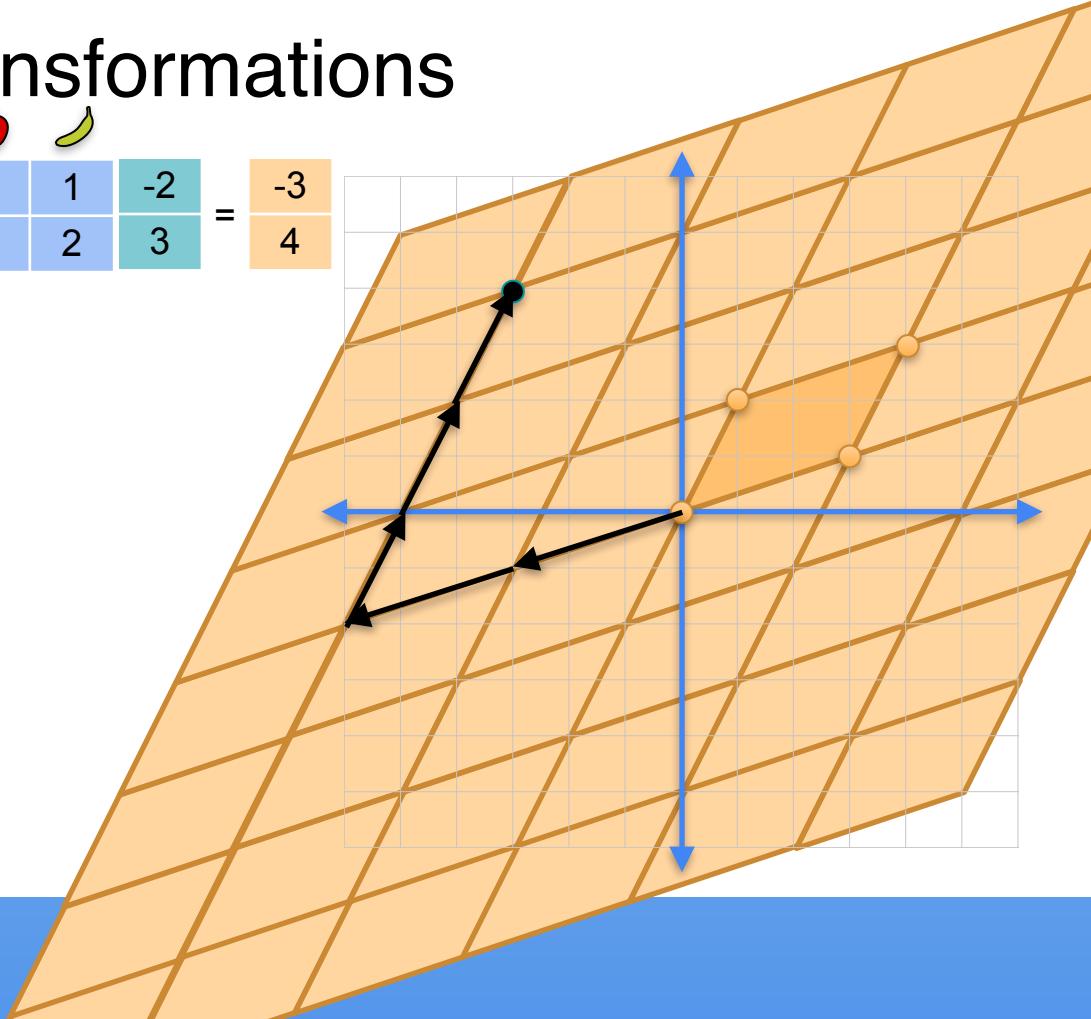
$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



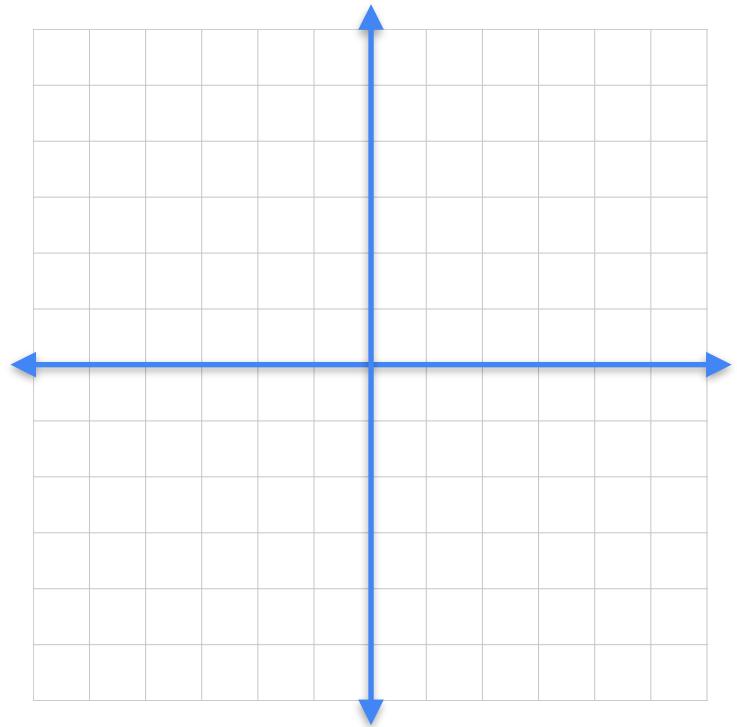
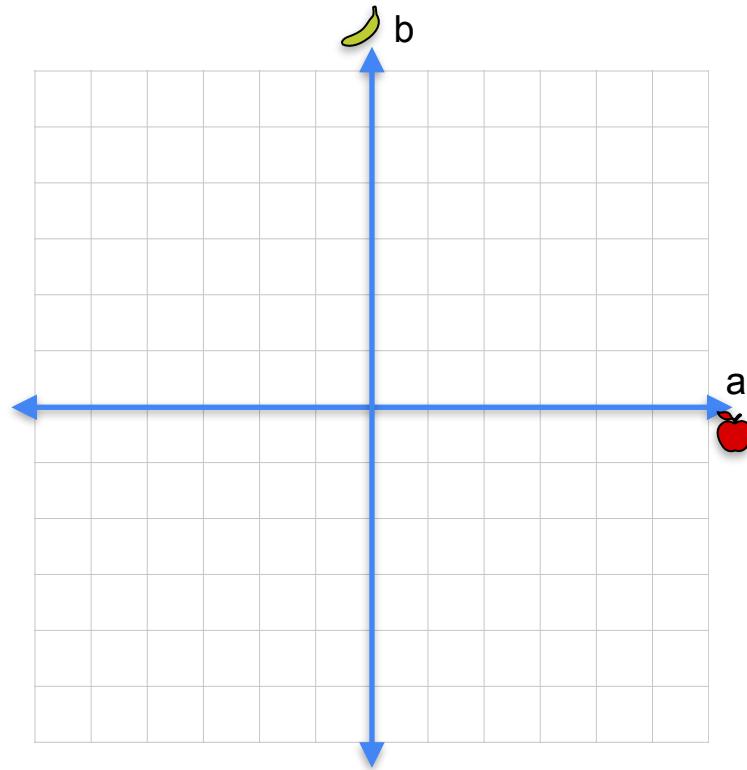
Matrices as linear transformations



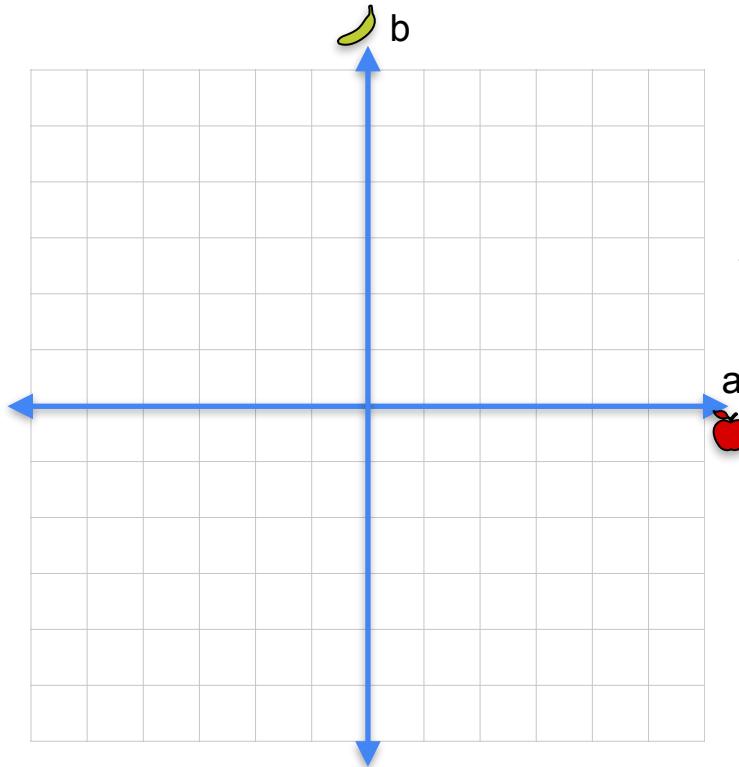
$$\begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix}$$



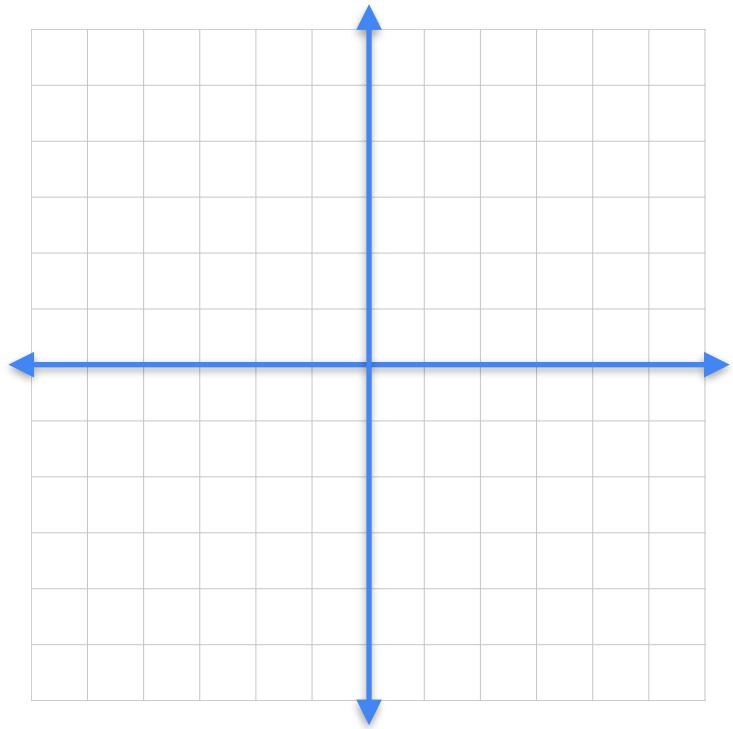
Systems of equations as linear transformations



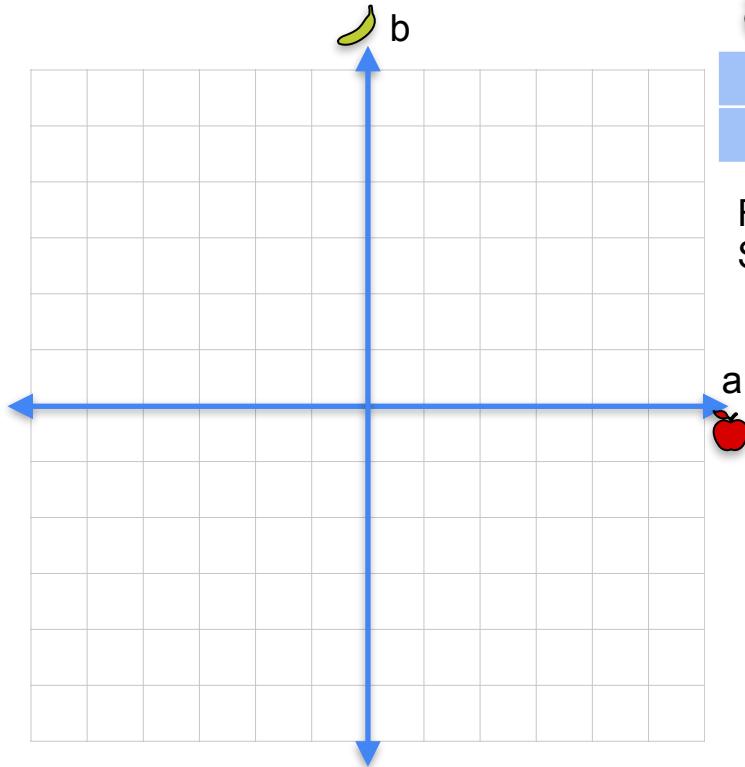
Systems of equations as linear transformations



First day: $3a + b$
Second day: $a + 2b$

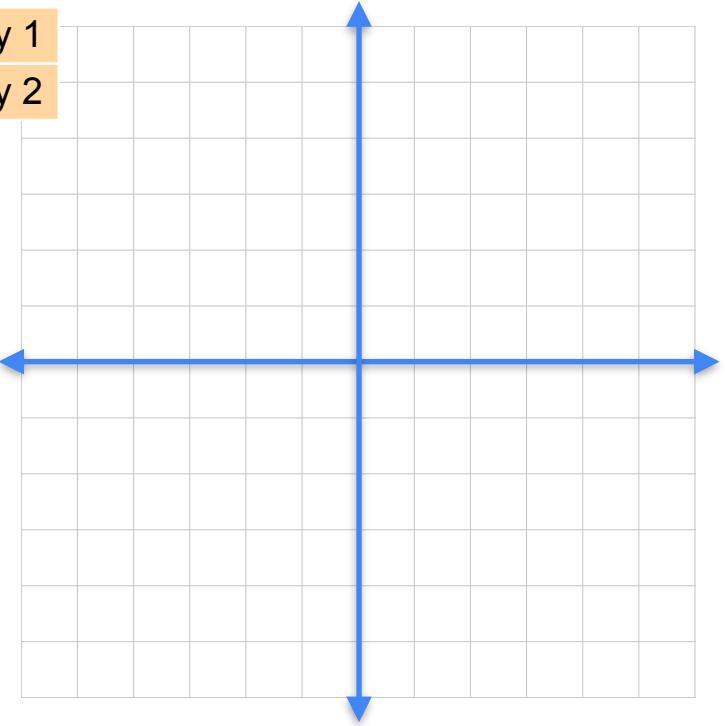


Systems of equations as linear transformations

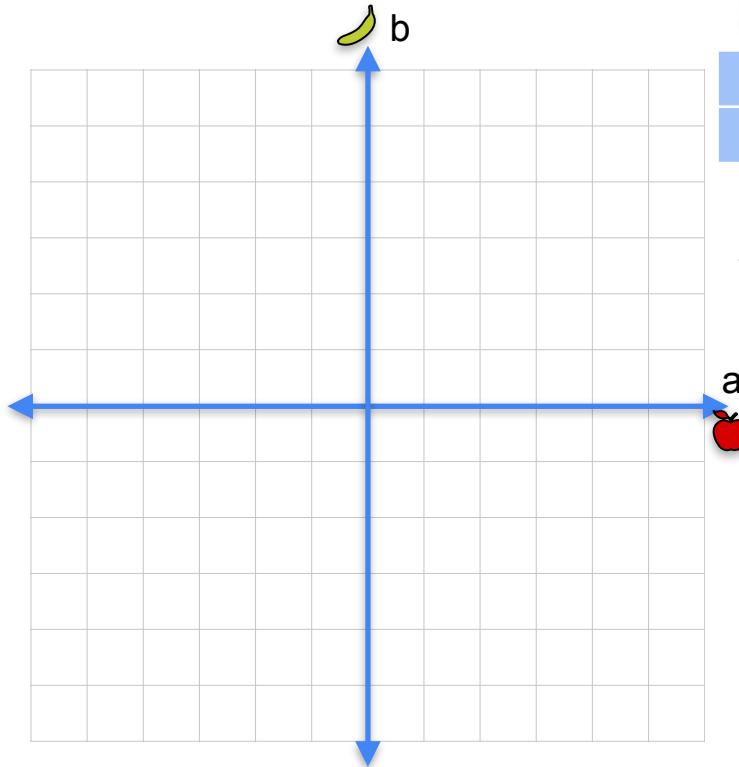


$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ b \end{matrix} = \begin{matrix} \text{Day 1} \\ \text{Day 2} \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

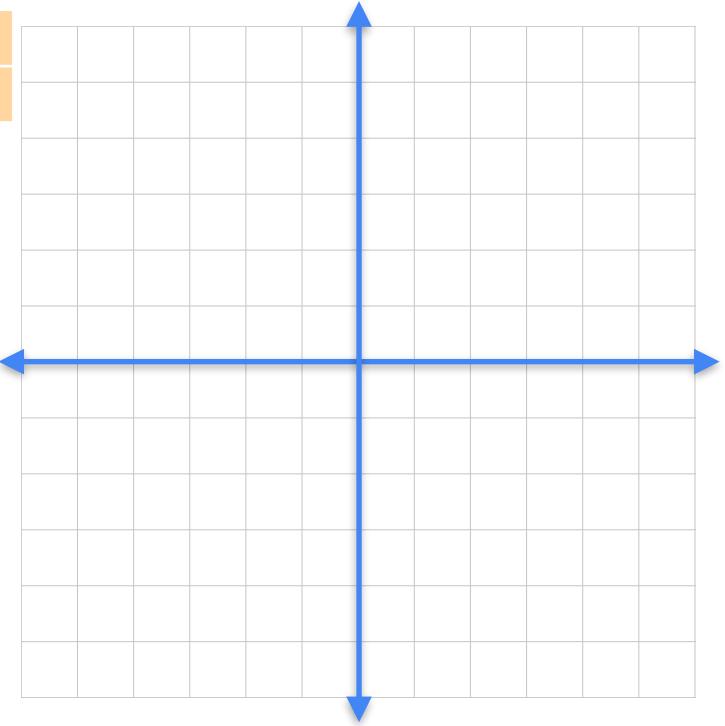


Systems of equations as linear transformations

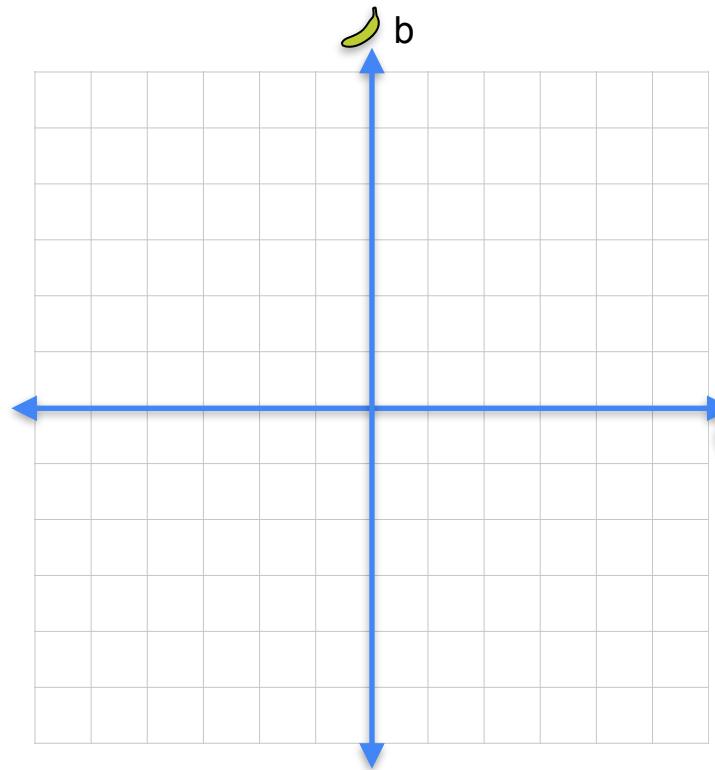


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{orange} \\ \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$



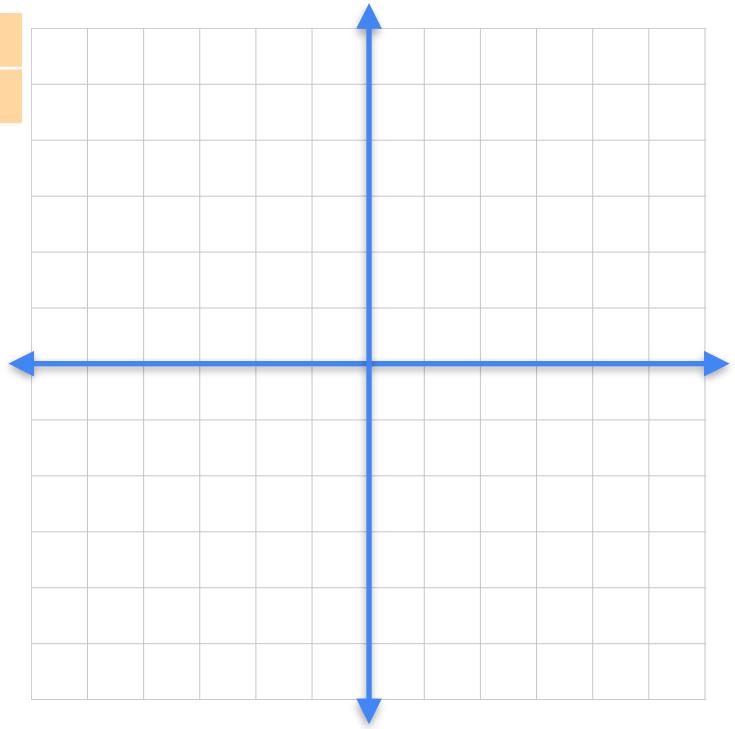
Systems of equations as linear transformations



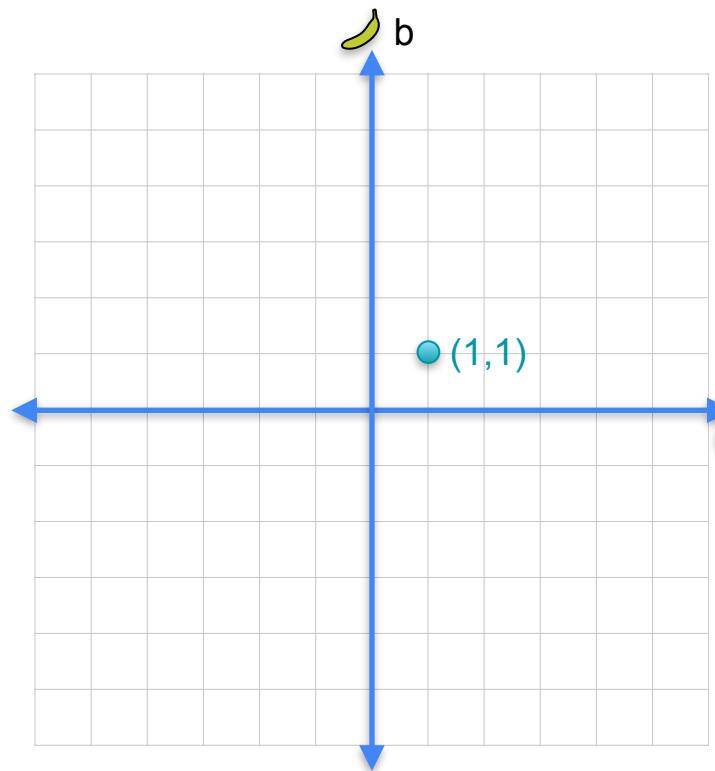
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

$$(1,1) \rightarrow (4,3)$$

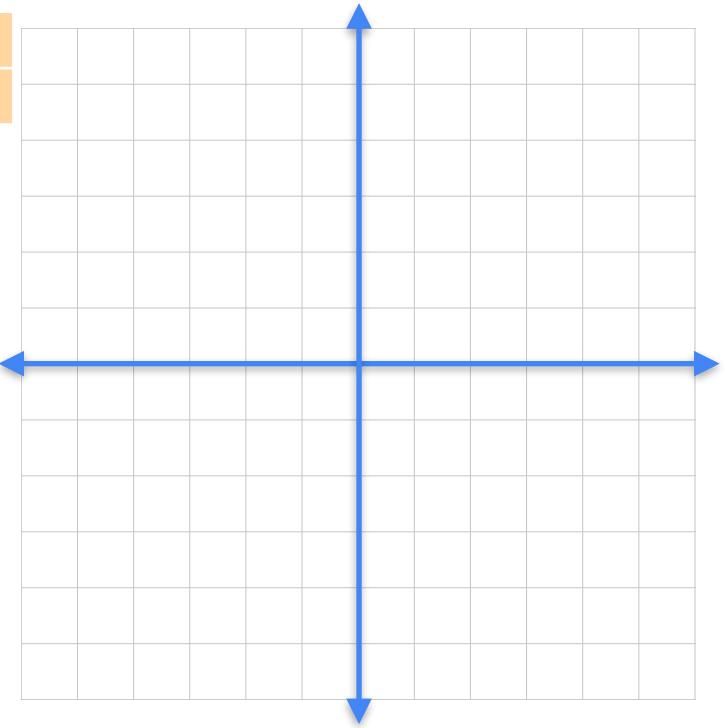


Systems of equations as linear transformations

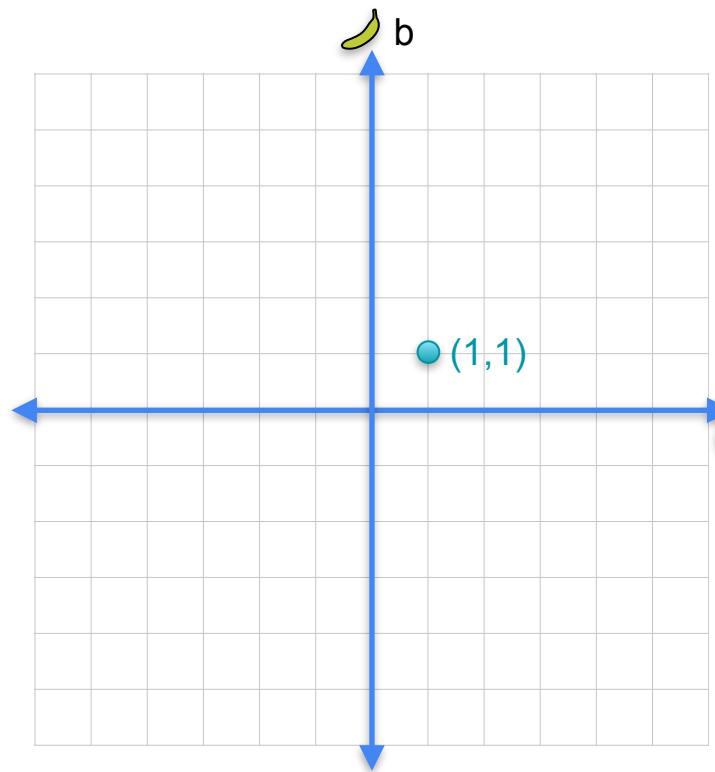


$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} \text{orange} \\ 4 \\ 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

