University of Zurich[UZH]

Complexity counts: the unusual case of Indo-Aryan numeral systems

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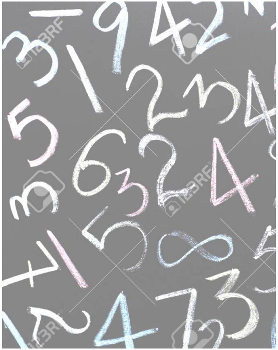
Numeral systems and numerical cognition

- Human languages share the property of referring to different quantities
- Numeral systems (i.e., elements referring to quantities) are related to this species-wide cognitive capacity (Xu and Regier 2014, Núñez 2017)
- Major question: what can be expressed (e.g., exact versus approximate systems)?
- Related question: how are quantities realized formally?



Numeral terms: word structure

- Numeral systems of the world's languages tend to consist of words which denote information regarding the tens and digits place in a transparent fashion
- E.g., Sanskrit numerals 1–49 (rows represent the tens place; columns represent the digits place)

	0	1	2	3	4
0	—	eka	dva	tri	catur
10	daśa	ekādaśa	dvādaśa	trayodaśa	caturdaśa
20	viṃśati	ekaviṃśati	dvaviṃśati	trayoviṃśati	caturviṃśati
30	triṃśat	ekatriṃśat	dvātriṃśat	trayastriṃśat	catustriṃśat
40	catvāriṃśat	ekacatvāriṃśat	dvacatvāriṃśat	trayaścatvāriṃśat	catuscatvāriṃśat

	5	6	7	8	9
0	pañca	ṣaṣ	sapta	aṣṭa	nava
10	pañcadaśa	ṣoḍaśa	saptadaśa	aṣṭādaśa	navadaśa
20	pañcaviṃśati	ṣaḍviṃśati	saptaviṃśati	aṣṭaviṃśati	navaviṃśati
30	pañcatriṃśat	ṣaṭtriṃśat	saptatriṃśat	aṣṭatriṃśat	navatriṃśat
40	pañcacatvāriṃśat	ṣaṭcatvāriṃśat	saptacatvāriṃśat	aṣṭacatvāriṃśat	navacatvāriṃśat

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Ca. 2500 years later...

Hindi/Urdu numerals 1–99

	0	1	2	3	4	5	6	7	8	9
0	—	ek	do	tin	cɑr	pɑ̃c	c^{h}ɛ	sɑt	ɑʈ^{h}	nɔ
10	dəs	gjɑrə	bɑrə	terə	cɔdə	pəndrə	solə	sətrə	əʈ^{h}ɑrə	ʊnnis
20	bis	ɪkkis	bɑis	teis	cɔbis	pəccis	c^{h}əbbis	səttɑis	əʈʈɑis	ʊntis
30	tis	ɪkəttis	bəttis	tæ̃tis	cɔ̃tis	pæ̃tis	c^{h}əttis	sæ̃tis	əɽtis	ʊntɑlis
40	cɑlis	ɪktɑlis	bəjɑlis	tæ̃tɑlis	cəʋɑlis	pæ̃tɑlis	c^{h}ɪjɑlis	sæ̃tɑlis	əɽtɑlis	ʊncɑs
50	pəcɑs	ɪkjɑʋən	bɑʋən	tɪrpən	cəuʋən	pəcpən	c^{h}əppən	səttɑʋən	əʈʈ^{h}ɑʋən	ʊnsəʈ^{h}
60	sɑʈ^{h}	ɪksəʈ^{h}	bɑsəʈ^{h}	tɪrsəʈ^{h}	cɔ̃səʈ^{h}	pæ̃səʈ^{h}	c^{h}ɪjɑsəʈ^{h}	sərsəʈ^{h}	əɽsəʈ^{h}	ʊnhəttər
70	səttər	ɪkhəttər	bəhəttər	tɪhəttər	cɔhəttər	pəchəttər	c^{h}ɪhəttər	səthəttər	əʈ^{h}həttər	ʊnjɑsi
80	əssi	ɪkjɑsi	bəjɑsi	tɪrɑsi	cɔrɑsi	pəcɑsi	c^{h}ɪjɑsi	səttɑsi	əʈʈ^{h}ɑsi	nəʋɑsi
90	nəʋe	ɪkjɑnʋe	bɑnʋe	tɪrɑnʋe	cɔrɑnʋe	pəcɑnʋe	c^{h}ɪjɑnʋe	səttɑnʋe	əʈʈ^{h}ɑnʋe	nɪnjɑnʋe

