Ejercicios: Álgebra lineal

→ $\boxed{\text{Ejercicio: sistema de ecuaciones}}$

$$\left.\begin{array}{l} x + y + 2z = 0 \\ ax - 3z = a \\ 2x + ay - z = a \end{array}\right\}$$

- Voy a discutir este sistema utilizando el teorema de Rouché - Fröbenius.

- $rg(A) = rg\begin{pmatrix} 1 & 1 & 2 \\ a & 0 & -3 \\ 2 & a & -1 \end{pmatrix}$

$$\begin{vmatrix} 1 & 1 & 2 \\ a & 0 & -3 \\ 2 & a & -1 \end{vmatrix} = -6 + 2a^2 + 3a + a = 2a^2 + 4a - 6;$$

$2a^2 + 4a - 6 = 0$

$a^2 + 2a - 3 = 0$

$$a = \frac{-2 \pm \sqrt{4+12}}{2} = < \begin{matrix} 1 \\ -3 \end{matrix}$$

- Si $\boxed{a \neq 1}$ y $\boxed{a \neq -3}$ $\Rightarrow$ $rg(A) = 3 = rg(A|B) = n^{\circ}$ incógnitas → $\boxed{SCD}$
- Si $\boxed{a = 1}$ $\Rightarrow$ $rg(A) = 2 = rg(A|B) \neq n^{\circ}$ incógnitas → $\boxed{SCI}$

- $rg(A) = rg\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ \cancel{2} & \cancel{1} & \cancel{-1} \end{pmatrix} = 2$ $\quad F_1 + F_2 = F_3$

- $rg(A|B) = rg\begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & -3 & 1 \\ \cancel{2} & \cancel{1} & \cancel{-1} & \cancel{1} \end{pmatrix} = 2$ $\quad F_1 + F_2 = F_3$

- Si $\boxed{a = -3}$ $\Rightarrow$ $rg(A) = 2 \neq rg(A|B) = 3$ → $\boxed{SI}$

- $rg(A) = rg\begin{pmatrix} 1 & 1 & \cancel{2} \\ -3 & 0 & \cancel{-3} \\ 2 & -3 & \cancel{-1} \end{pmatrix} = 2$ $\quad C_1 + C_2 = C_3$ $\qquad \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -3 \neq 0$

- $rg(A|B) = rg\begin{pmatrix} \cancel{1} & 1 & 2 & 0 \\ \cancel{-3} & 0 & -3 & -3 \\ \cancel{2} & -3 & -1 & -3 \end{pmatrix} = 3$ $\quad C_1 + C_2 = C_3$ $\qquad \begin{vmatrix} 1 & 2 & 0 \\ 0 & -3 & -3 \\ -3 & -1 & -3 \end{vmatrix} = 9 + 18 - 3 = 24 \neq 0$

- Resolvemos el sistema para $\boxed{a \neq 1}$ y $\boxed{a \neq -3}$ por Cramer:

$$x = \frac{\begin{vmatrix} 0 & 1 & 2 \\ a & 0 & -3 \\ a & a & -1 \end{vmatrix}}{2a^2+4a-6} = \frac{-3a+2a^2+a}{2a^2+4a-6} = \frac{2a^2-2a}{2a^2+4a-6} = \frac{2a(a-1)}{2(a-1)(a+3)} = \frac{a}{a+3}$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 2 \\ a & a & -3 \\ 2 & a & -1 \end{vmatrix}}{2a^2+4a-6} = \frac{-a+2a^2-4a+3a}{2a^2+4a-6} = \frac{2a^2-2a}{2a^2+4a-6} = \frac{a}{a+3}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 0 \\ a & 0 & a \\ 2 & a & a \end{vmatrix}}{2a^2+4a-6} = \frac{2a-a^2-a^2}{2a^2+4a-6} = \frac{2a-2a^2}{2a^2+4a-6} = \frac{2a(1-a)^{-1}}{2(a-1)(a+3)} = \frac{-a}{a+3}$$