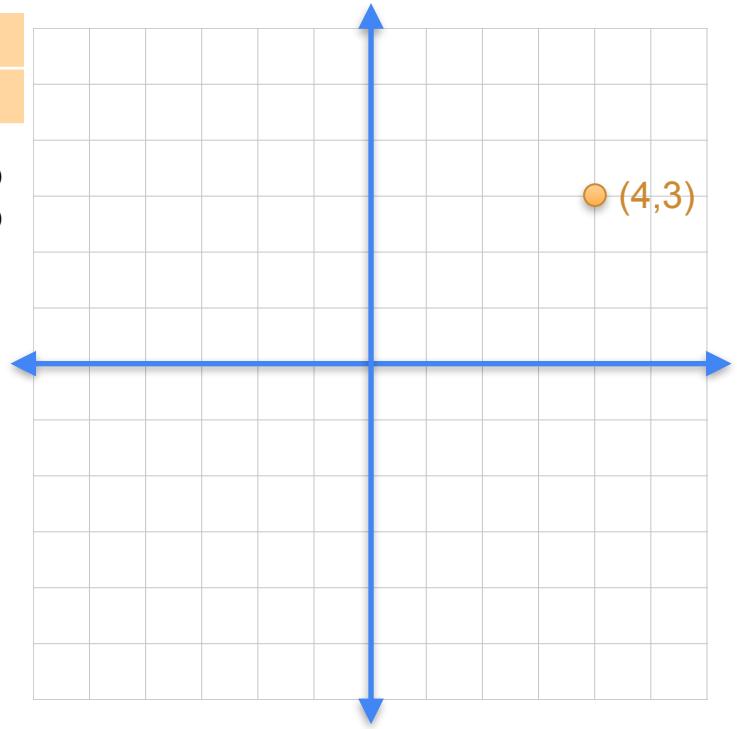


Systems of equations as linear transformations

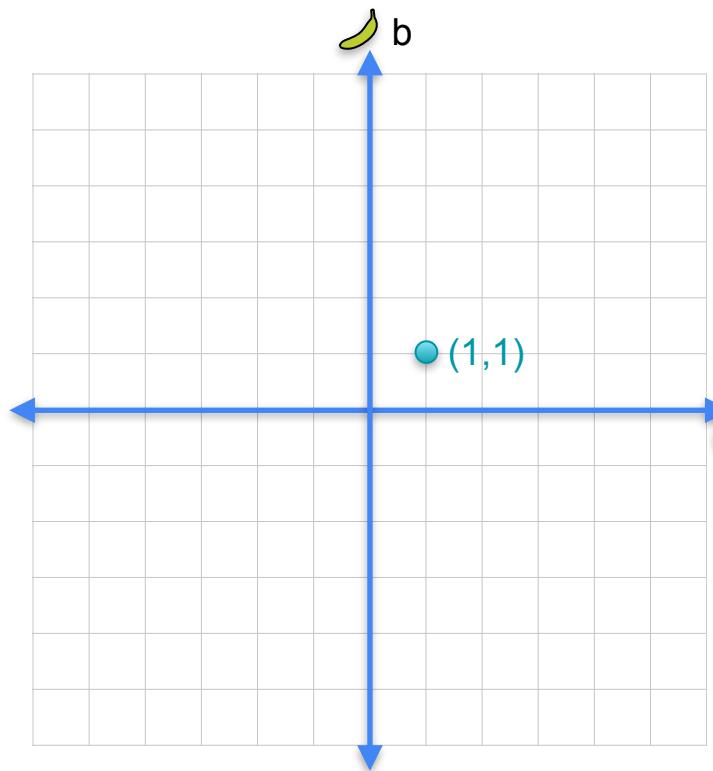


| | | | | |
|---|---|-------|--------|---|
| | | apple | banana | |
| 3 | 1 | | | 4 |
| 1 | 2 | | | 3 |

First day: $3a + b$
Second day: $a + 2b$

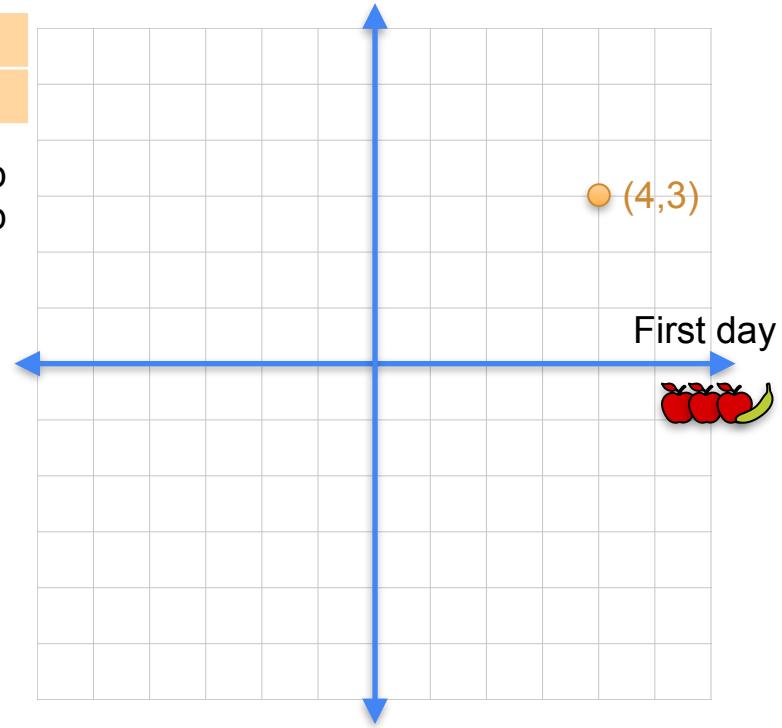


Systems of equations as linear transformations

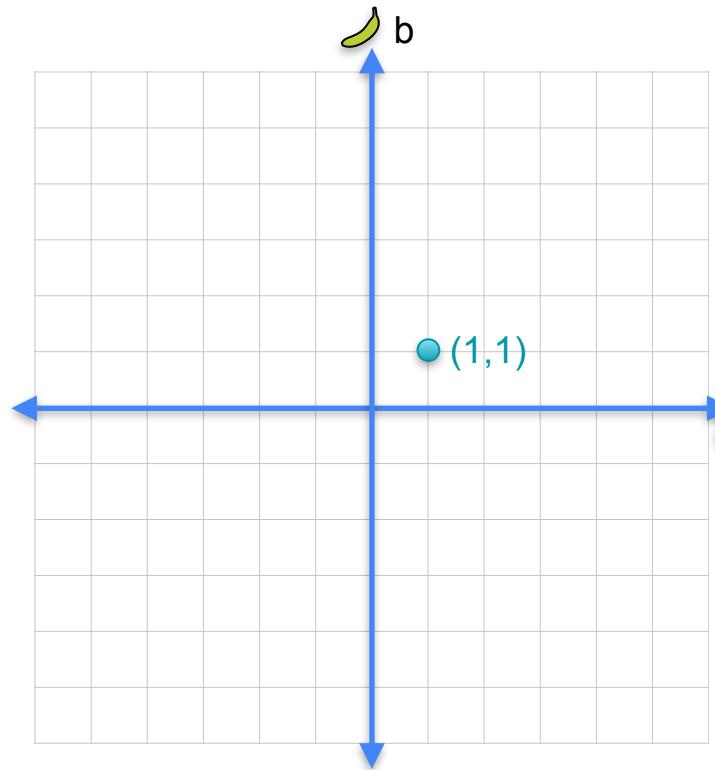


$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} \text{orange} \\ 4 \\ 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$



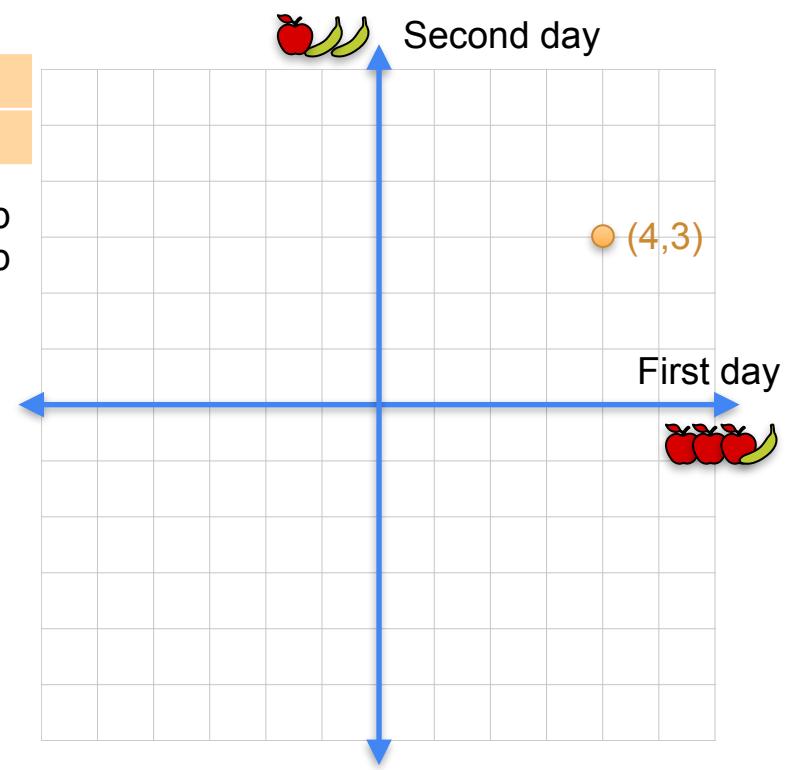
Systems of equations as linear transformations



| | | | |
|---|---|---|---|
| | | | |
| | | | |
| 3 | 1 | 1 | = |
| 1 | 2 | 1 | 4 |

| | | | |
|--|--|--|---|
| | | | |
| | | | |
| | | | = |
| | | | 3 |

First day: $3a + b$
Second day: $a + 2b$



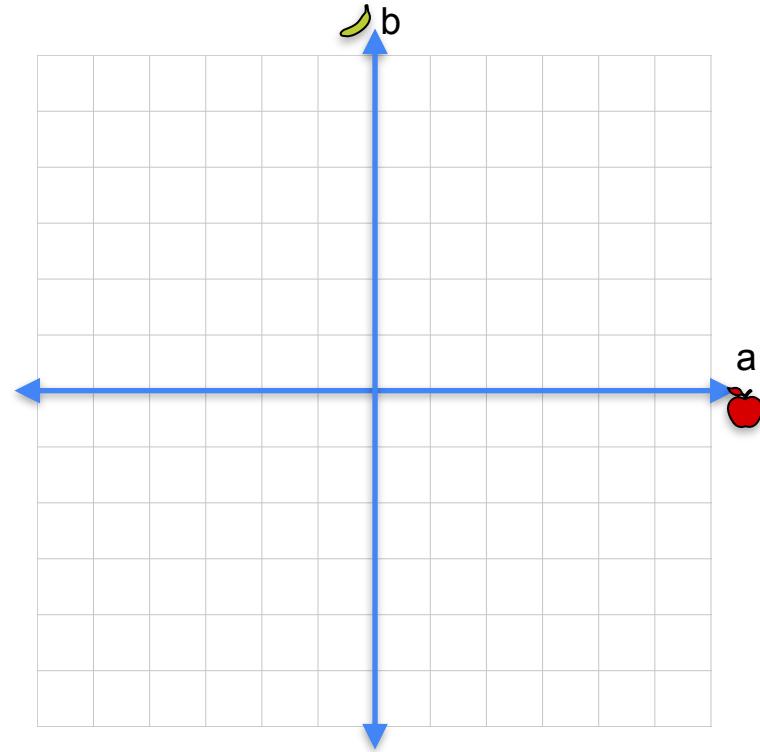
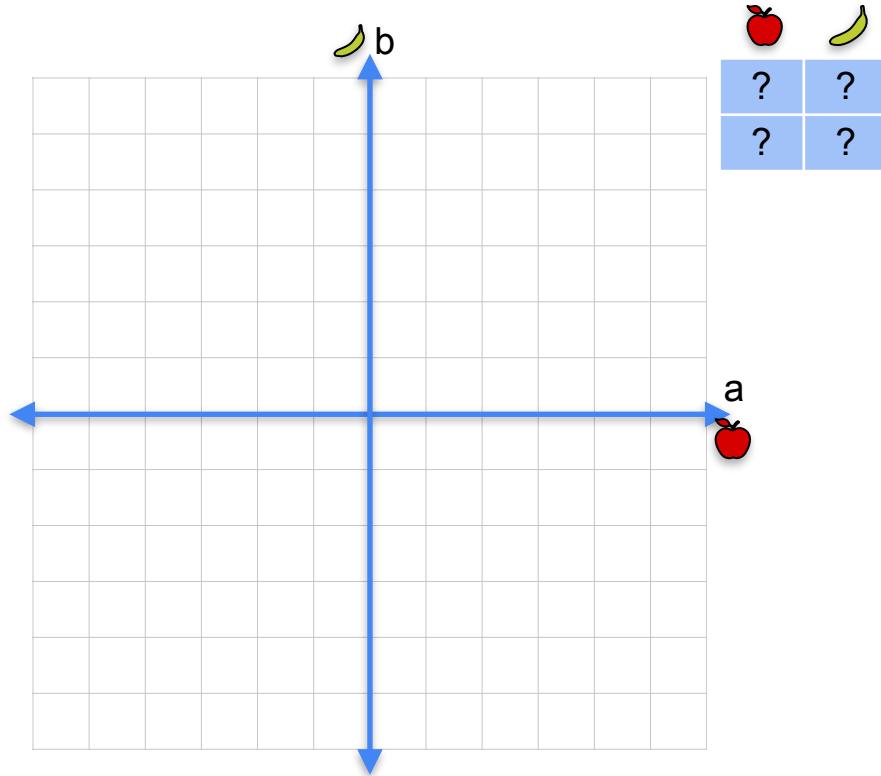


DeepLearning.AI

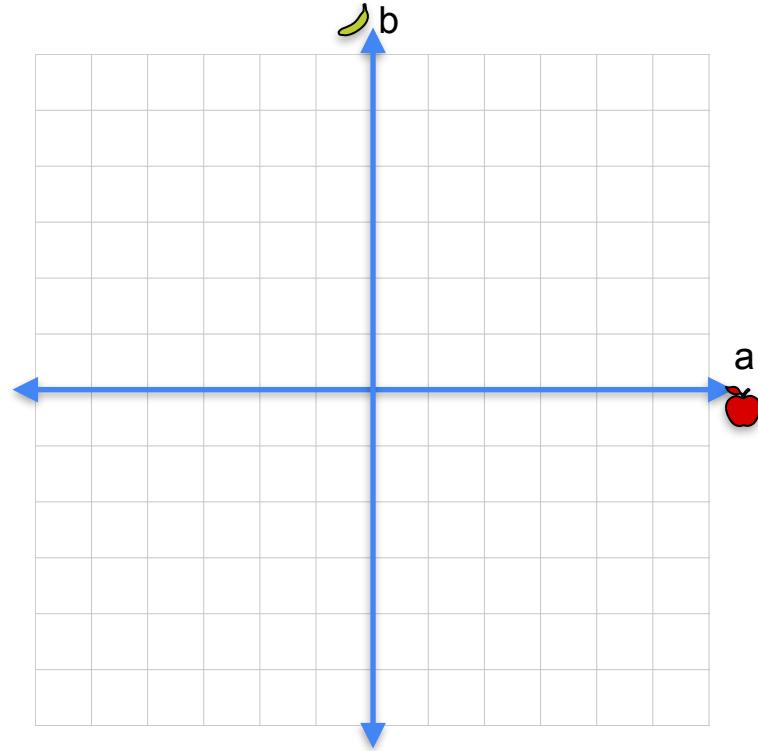
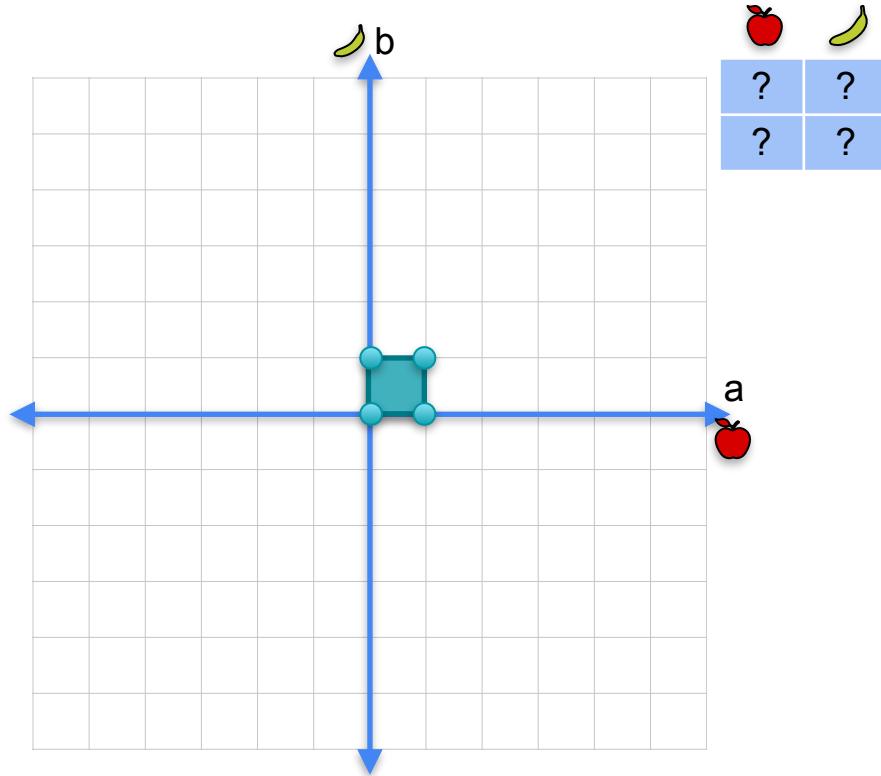
Vectors and Linear Transformations

**Linear transformations as
matrices**

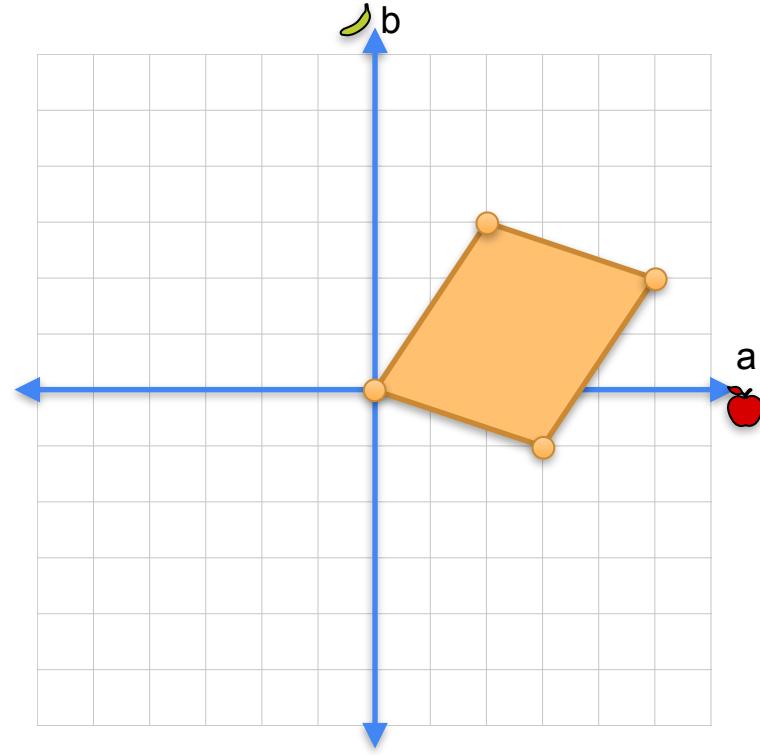
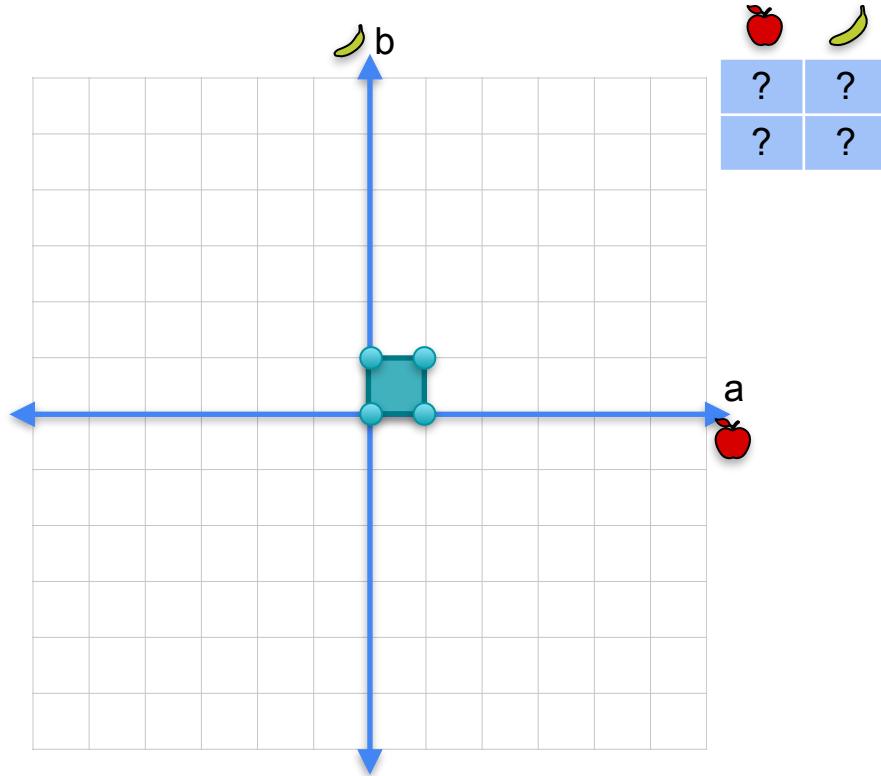
Linear transformations as matrices



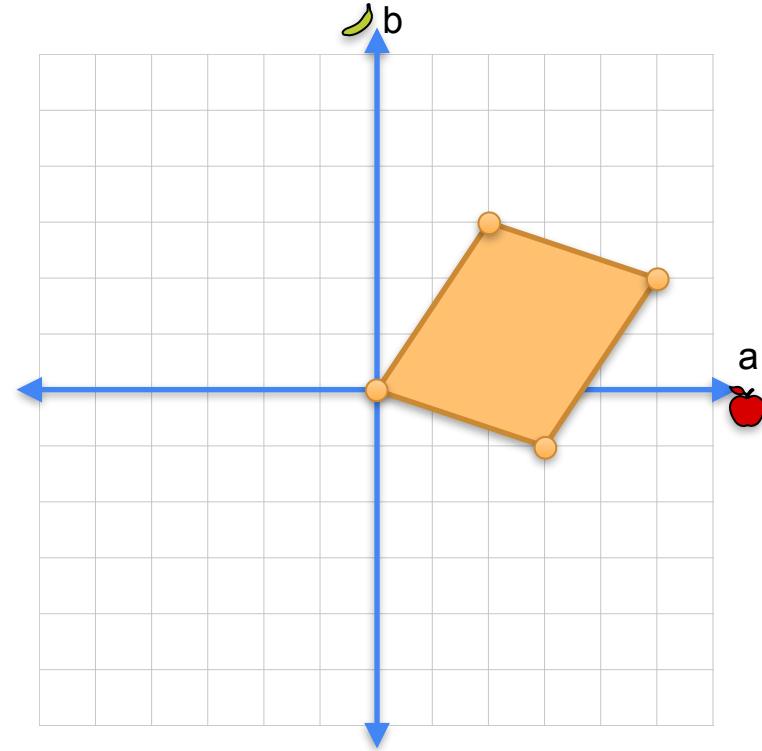
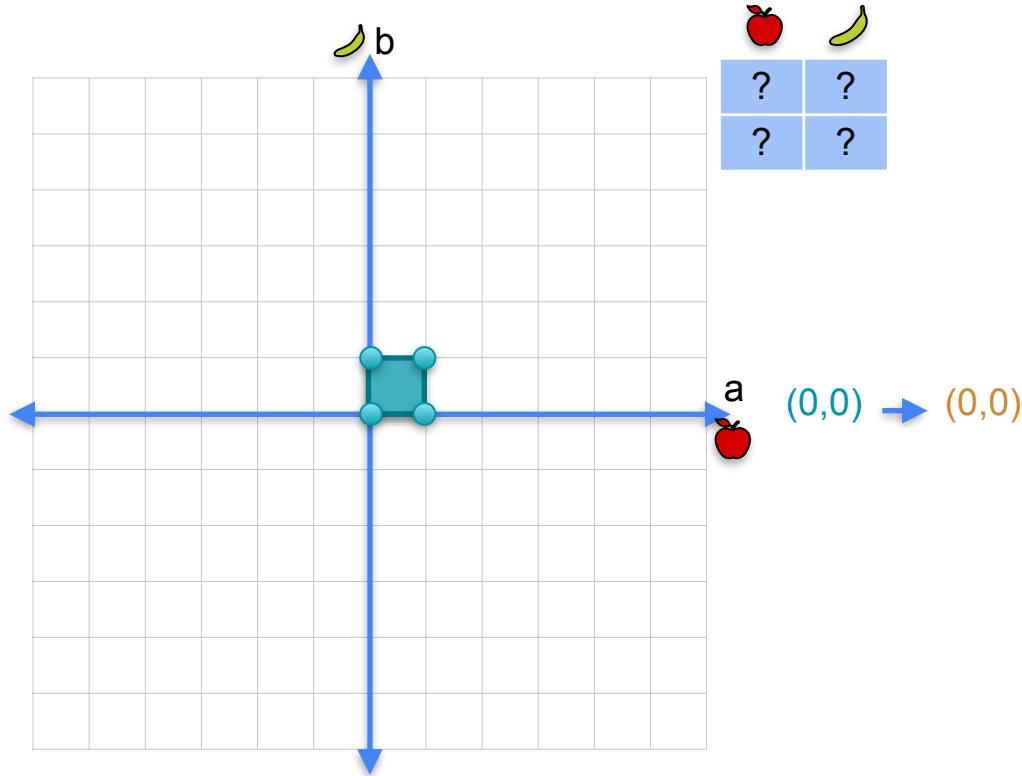
Linear transformations as matrices



