

Preliminary Report

Christian Ulfert

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1 Overview of the Project

In this project I will try to develop a 4 dimensional variant of Tetris. The game will try to capture the essence of the original Tetris gameplay as well as the endless replayability while enhancing it with innovative new features. To find a balance in between the increased complexity provided by moving into higher dimensional spaces and the actual playability and enjoyability of the game, I will be limiting the 4 dimensional aspect to the Tetris pieces only. The player will see a 3 dimensional playing field that pieces fall into. Once they start rotating the 3 dimensional pieces they will notice that they are actually 3 dimensional slices of 4 dimensional collections of hypercubes. This will essentially just increase the number of possible rotations of the pieces and remove the rotational symmetry that is present in classic Tetris. The goal is to create a fun but complex game that lends itself to endless replayability as well as scientific exploration, just as Tetris did.

2 Project Template Used

CM3030 Games Development Project Idea Title 1: Arcade Game

3 Existing work

3.1 Tetris

Introduction Tetris is one of the most popular games in the history of computer gaming. Developed in 1984 by Alexey Pajitnov, as a testbed for emerging computing hardware it has transcended its initial purpose and become a cultural icon. After initial struggles to leave the UDSSR the big break came when Nintendo bundled it with the original Gameboy in 1989. Through capturing a whole generation of newly computer-affine people it has cemented its place in gaming history. The total estimated sales of Tetris are over 500 million copies, most of them in digital form, which only came reality 3 decades after the initial game was developed. It has also captured the imagination of countless scientists and mathematicians, who have studied the game from a variety of perspectives (computer science, psychology, medicine, mathematics...).

Gameplay The staple of tetris is the tetrominoe. A collection of 4 squares arranged in different shapes. The tetrominoes fall from the top of the screen into a 10x20 playing field, they can not be stopped. The player rotates and moves these shapes to arrange them at the bottom of the playing field in a way such they cover a whole row. A full row will be removed from the playing field, giving the player more room to place blocks, and changing the arrangement of the blocks, potentially opening new positions for the next blocks. In addition the player receives a score for each row removed. The score is higher for removing multiple rows at once. The game ends when the playing field is full and no more blocks can be placed. Tetrominoes are a special case of polyominoes ($A(4)$), which are shapes made up of squares. This is another interesting field of study, and a possible route to increase complexity in tetris, as the number of possible shapes increases dramatically as discussed in Barequet et al. [2] and visualized in Figure ???. This path however will not be followed in this project.

Mathematics There are few computer games whose mathematical properties have been researched to the same degree as Tetris. The following section will give a short but incomplete overview.

Tiling is a fundamental problem in mathematics. It deals with the problem of covering a plane with a set of geometric shapes. While this relates directly to the gameplay, it has also found interest in scientific research such as robotics [17] or neuroscience [9]. While these articles do not require the existence of Tetris, it has certainly sparked interest in the domain and helped with visualizing and understanding the problem.

Complexity - One of the most renowned scientific papers in game-related computer science is a proof by Demaine et al. that Tetris is NP-complete [8]. Being NP-complete means that Tetris is at least as hard as the hardest problems in NP, which are the problems that can be solved in polynomial time by a non-deterministic Turing machine. This means Tetris is one of the hardest problems in computer science.

While all this does not influence the development of our 4D Tetris variant, it does inspire how far such a simple game can grow. And there is a hope that a variant in a more complex space will entice similar research.

