巨量資料管理學院碩士在職專班

統計分析

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陳光宏

假設檢定
(Hypothesis testing)

5個步驟

- 寫出假說
- 設定決策的規則
- 從母體取得樣本
- 計算檢定統計量
- 結論

寫出虛無假說與對立假說

猜測與否證的過程

虛無假說
(Null hypothesis)

希望被否定、放棄的假設
以H_0表示

對立假說
(Alternative hypothesis)

希望成立的假設
以H_1表示

範例

- 法院判決

H_0: The defendant is innocent

H_1: The defendant is guilty

- 禁藥檢測

H_0: No banned substance was used

H_1: Banned substance was used

- 指紋辨識

H_0: User is legitimate

H_1: User is not legitimate

範例

- 住家某放射性物質的安全含量

$$H_0: \mu \leq 4.0 \text{ pCiL}$$
$$H_1: \mu > 4.0 \text{ pCiL}$$

- 手機的冷啟動電流

$$H_0: \mu \geq 880 \text{ CCA}$$
$$H_1: \mu < 880 \text{ CCA}$$

練習

- 影印紙的寬度大約216mm，現有一批新的影印紙，欲檢定這批影印紙的寬度是否為216mm，請寫出虛無假說與對立假說。

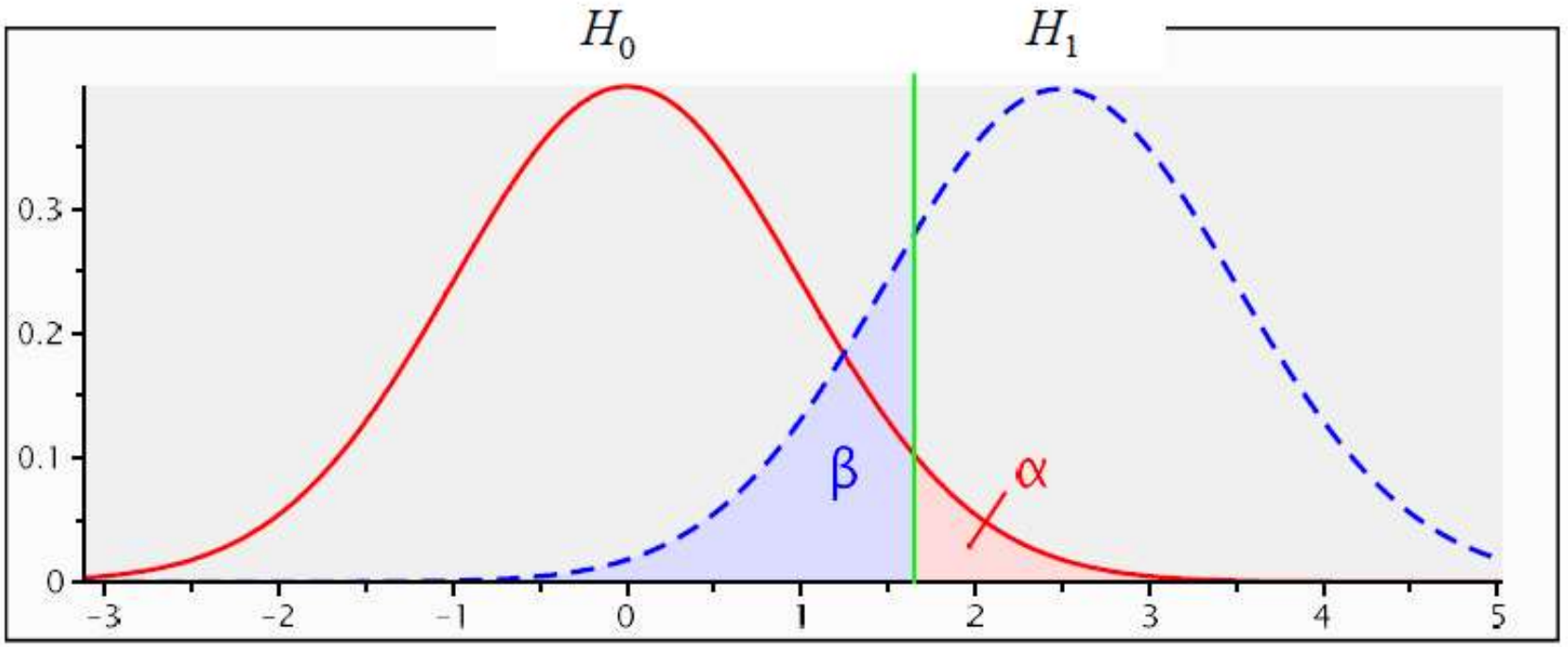
$H_0: \mu = 216$ mm ⇐ Assume H_0 is true unless evidence says otherwise