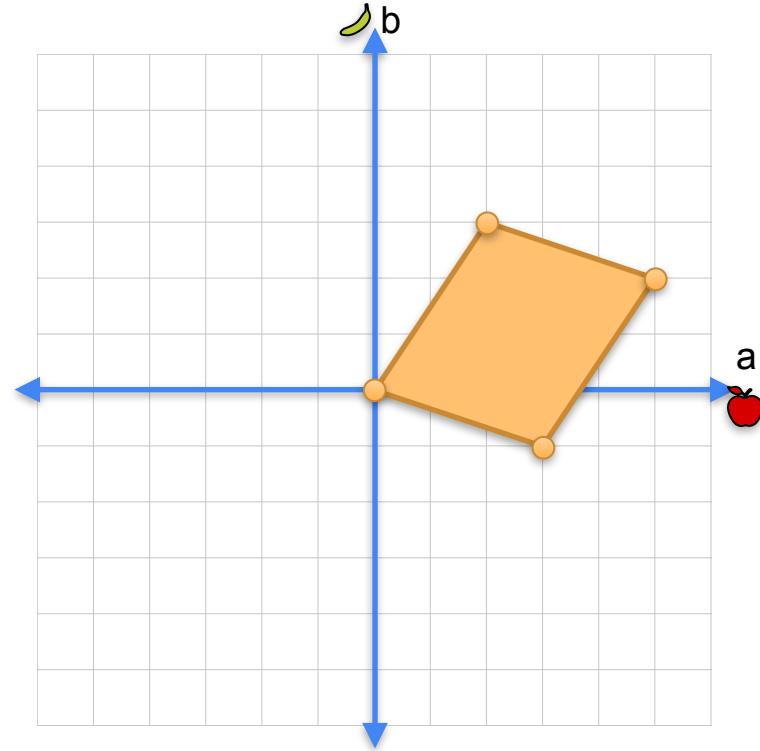
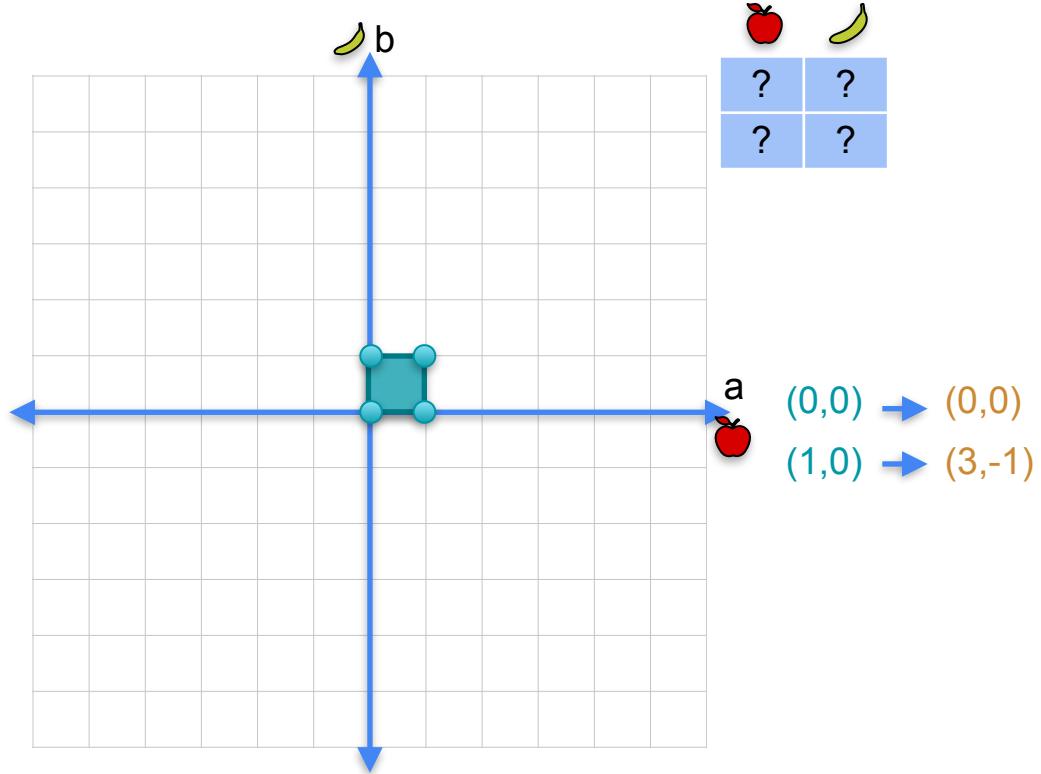
Linear transformations as matrices



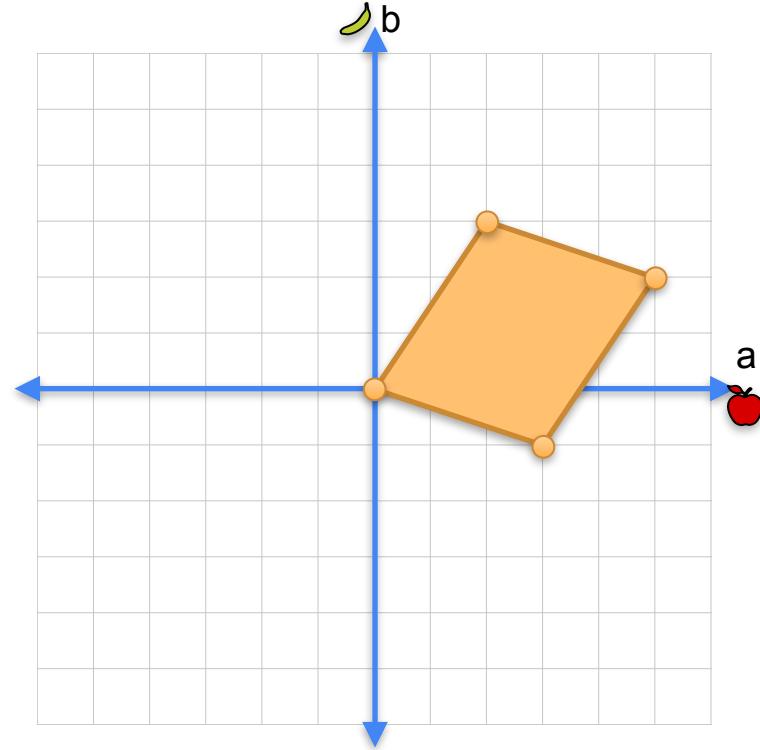
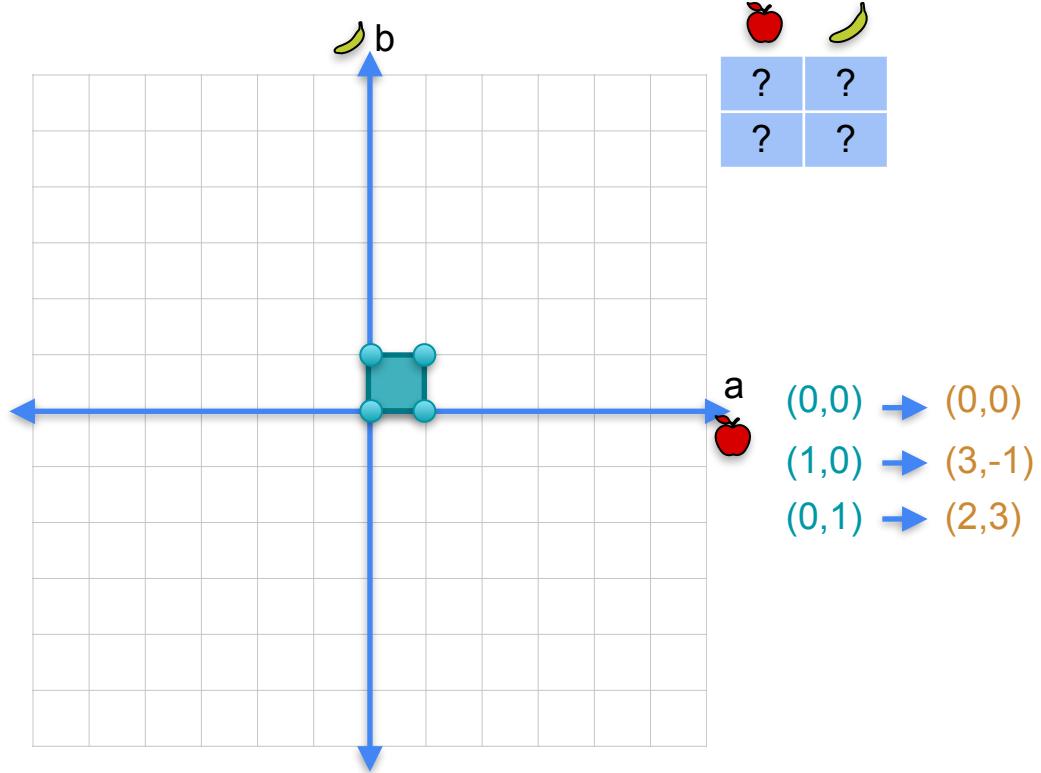
Linear transformations as matrices



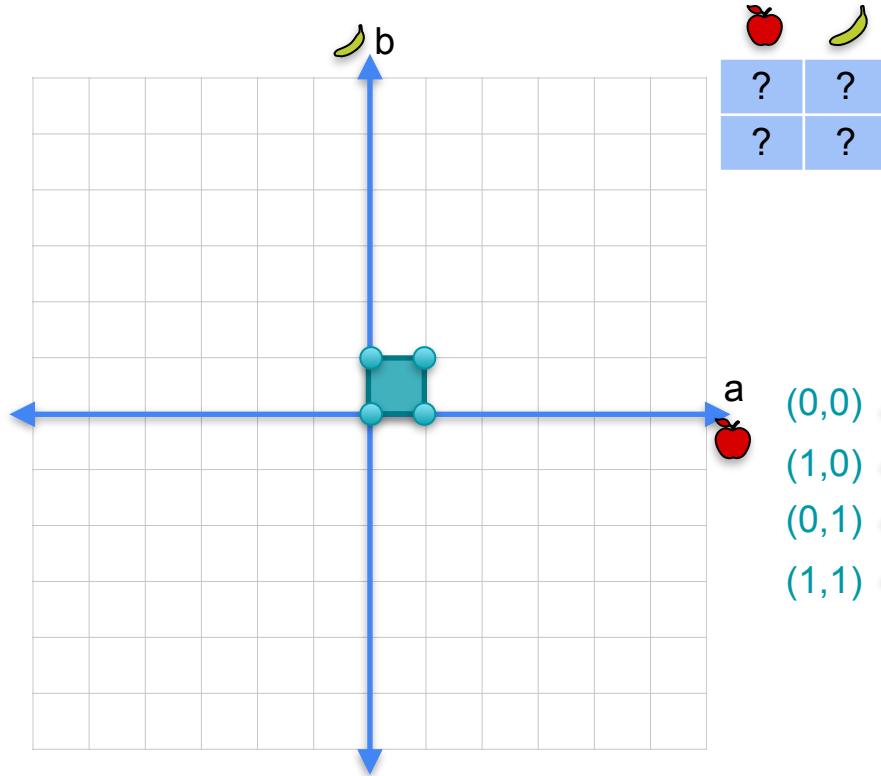
Linear transformations as matrices



Linear transformations as matrices

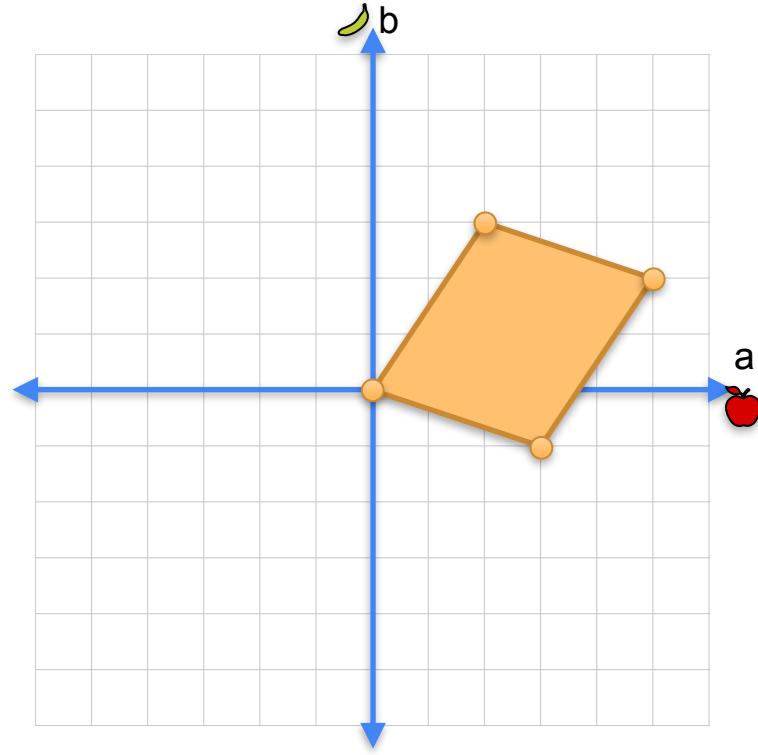


Linear transformations as matrices

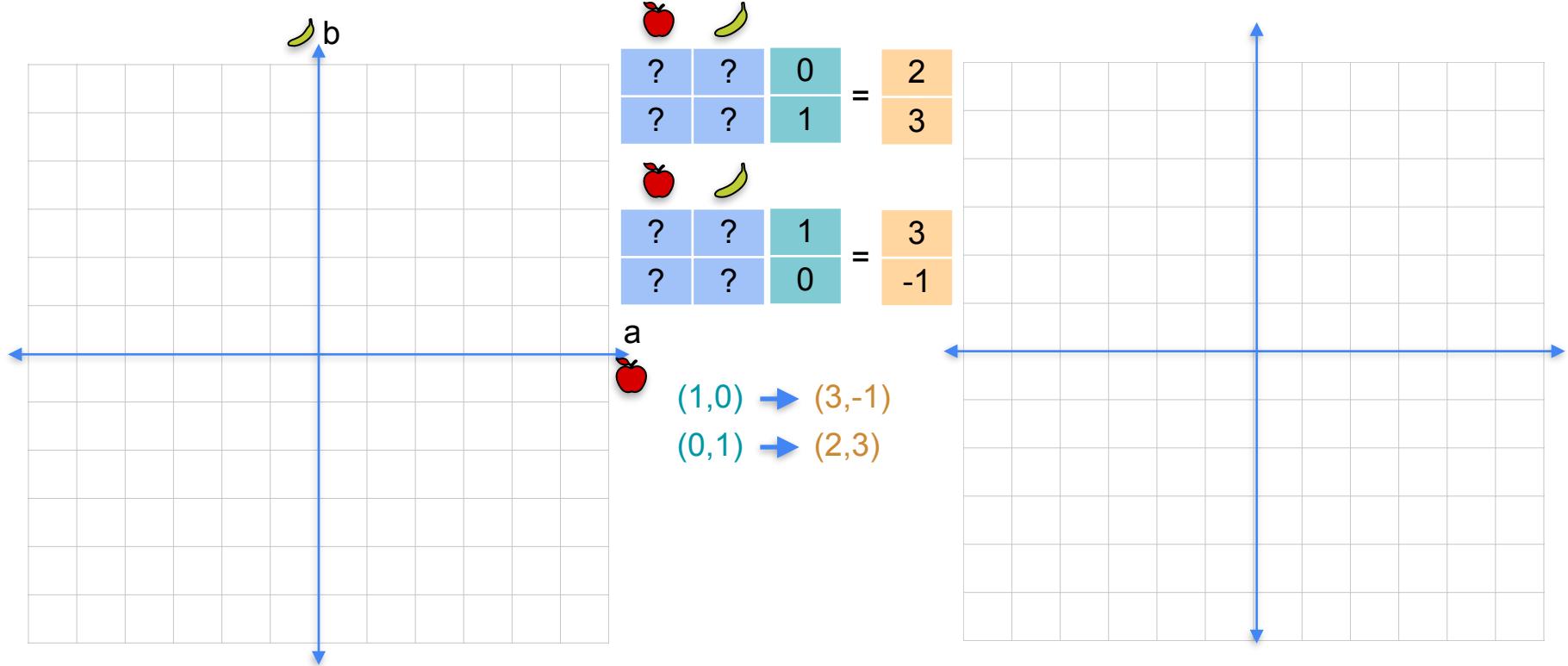


| | |
|---|---|
| | |
| ? | ? |
| ? | ? |

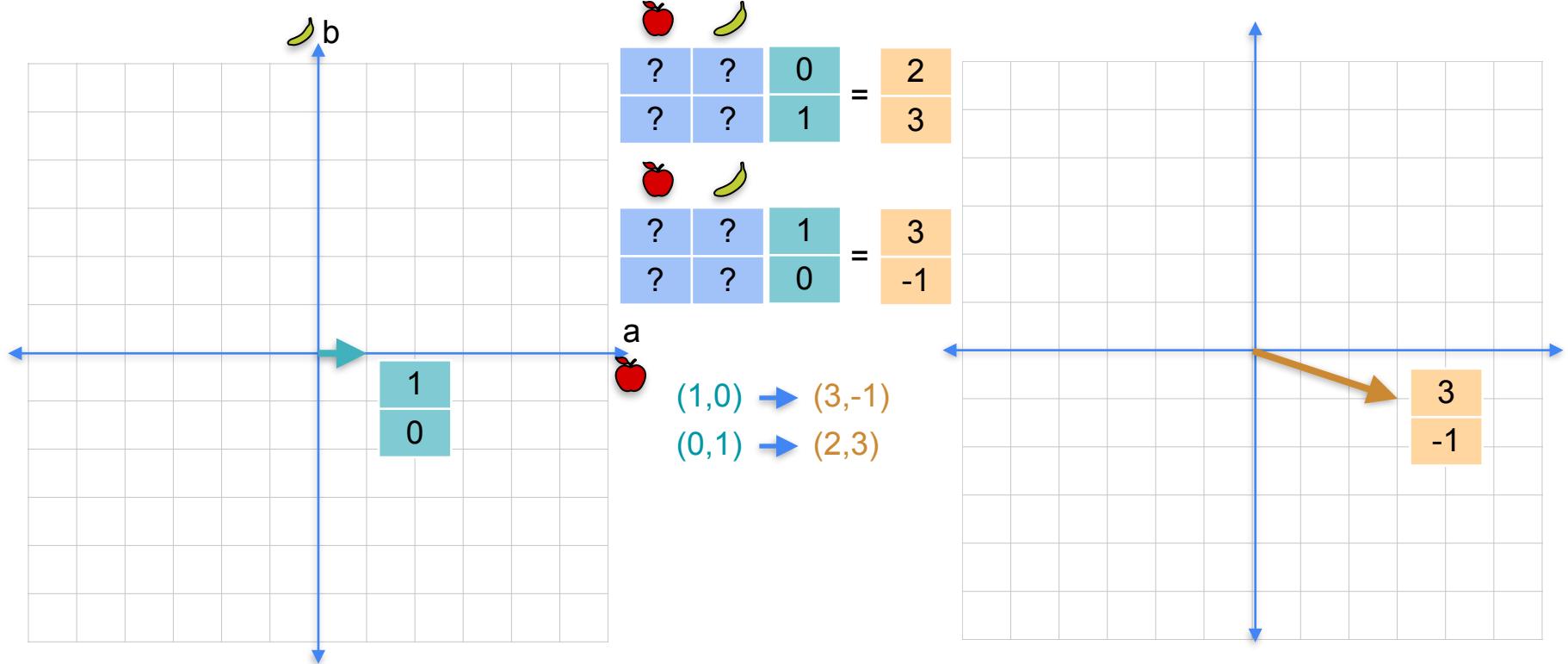
- (0,0) \rightarrow (0,0)
- (1,0) \rightarrow (3,-1)
- (0,1) \rightarrow (2,3)
- (1,1) \rightarrow (5,2)