- Hindi/Urdu representative of Indo-Aryan numeral systems
- High integrative complexity (Ackerman and Malouf 2013): hard to predict forms on the basis of each other
- Developed out of the relatively transparent Sanskrit numeral system

Indo-Aryan numeral systems

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- Cathcart (2017): computational models generally capable of classifying HU numerals on basis of phonological cues
- Schneider et al. (2020): children acquiring Hindi and Gujarati are less able to rely on successor functions in counting than children acquiring other languages

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- I aim to demonstrate that Indo-Aryan numeral systems exhibit higher integrative complexity
- I will show that Indo-Aryan numeral systems are subject to some of the same general communicative pressures as other systems
- Zooming in on South Asia, I will attempt to highlight factors that are involved in the maintenance and loss of high-complexity numeral systems

Large data sets and quantitative metrics provide new insights on these issues

Data used

UniNum (Ritchie et al. 2019)

- Collection of numerals ranging between 0 and 100000000000 (inclusive), provided by Google and language experts.
- Curated for text-to-speech purposes
- 186 speech varieties
- Representations are orthographic, not phonemic

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Eugene Chan's numeral data set (Chan et al. 2019)

- Collection of numerals 1–29, 30, 40, 50, 60, 70, 80, 90, 100, 200, 1000, 2000
- 5352 speech varieties
- Phonemic representations

Metrics used: Minimum description length (MDL)

- Information theoretic principle: seeks the shortest set of combinable elements needed to generate a code (Rissanen 1983)
- Can be inferred using several Bayesian algorithms (e.g., Goldwater et al. 2009)
- Simpler numeral systems have shorter descriptions

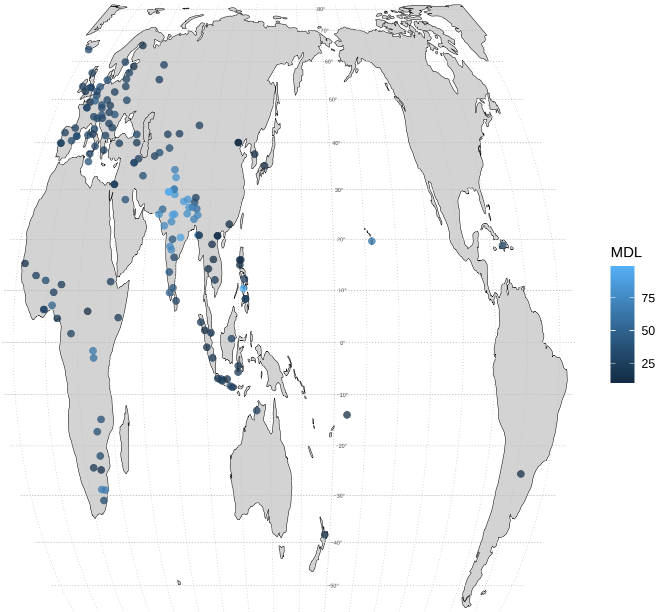
Metrics used: Phonotactic surprisal

- Represents the unpredictability of a phoneme or grapheme in context, i.e., given the two previous phonemes/graphemes (Piantadosi et al. 2012, Dautriche et al. 2017)
- Numeral systems containing more recurrent, predictable elements will exhibit lower surprisal