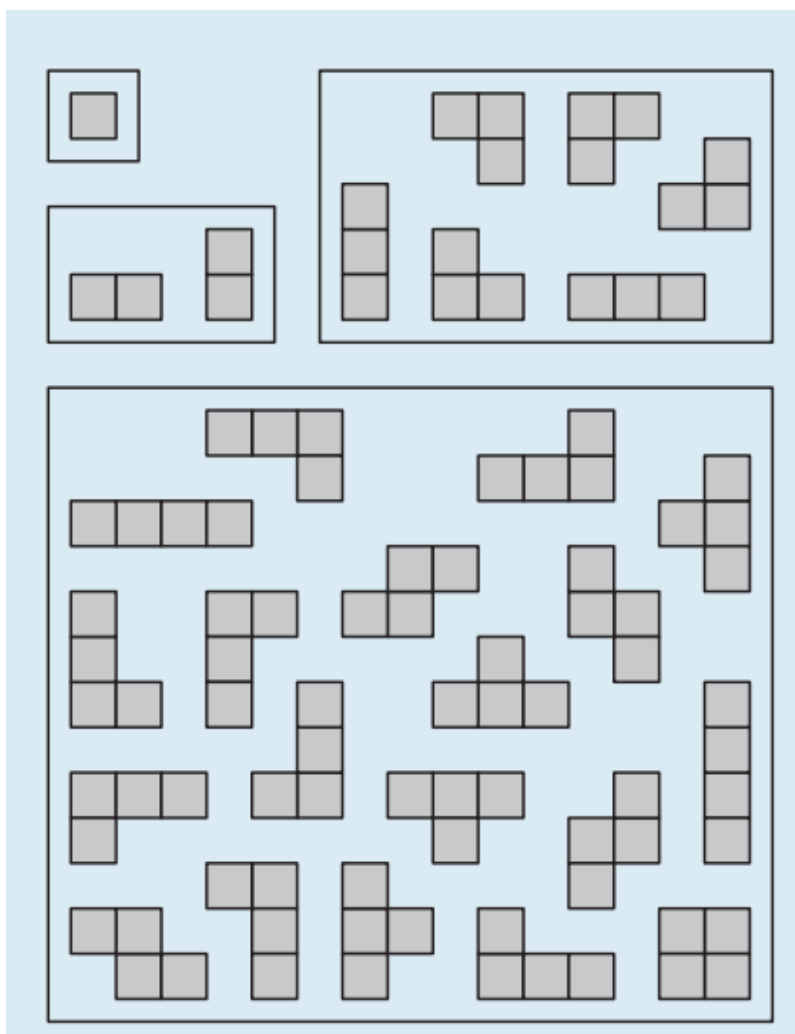


Figure 1: The single monomino ($A(1) = 1$), the two dominoes ($A(2) = 2$), the $A(3) = 6$ triominoes, and the $A(4) = 19$ tetrominoes (Tetris pieces) [2]

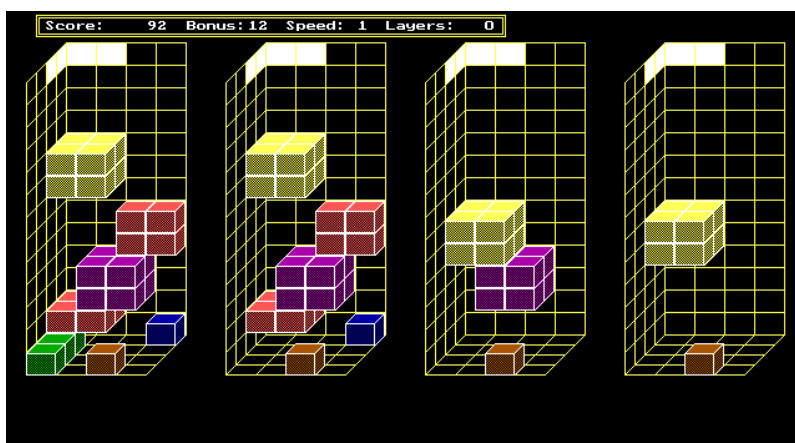


Figure 2: The 4 playing fields of Frac 4D. Image from myabandonware.com

User Interface Part of the success of Tetris is its accessibility. There is no need to explain anything. The main menu features a button named "Start" and the game is controlled. The game screen shows the playing field and the rest of the screen is used to display the next block, the score, the level and the lines cleared. There is no explanation needed. This is a feature that needs to be replicated to make the game accessible to a broad audience. The difficulty will be to translate the increased complexity of this project into an accessible product.