$H_1: \mu \neq 216$ mm ⇐ The $\neq$ in H_1 points to both tails (two-tailed test)

推論的誤差種類

	H_0 *is true*	H_0 *is false*
Reject H_0	Type I error	Correct decision
Fail to reject H_0	Correct decision	Type II error

型I誤差與型II誤差的關係

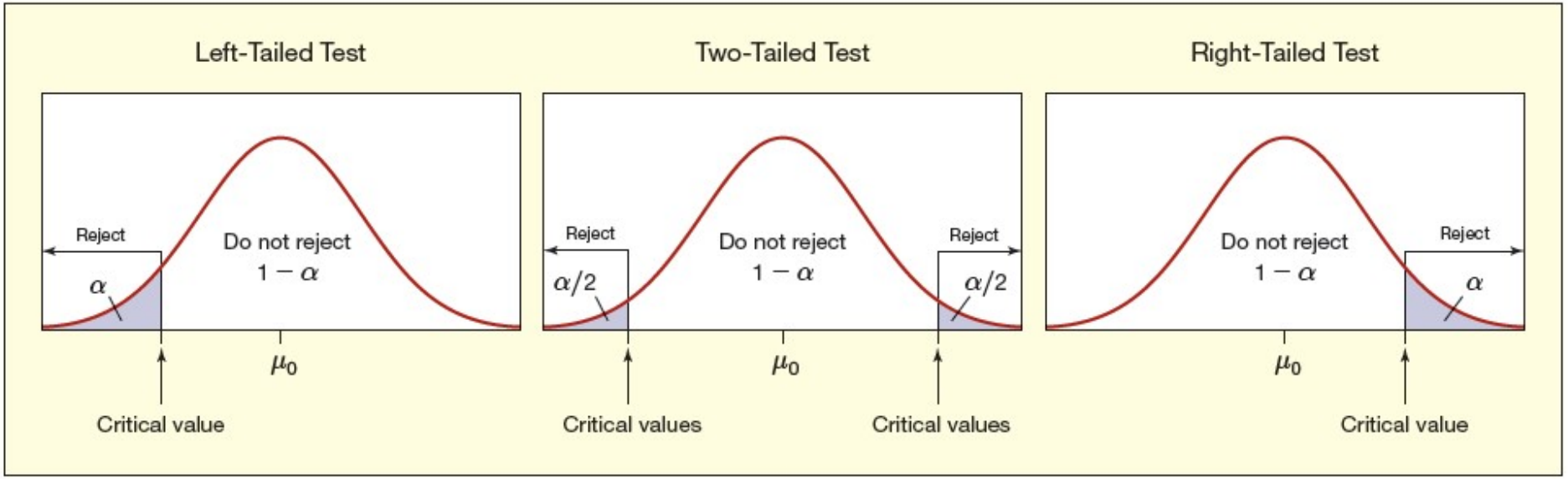


Key Term	*What Is It?*	*Symbol*	*Definition*	*Also Called*
Type I error	Reject a true hypothesis	α	$P(\text{reject } H_0 \mid H_0 \text{ is true})$	False positive
Type II error	Fail to reject a false hypothesis	β	$P(\text{fail to reject } H_0 \mid H_0 \text{ is false})$	False negative
Power	Correctly reject a false hypothesis	$1 - \beta$	$P(\text{reject } H_0 \mid H_0 \text{ is false})$	Sensitivity