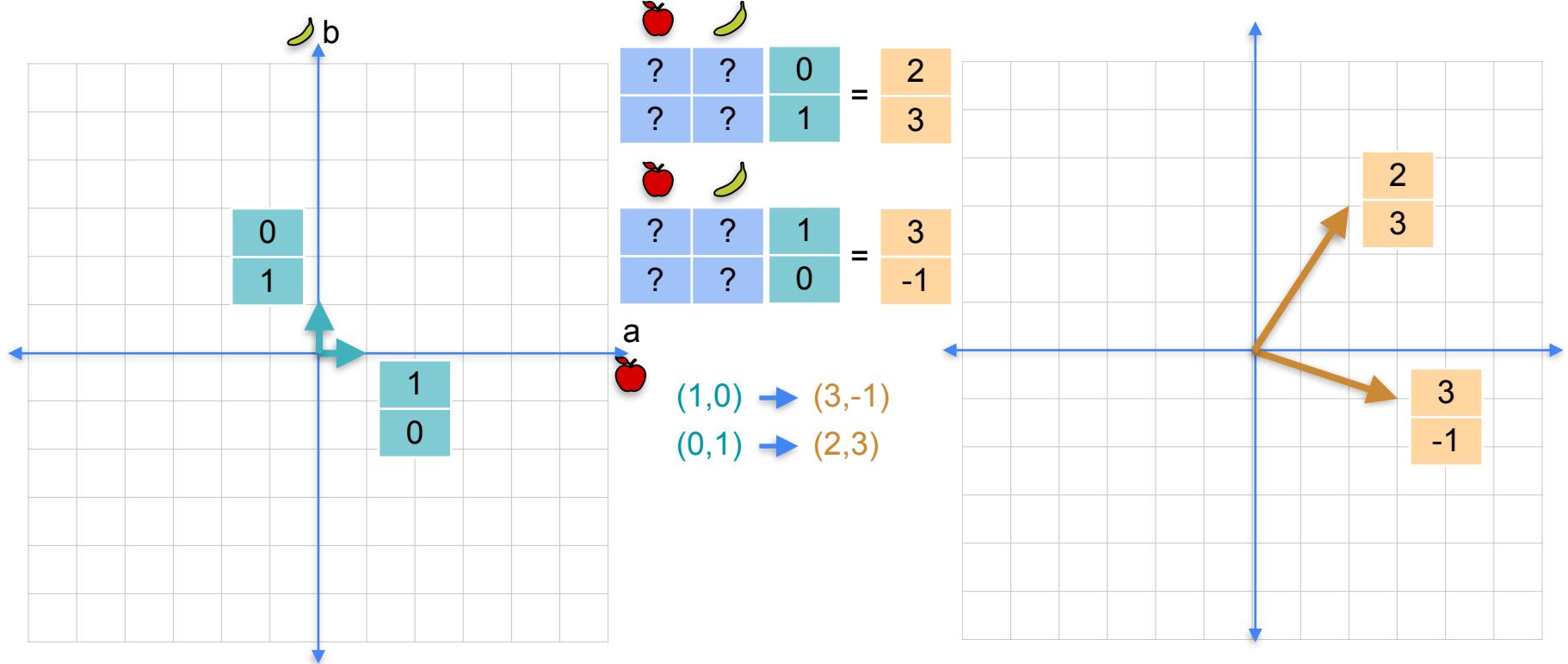
Linear transformations as matrices



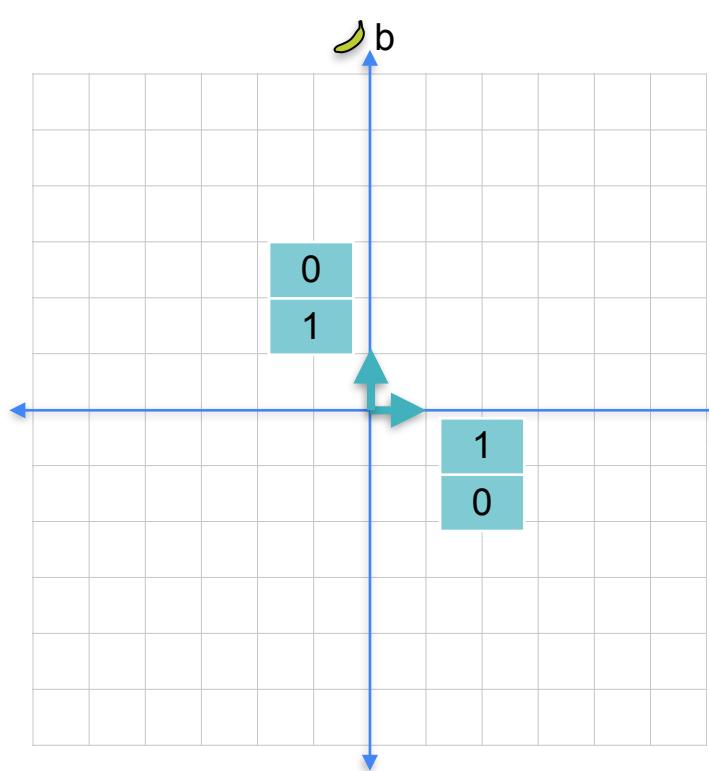
Linear transformations as matrices



Linear transformations as matrices



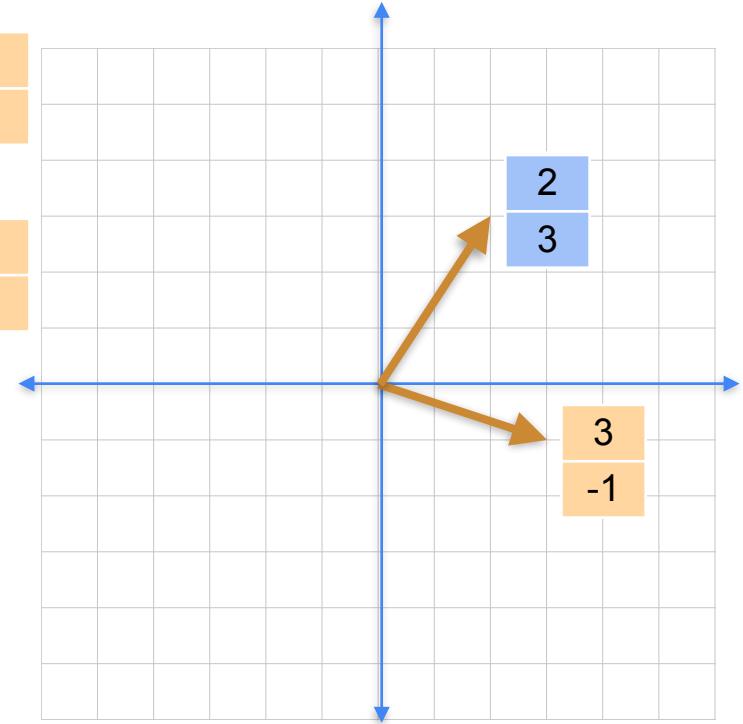
Linear transformations as matrices



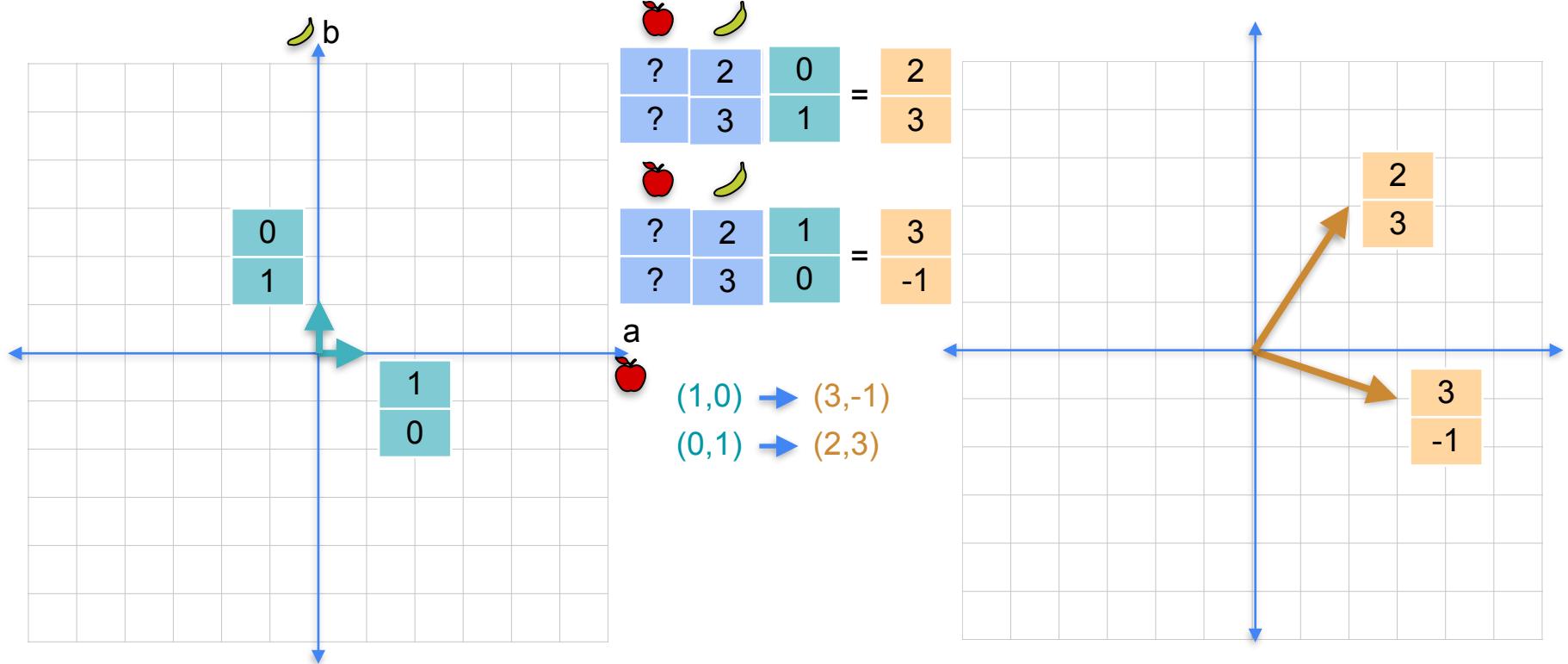
Two matrix equations illustrating the linear transformation:

$$\begin{matrix} \text{apple} & \text{banana} \\ ? & ? \\ ? & ? \end{matrix} \begin{matrix} 0 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 3 \end{matrix}$$
$$\begin{matrix} \text{apple} & \text{banana} \\ ? & ? \\ ? & ? \end{matrix} \begin{matrix} 1 \\ 0 \end{matrix} = \begin{matrix} 3 \\ -1 \end{matrix}$$

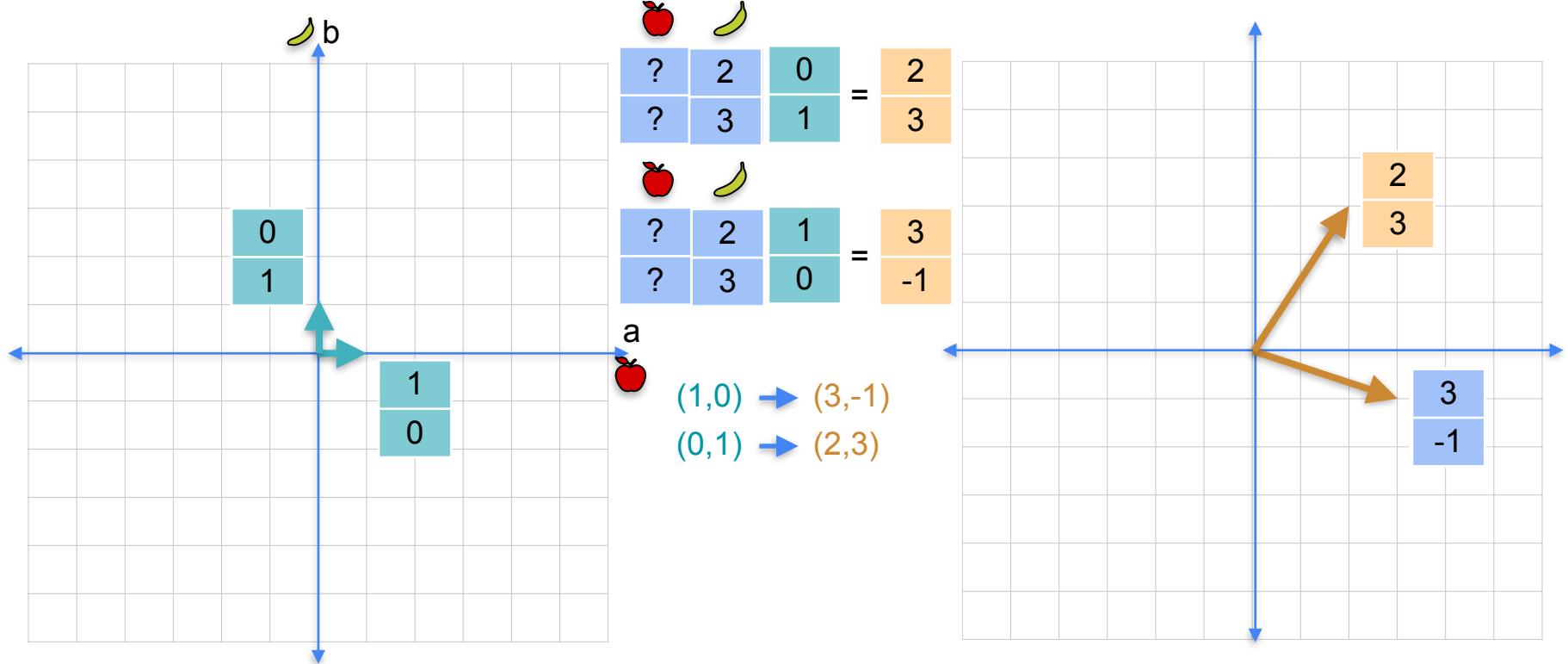
Coordinate pairs: $(1,0) \rightarrow (3,-1)$
 $(0,1) \rightarrow (2,3)$



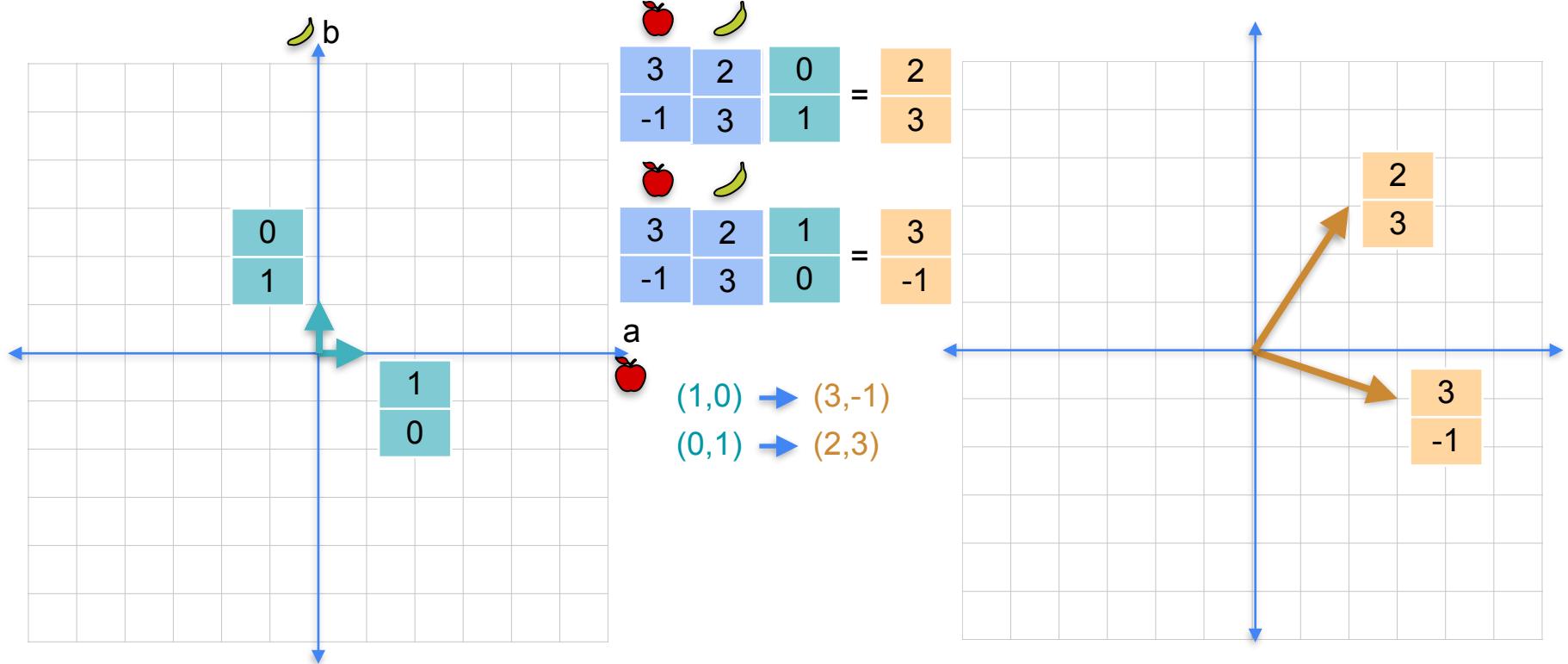
Linear transformations as matrices



Linear transformations as matrices



Linear transformations as matrices



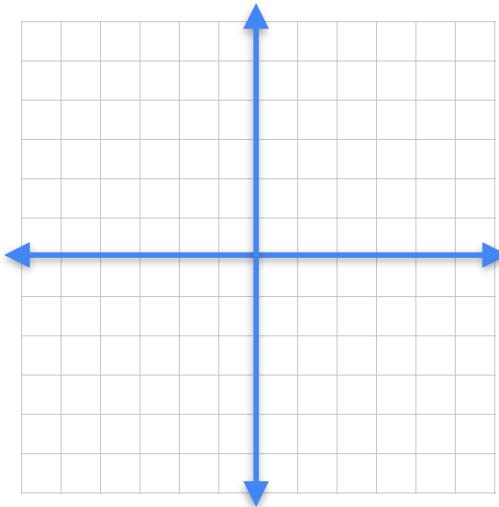
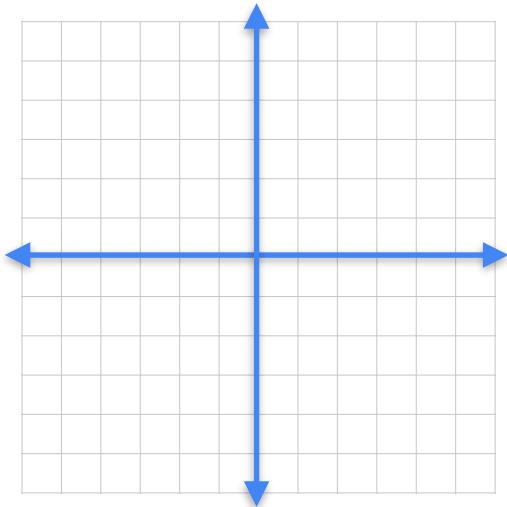


DeepLearning.AI

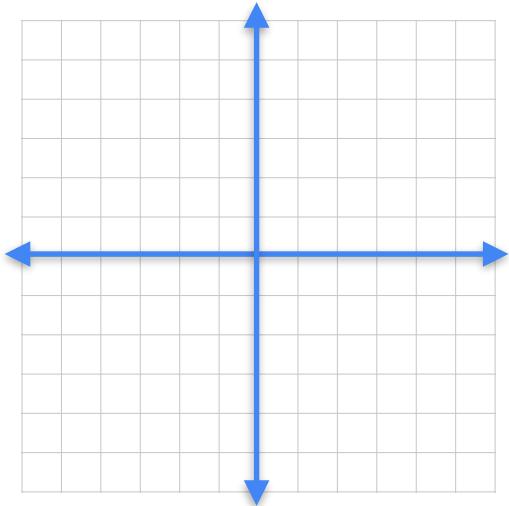
Vectors and Linear Transformations

Matrix multiplication

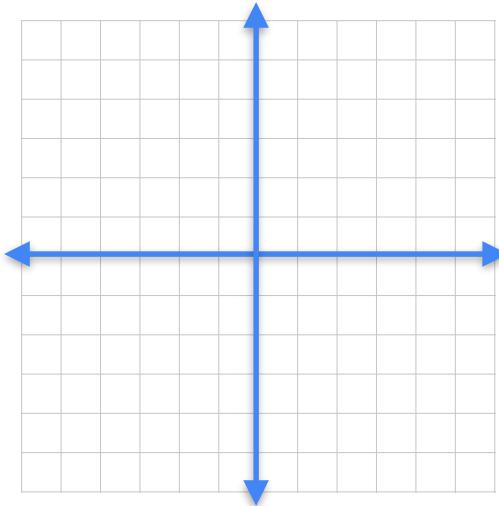
Combining linear transformations



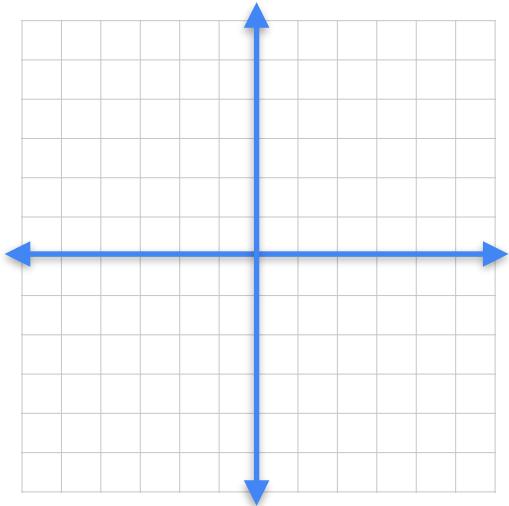
Combining linear transformations



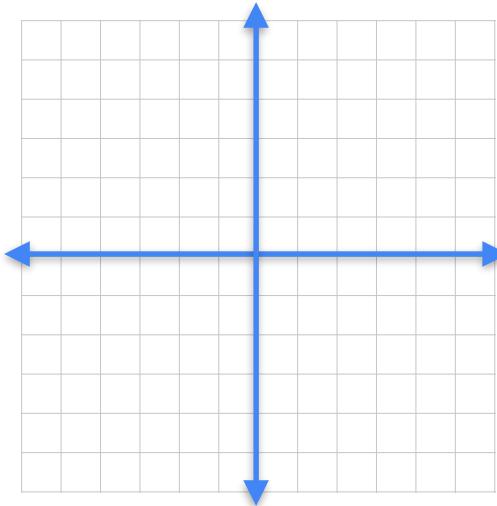
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$



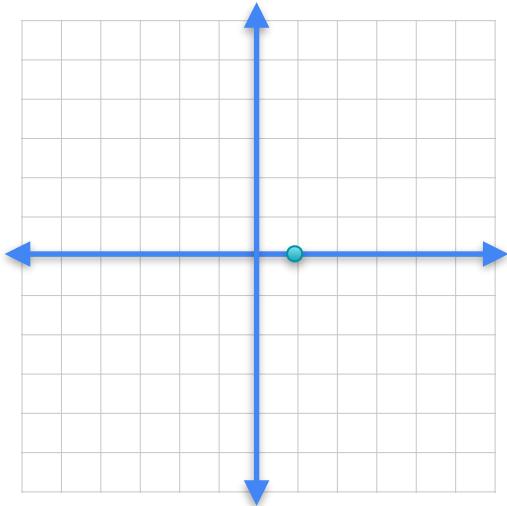
Combining linear transformations



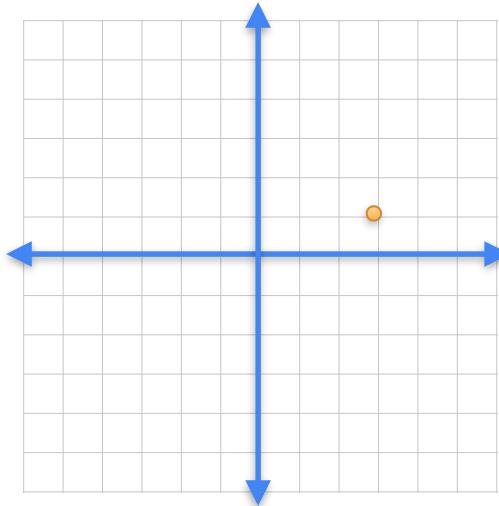
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



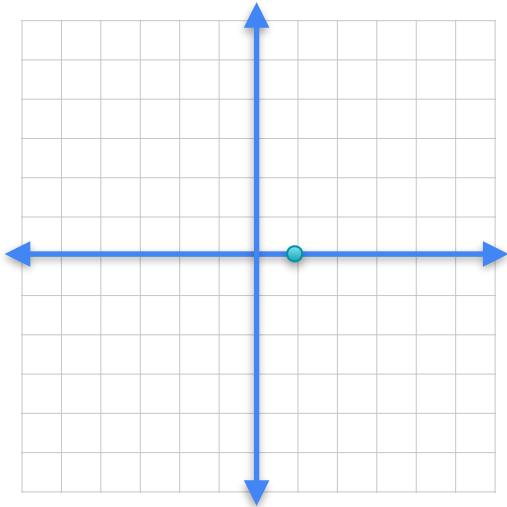
Combining linear transformations



$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

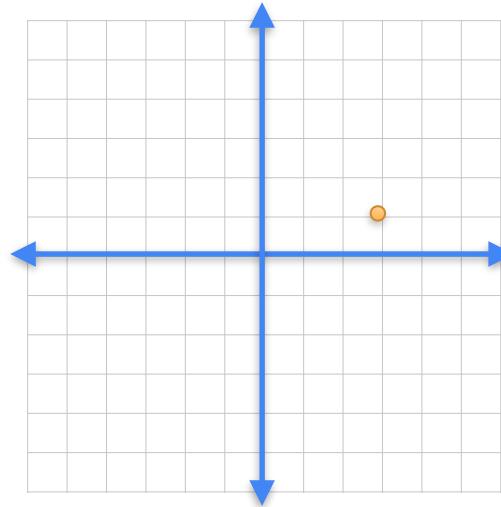


Combining linear transformations

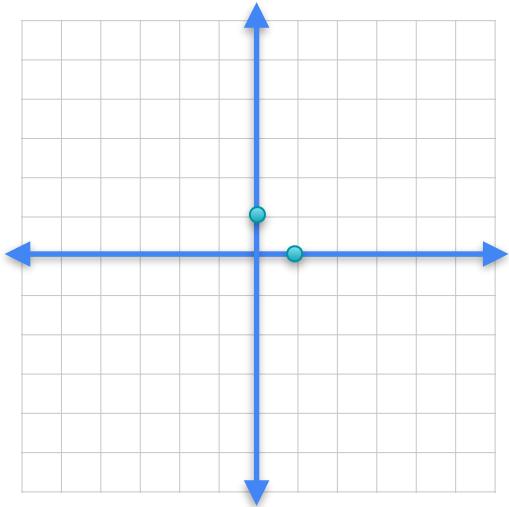


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

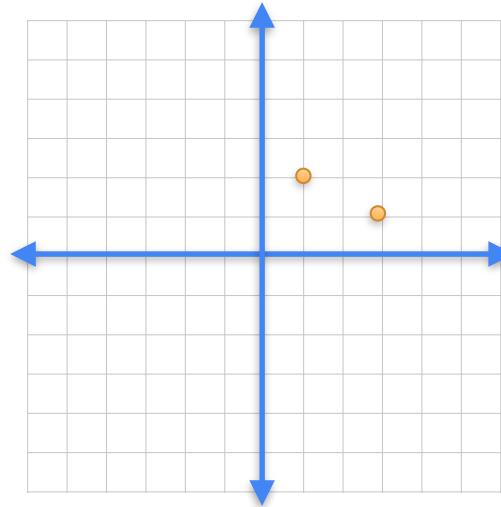


Combining linear transformations

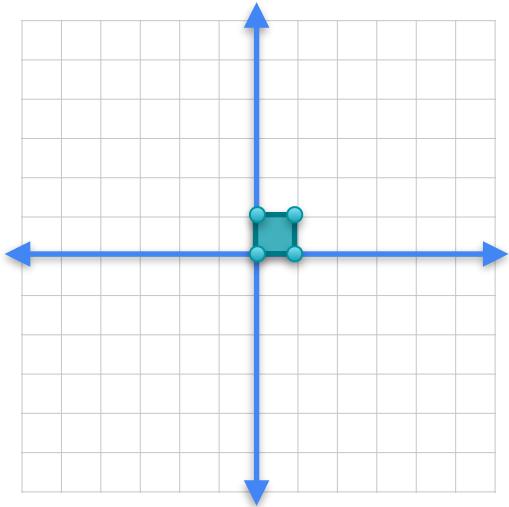


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

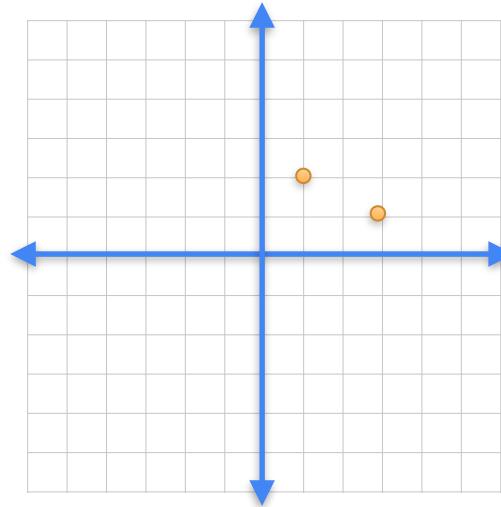


Combining linear transformations

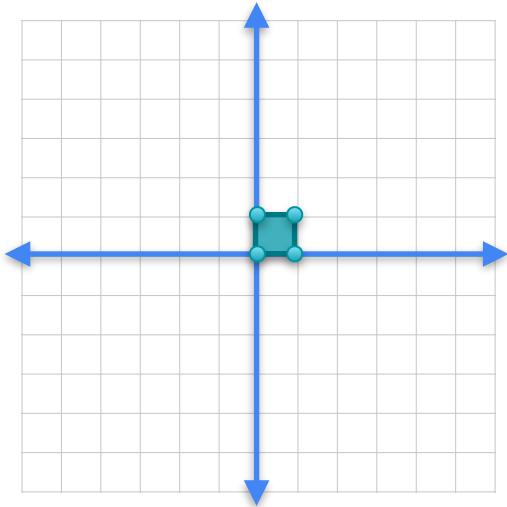


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

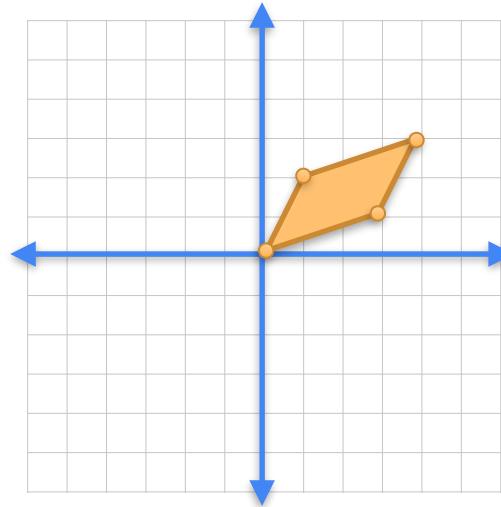


Combining linear transformations

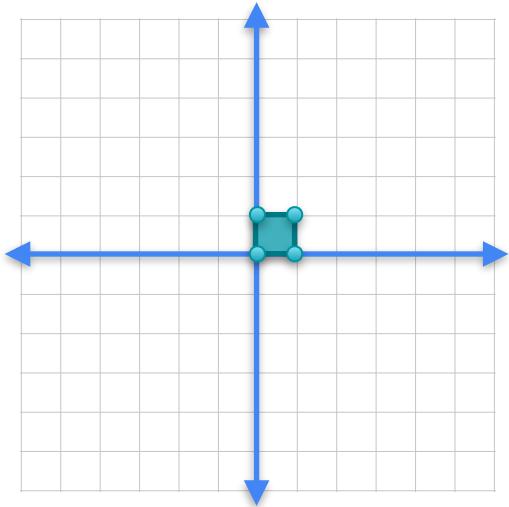


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

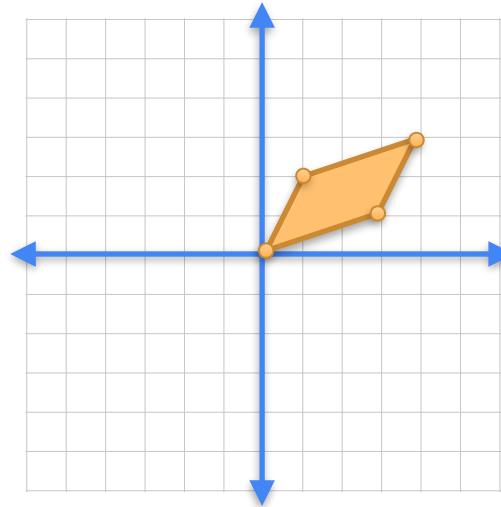


Combining linear transformations

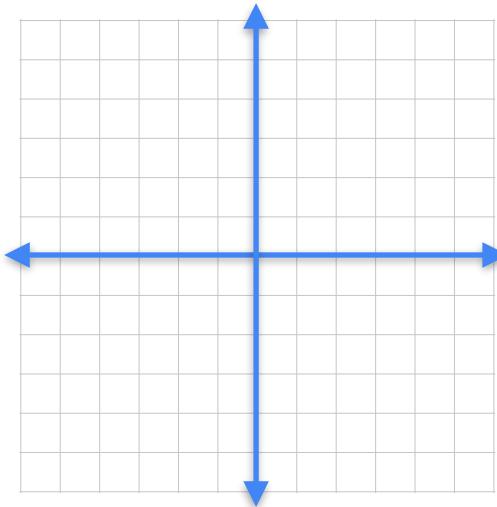
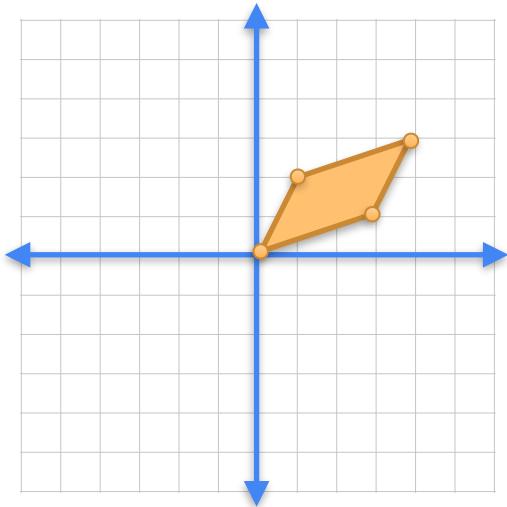


