Problem Set 7

1. Consider the following three stocks.

	Expected return	Standard deviation
A	5%	10%
B	20%	40%
C	20%	40%

The return on stock C is not correlated with A and B. Correlation of returns between A and B is -0.5.

a Suppose that an investor allocates 40% of her wealth to stock A, 60% to stock C, and nothing to stock B. What is the standard deviation and expected return of her portfolio?

b Suppose that an investor allocates 40% of her wealth to stock A, 60% to stock B, and nothing to stock C. What is the standard deviation and expected return of her portfolio?

c Suppose that an investor constructs an equally weighted portfolio out of stocks A, B, and C. What is the standard deviation and expected return of her portfolio?

2. Suppose there are 2 stocks in the market.

	Expected return	Standard deviation
A	5%	10%
C	20%	40%

The returns on stocks A and C are not correlated.

a Build a portfolio with an expected return of 10%

b Suppose that the maximum risk you can afford to take is 20%. Build a portfolio that achieves this target. What is the expected return of this portfolio?

c Suppose that there is an investor with low tolerance for risk. Construct a minimum variance portfolio (MPV) for this investor. What is the expected return of this minimum variance portfolio?

3. Use the same data from 2. In addition, there is a riskless asset with guaranteed return of 3%. Suppose there are two investors, X and Z, who maximize risk-return tradeoff and have mean-variance preferences. Investor X can only invest in stocks A and C. Investor Z, in addition to being able to invest in stocks A and C, can also invest in the riskless bond. Investor X targets standard deviation of 20% and investor Z targets standard deviation of 12%.

a What is the maximum Sharpe ratio investor X can achieve?

b What is the maximum Sharpe ratio investor Z can achieve?

c Describe the portfolio of investor Z. What is the expected return of this portfolio?

d If investor X had access to the risk-free asset, could her return have been improved? If yes, compute the expected return for a 20% target level of risk.

5. Suppose that the expected return on the market portfolio is 14% and the standard deviation of its returns is 25%. The risk-free rate is 6%. Consider a portfolio with expected return of 16% and assume that it is located on the Capital Market Line (CML).

a What is the beta of this portfolio?

b What is the standard deviation of this portfolio?

c What is its correlation with the market?

6. Suppose an endowment fund is currently investing 20% of its portfolio in a risk-free asset and investing the remaining 80% with a private equity fund, YZW LLC. The endowment fund analyzed holdings of YZW's portfolio and found that its expected returns over the next year will be 10%, with a standard deviation of 25%.

Research conducted by the endowment fund indicates that the market portfolio will have expected return of 10% and standard deviation of 20% during the next year. The correlation of returns between YZW's portfolio and the market portfolio is 0.8. Suppose that the risk-free asset will yield 5% return over the next year.

a Compute the CAPM beta of YZW's portfolio.

b Compute the CAPM beta and alpha of the endowment fund's portfolio.

c Suppose that you were retained as an advisor to the endowment fund. Would you recommend divesting part of the holdings in YZW and reallocating them to the market portfolio? (Hint: compare Sharpe ratios)

7. Suppose company XYZ is a publicly traded firm. Its current share price is \$22.5. The total number of outstanding shares is 20 million. XYZ has 525,000 bonds outstanding, each with a face value of \$100, maturing one year from now. The bonds do not pay a coupon and are considered risk-free. XYZ's stock has a market beta of 0.8. The expected market risk premium is 6%, and the risk-free rate is 5%.

a What is the current market value of the firm?

b What is the market beta of the firm?

c Suppose that the beta of the firm's assets and the expected interest rates are constant over the next two years. What is the expected price of the stock in two years if it does not pay dividends? Assume that the performance of the company in Year 1 does not predict its performance in Year 2. (Hint: the firm pays off its current debt after one year.)

d If the firm issues 551,250 2-year zero-coupon bonds with face value \$100 to buy back its stock, will it change the equity beta of the firm? If yes, compute the new equity beta. Suppose that investors consider these new bonds risk-free. (Hint: the firm's asset beta is not affected by capital structure)

8. Consider a two-factor model: $r_i = E[r_i] + \beta_{i,1} f_1 + \beta_{i,2} f_2 + \epsilon_i$ and $E[f_1] = 0, E[f_2] = 0, Cov(f_1, f_2) = 0$ Assume that the standard deviation of Factor 1 is 7% and that of Factor 2 is 5%. There are two stocks with the following properties (loadings mean betas):

	Expected return	Load on Factor 1	Load on Factor 2
A	10%	1	1.5
B	10%	2	3

Assume that the idiosyncratic component of returns of stocks A and B is uncorrelated with each other, as well as with both factor returns.

a Compute standard deviation of returns on Stocks A and B.

b Compute the correlation between returns on Stock A and Stock B.

9. Consider a two-factor model: $r_i = E[r_i] + \beta_{i,1} f_1 + \beta_{i,2} f_2 + \epsilon_i$. There are three well-diversified portfolios with the following properties:

	Expected return	Load on Factor 1	Load on Factor 2
A	8%	0.95	1.15
B	6%	0.85	0.70
C	10.5%	1.20	1.50

Assume that the current risk-free rate is 1.5%. Construct an arbitrage strategy that generates $1,000 today and zero payoff in the future. (Hint: Use the Arbitrage Pricing Theory. You don't want to take any factor risk.)

10. Consider a three-factor model: $r_i = E[r_i] + \beta_{i,1} f_1 + \beta_{i,2} f_2 + \beta_{i,3} f_3 + \epsilon_i$. There are four well-diversified portfolios with the following properties:

	Expected return	Load on Factor 1	Load on Factor 2	Load on Factor 2
A	21.65%	0.95	1.20	0.85
B	19.23%	1.00	0.75	1.35
C	21.48%	1.10	1.05	0.95
D	23.15%	1.25	1.05	1.15

Compute factor risk premia (also known as factor risk prices) as well as the risk-free rate.

11. Download the data contained in the file posted on course webpage. The data contains historical monthly returns on eight firms from January 2005 through December 2019. Four of these firms are from the gold mining industry. And the other four are from the technology industry.

We will consider a two-factor model that has the market and the price of gold as the risk. To proxy those factors, we will use factor-mimicking portfolios: For the market factor, we will use the returns on the market portfolio, and for the price of gold, we will use the returns on the Spider Gold Shares ETF. The data with the returns of those factors and the risk-free rates are also in the file.

The following equation describes the two-factor model:

$$r_i - r_f = \alpha_i + \beta^i_{mkt}(r_{mkt} - r_f) + \beta^i_{gld}(r_{gld} - r_f)$$

a Using the OLS regression over the full sample, estimate factor loadings, β_{mkt}, β_{gld}, and the alphas for each of the eight stocks.

  Report, separately, the arithmetic average of the factor loadings and alphas for the gold mining stocks and for the technology stocks.

b Assume that the returns on all stocks satisfy the exact 2 -factor APT model with the stock market and the gold price as the two factors. Take your estimated factor loadings as exact. Suppose you are given a forecast for the expected excess monthly returns going forward: 0.75% for the stock market portfolio, and 0.15% for the gold price. Using the average factor loadings you've computed in part (a), compute the expected excess return for the gold Mining sector, and the technology sector. Report both numbers as monthly, in %.

(Solution is provided in the file.)