CSC446/546 —OR II: Simulations
Assignment 2
Due date: Oct 26, 2023

This assignment walks you through various system modeling scenarios to conduct performance studies. Please use the MS Teams course channel to ask any questions related to this assignment.

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1. We compute the mean, variance, skewness and kurtosis of a given data set either theoretically if the distribution is known or from the raw data. Instead of using some raw data, we can also generate data (known as synthetic data) corresponding to a given distribution and use that as raw data. For this question, use Python (and associated packages such as scipy etc) to produce synthetic raw data of a given distribution and then use Python scipy/numpy to compute mean, variance, skewness and kurtosis. For example $x = np.random.normal(m1, m2, n)$ would produce Normal data with mean=$m1$, Standard deviation=$m2$ and number of samples=n. Compare it with theoretical values using two different sample sizes of n=10000 and 50000 to produce synthetic data. Do this for three distributions: a) Uniform (0,1), b) exponential(mean=1) and c) Normal (mean=1, standard deviation=1). Plot (visualize) the synthetic data using packages such as seaborn.

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2. A terminal at an airport has one queue to check-in for Terminal 1 passengers. After the check-in, Terminal 1 passengers join a security check queue along with Terminal 2 passengers as shown in Figure 1. The arrivals are Poisson distributed with rates λ_1 and λ_2 for Terminal 1 and 2 passengers respectively. The service times are exponentially distributed with service rates μ_1, μ_S for Terminal 1 check-in and Security-Check respectively. The arrival rates are 10 customers/hour for Terminal

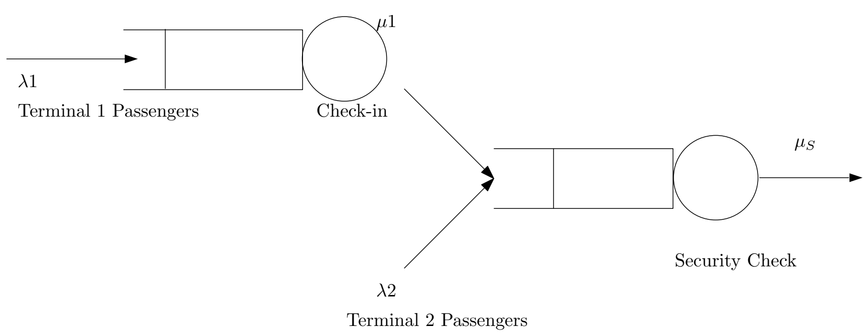

Figure 1: Queuing at an airport

1 passengers and 20 customers/hour for Terminal 2 passengers. The average service times at the Terminal 1 check-in is 2 minutes per customer. The Security-Check queue maintains an utilization of 80% all the time.

- What is the average service time at the Security-Check?
- What are the performance metrics (ρ, L, L_Q, w, w_Q) for Terminal 1 Check-in?

- What are the performance metrics (ρ, L, L_Q, w, w_Q) for Security-Check?
- What is the total average time (system time) spent by Terminal 1 passengers in the system?
- What is the total average number of Terminal 1 passengers in the system?
- What is the average time spent (System time) by Terminal 2 passengers in the system?

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3. Simulate the Security-Check queue in Problem 2, by re-using the sample code of single-queue and single-server queuing system (FIFO) that you extended in Assignment 1. Run the simulation 5 times with different seeds, with each run processing up to 50000 customers. Compute the average values across simulations for the performance metrics. Use a table to compare the simulation results with the theoretical results obtained from Problem 3. Please note that you have to collect the stats for the performance metrics (ρ, w, w_Q) in your code. You can use Little's law to obtain L and L_Q. Please submit this code along with your assignment.

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4. Study the effect of *pooling servers* (having multiple servers draw customers from a single queue, rather than each having its own queue) by comparing the performance measures (ρ, L, w, w_Q, L_Q) for two $M/M/1$ queues, each with arrival rate λ and service rate μ, to an $M/M/2$ queue with arrival rate 2λ and service rate μ for each server. Which one is better?

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5. A self-service car wash has 4 washing stalls. When in a stall, a customer may choose from among three options: rinse only; wash and rinse; and wash, rinse, and wax. Each option has a fixed time to complete: rinse only, 3 minutes; wash and rinse, 7 minutes; wash, rinse, and wax, 12 minutes. The owners have observed that 20% of customers rinse only; 70% wash and rinse; and 10% wash, rinse, and wax. There are no scheduled appointments, and customers arrive at a rate of about 34 cars per hour. There is room for only 3 cars to wait in the parking lot, so, currently, many customers are lost. The owners wants to know how much more business they will do if they add another stall. Adding a stall will take away one space in the parking lot. Develop a queuing model of the system. Estimate the rate at which customers will be lost in the current and proposed system. State any assumptions or approximations you make, noting that we only studied $M/M/c/N$.

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6. Patients arrive for a physical examination according to a Poisson process at the rate 1 per hour. The physical examination requires three stages, each one independently exponentially distributed with a service time of 15 minutes. A patient must go through all the three stages before the next patient is admitted to the treatment facility.

- What is the mean and variance of total examination time?
- Is this an exponential distribution?

- Compute the average number of delayed patients, L_Q for this system.
- Compute the total mean time a customer spends in the system.

(*Hint:* See problem 7).

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7. You are given a sequence of independent and identically distributed (IID) random variables $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$. Find the mean (expectation) and variance of $\mathbf{Y} = (\mathbf{X}_1 + \mathbf{X}_2 + \ldots + \mathbf{X}_n) = \sum_{i=1}^{n} \mathbf{X}_i$ given that mean of $\mathbf{X}_i = \eta$ and variance of $\mathbf{X}_i = \sigma^2_{\mathbf{X}_i} = \sigma^2$. Note that for two independent random variables X and Y, the expectation $E[XY] = E[X].E[Y]$ and $E[X+Y] = E[X] + E[Y]$. (**P.S.**:We need this result to solve Problem 7).

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8. ***(CSC 546 students only)*** Suppose a random variable X has the density function as shown in Figure 2.

a. Write the density equation $f(x)$ for this random variable.

b. Write the Cumulative Distribution Function (CDF) $F(x)$ of this random variable.

c. What is the value of c?

d. What is $E[X]$? What is the variance of X?

e. What will be the density of Y if we used a transformation $Y = 2X$?

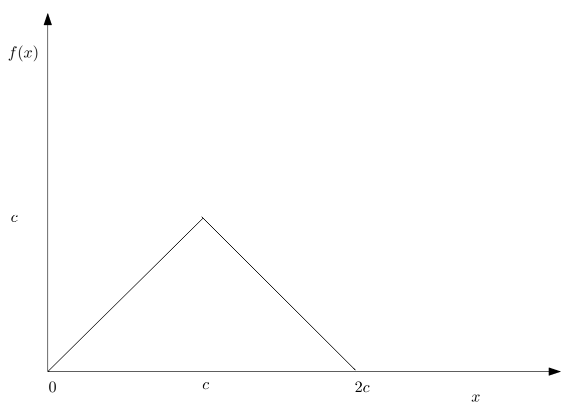

Figure 2: Density function of random variable X

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9. ***(CSC 546 students only)*** Compute the average number in the system (L) from the state probabilities $P(L=k) = P_k = (1-\rho)\rho^k$ of an $M/M/1$ queuing system. (*Hint:* Write $\sum_{k=0}^{\infty} k\rho^k$ as $\rho \frac{\partial}{\partial \rho} \sum_{k=0}^{\infty} \rho^k$. Note that $\sum_{k=0}^{\infty} \rho^k$ is a sum of geometric series).