Conclusion Tetris was a pivotal work. There is a reason it is displayed in the Museum of Modern Art in New York. It has inspired so many different fields and captured millions of players. This all needs to be kept in mind during development as I want to capture a similar feeling of simplicity and complexity in our game.

3.2 Frac 4D

This is a game developed in the early 90's and never completed. It featured 4 distinct, but connected 3 dimensional playing fields. Pieces can be rotated and due to their underlying 4 dimensional structure this will reveal different configurations, they can exist in some of the fields but not in others, which can lead to unexpected 'collisions'. The game was apparently very hard to play and in the end failed to provide an easy access into 4 dimensions.

Conclusion This game was probably the first 4D adaption of Tetris, and while featuring interesting concepts failed to succeed. There are however features that can be taken from this attempt, especially the highlighting of the corresponding position in the playing field to help the player as seen in Figure ??.

3.3 4DTris

This was another venture into the 4th dimension with Tetris. The concept however is completely different. This time the actual playing field exists in 4 dimensions and the pieces do as well. The playing field is projected into a hypercube, as are the pieces, which ends up looking at a cube that is being filled from the inside with different cubes. Development was stopped in 2012 and the author moved on, eventually trying to revive it in 2018, failing. The concept is very interesting, but very hard to imagine. While playing the game might make one proficient, the learning curve would be too steep for most.

Conclusion For me this is an example of a variant that went too far. While very interesting in concept, the sheer complexity of the gameplay automatically limits the addressable market. In this project falling into the same trap needs to be avoided by creating something fun for many different audiences.

3.4 BlockOut

BlockOut is a 3D Tetris variant. The playing field is a 3D cube and the pieces are 3D shapes. The playing field is projected from above. Individual layers are color coded. In any other way the game is very similar to Tetris. The game was moderately successful and can be played online for free nowadays.

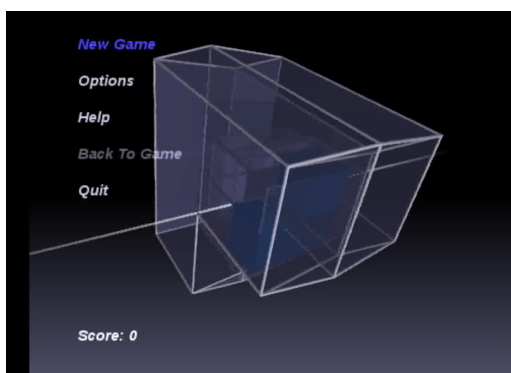


Figure 3: 4DTris in action. Still from a video on youtube, posted by the autor Simon Laszlo, <https://www.youtube.com/watch?v=WO9SNW5Tp7A>

Conclusion This is a great example of a Tetris variant and something that my version will largely be based on. I want to take large parts of the visualization style but enhance on UI/UX.

3.5 This project

will be an amalgamation of all the above. Taking the best features from every item. Looks from BlockOut, physics from 4DTris, UX features from Frac4D and general greatness from Tetris.

4 Literature

Understanding 4 dimensional space and its implications is a very challenging undertaking, however there is some literature out there that can help with the process.

4.1 The Fourth Dimension

by Rudy Rucker [16] takes us through a wild journey from ancient mathematics to contemporary philosophy and the other way round. This can create great insights on how it is possible to understand and even visualize 4 dimensional object within reality that is confined to 3 dimensions. This work does greatly help with becoming comfortable with the concept of 4 dimensions and will be a great help in the development of the game.

4.2 Geometric algebra for computer science

by Leo Dorst, Daniel Fontijne and Stephen Mann [10] serves as a great introduction into geometric algebra, which might be a solution to the problem of representing, and especially rotation 4 dimensional objects. While no final decision has been made on the technique that will be used, a decent understanding of the topic is necessary to finally choose the correct one.

4.3 An Introduction to Clifford Algebras and Spinors

by Jayme Vaz Jr. and Roldao da Rocha Jr. [12] is another great resource into the topic, especially as Clifford algebras are widely used in n-dimensional geometry. The chapter on spinors is especially interesting as they are one of the ways to represent rotations in higher dimensions and might be the foundation of the techniques we will use.