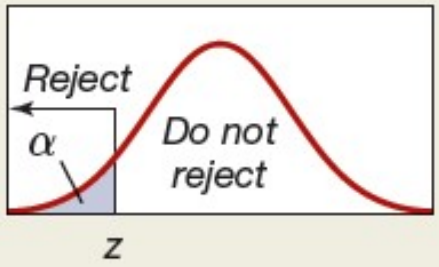
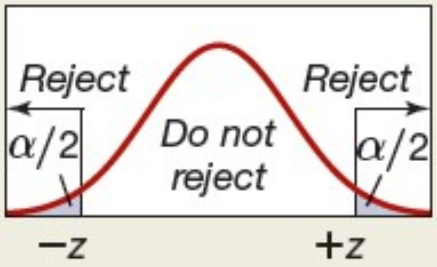
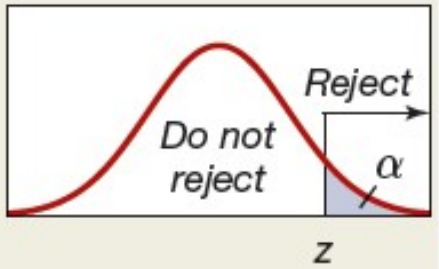
設定決策規則

1. 型I誤差 (通常設定5%，α =0.05)
2. 極端值 (critical value)
3. 拒絕域 (rejection region)

Tests for H_0: $\mu = \mu_0$



根據標準常態分布計算極端值

Level of Significance (α)	Left-Tailed Test	Two-Tailed Test	Right-Tailed Test
			
.10	$z_{.10} = -1.282$	$z_{.05} = \pm 1.645$	$z_{.10} = +1.282$
.05	$z_{.05} = -1.645$	$z_{.025} = \pm 1.960$	$z_{.05} = +1.645$
.01	$z_{.01} = -2.326$	$z_{.005} = \pm 2.576$	$z_{.01} = +2.326$

檢定統計量 (test statistics)

Test Statistic for a Mean: Known σ

Sample mean

Hypothesized mean

$$z_{\text{calc}} = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Standard error of the sample mean

範例：單一樣本的檢定

影印紙的寬度大約216 mm，現有一批新的影印紙50張，平均寬度是216.007 mm，假設母體的標準差為0.023 mm，欲檢定這批影印紙的寬度是否為216 mm