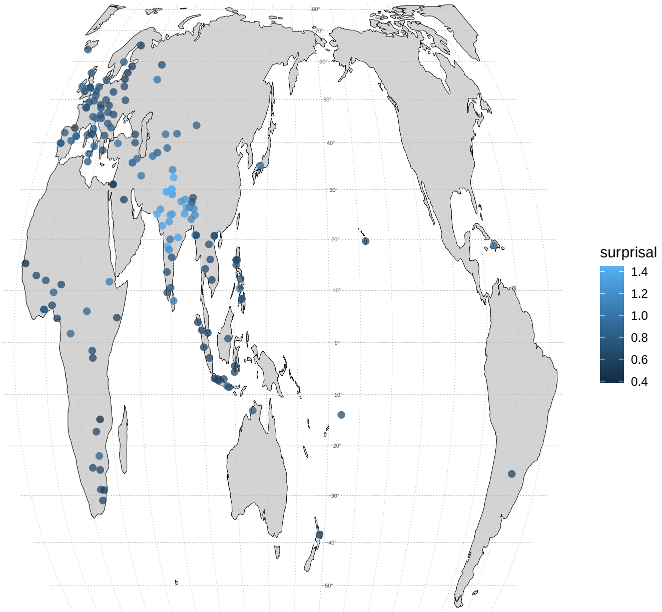
Metrics used: Linear discriminative learnability (LDL)

- Framework which learns mappings between semantic and phonological cues, e.g., trigram sequences of sounds (Baayen et al. 2018)
- Can be used to predict forms from semantic cues (e.g., TWENTY + ONE $\rightarrow$ *twenty-one*)