- Resolvemos para $\boxed{a = 1}$:

$$\left.\begin{array}{r} x+y+z=0 \\ x \quad -3z = 1 \\ 2x+y-z=1 \end{array}\right\}$$

$E_3 = E_1 + E_2$ ¡EC. REDUNDANTE!

$$\left.\begin{array}{r} x+y=-2z \\ x \quad =1+3z \end{array}\right\} \rightarrow 1+3z+y=-2z$$

$\boxed{y = -1-5z}$

$\boxed{x = 1+3z}$

- Solución del sistema:

$$\boxed{\begin{array}{l} z = \lambda \\ x = 1+3\lambda \\ y = -1-5\lambda \end{array}}$$

→ **Ejercicio: sistema de ecuaciones**

$$\left.\begin{array}{r} 2y-z=m \\ 3x \quad -2z = 11 \\ y+z=6 \\ 2x+y-4z=m \end{array}\right\}$$

- Voy a discutir este sistema utilizando el teorema de Rouché-Frobenius.

$$rg(A|B) = rg\begin{pmatrix} 0 & 2 & -1 & m \\ 3 & 0 & -2 & 11 \\ 0 & 1 & 1 & 6 \\ 2 & 1 & -4 & m \end{pmatrix}$$

$$\begin{vmatrix} 0 & 2 & -1 & m \\ 3 & 0 & -2 & 11 \\ 0 & 1 & 1 & 6 \\ 2 & 1 & -4 & m \end{vmatrix} \underset{\substack{F_1-2F_4 \\ F_3-F_4}}{=} \begin{vmatrix} -4 & 0 & 7 & -m \\ 3 & 0 & -2 & 11 \\ -2 & 0 & 5 & 6-m \\ 2 & 1 & -4 & m \end{vmatrix} = 1\cdot(-1)^{4+2}\begin{vmatrix} -4 & 7 & -m \\ 3 & -2 & 11 \\ -2 & 5 & 6-m \end{vmatrix} = 48-8m-154-15m+4m+220-126+21m = 2m-12;$$

$2m-12=0$; $m=6$

- Si $\boxed{m \neq 6} \Rightarrow rg(A|B) = 4 \neq rg(A) \rightarrow \boxed{SI}$

- Si $\boxed{m = 6} \Rightarrow rg(A|B) = 3 = rg(A) = n^{\circ}$ incógnitas $\rightarrow \boxed{SCD}$

$$\cdot\ rg(A|B) = rg\begin{pmatrix} 0 & 2 & -1 & 6 \\ 3 & 0 & -2 & 11 \\ 0 & 1 & 1 & 6 \\ 2 & 1 & -4 & 6 \end{pmatrix} \qquad \begin{vmatrix} 0 & 2 & -1 \\ 3 & 0 & -2 \\ 0 & 1 & 1 \end{vmatrix} = -3-6 = -9 \neq 0$$

- Resolvemos para $\boxed{m = 6}$:

$$\left.\begin{aligned} 2y - z &= 6 \\ 3x \quad - 2z &= 11 \\ y + z &= 6 \\ 2x + y - 4z &= 6 \end{aligned}\right\}$$

¡EC. REDUNDANTE!

$3x - 2\cdot 2 = 11;\quad 3x = 15;\quad \boxed{x = 5}$

$$\left.\begin{aligned} 2y - z &= 6 \\ 3x \quad - 2z &= 11 \\ y + z &= 6 \end{aligned}\right\} \rightarrow \begin{aligned} 2y - z &= 6 \\ y + z &= 6 \\ \hline 3y &= 12 \end{aligned}$$

$\boxed{y = 4}$

$\rightarrow 4 + z = 6 \qquad \boxed{z = 2}$

$\rightarrow$ **Ejercicio: sistema de ecuaciones**

$$\left.\begin{aligned} x + y - z &= 0 \\ -4x - 2y + mz &= 0 \\ 3x + y + 2z &= 0 \end{aligned}\right]$$

- ¿Hay algún valor de m que haga que sea un S.I.?