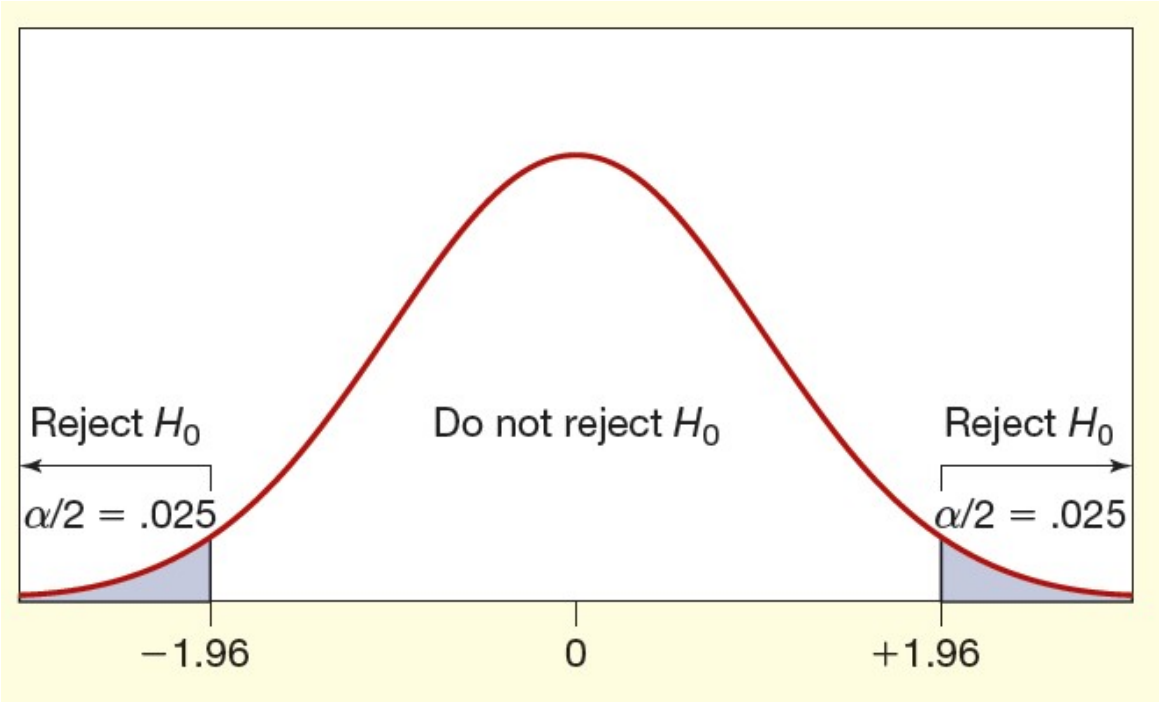
步驟一：寫出假說

H_0: $\mu = 216$ mm ⇐ Assume H_0 is true unless evidence says otherwise

H_1: $\mu \neq 216$ mm ⇐ The $\neq$ in H_1 points to both tails (two-tailed test)