4.4 On Rotations in Space of Four Dimensions

by Cole [4] is an article from 1888 that deals with the problem of rotations in 4 dimensions. The article proves a general rotation matrix for 4 dimensions. The most important feature however is the date of publication, proving the persistent human interest in the topic of higher dimensions and therefore (somewhat) validating the project.

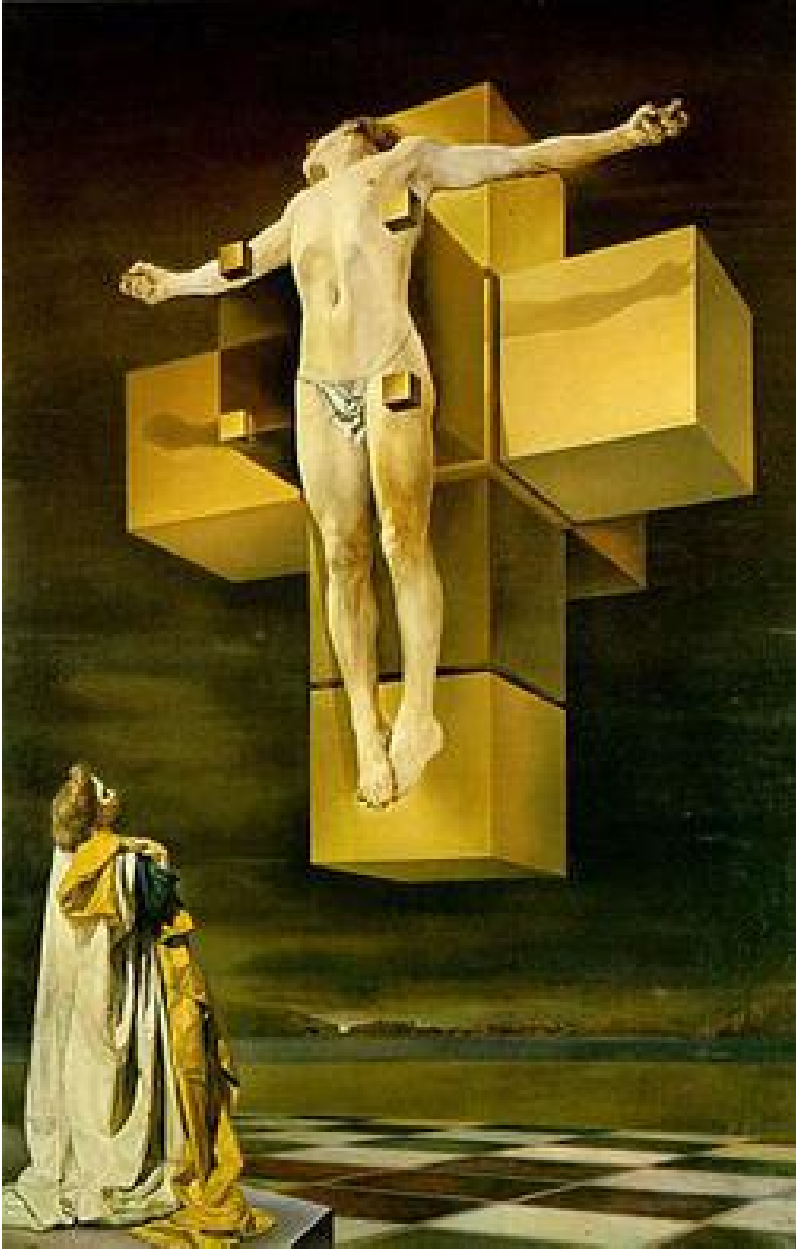


Figure 4: The Crucifixion (Corpus Hypercubus) by Salvador Dali. A representation of a hypercube unfolded into 3 dimensions.

4.5 Conclusion

This is just a small outtake on literature that can help understand the topic of higher dimensions which is necessary for this project. Especially the decision on which mathematical representation and technique will be chosen is heavily informed by these books/articles.

