CSC446/546 —Operations Research II: Simulations

Assignment 1

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Useful Tip: For a single channel queuing system, if λ is the average arrival rate and μ is the average service (departure) service rate, then the "utilization" of the system is defined as $\rho = \lambda/\mu$. Please note that $1/\lambda$ is the average inter-arrival (time between arrivals) time and $1/\mu$ is the average service time.

You can use any other alternate tools if you wish, but you have to code it. If so, please share your code with me. Please submit your solutions on Brightspace by the deadline. Thanks.

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1. (Simulation Table) A courier service receives shipping packages from both business and home customers with *equal probability*. Let us simulate the package arrivals by tossing a coin. Assume that a home customer is represented by the outcome "Heads" and a business customer by the outcome "Tails". The distribution of random time between package arrivals is specified as:

Toss	*Time between arrivals (in minutes)*
Head *(H)*	20
Tail *(T)*	10

Table 1: Distribution of time between Arrivals

Packages are processed by a single server in first-come-first-serve order. The processing time for each package is assumed to be *constant* at 5 minutes for each home customer and 10 minutes for each business customer. Construct a simulation table (similar to the one on Slide 26, Chapter 2 slides of class notes) for up to 20 customers. Assume the sequence for the 20 customers is: *HHTHTHTHHTHHHTH*. Start with simulation clock at *t=0* and assume that system starts empty (i.e, the first customer H arrives to the system after 10 mins as specified by the toss). Create the event list in the *chronological* order of events in a table format. Include customer number (type) and whether the event is an arrival or a departure as well as simulation clock when that event will be processed. You can write a small program in any language of your choice or construct it manually.

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2. (Performance Variables and Stats) For the Problem 1 above, compute the following:

a. Average service time as computed from your simulation table

b. Average inter-arrival time as computed from your simulation table

c. Server utilization as computed from your simulation table

d. Theoretical Server utilization (calculate from theoretical average inter-arrival time and service times). Is there any discrepancy between this and the simulated results? If so why?

e. Average time customer spends in the system

f. Assume that the packages received from home and business customers are not of equal probability, but say, with a probability p a package is received from the home customers and with a probability $1 - p$ it is from the business customers.

    1. What will be the average service time in terms of p?
    2. What are the bounds of p so that the system is stable?

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3. (Single Server Queue) This problem is to study the Java example code of the single server queue provided on the course Brightspace site. `Single_Server_Java.zip` is in the Code Samples folder. Please note that you may need to compile the java files in this file set as per your platform. The simulation is run by: "java Sim xxxx", where xxxx is the initial seed number. Indicate the seed numbers that you used. **Note that the mean service times and arrival times are just printed off from the input in the code provided. You need to modify the code so that you actually measure the actual inter-arrival and service times observed and then print the final averages for this exercise**.

a. What is the mean and type of distribution used for the inter-arrival time?

b. What is the mean and type of distribution used for the service time?

c. What is the theoretical server Utilization?

d. Run the simulation for 10000 customers with two different seeds (two runs) and note the outputs: mean inter-arrival time, mean service time, server utilization and mean response time

e. Run the simulation for 50000 customers with two different seeds (two runs) and note the outputs: mean inter-arrival time, mean service time, server utilization and mean response time.

f. What do you infer from the above runs?

g. Change the service distribution to *exponential* with the same mean and note the outputs: mean inter-arrival time, mean service time, server utilization and mean response time for 10000 customers with two different seeds

h. Run the above simulation with 5 different seeds (five runs) for 50000 customers and note the mean inter-arrival time, mean service time, server utilization and mean response time in the program output. Calculate the mean and variance of each of these observed variables across the runs. What do you infer from the results? Are they any good?

i. **(For CSc 546 students only)**This Java simulation code does not include any warm up period. Modify the code to start collecting statistics after a specified "warm up period". As discussed in the class, warm up period is used to establish a steady state behavior of the system. Instead of time, you can use a specified number of customers for the warm up period. Repeat the simulation listed just above (five runs of 5000 customers each) with two different sets of warm up periods: 1) warm up period ends after 100 customers; 2) warm up period ends after 1000 customers. What do you infer from these results?

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4. Consider the single-channel (single-server) queue (example 2.5, Slides 24 to 30 of chapter 2 simulation examples). Let us suppose that the service distribution is changed to uniform (all values equally likely to occur) between 1 and 6 minutes. What is the expected service time? What is the system utilization? Re-run the excel simulation with this service distribution and compare the results with the original example.

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5. (What if Scenario) Let us re-visit the Able-Baker call center problem (Slides 31 to 39 of Chapter 2 class notes). Change the policy to "Baker" on who gets the call when the servers are idle and when a new call arrives; i.e., Baker is chosen instead of Able.

a. Modify the posted spreadsheet of Call center problem to include this policy

b. Run one trail with 50 customers and report the performance metrics that you obtained through this simulation.

c. Run an experiment with 200 trails and note the average caller delay with the modified policy. Compare this by repeating this experiment with the original policy. Which policy is better and why? Justify your analysis.

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6. (What if Scenario) This problem looks at the effect of variance ***(what if scenarios)*** in the air-supply expedition:

a. Set σ_x = 650 meters and σ_y =300 meters in the spread sheet of target hitting example. Conduct a simulation of 200 trials. What was the average number of hits?

b. Repeat above with a simulation of 400 trials. What was the average number of hits?

c. Set $\sigma_x = 50$ meters and σ_y =250 meters in the spread sheet of target hitting example. Conduct a simulation of 200 trials. What was the average number of hits?

d. Set $\sigma_x = 50$ meters and σ_y =500 meters in the spread sheet of target hitting example. Conduct a simulation of 200 trials. What was the average number of hits?

e. Set $\sigma_x = 2\sigma_y$. What is the value of σ_x if the average number of hits is to be about 0.6 based on on experiment of 400 trials?

f. What do you infer from these simulations?

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7. (Parameter Estimation) This problem is to use simulation for "parameter estimation". Parameter estimation is the technique in which you conduct an experiment, observe a given variable and draw conclusions about the validity of that variable. Assume that you are given a weighted coin. When you toss the coin, the outcome is heads with probability p and tails with probability $1 - p$. Write a simple program (in any language or tool of your choice) to estimate the parameter p. That is, conduct the experiment and observe the number of heads. Then p is estimated as $\hat{p}$=*number of heads observed/number of tosses*. Run the simulation until it simulates 10000 tosses.

a. When $p = 0.3$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

b. When $p = 0.1$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

c. when $p = 0.001$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

d. when $p = 0.0001$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

e. What can you conclude from this experiment? What is your suggestion to reduce the normalized estimation error? Please try your solution to see if it works well.

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8. (Queue behavior) **(For CSc 546 students only)** The single-channel Java simulation code that is provided to you uses FIFO (First-In-First-Out) queuing structure to en-queue and de-queue customers. The purpose of this problem is to study performance of the system if the queue is changed to a LIFO (Last-In-First-Out). Modify the code provided to achieve the LIFO order to service customers. Compare the performance of LIFO order of service with the FIFO order by running *appropriate* simulations.