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

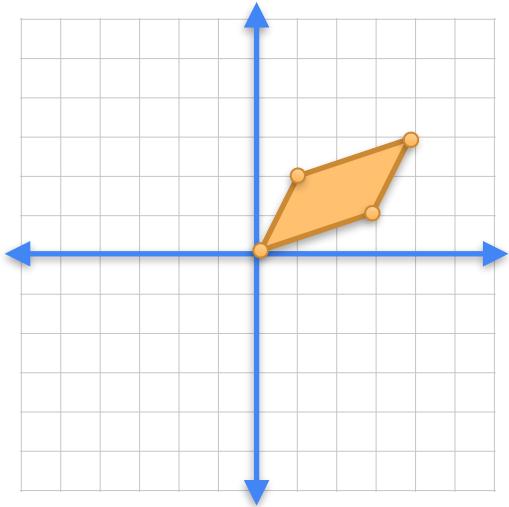
$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



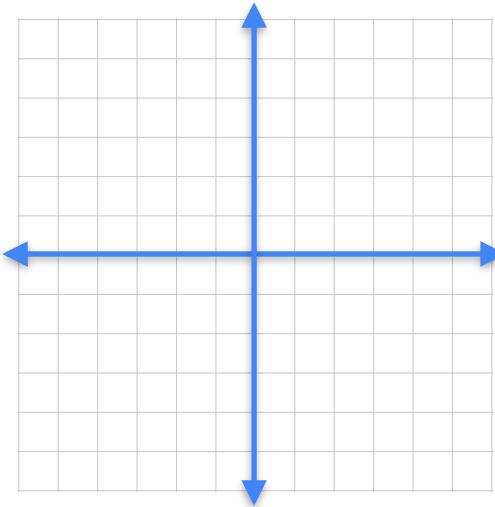
Combining linear transformations



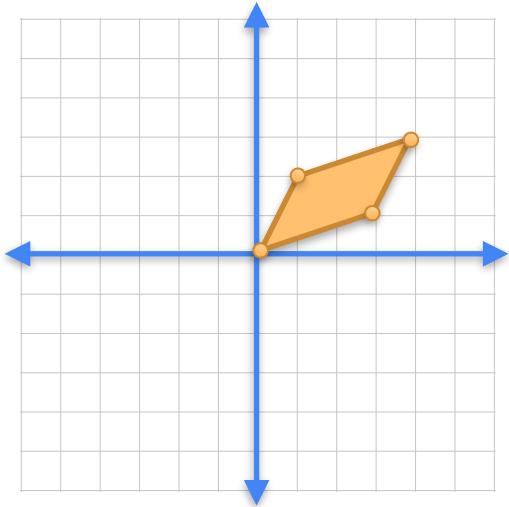
Combining linear transformations



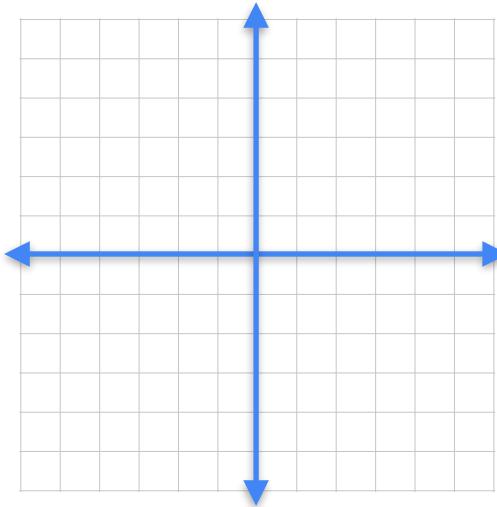
$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix}$$



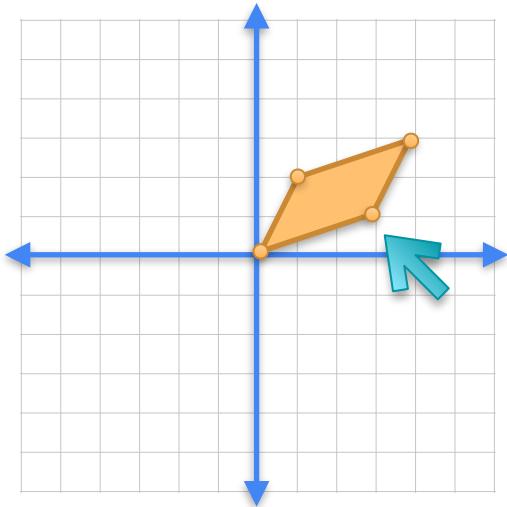
Combining linear transformations



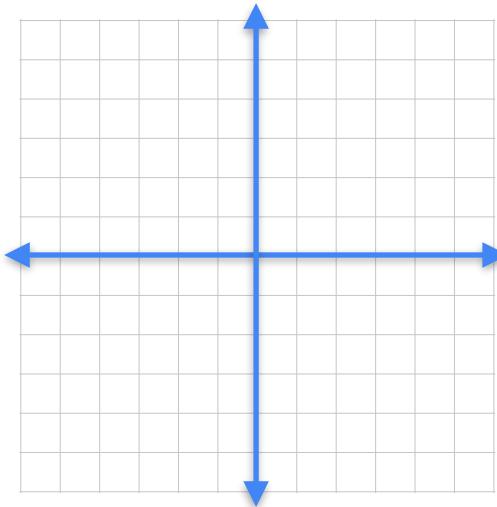
$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$



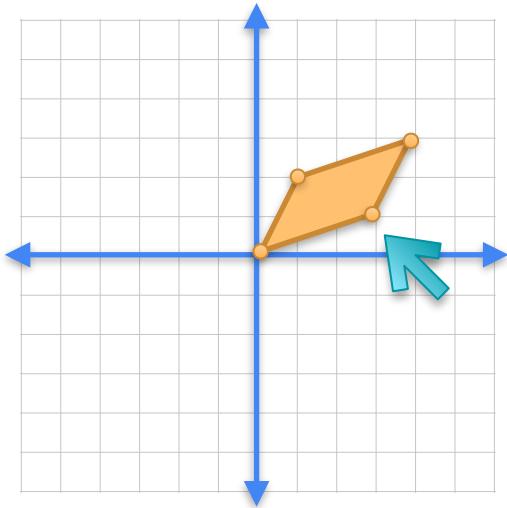
Combining linear transformations



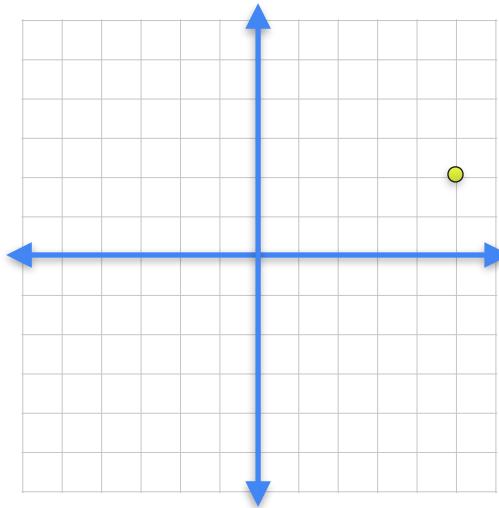
$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$



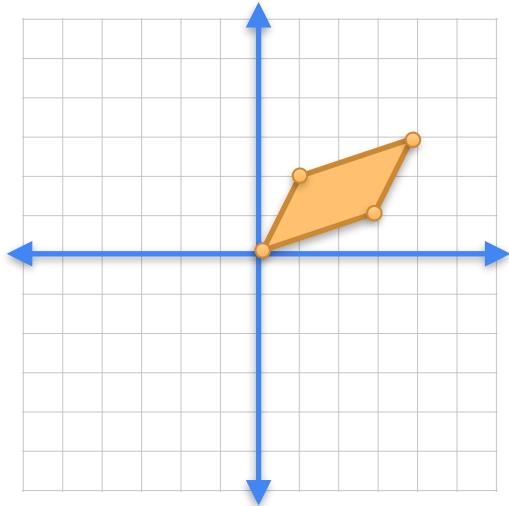
Combining linear transformations



$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

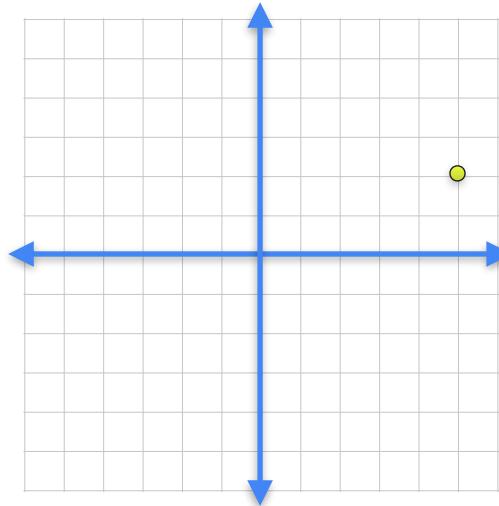


Combining linear transformations

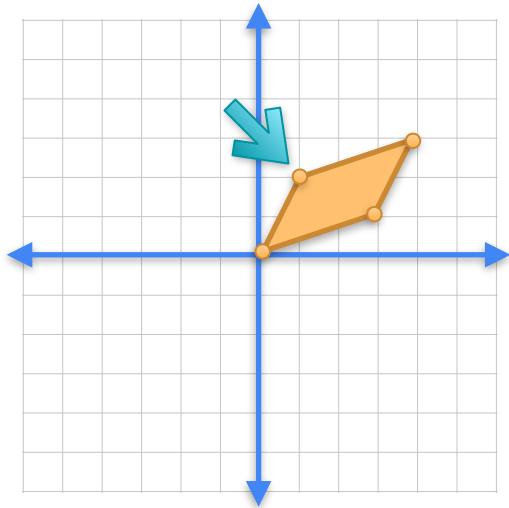


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

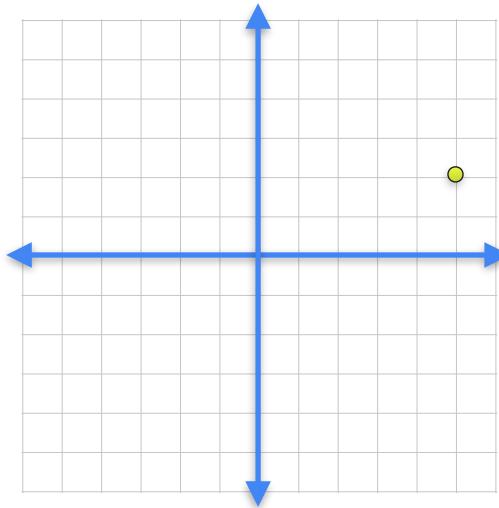


Combining linear transformations

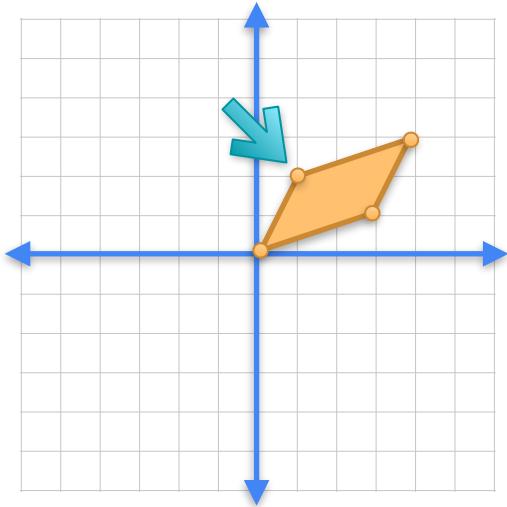


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

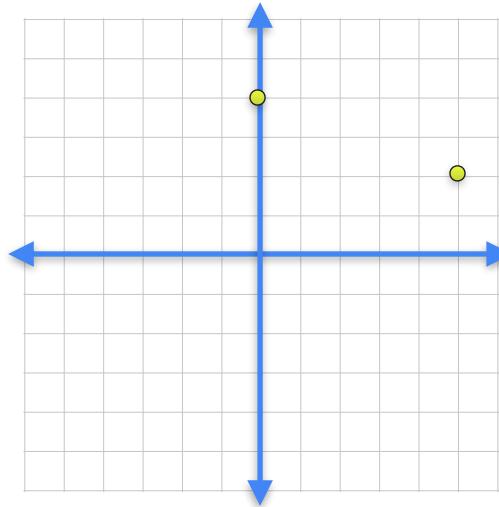


Combining linear transformations

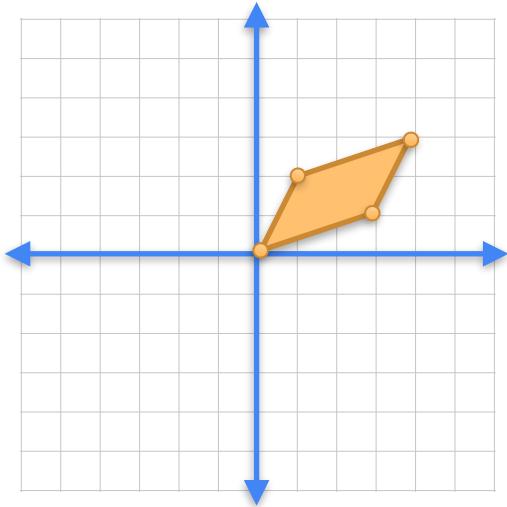


$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

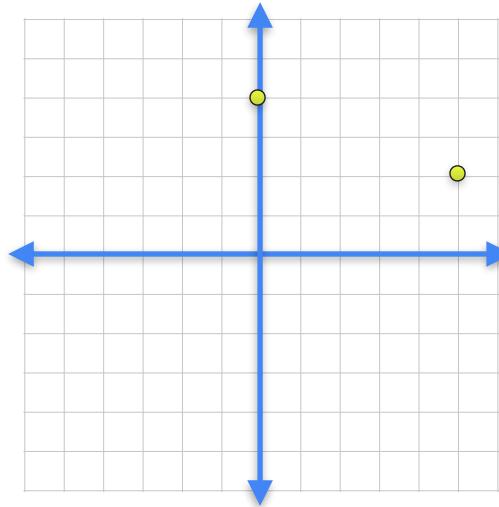


Combining linear transformations

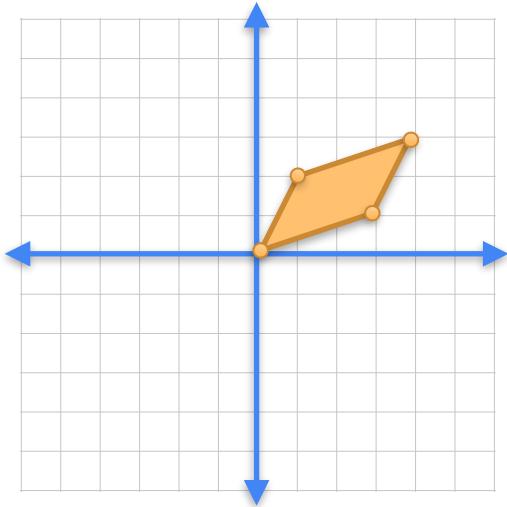


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

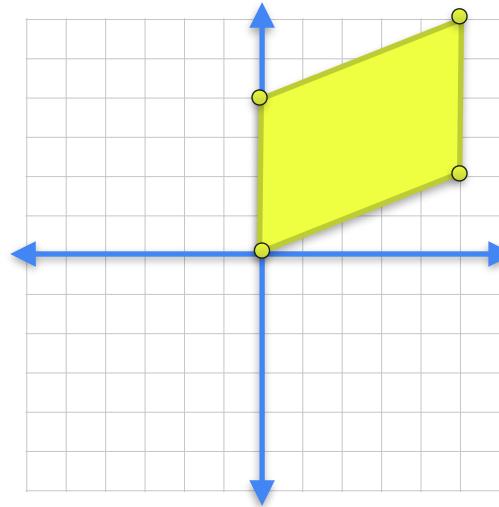


Combining linear transformations

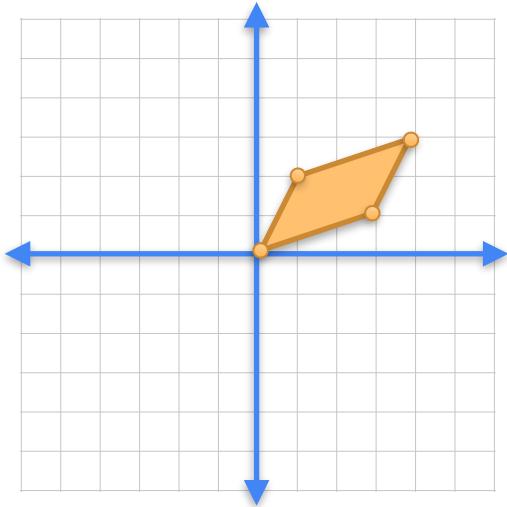


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

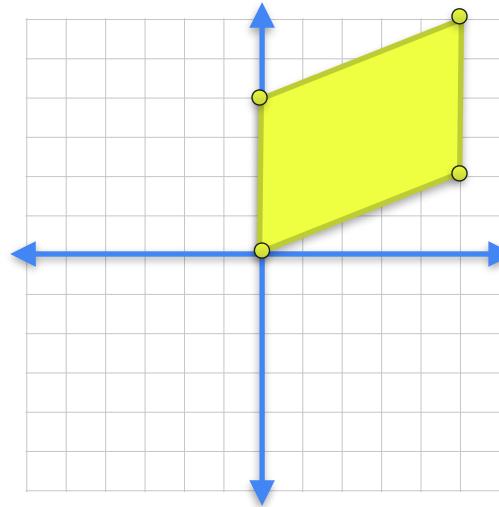


Combining linear transformations

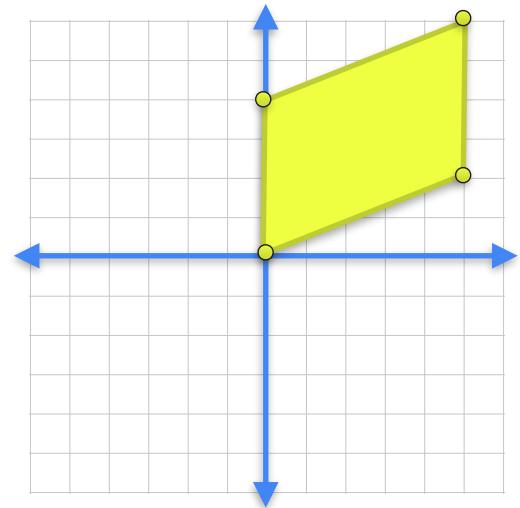
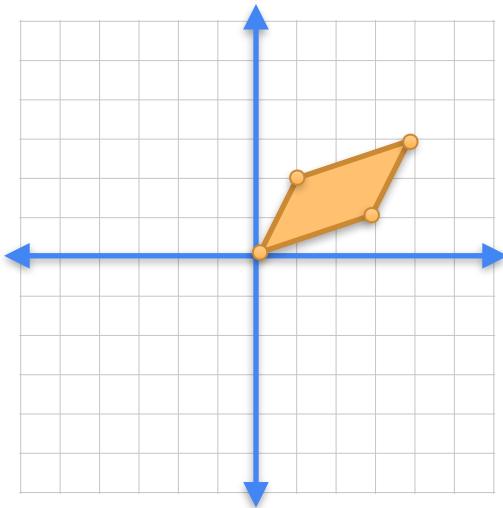
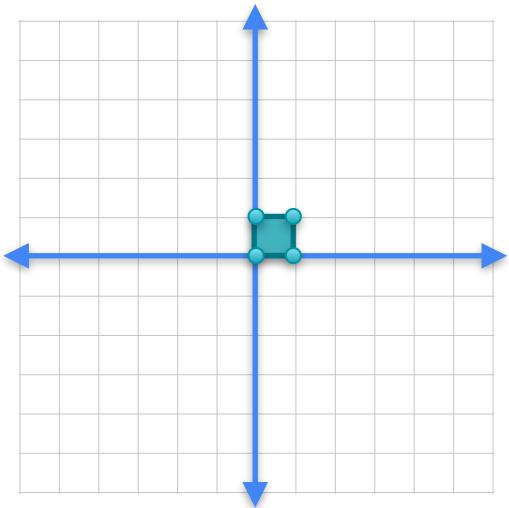


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

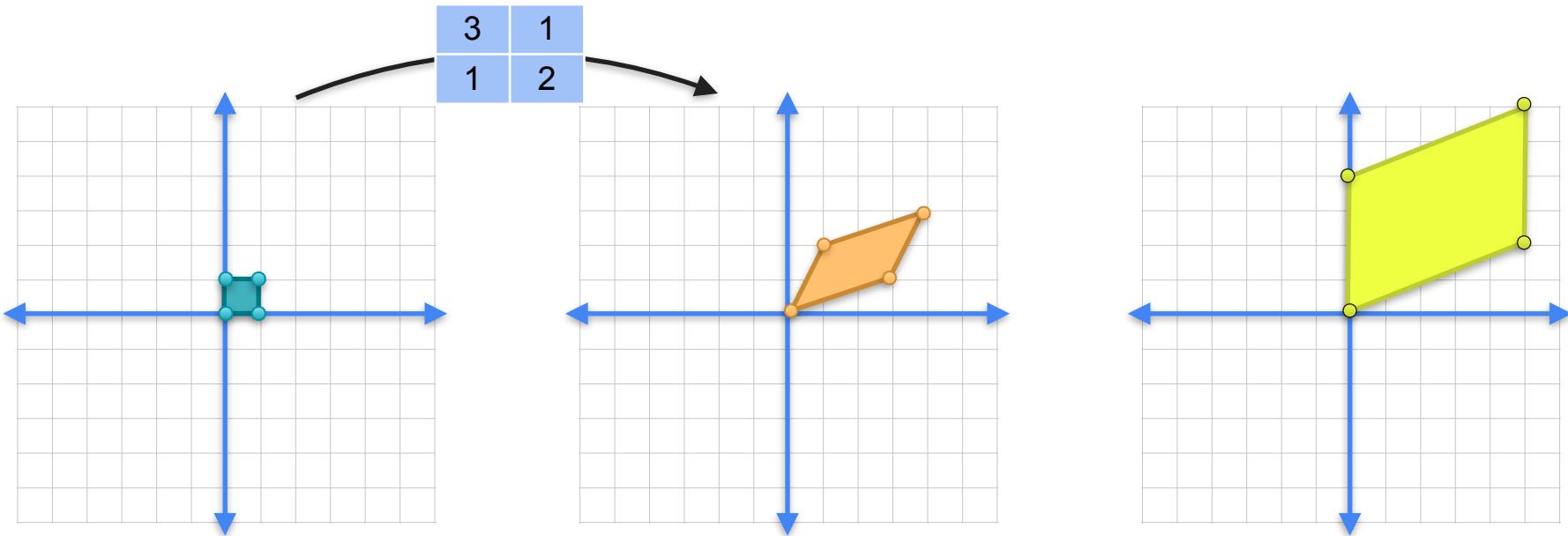
$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$