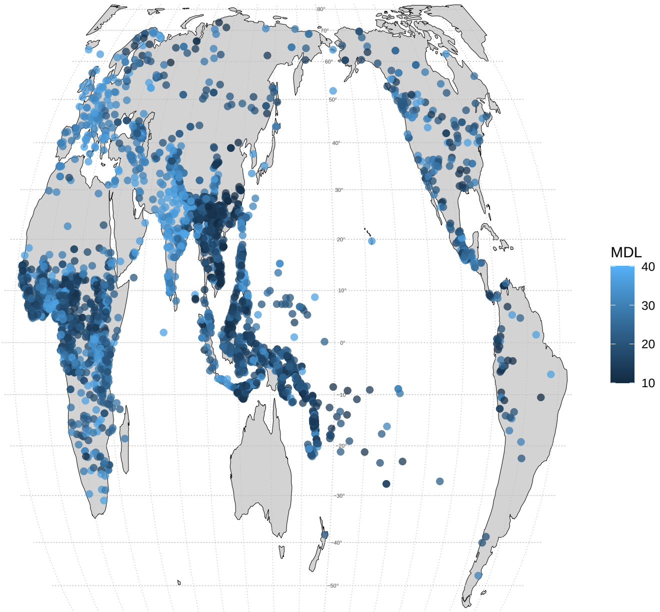
UniNum, minimum description length



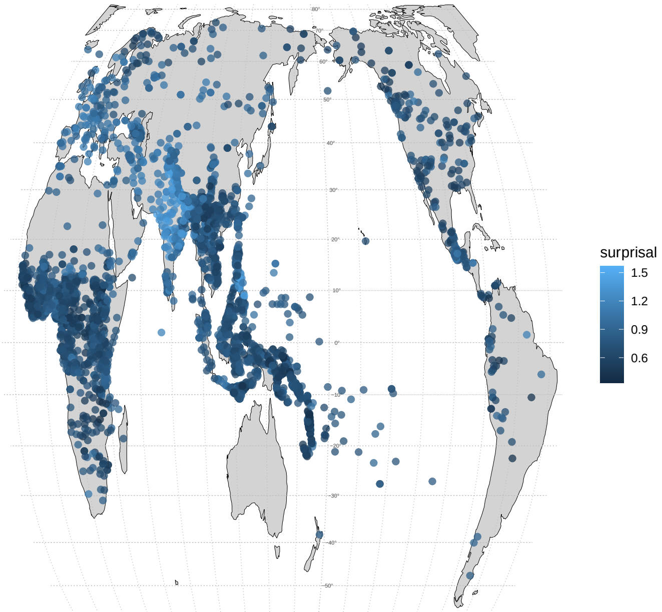
UniNum, phonotactic surprisal



Chan numerals, minimum description length



Chan numerals, phonotactic surprisal



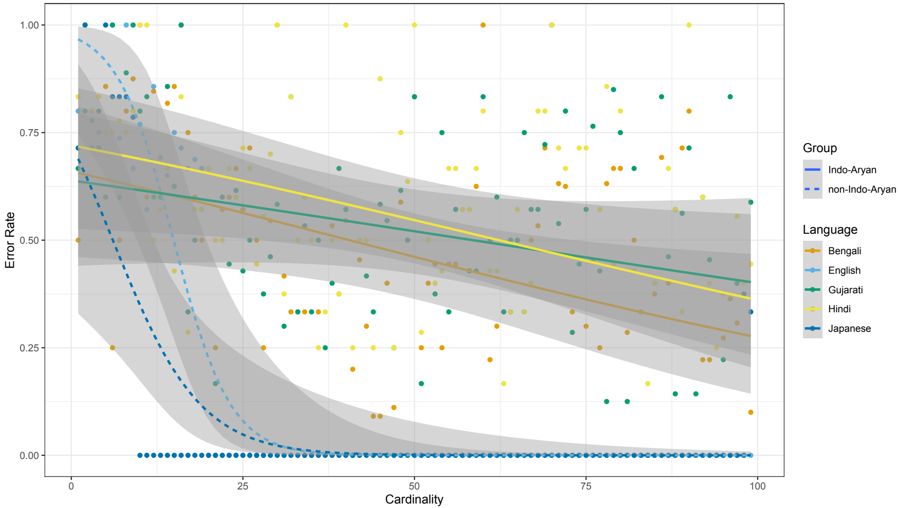
Interim summary

- Different metrics capture different properties of numeral systems (e.g., min. desc. length may be sensitive to mixed systems, etc., as well as integrative complexity)
- Small sample sizes in the Chan data set may cause issues in situating languages according to metrics
- At the same time, South Asia consistently emerges as a hotbed of complexity for all metrics and data sets considered

Transparency and frequency in numeral systems

- In morphological systems (e.g., noun, verb paradigms), less frequent forms tend to be more regular and transparent (Blevins et al. 2017), e.g., *go* $\sim$ *went* vs. *ambulate* $\sim$ *ambulated*
- Same principles may apply to numeral systems: e.g., English *twelve* is more frequent than *ninety-nine* (Brysbaert 2005)
- If this is a universal principle underlying numeral systems, then Indo-Aryan systems should exhibit higher predictability for items of higher cardinality (and lower frequency; Dehaene and Mehler 1992)
- Predictive accuracy modeled via linear discriminative learning (error rate)

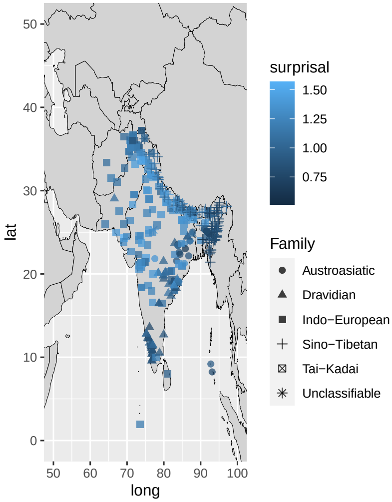
Cardinality vs. predictability



Predictive errors decrease as cardinality increases in all languages; trend is more gradual for Indo-Aryan languages

Numeral complexity in South Asia

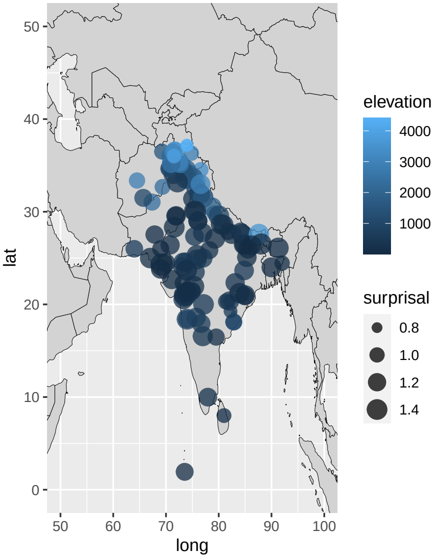
- S. Asia is a hotbed of enumerative complexity in numeral systems
- At the same time, there is variation in the degree of complexity exhibited by individual languages, within and across families
- Complexity is a largely Indo-Aryan phenomenon; complex systems in non-IA languages are often due to borrowing from IA