No, ya que es un sist. homogéneo y por tanto siempre compatible

- Voy a discutir este sistema utilizando el teorema de Rouché - Frobenius.

$$rg(A) = rg\begin{pmatrix} 1 & 1 & -1 \\ -4 & -2 & m \\ 3 & 1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 & -1 \\ -4 & -2 & m \\ 3 & 1 & 2 \end{vmatrix} = -4 + 3m + 4 - (6 + m - 8) = -4 + 3m + 4 - 6 - m + 8 = 2m + 2;$$

$$2m + 2 = 0;\quad m = -1$$

- Si $\boxed{m \neq -1} \Rightarrow rg(A) = 3 = rg(A|B) = n^{\circ}$ incógnitas $\rightarrow \boxed{SCD}$ (solución trivial $\begin{matrix} x = 0 \\ y = 0 \\ z = 0 \end{matrix}$)

- Si $\boxed{m = -1} \Rightarrow rg(A) = 2 = rg(A|B) \neq n^{\circ}$ incógnitas $\rightarrow \boxed{SCI}$

$$\cdot\ rg(A) = rg\begin{pmatrix} 1 & 1 & -1 \\ -4 & -2 & -1 \\ \cancel{3} & \cancel{1} & \cancel{2} \end{pmatrix} = 2$$

$$-F_1 - F_2 = F_3$$

$$\begin{vmatrix} 1 & 1 \\ -4 & -2 \end{vmatrix} = -2 + 4 = 2 \neq 0$$

• Resolvemos para $\boxed{m=1}$ por Cramer:

$$\left.\begin{array}{r} x+y-z=0 \\ -4x-2y-z=0 \\ \cancel{3x+y+2z=0} \end{array}\right\} \qquad \left.\begin{array}{r} y-z=-x \\ -2y-z=4x \end{array}\right\}$$

¡EC. REDUNDANTE!

$-Ec_1-Ec_2=E_3$

• Solución: $\boxed{\begin{array}{l} x=\lambda \\ y=-\frac{5}{3}\lambda \\ z=-\frac{2}{3}\lambda \end{array}}$

$$\cdot\ y=\frac{\begin{vmatrix} -x & -1 \\ 4x & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix}}=\frac{x+4x}{-1-2}=\frac{5x}{-3}$$

$$\cdot\ z=\frac{\begin{vmatrix} 1 & -x \\ -2 & -4x \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix}}=\frac{4x-2x}{-3}=\frac{2x}{-3}$$

→ $\boxed{\text{Ejercicio: sistema de ecuaciones}}$

$$\left.\begin{array}{r} x+y+z=0 \\ x+2y+3z=0 \\ mx+(m+1)y+(m-1)z=m-2 \\ 3x+(m+3)y+4z=m-2 \end{array}\right\}$$

• Voy a discutir este sistema utilizando el teorema de Rouché-Fröbenius.

$$rg\,(A|B)=rg\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ m & m+1 & m-1 & m-2 \\ 3 & m+3 & 4 & m-2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ m & m+1 & m-1 & m-2 \\ 3 & m+3 & 4 & m-2 \end{vmatrix} \underset{\substack{F_2-F_1 \\ F_3-mF_1 \\ F_4-3F_1}}{=} \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & -1 & m-2 \\ 0 & m & 1 & m-2 \end{vmatrix} = 1\cdot(1)^{1+1}\begin{vmatrix} 1 & 2 & 0 \\ 1 & -1 & m-2 \\ m & 1 & m-2 \end{vmatrix} = 2m^2-8m+8;$$