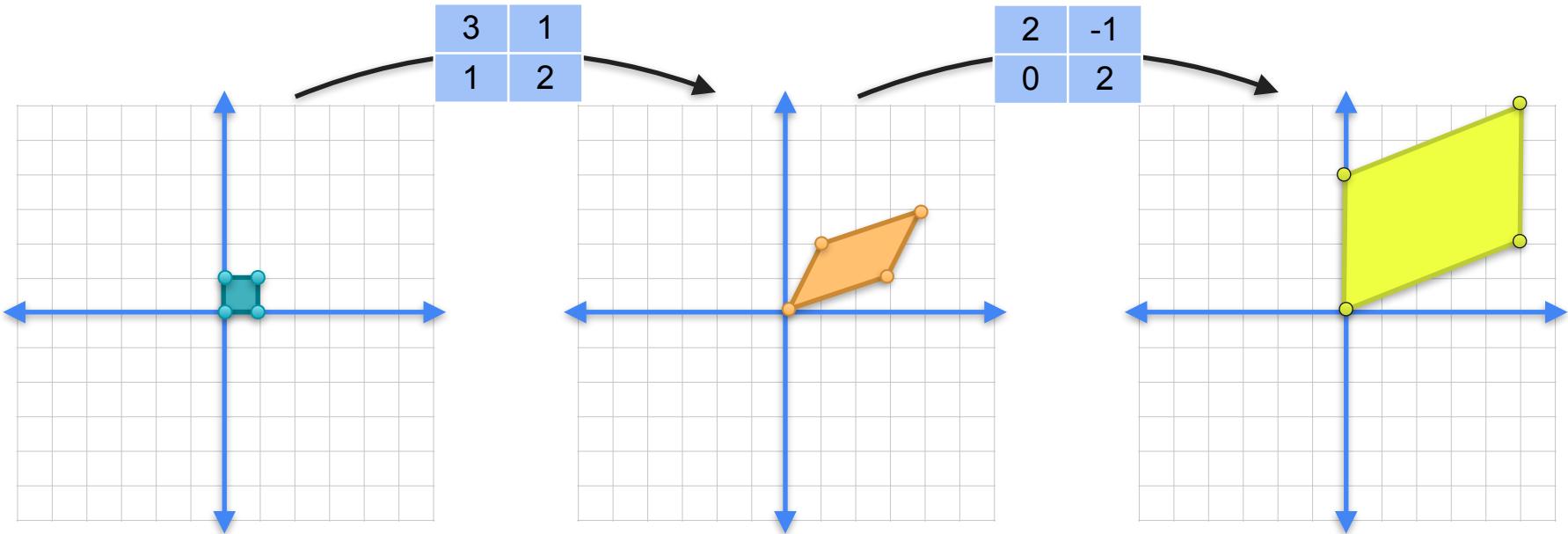
Combining linear transformations



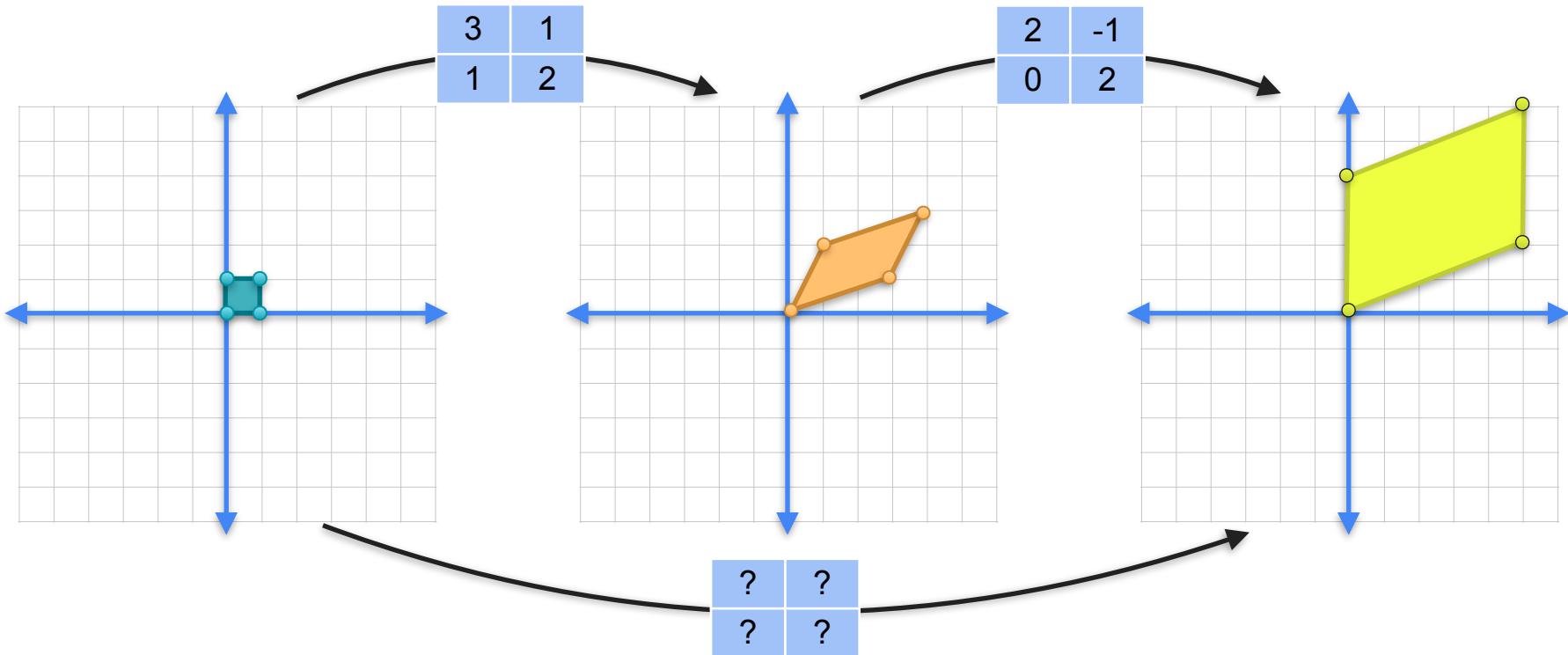
Combining linear transformations



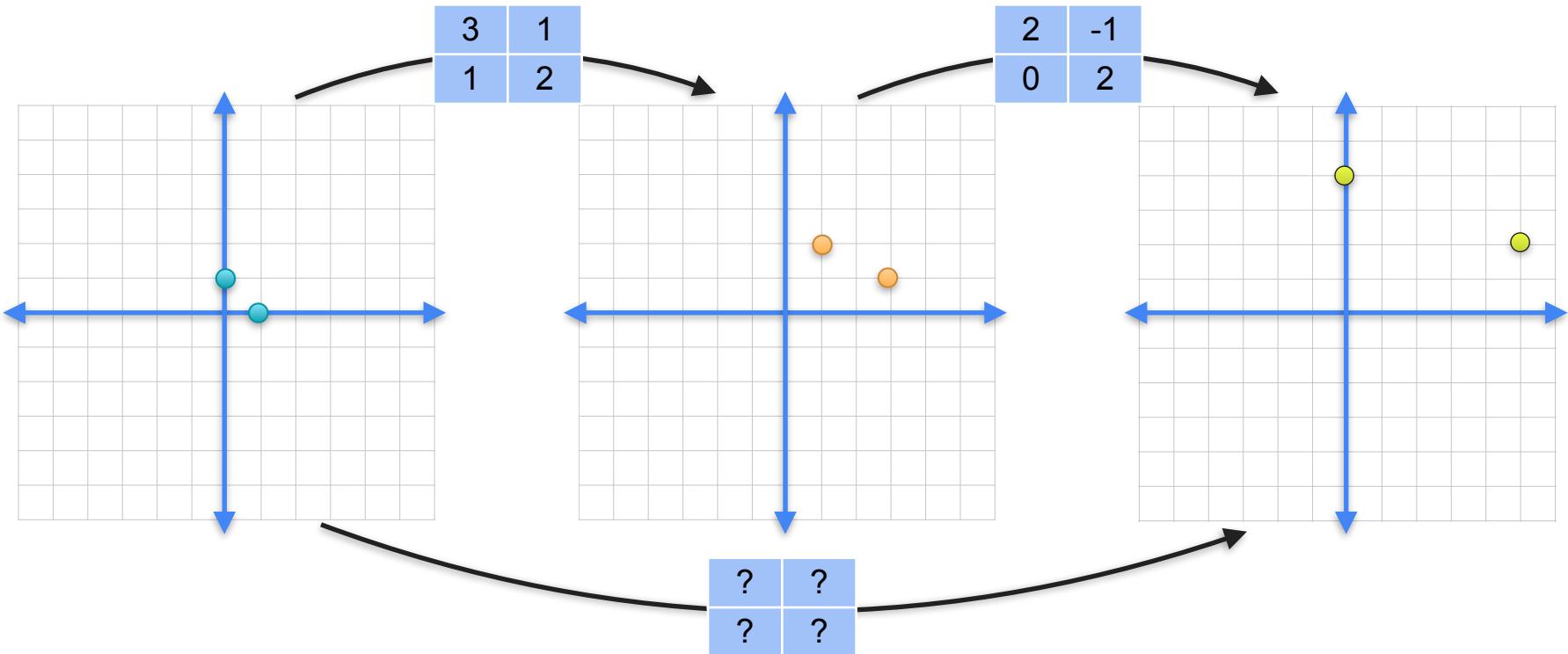
Combining linear transformations



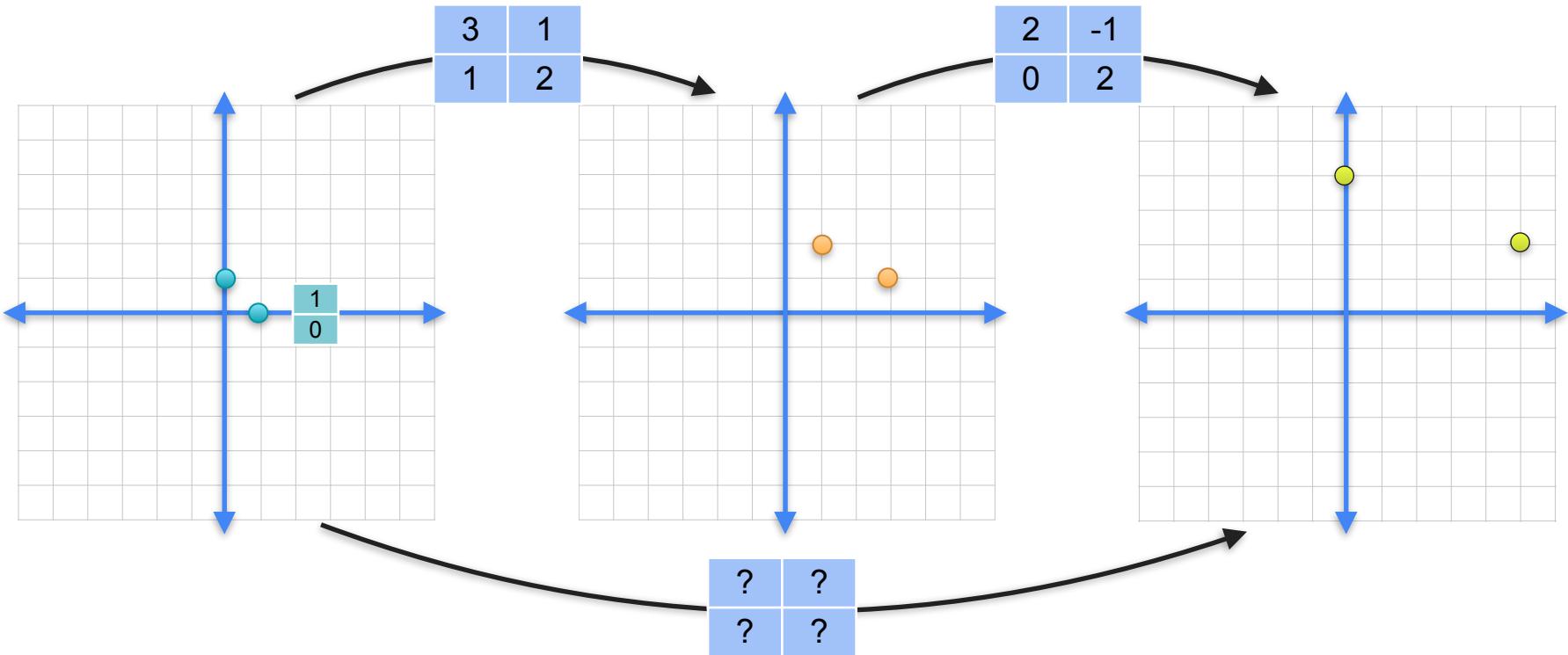
Combining linear transformations



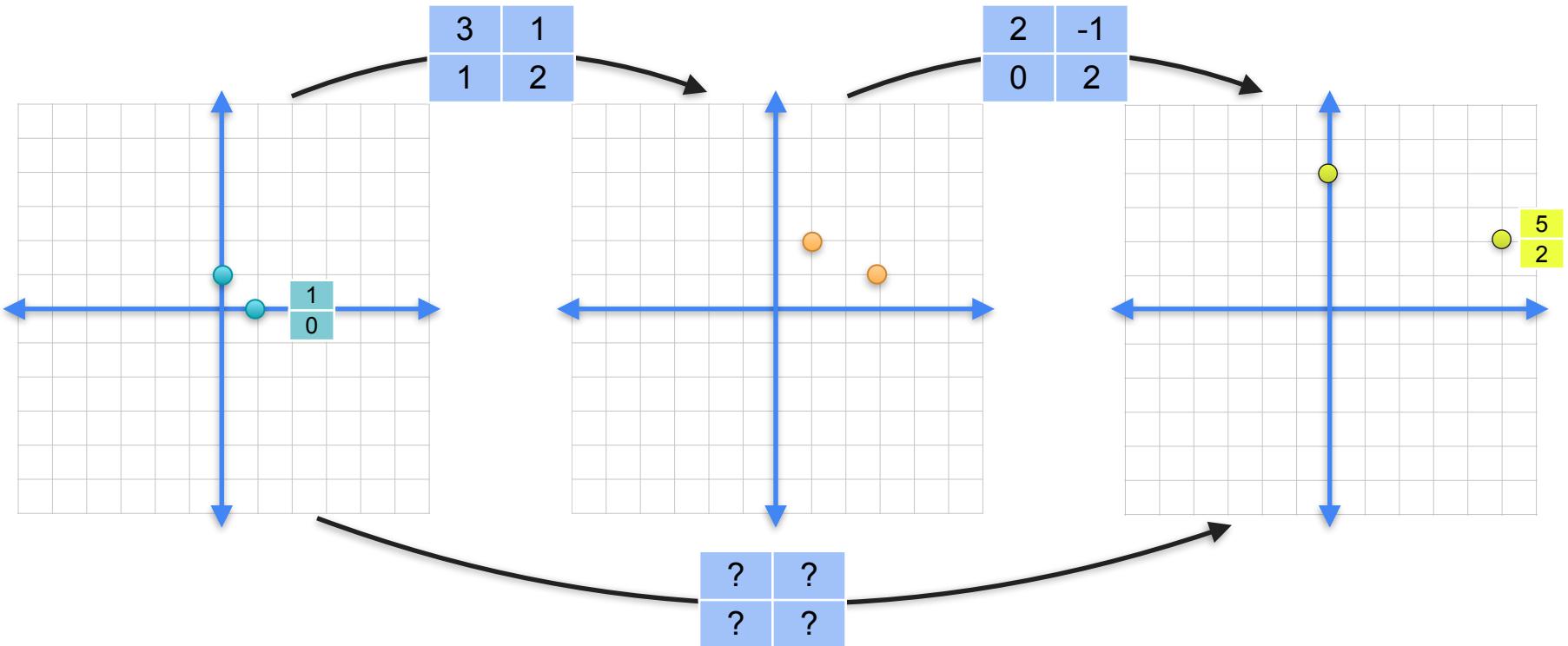
Combining linear transformations



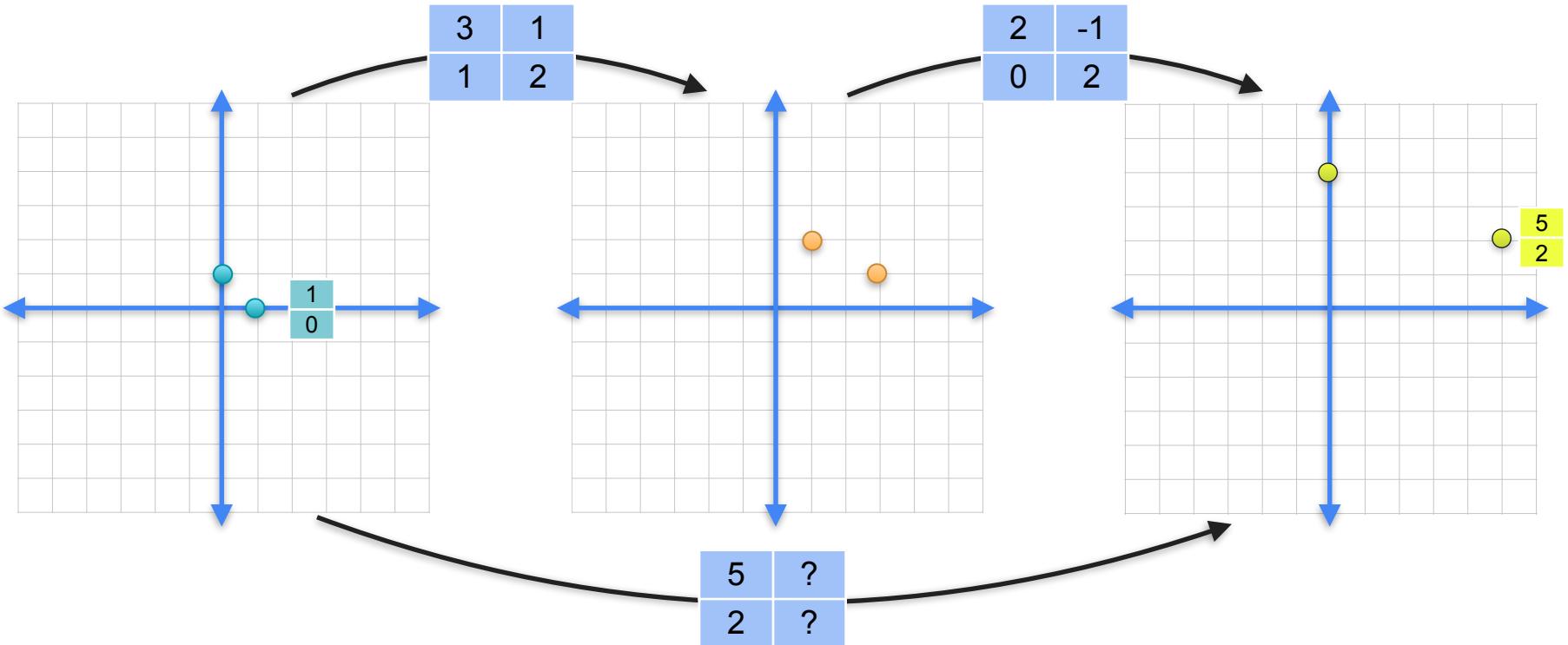
Combining linear transformations



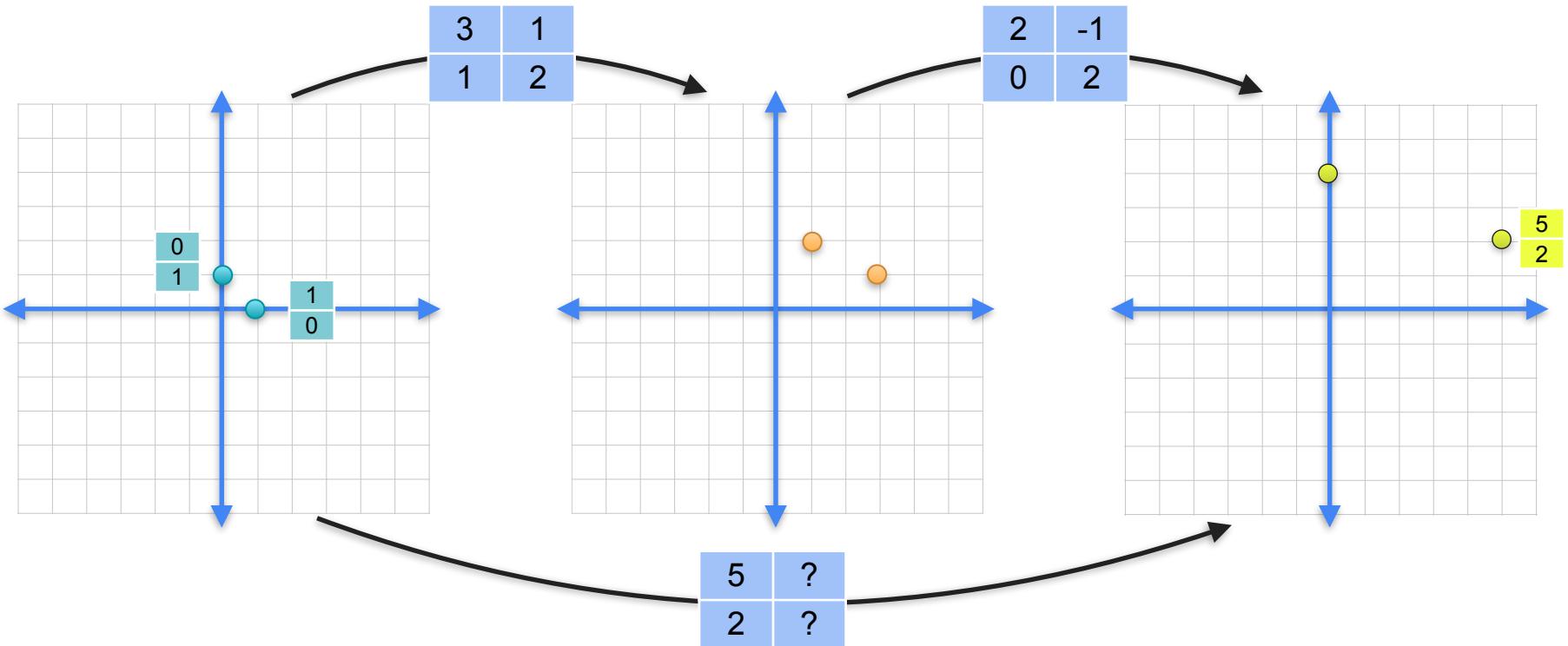
Combining linear transformations



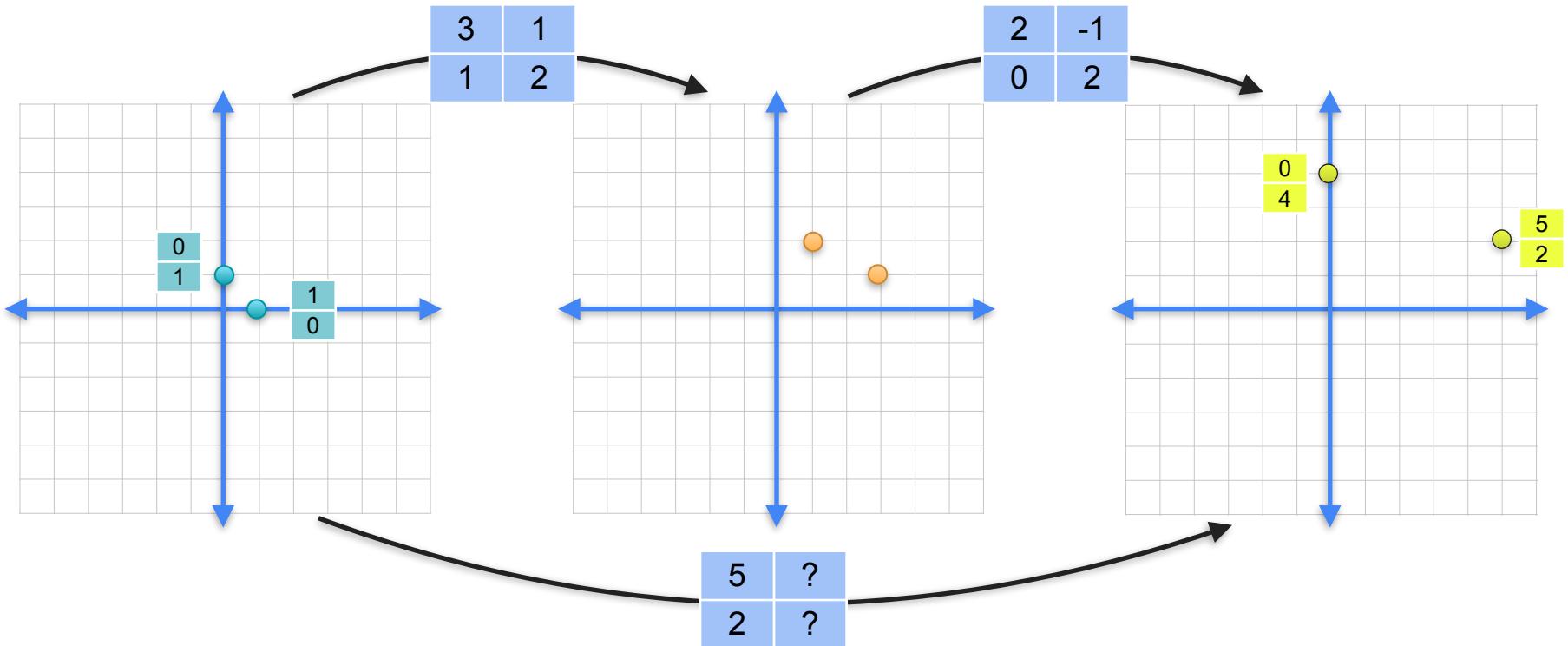
Combining linear transformations



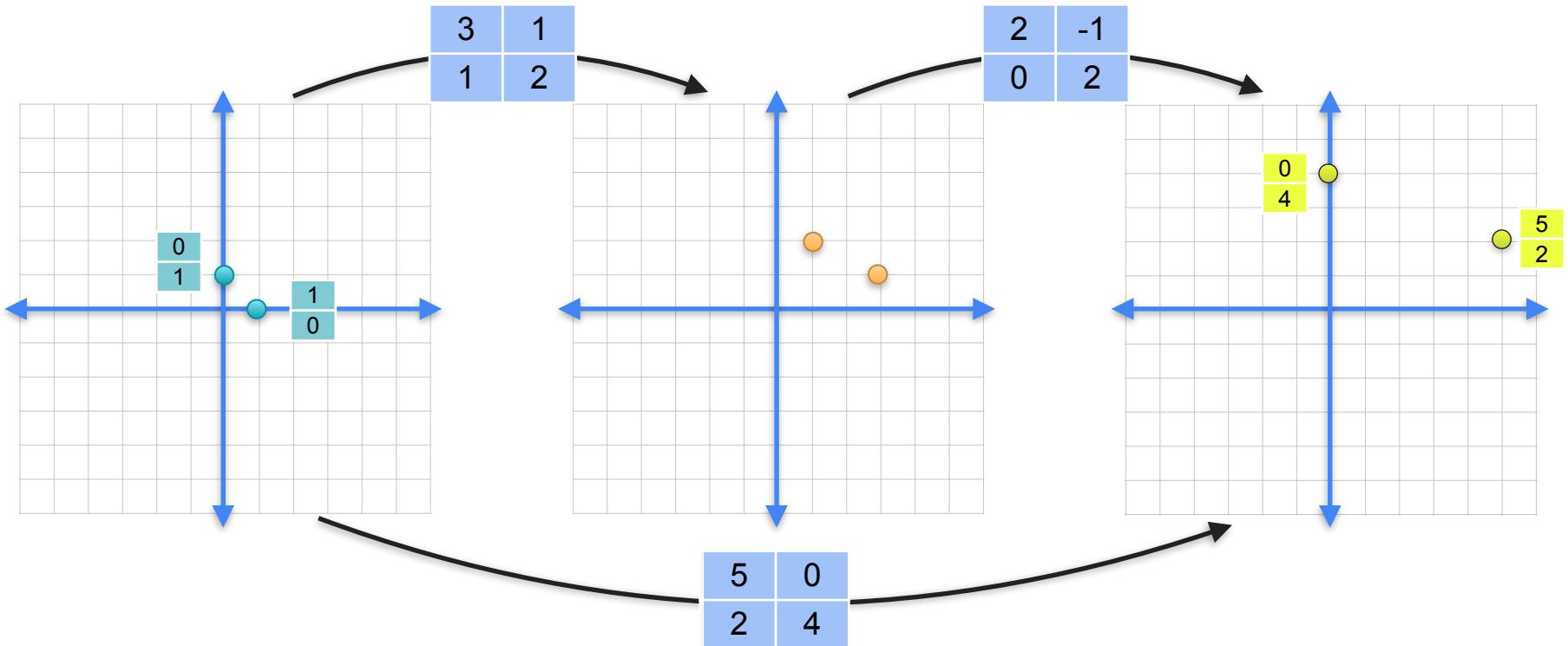
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$

Combining linear transformations

First
↓

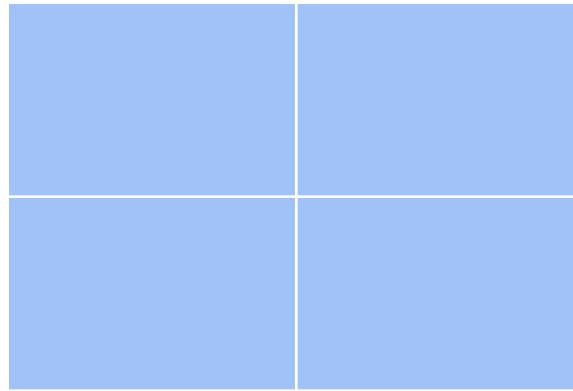
$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$

Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 5 & 0 \\ 0 & 4 \end{matrix} =$$



Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} & \begin{matrix} 3 & 1 \end{matrix} \\ \begin{matrix} 2 & -1 \end{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ & \begin{matrix} 3 & 1 \end{matrix} \\ \begin{matrix} 0 & 2 \end{matrix} & \begin{matrix} 1 & 2 \end{matrix} \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & & & \\ & \begin{matrix} 2 & -1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 2 \end{matrix} & \\ & \begin{matrix} 0 & 2 \\ 3 & 1 \end{matrix} & \begin{matrix} 1 \\ 2 \end{matrix} & \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 0 & 2 \end{matrix}$$

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix has cyan cells at positions (1,1), (1,2), and (2,1), and an orange cell at (2,2). The second matrix has orange cells at (1,1), (1,2), (2,1), and a cyan cell at (2,2). The result is a 2x2 matrix where the top-left cell is 5 (cyan), the top-right cell is 0 (orange), the bottom-left cell is 0 (cyan), and the bottom-right cell is 2 (orange). The result matrix is shown with its top row and left column highlighted in cyan, while the bottom row and right column are highlighted in orange.