5 Visualization

Visualization of 4 dimensional objects is a very challenging but interesting task.

There is a large amount of great resources available to help understanding how this can be done. A very good overview over the techniques is presented in the PhD thesis by Hallasch [1]. In extension to this the website "<https://baileysnyder.com/interactive-4d/4d-cubes/>" [15] offers great interactive tools to generate an understanding of how to get to 4d dimensions and back.

5.1 Illuminating the fourth dimension

Illuminating the fourth dimension by Hanson et al. [11] describes multiple techniques on how to render 4d objects into 3d space onto a 2d screen. This includes advanced shadowing and shading techniques which exceed the necessity of this project since unity will be used.

6 Domain and Users of the Project

6.1 Gamers

I will try to capture a wide audience for this game. Tetris is played by young and old, people from different educational backgrounds and different cultures. As described by Csikszentmihalyi [7] this can be done by creating a state of flow. To induce this state several things must be true.

1. A challenging activity that requires skill
2. Clear goals and feedback
3. The player must be able to concentrate on the task at hand
4. Direct and immediate feedback
5. A sense of control
6. A loss of self-consciousness
7. An altered sense of time

When looking at this list, it is clear that Tetris succeeded in all of them. The biggest challenge for this project will be to make it easy enough so users of all backgrounds can achieve 'flow'. As Chen [3] stated this can be achieved, by unlocking additional choices (pieces) when the player has obtained some sufficiency. Some users will struggle with the basic concept of higher dimensions, and the game is required to ease them into a basic understanding of how the rotations work. Others will have no issues getting into the game but will require additional challenges to be sucked into the game and keep engaged. The game needs to create a sense of reward in users playing it for 5 minutes during a short break as well as users that are willing to invest several hours to achieve higher scores. Due to the nature of the game it is expected that it will have a somewhat larger appeal on users with advanced education, so these will be the primary focus group during evaluation, however, previous projects into the same realm have shown that focussing solely on this group will not be enough to create a successful game. Targeted testing will be carried out, and I will try to include at least 25% of the testers from a group with a lower educational background.

While these are the target audiences for the game we have to accommodate the fact that the main group of users will be male, between 20-40 years old and most likely with a higher educational background. This will be the primary target group and affect marketing and pricing strategies.

6.2 Research

A secondary target user group will be the academic community. Tetris has been widely used as a research subject. Our variant will rely heavily on mental rotation and spatial reasoning, both have been an important part of psychological research [5] [6] [13] [14]. This could be a very interested target audience if the game itself implements features that aid researchers in experiment design. Voice of customer will be conducted in advance of development to gain insights into what specific features would be suitable to engage the scientific community. This might be as simple as exporting pieces spawned + keystrokes into a csv file.

7 Justification of Design Choices

Several choices had to be made during envisioning.

7.1 4D

While it was very enticing to try to implement a fully 4 dimensional game and or move the play field into 4D as well. Preliminary user research as well as an extensive review of previous work has shown that this would increase the complexity to a level where the likelihood of overall commercial success for such a project would decrease dramatically due to a decrease in target users and the total obtainable market. I will therefore limit the playingfield to 3 dimensions and the pieces to 3 dimensional slices of 4 dimensional objects.

7.2 Gameplay

Due to limitations when it comes to user input the interactions within the 4th dimension will be limited. To be precise there will be no way to move the w plane and change the slice we are projecting into 3d space. The w plane will always be in the middle. This does however leave and opening for later expansion/enhancement of the gameplay.

7.3 User Interface

I feel that it is necessary to create a very informative user interface due to the complexity of the game and the fact that the arrow keys will have different functionalities depending on which action key is pressed. This shall be achieved through graphical overlays that will inform the player of the type of rotation/movement that will be performed in the current state.

7.4 Scoring

For the prototype multiline tetrises will be disabled. The primary goal of the MVP is to establish fun and engaging gameplay.

7.5 Art Style

The art style will be very important for the game. While people with higher educational background will likely be enticed by the mere concept of a 4 dimensional game, other user groups will need appealing and contemporary visuals to be kept engaged.