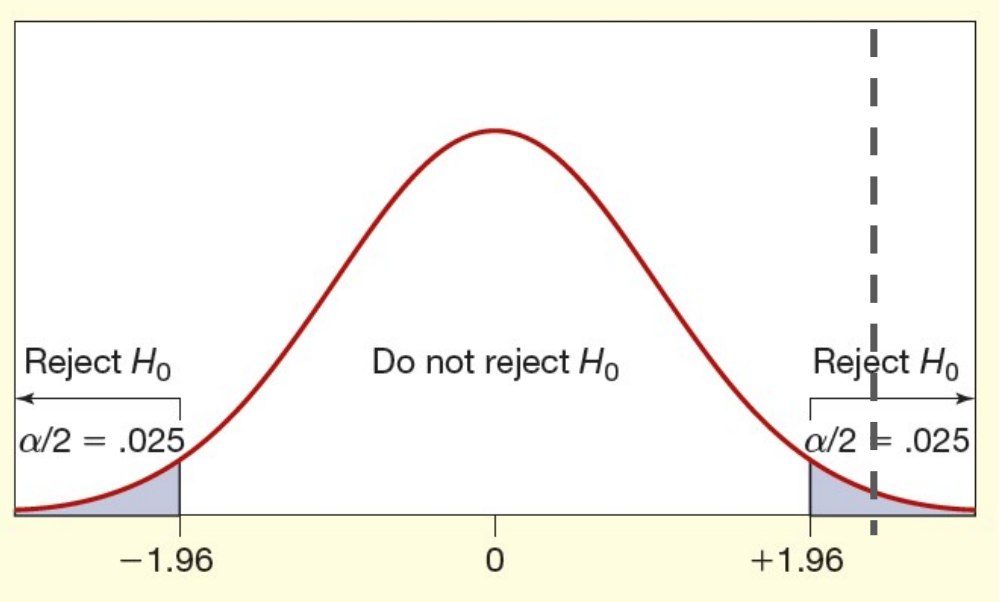
步驟二：決策規則

1. $\alpha = 0.05$
2. 假設標準常態分布，計算臨界值為± 1.96



步驟三：收集資料
步驟四：計算檢定統計量

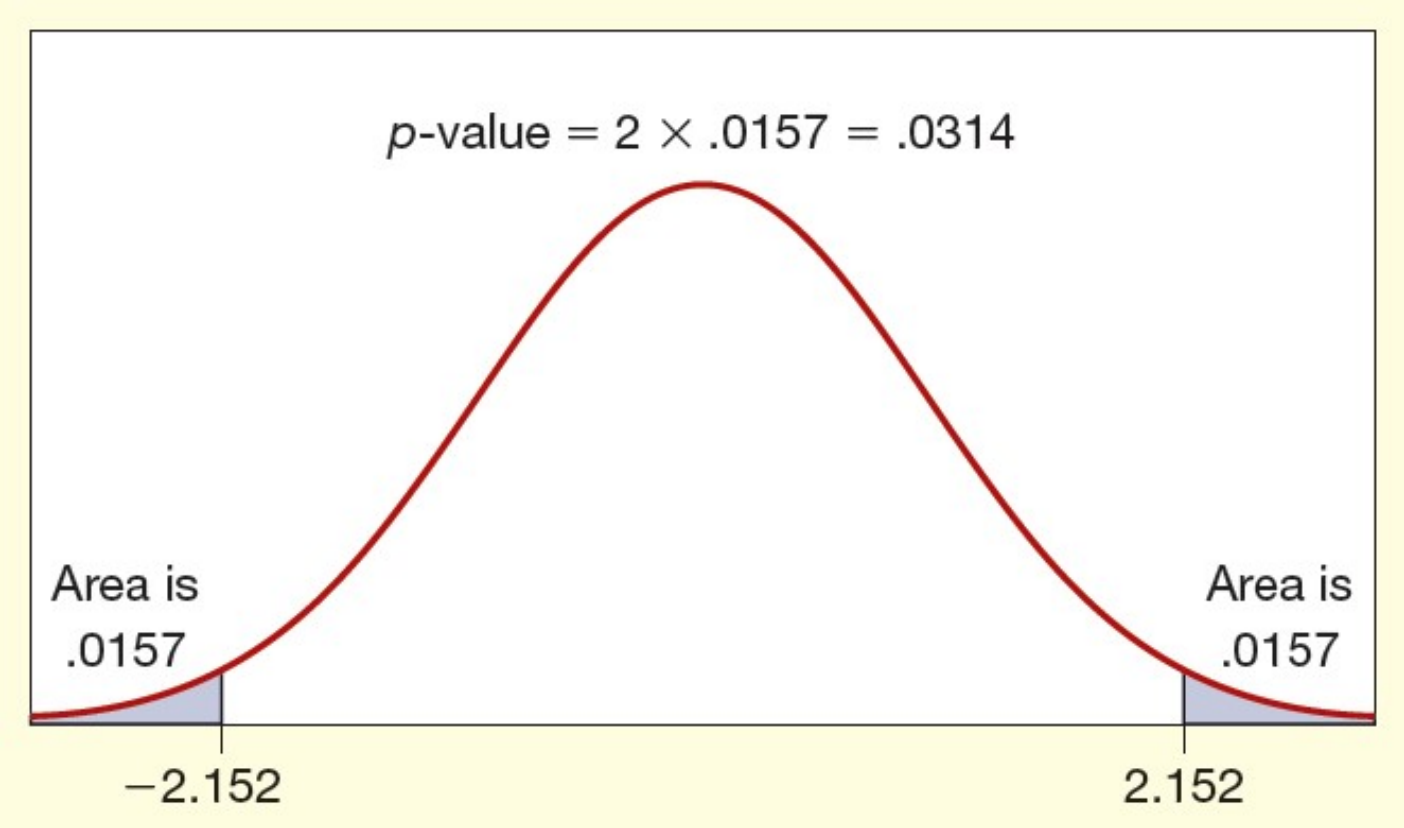
$$z_{\text{calc}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{216.0070 - 216.0000}{\frac{.0230}{\sqrt{50}}} = \frac{.0070}{.00325269} = 2.152$$