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & \begin{matrix} 0 & 2 \\ 1 & 2 \end{matrix} \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$



DeepLearning.AI

Vectors and Linear Transformations

The identity matrix

The identity matrix

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

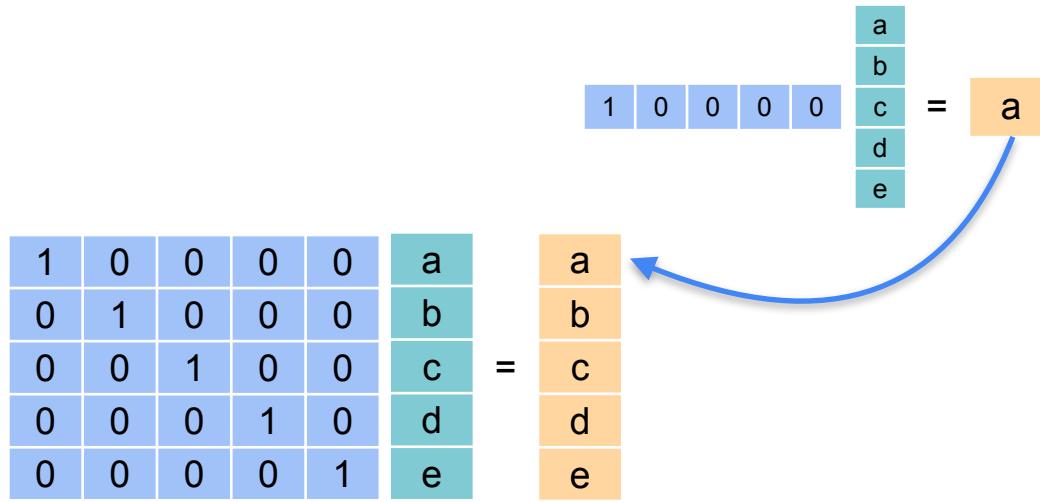
The identity matrix

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | a |
| 0 | 1 | 0 | 0 | 0 | b |
| 0 | 0 | 1 | 0 | 0 | c |
| 0 | 0 | 0 | 1 | 0 | d |
| 0 | 0 | 0 | 0 | 1 | e |

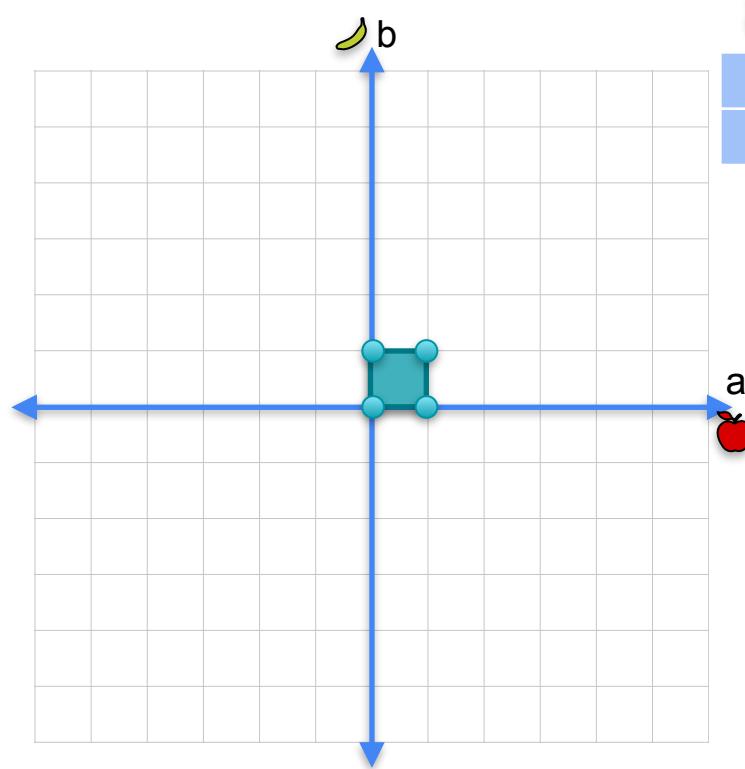
The identity matrix

$$\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & a \\ 0 & 1 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & 0 & 0 & c \\ 0 & 0 & 0 & 1 & 0 & d \\ 0 & 0 & 0 & 0 & 1 & e \end{array} = \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array}$$

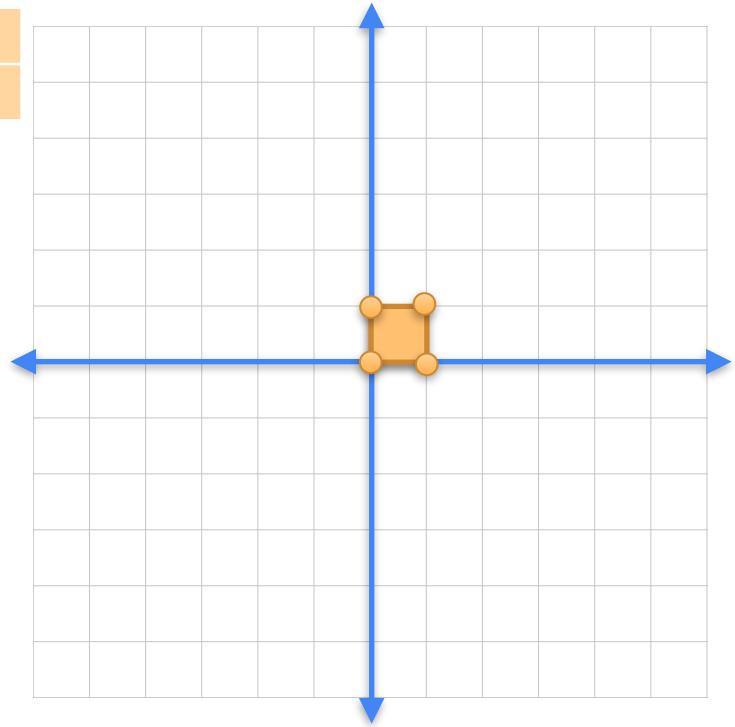
The identity matrix



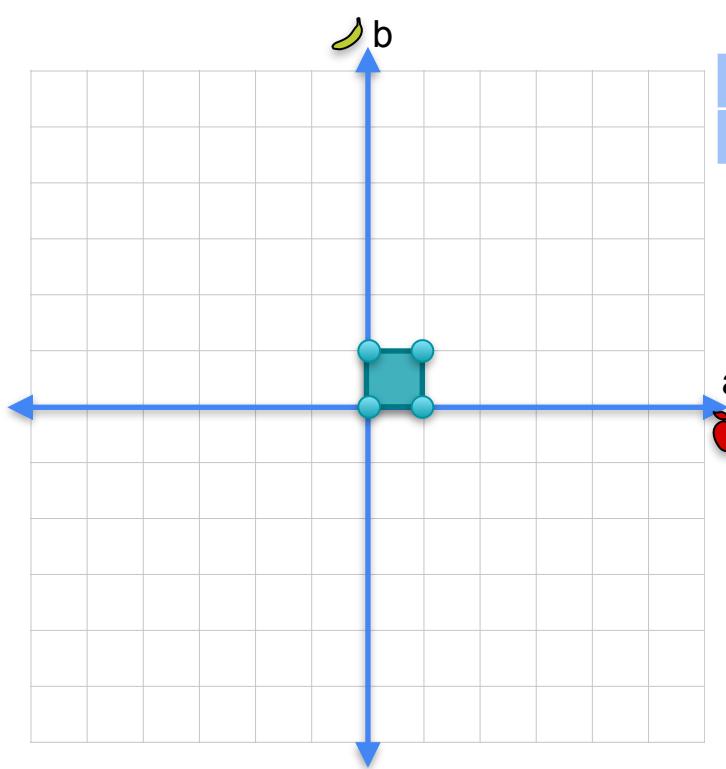
The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} x \\ y \end{matrix} \end{matrix} = \begin{matrix} \text{banana} & \text{apple} \\ \begin{matrix} x \\ y \end{matrix} & \end{matrix}$$

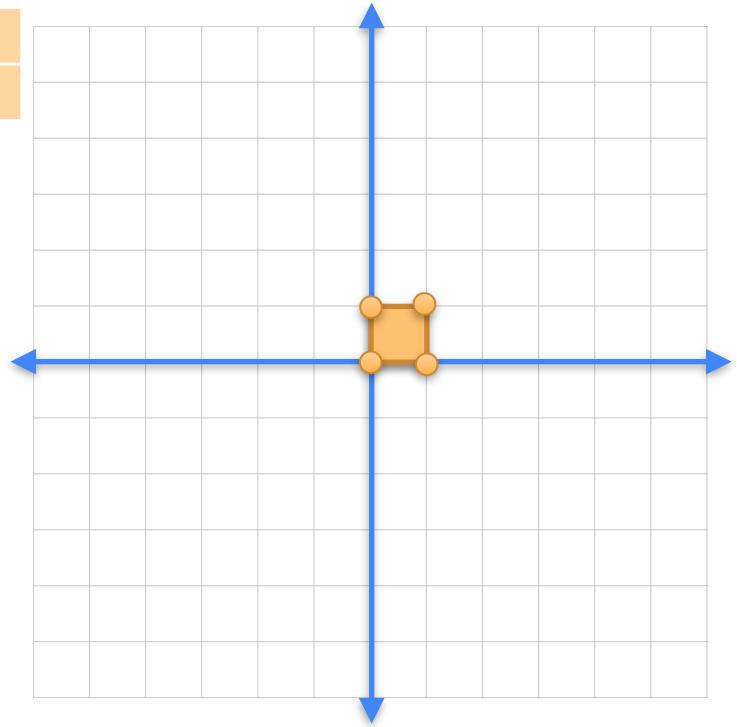


The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ 1 & 0 \\ 0 & 1 \end{matrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,0)$
 $(0,1) \rightarrow (0,1)$
 $(1,1) \rightarrow (1,1)$





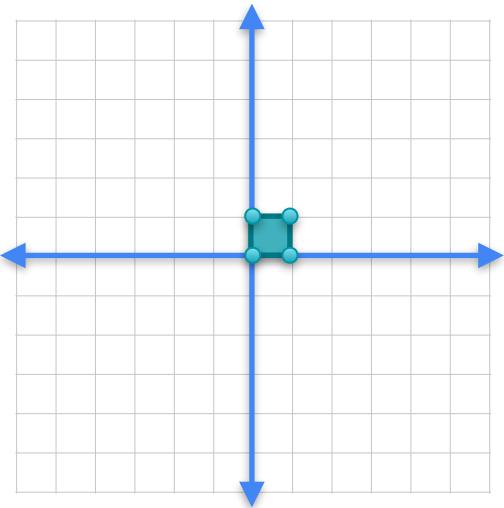
DeepLearning.AI

Vectors and Linear Transformations

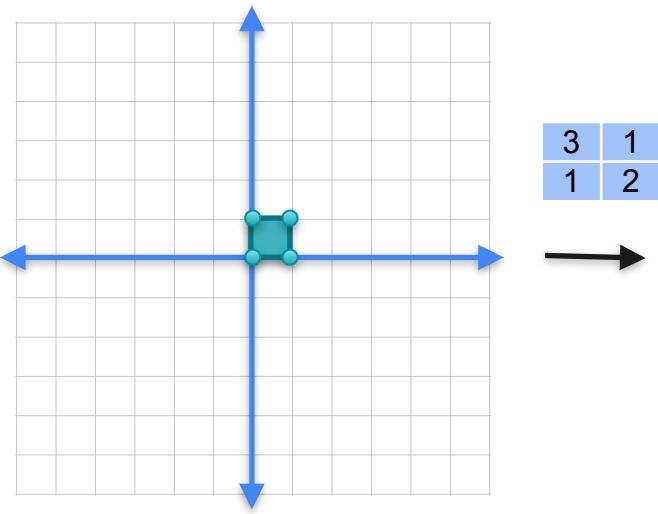
Matrix inverse

Matrix inverses

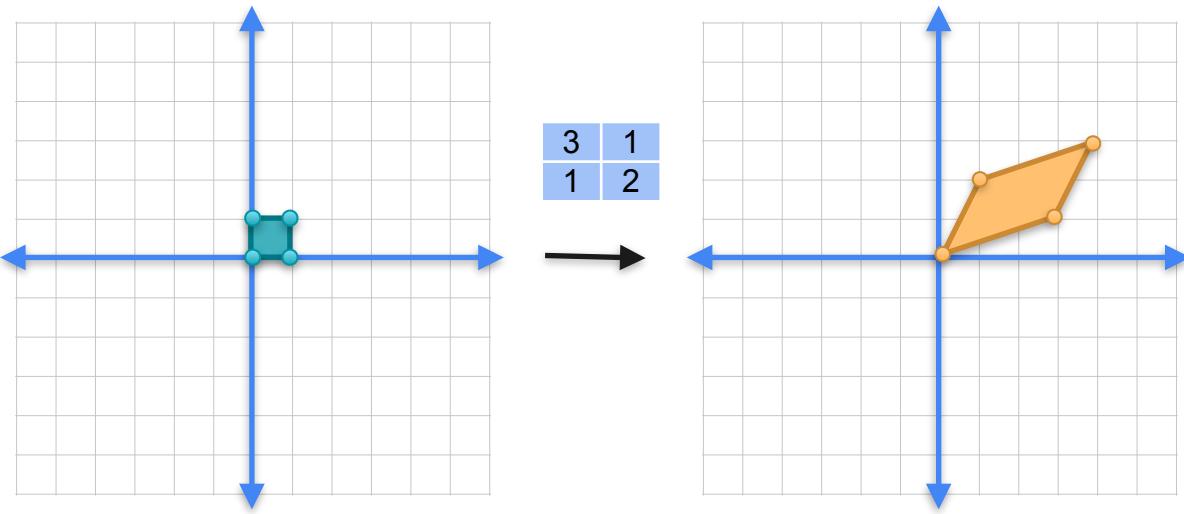
Matrix inverses



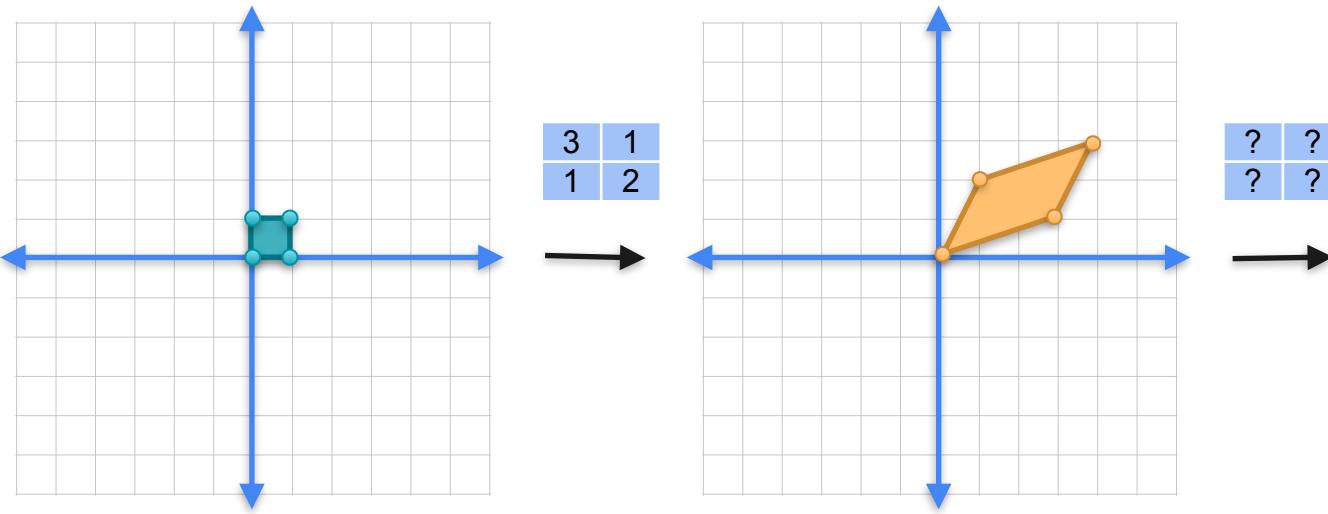
Matrix inverses



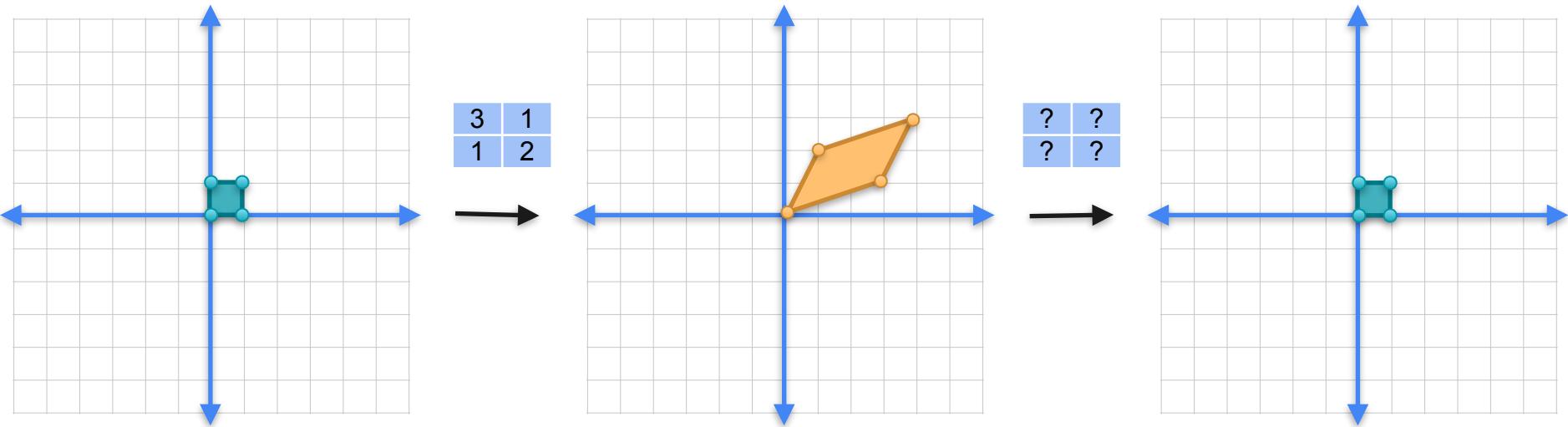
Matrix inverses



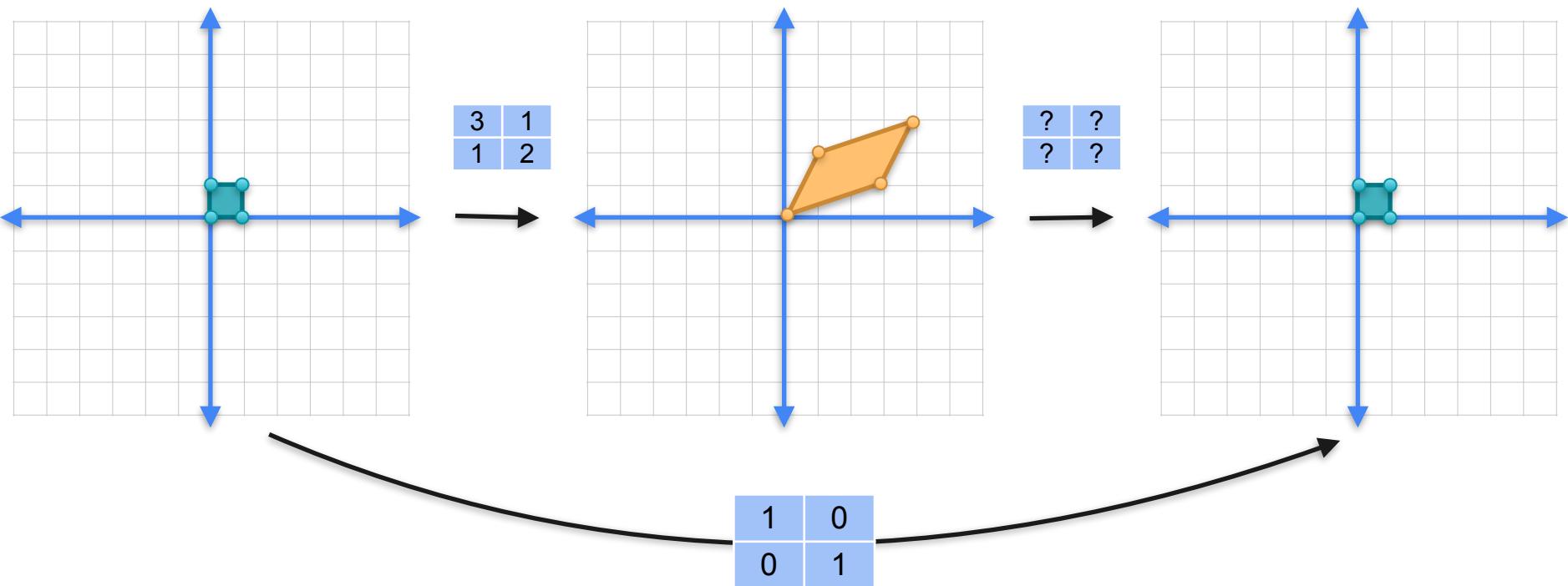
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

| | |
|---|---|
| a | b |
| c | d |

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$
$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{matrix}$$


How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1b = 1$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2b = 0$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3c + 1d = 0$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1c + 2d = 1$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1b = 1 \quad a = \frac{2}{5}$$
$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2b = 0 \quad b = -\frac{1}{5}$$
$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3c + 1d = 0 \quad c = -\frac{1}{5}$$
$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1c + 2d = 1 \quad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

| | |
|---|---|
| 5 | 2 |
| 1 | 2 |

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \\ 0 \end{matrix} \quad \bullet \quad 5a + 2c = 1$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \bullet \quad 5b + 2d = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \bullet \quad a + 2c = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \\ 0 \end{matrix} \quad \bullet \quad b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.



