CSC446/546 —OR II: Simulations
Assignment 3
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Due date: November 16, 2023

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1. This problem is to test the random number generator used in either java, python or C programs. Use standard libraries in either of these languages and produce ten random numbers. Also try to find what method they use for Random Number generation. Then use Kolmogorv-Smirnov method with a 0.05 level of significance (α) to test whether the random number generator passes the uniformity test or not by generating ten numbers from your program. What can you conclude from this? Generate 2000 numbers next and perform a uniformity test by conducting a Chi-Square test for the same level of significance. What are your test outcomes if we changed the level of significance to 0.01 for both of these tests?

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2. A random number generator uses the following algorithm to produce random numbers $X_i, i = 0, 1, 2, \ldots$. Write a program to generate 1000 (=N) numbers using this algorithm and test the generator for uniformity (using Chi-square test) for 0.05 level of significance (α). The test uses $n = 10$ intervals. Discuss any logical problems you might have encountered using this algorithm and suggest any fixes for it. Discuss whether this algorithm is a viable random number generator or not. The steps in the algorithm are:

Initialization:

Set X_0=a real number between 0 and 1. Use at least 8 decimal places;

Set X_1= another real number between 0 and 1. Use at least 8 decimal places;

while $(i \leq N)\{$

$X_i = X_{i-1} + X_{i-2};$

if $X_i \geq 1, \{X_i = X_i - 1.0\};$

i++;}

Submit your code on Brightspace.

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3. Consider the multiplicative congruential generator under the following circumstances:

a. $a = 11, m = 16, X_0 = 7$

b. $a = 11, m = 16, X_0 = 8$

c. $a = 7, m = 16, X_0 = 7$

d. $a = 7, m = 16, X_0 = 8$

Generate enough values to complete a cycle and analyze each case. What inferences can be drawn? Is the maximum period achieved?

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4. Develop a random-variate generator for a random variable X with the pdf

$$f(x) = \begin{cases} e^{2x}, & -\infty < x \leq 0 \\ e^{-2x}, & 0 < x < \infty \end{cases}$$

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5. Begining with the first number, test for the auto correlation between every third number for the following sequence of numbers (use $\alpha = 0.05$): 0.594, 0.928, 0.515, 0.055, 0.507, 0.351, 0.262, 0.797, 0.788, 0.442, 0.097, 0.798, 0.227, 0.127, 0.474, 0.825, 0.007, 0.182, 0.929, 0.852.

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6. Write a program to generate exponential random variate with $\lambda = 1$ using Python. Then generate $N = 1000$ samples for the exponential random variate (X) that you developed in the problem above (Problem 6). Our goal is to see the behavior of the density function for this random variate. For this purpose, you will produce a histogram by observing the number of occurences for X in an interval. Use 11 bins in increments of 0.5 from 0 up to a max value of 5.5(i.e., [0,0.5),[0,5,1.0),...,[5.0,5.5)) to keep these counts. Count all the $X \geq 5$ into the last bin [5.0, 5,5). Normalize the counts to the number of samples (N). This gives the relative frequency and would be used as an approximation to the actual exponential density function $f(x) = exp(-x) = e^{-x}$ as $\lambda = 1$. Compare the developed histogram with actual density at the bin midpoints. Build the cumulative distributon from the histogram data and compare it with exponential CDF. *Note: As discussed in the class, the histograms have to be scaled appropriately in order for them to represent as density functions*