$2m^2-8m+8=0$; $m=2$

• Si $\boxed{m\neq 2}$ ⇒ $rg\,(A|B)=4\neq rg\,(A)$ → $\boxed{SI}$

• Si $\boxed{m=2}$ ⇒ $rg\,(A|B)=3=rg\,(A)=$ nº incógnitas → $\boxed{SCD}$ (solución trivial $\begin{array}{l} x=0 \\ y=0 \\ z=0 \end{array}$)

$$\cdot\ rg\,(A|B)=rg\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 1 & 0 \\ \cancel{3} & \cancel{5} & \cancel{4} & \cancel{0} \end{pmatrix}=3 \quad F_4=F_2+F_3 \qquad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}=-3\neq 0$$

$$\cdot\ rg\,(A)=rg\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \\ \cancel{3} & \cancel{5} & \cancel{4} \end{pmatrix}=3 \quad F_4=F_2+F_3$$

→ **Ejercicio: subespacios vectoriales**

- $B = \{(x, y, z) \in \mathbb{R} / x+y = 2\}$

- 1ª condición = $\vec{u}, \vec{v} \in B \Rightarrow \vec{u} + \vec{v} \in B$

  - $\vec{u} = (x_1, y_1, z_1)$
  - $\vec{v} = (x_2, y_2, z_2)$

$$\vec{u} + \vec{v} = (x_1 + x_2, y_1 + y_2, z_1 + z_2) \notin B$$

$$x_1 + x_2 + y_1 + y_2 = 2$$

$$\underbrace{x_1 + y_1}_{2} + \underbrace{x_2 + y_2}_{2} = 2$$

$$2 + 2 \neq 2$$

- Como no se verifica la 1ª condición, B no es un subespacio vectorial.

→ **Ejercicio: subespacios vectoriales**

- $C = \{(x, y, z) \in \mathbb{R}^3 / x+y+z = 0\}$

- 1ª condición = $\vec{u}, \vec{v} \in C \Rightarrow \vec{u} + \vec{v} \in C$

  - $\vec{u} = (x_1, y_1, z_1)$
  - $\vec{v} = (x_2, y_2, z_2)$

$$\vec{u} + \vec{v} = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

$$x_1 + x_2 + y_1 + y_2 + z_1 + z_2 = 0$$

$$\underbrace{x_1 + y_1 + z_1}_{0} + \underbrace{x_2 + y_2 + z_2}_{0} = 0$$

$$0 + 0 = 0$$

- 2ª condición: $\vec{u} \in C, \lambda \in \mathbb{R} \Rightarrow \lambda \vec{u} \in C$

$$\lambda \vec{u} = \lambda (x_1, y_1, z_1) = (\lambda x_1, \lambda y_1, \lambda z_1)$$

$$\lambda x_1 + \lambda y_1 + \lambda z_1 = 0$$

$$\lambda \underbrace{(x_1 + y_1 + z_1)}_{0} = 0$$

$$\lambda \cdot 0 = 0$$

- Como se cumplen las dos condiciones, C es un subespacio vectorial de $\mathbb{R}^3$.

→ **Ejercicio: matriz cambio de base**

- Dadas las bases $B = \{(2,0,1), (0,1,0), (-2,0,4)\}$ y $B' = \{(0,1,5), (2,1,-4), (2,3,1)\}$ halla $M_{B,B'}$:

• $(2,0,1) = \lambda_1 (0,1,5) + \lambda_2 (2,1,-4) + \lambda_3 (2,3,1)$

$$\left.\begin{aligned} 2\lambda_2 + 2\lambda_3 &= 2 \\ \lambda_1 + \lambda_2 + 3\lambda_3 &= 0 \\ 5\lambda_1 - 4\lambda_2 + \lambda_3 &= 1 \end{aligned}\right\} \quad \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & 2 & 2 & 2 \\ 5 & -4 & 1 & 1 \end{pmatrix} \underset{E_3 - 5E_1}{\sim} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & -9 & -14 & 1 \end{pmatrix} \underset{2E_3 + 9E_2}{\sim} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & -10 & 20 \end{pmatrix}$$