DeepLearning.AI

Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.25 & -0.25 \\ \hline -0.125 & 0.625 \\ \hline \end{array}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix

Non-singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix

Non-singular matrix

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Det = 8

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 5$$

Non-singular matrix
Invertible

$$\text{Det} = 8$$

Singular matrix
Non-invertible

$$\text{Det} = 0$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

$$\text{Det} = 5 \quad \text{Det} = 8$$

Non-zero determinants

Det = 0
Zero determinant



DeepLearning.AI

Vectors and Linear Transformations

**Neural networks and
matrices**

AI , ML , DL , RL



Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Scores:

Lottery: ____ points

Win: ____ points

Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Goal: Find the best points and threshold

Lottery: ____ point

Win: ____ point

Threshold: ____ points

Quiz: Natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Score | > 1.5? |
|-------|--------|
| 2 | Yes |
| 3 | Yes |
| 0 | No |
| 2 | Yes |
| 1 | No |
| 1 | No |
| 4 | Yes |
| 2 | Yes |
| 3 | Yes |

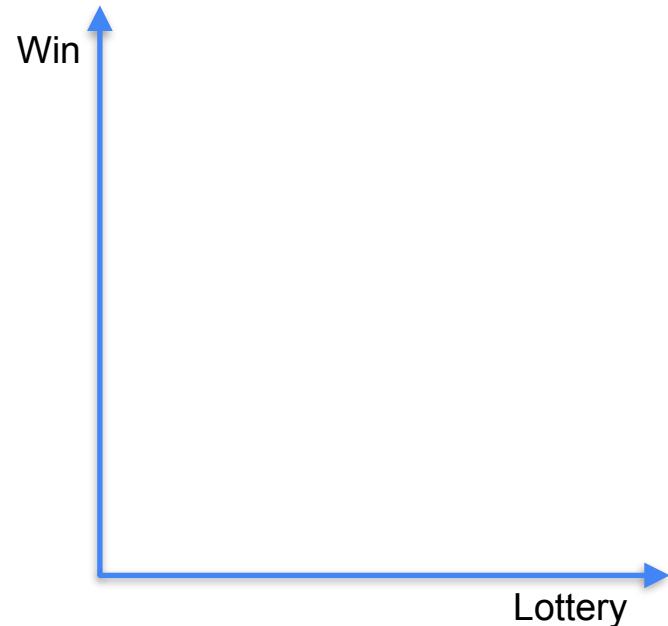
Solution:
Lottery: 1 point
Win: 1 point
Threshold: 1.5 points

Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

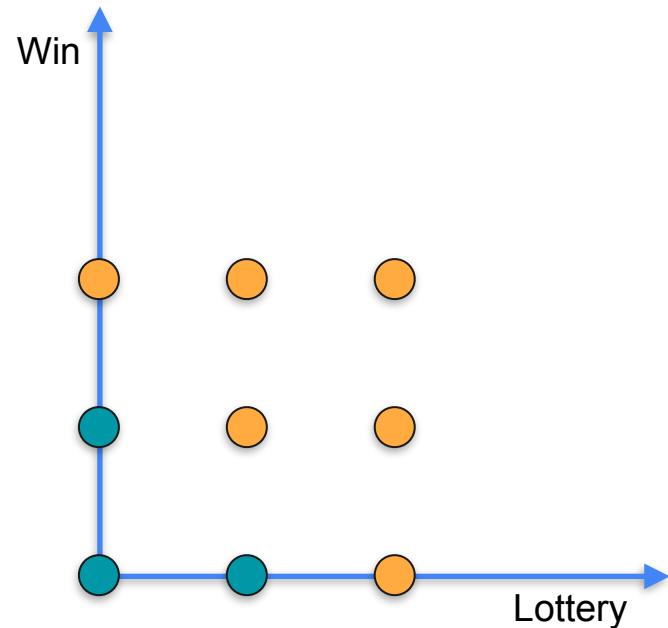
Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



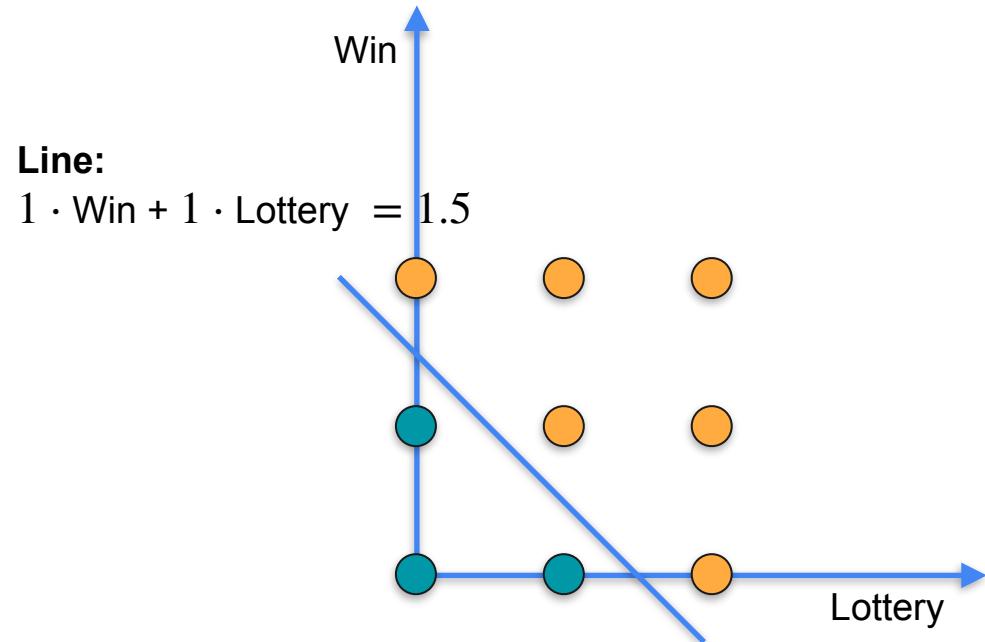
Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



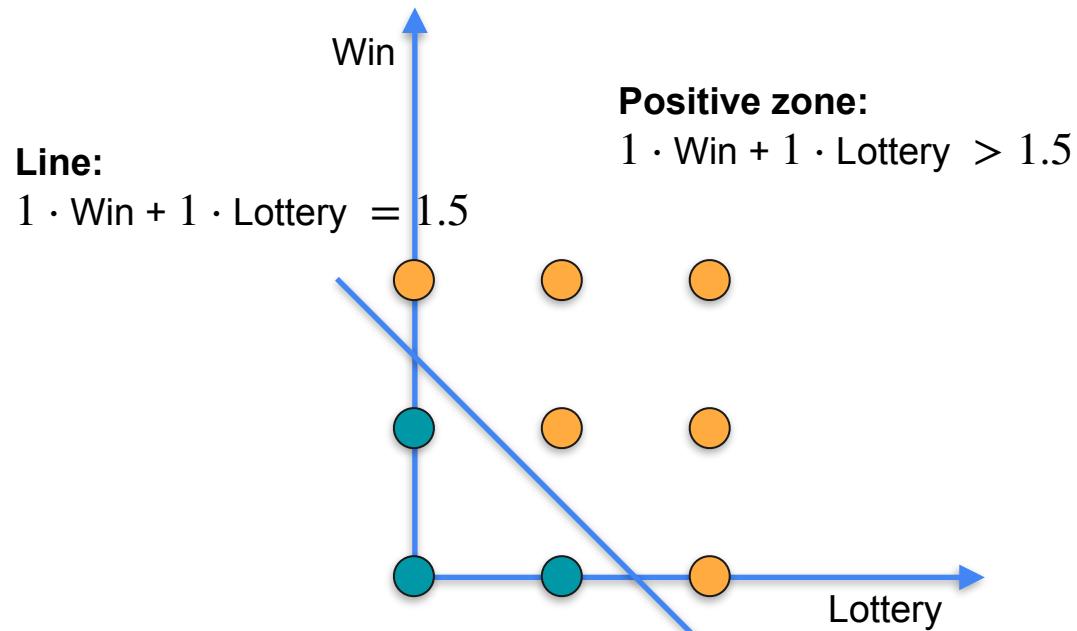
Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



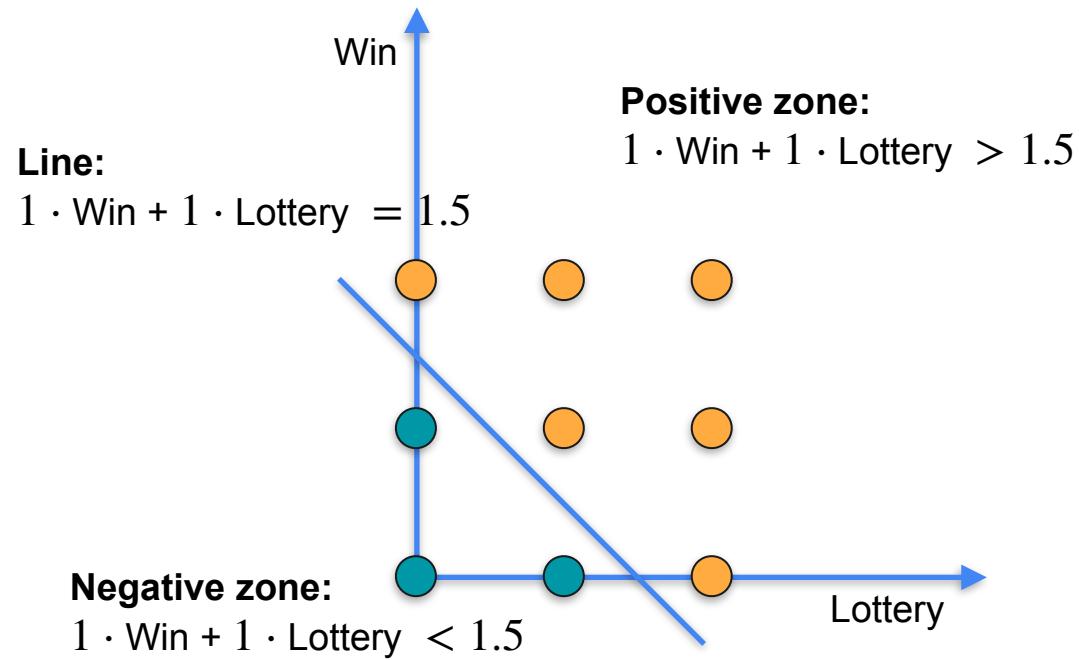
Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Graphical natural language processing

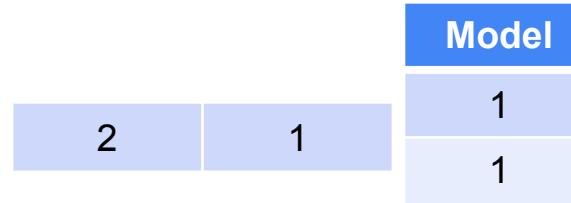
| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Model |
|-------|
| 1 |
| 1 |

Check: > 1.5?

Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Check: > 1.5?

Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Graphical natural language processing

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Dot product between vectors

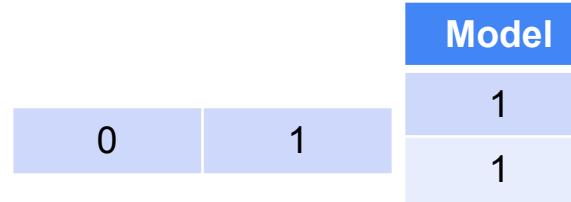
| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Model |
|-------|
| 1 |
| 1 |

Check: > 1.5?

Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |



Check: $> 1.5?$

Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Check: $> 1.5?$

$$0 \quad | \quad 1 = \begin{matrix} \text{Model} \\ 1 \\ 1 \end{matrix}$$

Dot product between vectors

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

$$\begin{matrix} & & \text{Model} \\ & 0 & 1 \\ \hline & 1 & 1 \end{matrix} = 1$$

Check: $> 1.5?$



Not spam

Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Model |
|-------|
| 1 |
| 1 |

Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

$$\begin{matrix} & \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} & = \begin{matrix} 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix} \end{matrix}$$

The diagram illustrates matrix multiplication. On the left, a 9x3 matrix is multiplied by a 3x9 matrix. The result is a 9x9 matrix labeled "Prod". The first row of the "Prod" matrix is [2, 3, 0]. The second row is [2, 1, 1]. The third row is [4, 2, 3]. The fourth row is [1, 1, 1]. The fifth row is [0, 2, 1]. The sixth row is [1, 1, 1]. The seventh row is [2, 1, 1]. The eighth row is [3, 2, 1]. The ninth row is [1, 1, 1].

Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

$$\begin{matrix} \text{Model} \\ \hline 1 \\ 1 \end{matrix} = \begin{matrix} \text{Prod} \\ \hline 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix}$$

Check: >1.5?



Matrix multiplication

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

$$\begin{matrix} \text{Model} \\ \hline 1 \\ 1 \end{matrix} = \begin{matrix} \text{Prod} \\ \hline 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix}$$

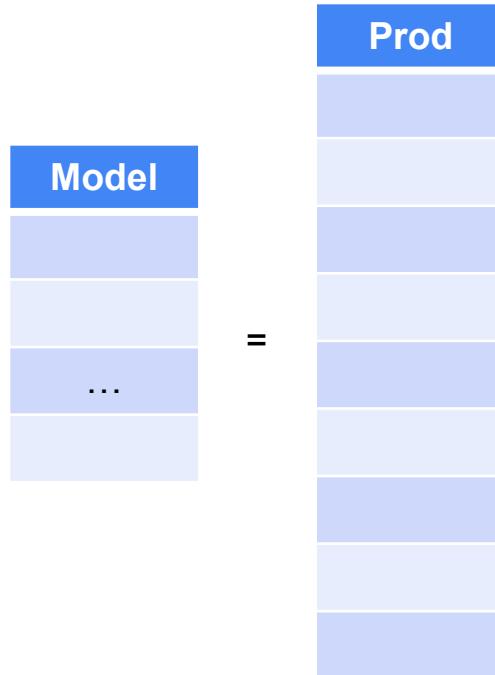
Check: >1.5?



| Check |
|-------|
| Yes |
| Yes |
| No |
| Yes |
| No |
| No |
| Yes |
| Yes |
| Yes |

Perceptrons

| Spam | Word1 | Word2 | ... | WordN |
|------|-------|-------|-----|-------|
| Yes | | | | |
| Yes | | | | |
| No | | | | |
| Yes | | | | |
| No | | | | |
| No | | | | |
| Yes | | | | |
| Yes | | | | |
| Yes | | | | |



Check:



| Check |
|-------|
| Yes |
| Yes |
| No |
| Yes |
| No |
| No |
| Yes |
| Yes |
| Yes |

Threshold and bias

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

| Model |
|-------|
| 1 |
| 1 |

Check: > 1.5?

Threshold and bias

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

| Model |
|-------|
| 1 |
| 1 |

Check: $> 1.5?$

Threshold and bias

| Spam | Lottery | Win |
|------|---------|-----|
| Yes | 1 | 1 |
| Yes | 2 | 1 |
| No | 0 | 0 |
| Yes | 0 | 2 |
| No | 0 | 1 |
| No | 1 | 0 |
| Yes | 2 | 2 |
| Yes | 2 | 0 |
| Yes | 1 | 2 |

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

| Model |
|-------|
| 1 |
| 1 |

Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes | 1 | 1 | 1 |
| Yes | 2 | 1 | 1 |
| No | 0 | 0 | 1 |
| Yes | 0 | 2 | 1 |
| No | 0 | 1 | 1 |
| No | 1 | 0 | 1 |
| Yes | 2 | 2 | 1 |
| Yes | 2 | 0 | 1 |
| Yes | 1 | 2 | 1 |

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

| Model |
|-------|
| 1 |
| 1 |

Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes | 1 | 1 | 1 |
| Yes | 2 | 1 | 1 |
| No | 0 | 0 | 1 |
| Yes | 0 | 2 | 1 |
| No | 0 | 1 | 1 |
| No | 1 | 0 | 1 |
| Yes | 2 | 2 | 1 |
| Yes | 2 | 0 | 1 |
| Yes | 1 | 2 | 1 |

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

| Model |
|-------|
| 1 |
| 1 |
| -1.5 |

Bias

Threshold and bias

| Spam | Lottery | Win | Bias |
|------|---------|-----|------|
| Yes | 1 | 1 | 1 |
| Yes | 2 | 1 | 1 |
| No | 0 | 0 | 1 |
| Yes | 0 | 2 | 1 |
| No | 0 | 1 | 1 |
| No | 1 | 0 | 1 |
| Yes | 2 | 2 | 1 |
| Yes | 2 | 0 | 1 |
| Yes | 1 | 2 | 1 |

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: > 0?

| Model |
|-------|
| 1 |
| 1 |
| -1.5 |

Bias

The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

| Model |
|-------|
| 1 |
| 1 |

The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

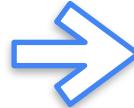
$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

Check: >1.5?



The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

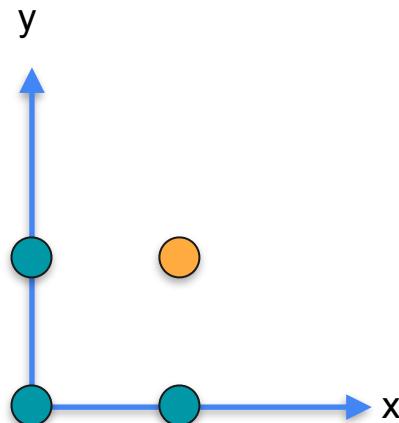
Check: $>1.5?$



| Check |
|-------|
| No |
| No |
| No |
| Yes |

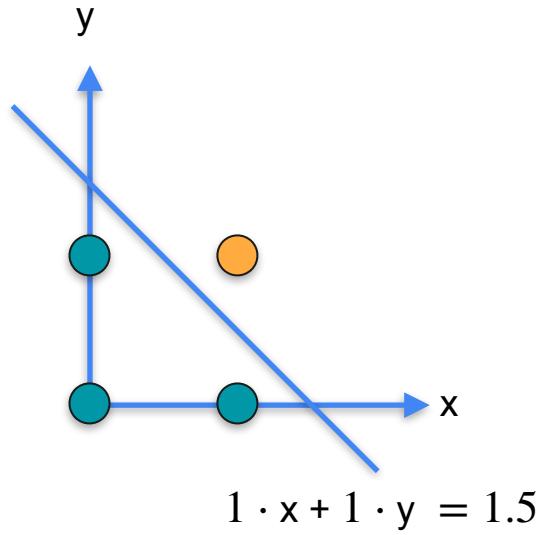
The AND operator

| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

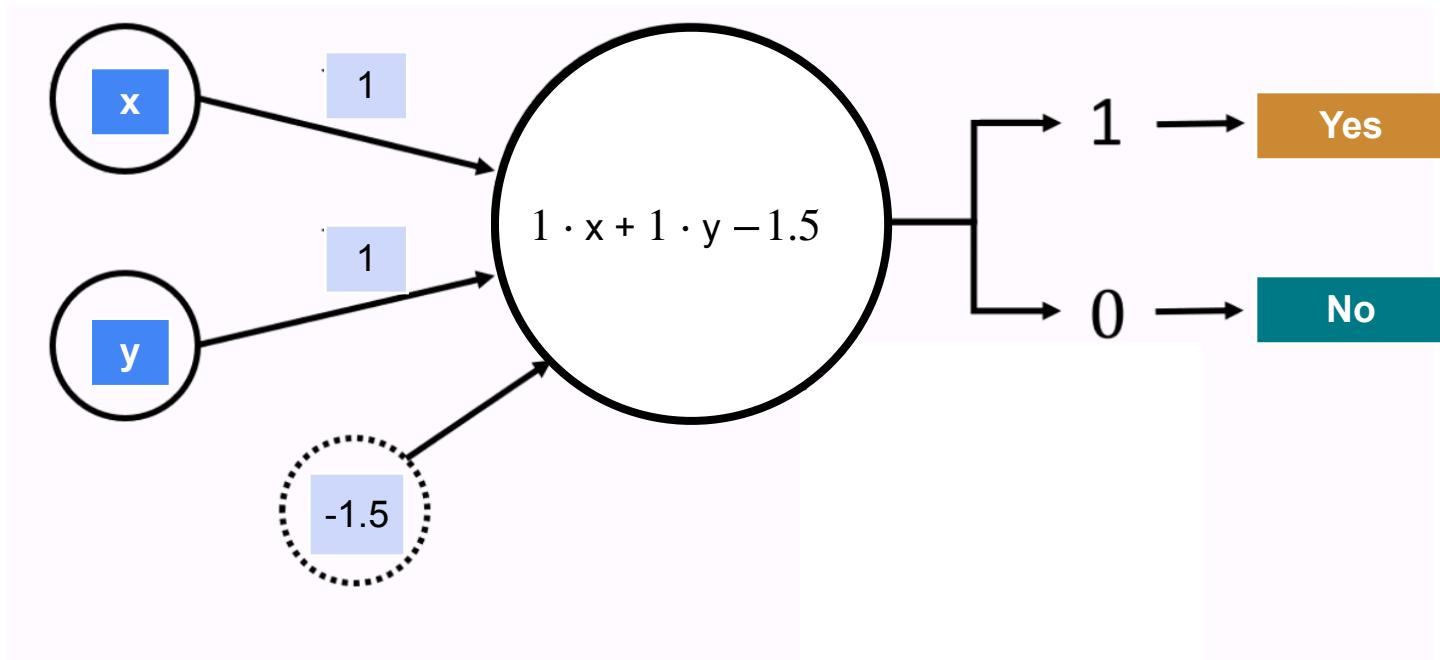


The AND operator

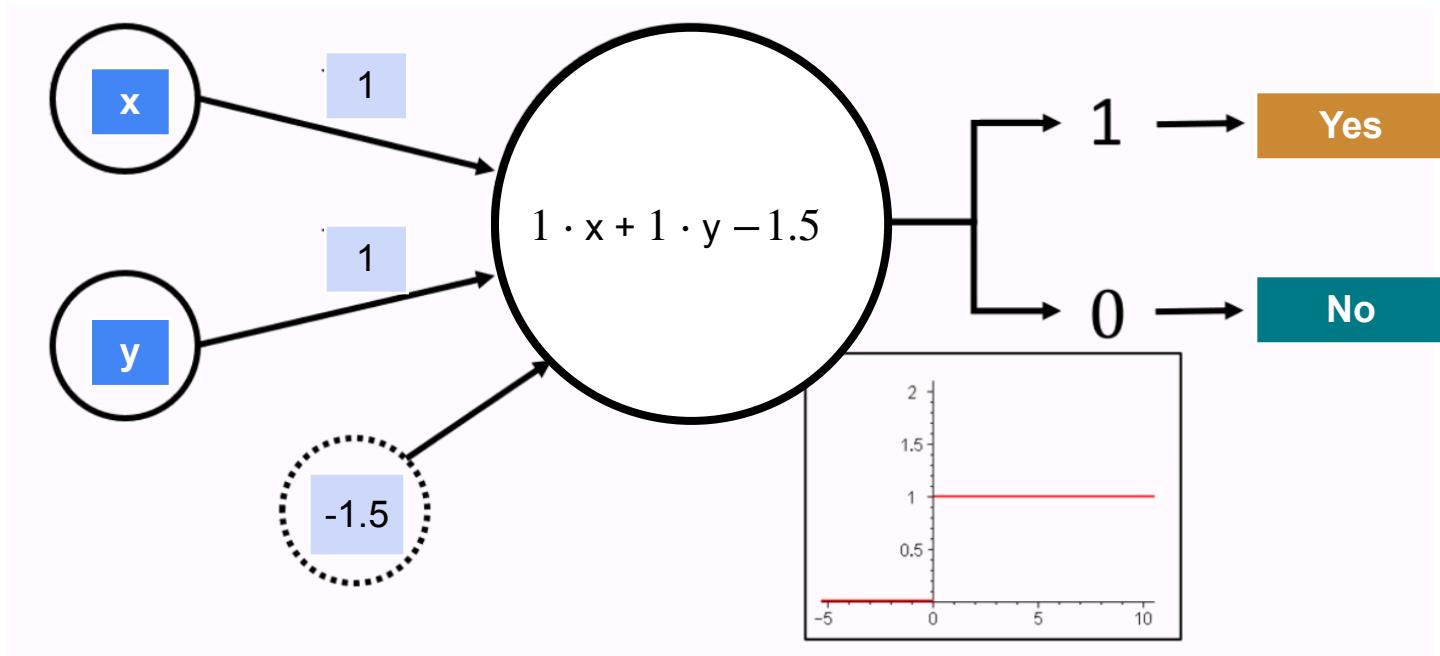
| AND | x | y |
|-----|---|---|
| No | 0 | 0 |
| No | 1 | 0 |
| No | 0 | 1 |
| Yes | 1 | 1 |

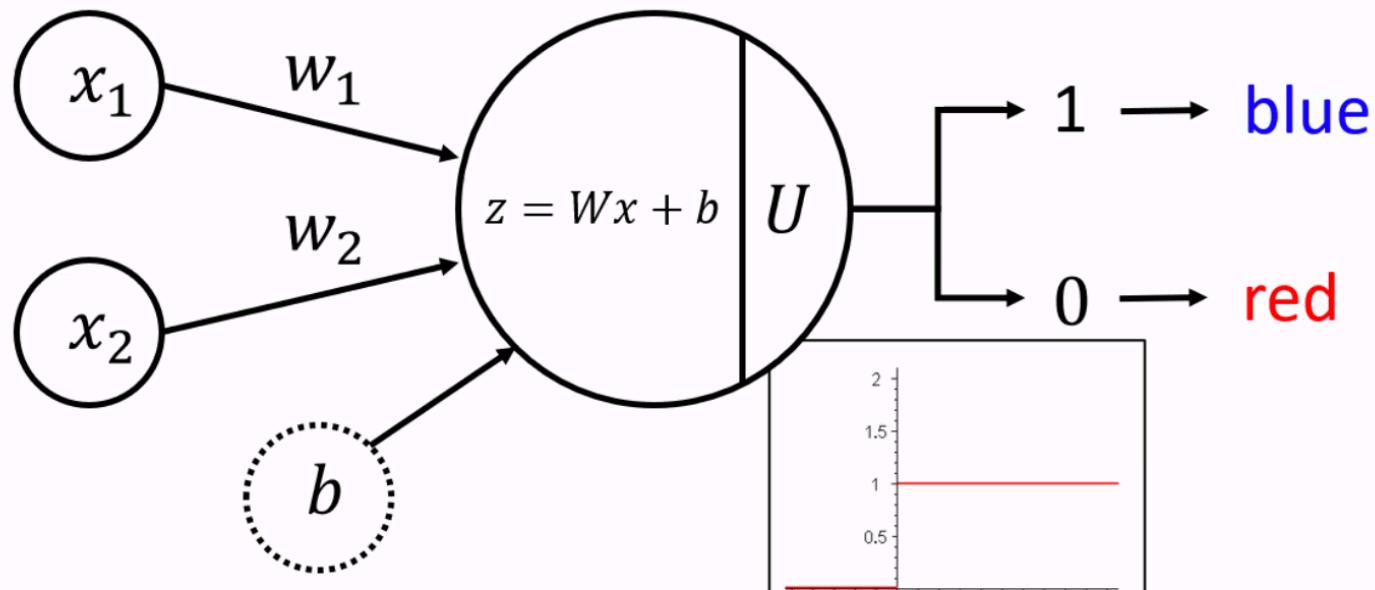


The perceptron



The perceptron







DeepLearning.AI

Vectors and Linear Transformations

Conclusion