8 Overall Structure of the Project

The game will be made out of several components that will interact with each other. The main components are:

8.1 GameManager

This will take care of the overall game state. It will keep track of the current score, the current level, the current piece, the next piece, the playing field and the current state of the game.

8.2 HyperCube

The basic building block of the game. This will be a 4 dimensional extension of a cube. It will be used to procedurally generate the mesh used for rendering the pieces as well as basic rotation logic. Collision detection will be done in 3 dimensional space.

8.3 Piece

This will be a 4 dimensional collection of hypercubes. the main rotational logic be implemented here. The current plan is to decompose the piece once it is resting on the playing field. In addition the rendering will switch from wireframe to solid. With individual cubes rendered in a color according to the row they are on.

8.4 Block

This will be the collection of all resting cubes and will be used for the detection of completed planes.

8.5 Level

This is the playing field. Will generate the mesh, keep track of the block and give options for later enlargement of the playing field (not in MVP).

8.6 User Interface

Will be a column on the right containing scores, level etc. as well as an overlay that will help the user understand the rotations.

10 Plan of Work

The Main activity in developing the project are going to be testing and developing. The additional task of writing the report will be done in parallel throughout the development. The most important features are outlined in the following Gantt chart, with ample amount of time included for unexpected challenges. In addition, due to a full time job, weeks that require business travel are included as well, as it is highly unlikely that any development can happen during those times.

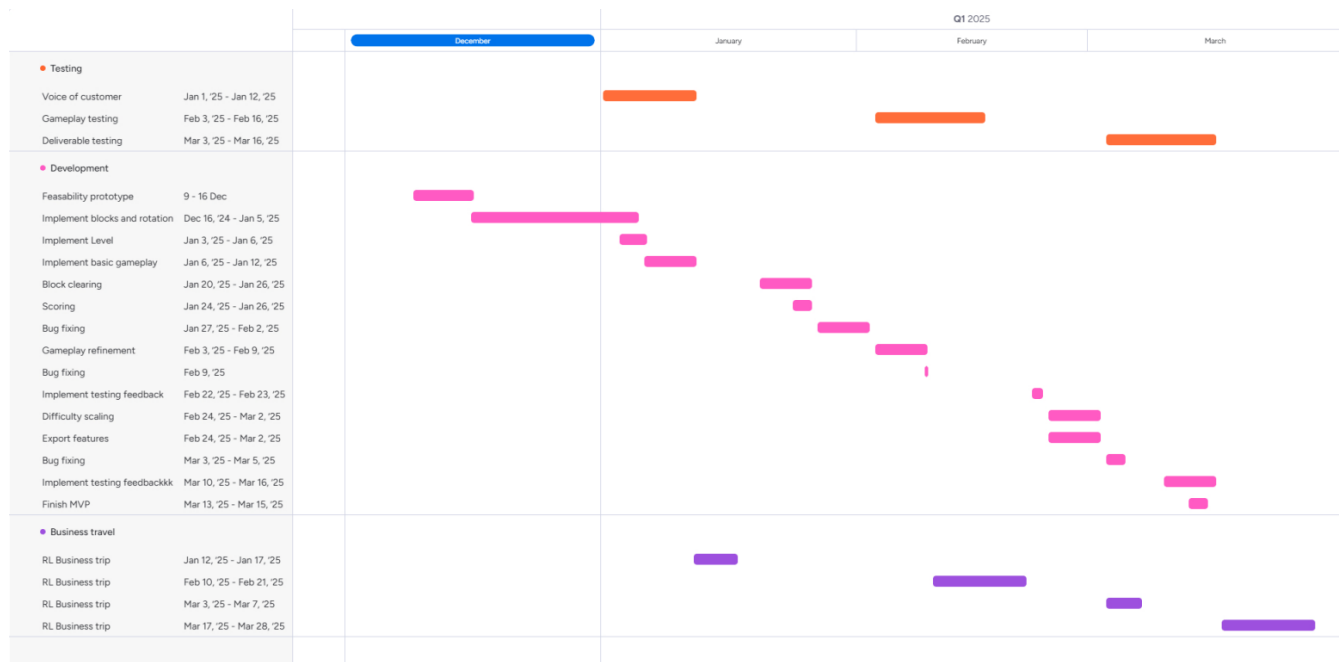


Figure 6: Project Timeline Gantt Chart

11 Testing and Evaluation Plan

Recurring Testing is key to a successful completion of this project. Given the limited time available for development there will be a total of 3 main test phases.