$$\left.\begin{aligned} \lambda_1 + \lambda_2 + 3\lambda_3 &= 0 \\ 2\lambda_2 + 2\lambda_3 &= 2 \\ -10\lambda_3 &= 20 \end{aligned}\right\} \begin{aligned} &\longrightarrow \lambda_1 - 2 + (-6) = 0 \rightarrow \lambda_1 + 3 - 6 = 0;\ \boxed{\lambda_1 = 3} \\ &\longrightarrow 2\lambda_2 - 4 = 2;\ \boxed{\lambda_2 = 3} \\ &\longrightarrow -10\lambda_3 = 20;\ \boxed{\lambda_3 = -2} \end{aligned}$$

• $(0,1,0) = \lambda_1 (0,1,5) + \lambda_2 (2,1,-4) + \lambda_3 (2,3,1)$

$$\left.\begin{aligned} 2\lambda_2 + 2\lambda_3 &= 0 \\ \lambda_1 + \lambda_2 + 3\lambda_3 &= 1 \\ 5\lambda_1 - 4\lambda_2 + \lambda_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} &\boxed{\lambda_1 = -1} \\ &\boxed{\lambda_2 = -1} \\ &\boxed{\lambda_3 = -1} \end{aligned}$$

• $(-2,0,4) = \lambda_1 (0,1,5) + \lambda_2 (2,1,-4) + \lambda_3 (2,3,1)$

$$\left.\begin{aligned} 2\lambda_2 + 2\lambda_3 &= -2 \\ \lambda_1 + \lambda_2 + 3\lambda_3 &= 1 \\ 5\lambda_1 - 4\lambda_2 + \lambda_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} &\boxed{\lambda_1 = -1} \\ &\boxed{\lambda_2 = -2} \\ &\boxed{\lambda_3 = 1} \end{aligned}$$

$$M_{B,B'} = \begin{pmatrix} 3 & -1 & -1 \\ 3 & -1 & -2 \\ -2 & -1 & 1 \end{pmatrix}$$

$\longrightarrow$ **Ejercicio: matriz cambio de base**

• Hallar $M_{B,B'}$: $B = \{(-1,-2), (1,5)\}$ $B' = \{(0,-3), (1,-1)\}$

• $(-1,-2) = \lambda_1 (0,-3) + \lambda_2 (1,-1)$

$$\left.\begin{aligned} \lambda_2 &= -1 \\ -3\lambda_1 - \lambda_2 &= -2 \end{aligned}\right\} \quad \begin{aligned} &\boxed{\lambda_2 = -1} \\ &\boxed{\lambda_1 = 1} \end{aligned}$$

• $(1,5) = \lambda_1 (0,-3) + \lambda_2 (1,-1)$

$$\left.\begin{aligned} \lambda_2 &= 1 \\ -3\lambda_1 - \lambda_2 &= 5 \end{aligned}\right\} \quad \begin{aligned} &\boxed{\lambda_2 = 1} \\ &\boxed{\lambda_1 = -2} \end{aligned}$$

$$M_{B,B'} = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$$

→ **Ejercicio : matriz cambio de base**

• Dados los vectores $B = \{(5,-2,-1), (-2,t,3), (1,-1,0)\}$

a) Estudiar para que valores de t B es una base de $\mathbb{R}^3$.