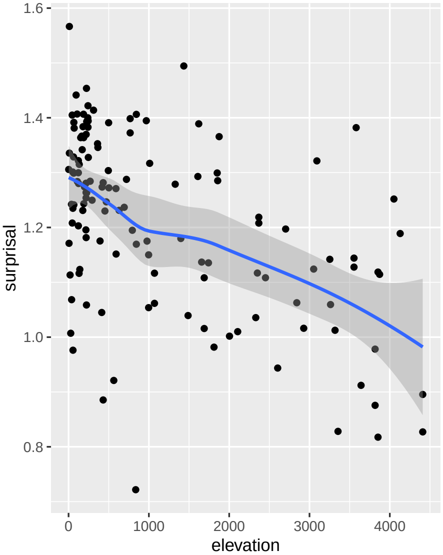


Origins and maintenance of IA numeral complexity

- Unusual properties of Indo-Aryan languages (e.g., retroflexion) are often ascribed to language contact (e.g., with Dravidian, Hock 1996) — impossible here
- Somewhat controversial ideas relate some grammatical properties of South Asian languages to social hierarchy (Emeneau 1974)
- It is possible that some form of social pressure led to the emergence and maintenance of complexity in the core area, which was lost in more peripheral areas
- I focus on two variables: altitude and vigesimality (i.e., presence of a base-20 system)

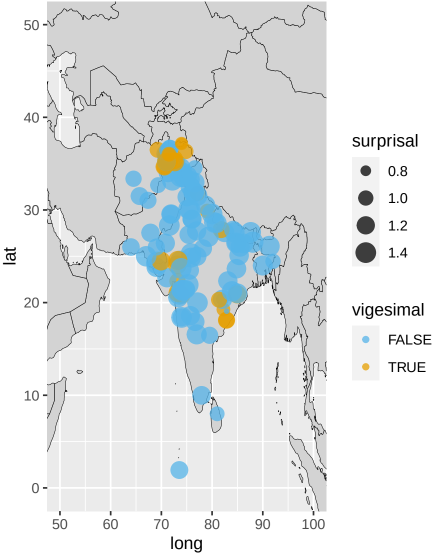
Altitude vs. complexity (Indo-Aryan)

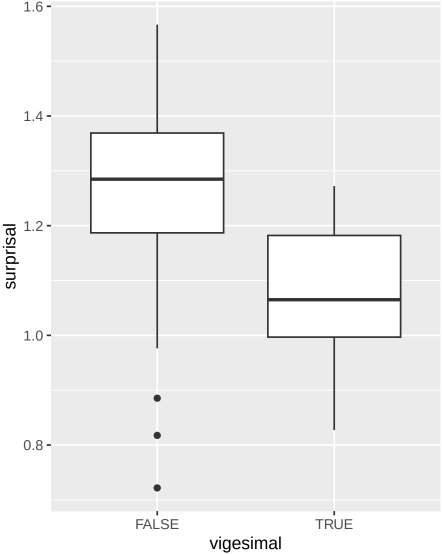




Complexity decreases as altitude increases

Vigesimality vs. complexity (Indo-Aryan)





Languages which have developed base-20 systems have lower complexity

- Higher altitude involves greater isolation, thought to foster complexity (Urban 2020)
- However, here, higher altitude coincides with simplification
- We are seeing the effects of at least two networks of language contact
  - Complexity is maintained in the core Indo-Aryan area
  - Higher-altitude languages of the Hindu Kush area (Weinreich 2015, Liljegren 2020) shifted to a vigesimal system, which involved simplification
- In line with the idea that properties of language involved in counting (e.g., numeral classifiers) are sensitive to contact (Grinevald 2000, Allassonnière-Tang et al. 2021)

Conclusion and outlook

- South Asia is a hotbed of integrative complexity in numeral systems; this is borne out by a number of metrics
- This is a largely Indo-Aryan phenomenon
- Despite this complexity, IA numeral systems appear to obey general principles of efficient communication: higher numbers are easier to realize and recognize
- Emergence seems to be due largely to local historical contingencies
- Social networks and networks of contact may be responsible for maintenance in the IA core area

Future directions: information regarding usage needed to better understand how such systems are maintained

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