- Voice of customer** - This primary testing phase will not test the actual software, but the assumptions I have taken so far. This is meant to inform me on different aspects before starting the development phase:
 - Is the idea interesting
 - Are the keybindings to complicated
 - Is the chosen graphical style appropriate
 - Does the concept sound interesting
 - How would you like level progression to go
 - faster speed
 - more complicated pieces
 - ...
- Gameplay testing** - After having created a feasible prototype, most likelz with limited audiovisual appeal. We will try to get some information from the users whether the gameplay eorks.
 - Do you understand what you have to Do
 - Do you understand the basic visuals
 - Are you aware how to line up the pieces

- (d) Is the speed ok
- (e) Is the point system rewarding

In addition we will have some UX/UI related questions

3. **Deliverable testing** - This final testing run will be used to test if the deliverable is up to spec. I will use the 'outstanding' criteria in the template as a reference and get metrics on how close we are to that goal. In addition users will be asked to rate different features of the game on a scale from 0 to 5 to identify any outliers. The goal is not to reach 5 in every category, but to identify components that perform below the average of the game. These might include:

- (a) Visual appeal
- (b) Audio effects
- (c) enjoyability
- (d) Difficulty
- (e) complexity
- (f) Ease of use (UI)

12 Prototype

Description Prototyping will go through several stages. Some are performed in Python, to quickly check my reasoning. Other features will be tested in Unity directly.

Rotations The first feasibility issue was the rotation of the 4 dimensional pieces and the projections into 3D space. To check if I was able to generate the correct rotations and the vertices I implemented a very basic simulation in Python. The reasoning behind this was, that Python support higher dimensional matrices out of the box and it seemed like a fast approach. In this round of prototyping I was able to show that I am able to produce the rotations as well as the projection, and in addition+ that the rotation of the pieces looks interesting and can lead to unexpected piece configurations just as planned.

Basic movement Basic movement prototyping was done in unity to learn how to simulate the tetris style movement of 1 tile per button press, but gradually interpolate the position of the pieces. In addition rotation of 4d hypercubes was also implemented in Unity to check general feasibility.

Conclusion While the prototypes are very rough, and not as far progressed as hoped to due to time constraints, they do show that the underlying concept is feasible and the project can be attempted. It did also inform me about important watchouts for the development phase, such as handling disconnected edges/vertices. Since in between rotations we return to 3D space, things like collisions and line clearing will be much easier to implement.

13 Implementation

Hypercubes These are the basic building blocks of my game. It was the first module that was implemented. All the hypercube class does is instantiate The first thing this class does is create the vertices of the hypercube. This is done through 4 nested for loops that store 4 dimensional vectors in a list. From the vertices we can create the edges, a special property of 4d hypercubes is that two vertices form an edge if and only if they differ in exactly one coordinate. Since this is done only once on startup it is also implemented thorough nested for loops.

```
for (int i = 0; i < baseVertices.Length; i++)
{
    for (int j = i + 1; j < baseVertices.Length; j++)
    {
        Vector4 vi = baseVertices[i];
        Vector4 vj = baseVertices[j];
```

```

        // Count how many coordinates differ
        int diffCount = 0;
        if (Mathf.Abs(vi.x - vj.x) > 0.001f) diffCount++;
        if (Mathf.Abs(vi.y - vj.y) > 0.001f) diffCount++;
        if (Mathf.Abs(vi.z - vj.z) > 0.001f) diffCount++;
        if (Mathf.Abs(vi.w - vj.w) > 0.001f) diffCount++;

        if (diffCount == 1)
        {
            edges.Add(new int[] { i, j });
        }
    }
}