• $rg \begin{pmatrix} 5 & -2 & 1 \\ -2 & t & -1 \\ -1 & 3 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 5 & -2 & 1 \\ -2 & t & -1 \\ -1 & 3 & 0 \end{vmatrix} = -2-6+t+15$

$t+7=0$

$\boxed{t=-7}$

• Si $\boxed{t \neq -7}$ ⟹ los vectores son l. indep, ademas al ser 3 vectores l. indep en $\mathbb{R}^3$ tb son sistema generador de dicho espacio. Por tanto si $t \neq -7$, B es una base de $\mathbb{R}^3$

b) Si $t=0$; calcular $M_{B,B'}$ a la base $B' = \{(4,-3,2), (-1,-1,3), (1,-1,0)\}$

• $(5,-2,-1) = \lambda_1(4,-3,2) + \lambda_2(-1,-1,3) + \lambda_3(1,-1,0)$

$$\left.\begin{aligned} 4\lambda_1 - \lambda_2 + \lambda_3 &= 5 \\ -3\lambda_1 - \lambda_2 - \lambda_3 &= -2 \\ 2\lambda_1 + 3\lambda_2 \quad &= -1 \end{aligned}\right\} \quad \boxed{\lambda_1 = 1} \; \boxed{\lambda_2 = -1} \; \boxed{\lambda_3 = 0}$$

• $(-2,0,3) = \lambda_1(4,-3,2) + \lambda_2(-1,-1,3) + \lambda_3(1,-1,0)$

$$\left.\begin{aligned} 4\lambda_1 - \lambda_2 + \lambda_3 &= -2 \\ -3\lambda_1 - \lambda_2 - \lambda_3 &= 0 \\ 2\lambda_1 + 3\lambda_2 \quad &= 3 \end{aligned}\right\} \quad \boxed{\lambda_1 = 0} \; \boxed{\lambda_2 = 1} \; \boxed{\lambda_3 = -1}$$

• $(1,-1,0) = \lambda_1(4,-3,2) + \lambda_2(-1,-1,3) + \lambda_3(1,-1,0)$

$$\left.\begin{aligned} 4\lambda_1 - \lambda_2 + \lambda_3 &= 1 \\ -3\lambda_1 - \lambda_2 - \lambda_3 &= -1 \\ 2\lambda_1 + 3\lambda_2 \quad &= 0 \end{aligned}\right\} \quad \boxed{\lambda_1 = 0} \; \boxed{\lambda_2 = 0} \; \boxed{\lambda_3 = 1}$$

$$M_{B,B'} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

→ **Suma e intersección s.v. : Ejercicio**

• $A = \{(x,y,z) \in \mathbb{R}^3 \,/\, x=0\}$ → $x=0$ $\quad \begin{aligned} x &= 0 \\ y &= \alpha \\ z &= \beta \end{aligned}$

$A = \{(0, \alpha, \beta) / \alpha, \beta \in \mathbb{R}\}$

- Base de A $= \{(0,1,0),(0,0,1)\}$
- dim A = 2

- B $= \{(x,y,z) \in \mathbb{R}^3 / y-z=0\}$ $\rightarrow y-z=0 \rightarrow z=y$

$y=\alpha$
$z=\alpha$
$x=\beta$

$B = \{(\beta, \alpha, \alpha) / \alpha, \beta \in \mathbb{R}\}$

$\left.\begin{matrix} \alpha = 1 \\ \beta = 0 \end{matrix}\right\} (0,1,1)$ $\quad$ $\left.\begin{matrix} \alpha = 0 \\ \beta = 1 \end{matrix}\right\} (1,0,0)$

- Base de B $= \{(0,1,1),(1,0,0)\}$
- dim B = 2

$\hookrightarrow$ $\boxed{\text{Suma}}$ $A + B = \mathbb{R}^3$

- $rg \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} = 3$ $\quad$ $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = -1 \neq 0$

- $\dim(A+B) = \dim A + \dim B - \dim(A \cap B)$

$3 = 2 + 2 - \dim(A \cap B)$

$\hookrightarrow$ $\boxed{\text{Intersección:}}$

$\left.\begin{matrix} x = 0 \\ y-z=0 \end{matrix}\right\} \rightarrow z = y$ $\quad$ $y=\alpha$, $z=\alpha$, $x=0$