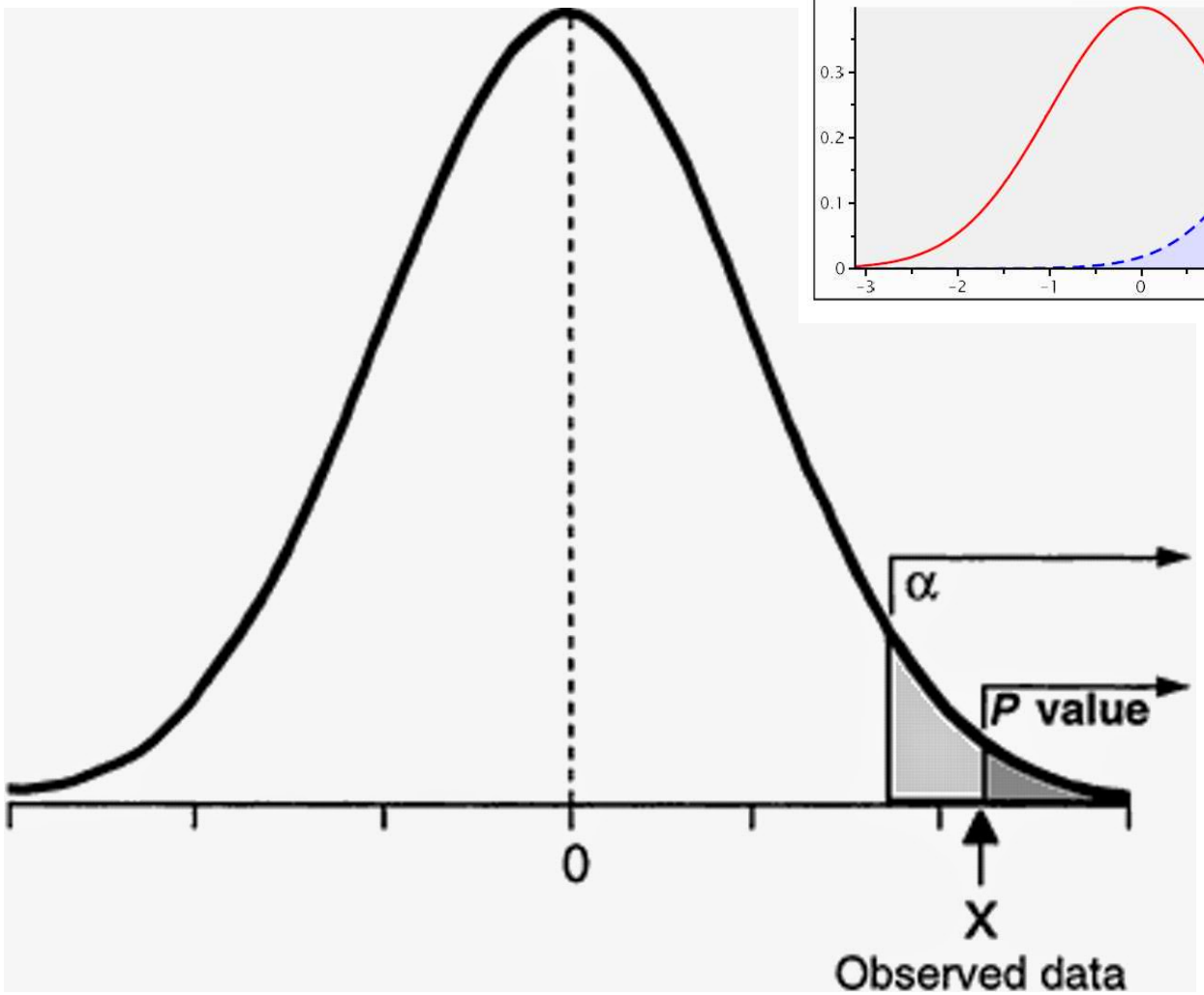


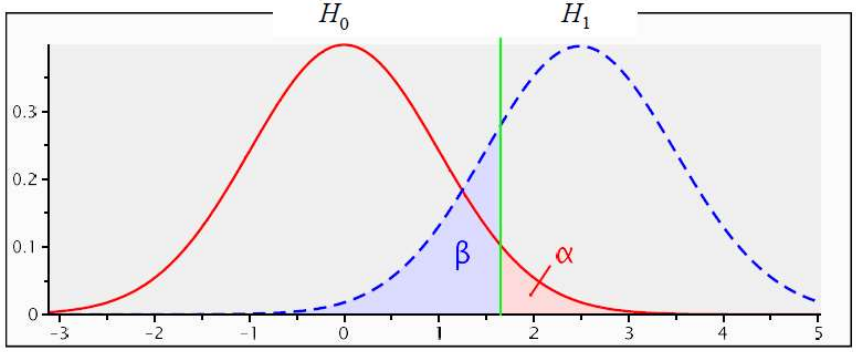
步驟五：結論

1. 統計檢定量的值超出臨界值，表示拒絕虛無假說 H_0
2. 表示這50張影印紙的平均寬度不等於216 mm

利用P值做為決策規則

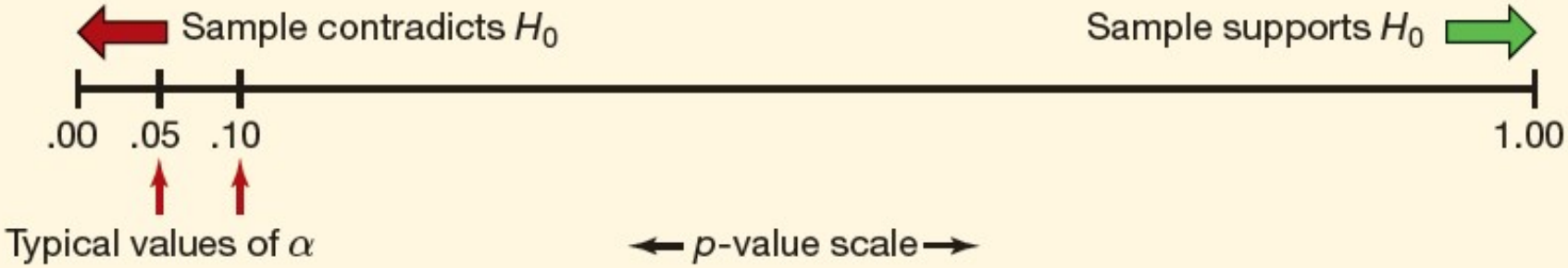



α
P value
0
X
Observed data


H_0
H_1
0.3
0.2
0.1
0
β
α
−3
−2
−1
0
1
2
3
4
5

What Is a *p*-Value?

A sample statistic is a random variable that may differ from the hypothesized value merely by chance, so we do not expect the sample to agree *exactly* with H_0. The *p*-value is the probability of obtaining a test statistic as extreme as the one observed, assuming that the null hypothesis is true. A large *p*-value (near 1.00) tends to support H_0, while a small *p*-value (near 0.00) tends to contradict H_0. If the *p*-value is less than the chosen level of significance (α), then we conclude that the null hypothesis is false.