```

To make the system easier, edges are not recalculated, but stay connected to the same vertices through the whole play. During initial testing this has not produced any critical issues, but might need to change in the future. The Hypercubes are also in charge of creating their own mesh. For now we rely on a wireframe mesh, but this is still planned to change in the future. We simply store the current position of the vertices in a list and reference them in the index array. From this we create a new mesh each update (there is a lot of room for optimization here). This is necessary since we are not simply creating an object in 3D space, but we are rotating in 4 dimensions and create a different 3D cut each update, in addition we are interpolating the rotations to make the movement look smooth. To obtain the positions of the vertices we use a helper class

```

    public static Vector4 MultiplyPoint4x4(this Matrix4x4 m, Vector4 v)
    {
        Vector4 result;
        result.x = m.m00 * v.x + m.m01 * v.y + m.m02 * v.z + m.m03 * v.w;
        result.y = m.m10 * v.x + m.m11 * v.y + m.m12 * v.z + m.m13 * v.w;
        result.z = m.m20 * v.x + m.m21 * v.y + m.m22 * v.z + m.m23 * v.w;
        result.w = m.m30 * v.x + m.m31 * v.y + m.m32 * v.z + m.m33 * v.w;
        return result;
    }
}

```

that lets us easily rotate a point by using a rotation matrix. Then we use a very trivial orthographic projection of the 4D space into 3D space. This is done by simply ignoring the w coordinate. This is a very simple approach, but it is sufficient for the current state of the project, and still gives nice visuals and interesting rotations. Future testing might compare a mathematically more correct approach against this in some sort of AB testing.

Polynominoes This module handles the actual pieces. It randomly picks from a list of pre-created polynomino layouts and creates the necessary hypercubes as well as rotates the piece into a random (snapped to 90°) state. The class also handles the rotations of the polynomino. Every frame we interpolate between the current position and rotation as well as the target position and rotation. This is fortunately all baked into C#.

```

    // Interpolate between current and target position
    transform.position = Vector3.Lerp(transform.position, targetPosition, Time.deltaTime * 2);

    // Interpolate between current and target rotation
    for (int i = 0; i < 6; i++)
    {
        currentRotation[i] = Mathf.Lerp(currentRotation[i], targetRotation[i], Time.deltaTime * 2);
    }
}

```

This provides us with smooth movements. This class has seen some revisions already, especially when it comes to collision detection. The first plan was to use the built in Unity collision detection system. I created a cube on top of every hypercube and attached a rigidbody to act as a collider and have constant collision detection. After thinking about it however, I discovered this would be quite wasteful, and instead we keep track of the extends of the playing field and the location of all walls and all blocks, and every time the player asks for any kind of movement we query if the final position after that move, not the interpolated one, is even possible. If not, in case of a wall we

simply ignore the move, in case of a block that prevents further falling, we start a timer, that unless the situation is resolved, will lock the piece in place. For now this does not consider the edge case, where a rotation leads to an out of bounds situation, which is however planned to be mitigated in the future.

Locking in place will not be handles by the polynominue. Instead it will hand itself of to the board, and be disassembled.

Input manager This class handles all the input provided by the player. For this assignement the input is very limited as we are bound to the arrow keys, plus 3 modifiers. The chosen layout table is below:

	LEFT	UP	RIGHT	DOWN
	Move left	Move up	Move right	Move down
A	XY-	XZ+	XY+	XZ-
S	XW-	YZ+	XW+	YZ-
D	YW-	ZW+	YW+	ZW-

Table 1: Input Layout: The arrow keys are used to move the piece, the A, S, D keys are used to modify the plane of rotation.

To make it easier for the user to understand, for the moment a text field on the UI layer is updated whenever a modifier is pressed to inform the user what planes of rotation are currently beeing used. In the future this will be modified to a graphical overlay. For the moment this class also handles the pieces falling towards the bottom, however this is not properly decoupled and will be moved in the future.

BoardState

Room Renderer

14 Evaluation

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