- $A \cap B = \{(0, \alpha, \alpha) / \alpha \in \mathbb{R}\}$ $\quad$ $A \cap B = \{(x,y,z) \in \mathbb{R}^3 / \begin{matrix} x=0 \\ y-z=0 \end{matrix}\}$

- Base $(A \cap B) = \{(0,1,1)\}$
- $\dim(A \cap B) = 1$

- A, B en este caso no son suma directa

$\rightarrow$ $\boxed{\text{Ejercicio: matriz asociada}}$

- Calcular la m. asociada a la aplicación $f(x,y) = (2y-x, x+3y, 2x-y)$ respecto a las bases $B = \{(-1,1),(1,0)\}$ y $B' = \{(-1,1,2),(0,1,0),(3,2,-3)\}$

$\mathbb{R}^2 \rightarrow \mathbb{R}^3$

- $f(-1,1) = (3,2,-3)$ $\quad$ - $f(1,0) = (-1,1,2)$

- $(3,2,-3) = \lambda_1(-1,1,2) + \lambda_2(0,1,0) + \lambda_3(3,2,-3)$

$\boxed{\lambda_1 = 0}$ $\boxed{\lambda_2 = 0}$ $\boxed{\lambda_3 = 1}$

NOTA: porq el vector $(3,2,-3)$ es $= \lambda_3$

- $(-1,1,2) = \lambda_1(-1,1,2) + \lambda_2(0,1,0) + \lambda_3(3,2,-3)$

$\boxed{\lambda_1 = 1}$ $\boxed{\lambda_2 = 0}$ $\boxed{\lambda_3 = 0}$

NOTA: porq el vector $(-1,1,2)$ es $= \lambda_1$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \rightarrow \text{matriz asociada}$$

→ Ejercicio: Dada la aplicación lineal $f: \mathbb{R}^2 \to \mathbb{R}^3$ definida por $f(x,y) = (0, 2x+2y, y-x)$

a) Hallar la m. asociada a f respecto a las bases canónicas:

- $f(1,0) = (0,2,-1)$
- $f(0,1) = (0,2,1)$

$$A = \begin{pmatrix} 0 & 0 \\ 2 & 2 \\ -1 & 1 \end{pmatrix} \rightarrow \text{matriz asociada}$$

b) Dadas las bases $B = \{(2,1),(1,-1)\}$ y $B' = \{(1,0,1),(0,1,-1),(1,1,1)\}$ de $\mathbb{R}^2$ y $\mathbb{R}^3$ respectivamente determinar la matriz asociada a la aplicación f respecto a dichas bases:

- $f(2,1) = (0,6,-1)$
- $f(1,-1) = (0,0,-2)$

$$A = \begin{pmatrix} -5 & 2 \\ 1 & 2 \\ 5 & -2 \end{pmatrix} \rightarrow \text{m. asociada}$$

- $(0,6,-1) = \lambda_1(1,0,1) + \lambda_2(0,1,-1) + \lambda_3(1,1,1)$

$$\left.\begin{aligned} \lambda_1 \quad\quad + \lambda_3 &= 0 \\ \lambda_2 + \lambda_3 &= 6 \\ \lambda_1 - \lambda_2 + \lambda_3 &= -1 \end{aligned}\right\}$$

$\boxed{\lambda_1 = -5}$ $\boxed{\lambda_2 = 1}$ $\boxed{\lambda_3 = 5}$

- $(0,0,-2) = \lambda_1(1,0,1) + \lambda_2(0,1,-1) + \lambda_3(1,1,1)$

$$\left.\begin{aligned} \lambda_1 \quad\quad + \lambda_3 &= 0 \\ \lambda_2 + \lambda_3 &= 0 \\ \lambda_1 - \lambda_2 + \lambda_3 &= -2 \end{aligned}\right\}$$

$\boxed{\lambda_1 = 2}$ $\boxed{\lambda_2 = 2}$ $\boxed{\lambda_3 = -2}$