利用信賴區間作為決策規則

判斷95%信賴區間是否包含216 mm

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$216.0070 \pm 1.960 \frac{.0230}{\sqrt{50}}$$

$$[216.001, 216.013]$$

問題

我們把題目修改如下

影印紙的寬度大約216 mm，現有一批新的影印紙50張，平均寬度是216.007 mm，**樣本的標準差為0.023 mm**，欲檢定這批影印紙的寬度是否為216 mm

請問結論是否相同？

檢定統計量 (test statistics)

未知母體變異數/標準差，使用樣本變異數/標準差

t分布

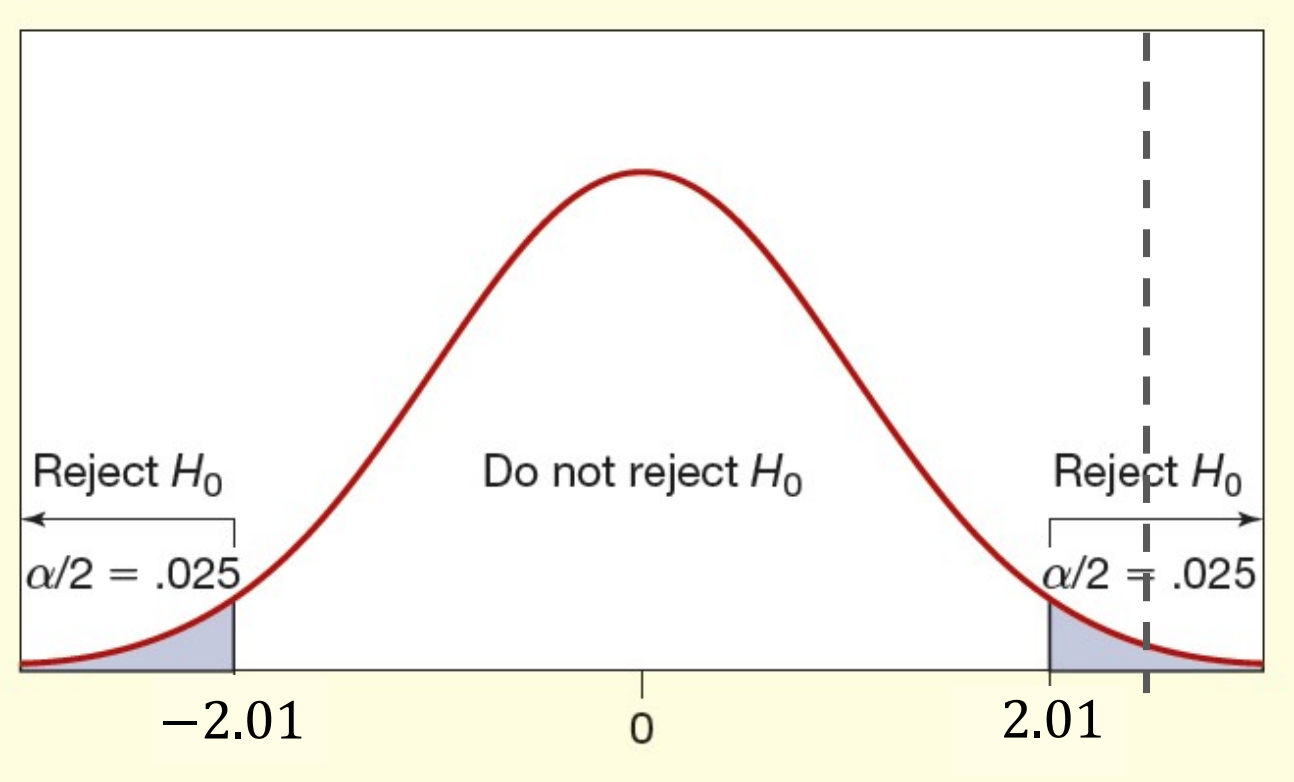
Test Statistic for a Mean: σ Unknown

Sample mean

Hypothesized mean

$$t_{\text{calc}} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \quad \text{if } \sigma \text{ is unknown}$$

Sample st. dev.

改用t分布，臨界值改變

$$t_{\text{calc}} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$