Introduction to Derivatives

BUSS284 Investments

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1. The current level of the S&P500 index is 3000, the futures price of a contract on S&P500 maturing 3 months from now is $F = 3009$. The futures contract is settled in cash with the payout at maturity equal to the closing value of the S&P500 index on that day. The 1-year continuously compounded spot risk-free rate is $r = 2.7\%$. What value of the S&P500 dividend yield is implied by this data? Use continuous compounding for the dividend yield.

Solution:

$F_0 = S_0 e^{(r-d)T} \rightarrow 3,009 = 3,000e^{(2.7\%-d)(1/4)}$. Therefore, $d = 2.7\% - 4 \times ln(3,009/3,000) = 1.5\%$.

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2. The 1-year risk-free interest rate of investments in US dollars is $r_{USD} = 1.22\%$. The 1-year risk-free interest rate of investments in Canadian dollars is $r_{CAD} = 4.00\%$. Use annual compounding. The current (spot) exchange rate between the two currencies is 1.41, i.e., the price of 1 USD is 1.41 CAD. The 1-year forward price of 1 USD is 1.41 CAD. You can trade in 1-year risk-free bonds denominated in both US and Canadian dollars, in the forward contract to buy 1 USD 1 year from now, and in the spot foreign exchange market, where you can buy and sell USD. Consider the following strategy:

   a. Borrow x USD at 1.22% today, which means that the total loan repayment obligation after a year would be $(1 + 1.22\%)x$ USD.

   b. Convert y USD into CAD at the spot rate of 1.41.

   c. Lock in the 4.00% rate on the deposit amount of $1.41y$ CAD, and simultaneously enter into a forward contract that converts the full maturity amount of the deposit into USD at the one-year forward rate of USD = 1.41 CAD.

   d. After one year, settle the forward contract at the contracted rate of 1.41. Suppose the above arbitrage strategy generates 100 USD today and nothing otherwise, solve for x and y.

Solution:

$x = 3,741.01, y = 3641.01$ from below:

	0	1
Borrow x USD	+x (USD)	-(1.0122)x (USD)

	0	1
Convert y USD	-y (USD) = -1.41y (CAD)	+1.41y(1.04)/1.41 (USD)
Total	100	0

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1. Suppose that on Day 0 you take a long position in a futures contract on copper maturing on Day 3. Each contract is on 2,000 kilograms of copper, with the closing futures price on Day 0 equal to 94 per kilogram. Suppose the closing futures prices on Day 1, 2, and 3 are 104, 98, 101 respectively. The futures contract is marked to market daily, at the end of the trading day, with resulting gains and losses settled using a margin account. Compute the cash flow into/from the margin account resulting from marking to market a long position in 1 futures contract at the end of Day 2.

Solution:

$2,000(98 - 104) = -12,000$

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4. Suppose the 1-month and 3-month future prices for oil are $F_1 = 36.1$ and $F_3 = 35.7$ per barrel. Assume the spot interest rates are the same for the first three months and equal to $r = 2.7\%$ (annualized, continuously compounded). What is the net convenience yield on storing oil implied by the market data?

Solution:

$F_1=S_0 e^{(2.7%-y)(1/12)} $ and $F_3 = S_0 e^{(2.7\% - y)(3/12)}$. Hence, $ln(F_3/F_1) = e^{(2.7\% - y)(2/12)}$. $y = 2.7\% - 6ln(35.7/36.1) = 9.385\%$

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5. Consider a European Call option of stock XYZ. The current price of the option is 7.89. This option has 6 months to maturity and the strike price is 100. Currently, the price of XYZ stock is 70. The 6-month interest rate (annualized, continuously compounded) is 5%. Compute the net payoff, considering the time value of money, to the buyer of the option at the expiration of the options, assuming that:

   a. the price of XYZ at maturity is 125

   b. the price of XYZ at maturity is 80

Solution:

a) $max(125 - 100, 0) - 7.89e^{0.05 \times 6/12} = 16.91$, b) -8.09

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6. Consider a stock, XYZ, which pays no dividends over the next year. The current stock price of XYZ is 105. You observe prices of two European options, both maturing in one year from now. One is a European call option with the strike price of 130, which currently trades at 17.

The other is a European put option with the strike price of 130, which currently trades at 40. What value of the one-year risk-free interest rate (continuously compounded) is consistent with absence of arbitrage?

Solution:

Put-call parity, $p + S_0 = c + Ke^{-rT}$. Hence, $r = 1.55\%$

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7. Stock AAA is currently traded at 170 per share. The stock price will either go up or go down by 10% in each of the next two years. The annual interest rate is 1%. Assume the stock pays no dividends the next two years. Bank XYZ issues a two-year European call option on the stock with the strike price 180.

   a. Suppose the price goes up at Year 1. How many shares of the stock and cash do you need to replicate the payoff of the call option? (in Year 1)

   b. Suppose the price goes down at Year 1. What is the price of the option?

   c. What is the price of the option at Year 0?

Solution:

a. Cash=-115.65 (borrow), Shares=0.687

b. The stock price is \$153.0 and it can go to \$168.3 or \$137.7. Hence, the payoff of the option will be \$0 in both following states. The value of the option at Year 1 is \$0.

c. u=1.1, d=0.9, p=0.55, f(uu)=25.7, and f(ud)=f(du)=f(dd)=0. Hence, the option price is \$7.62

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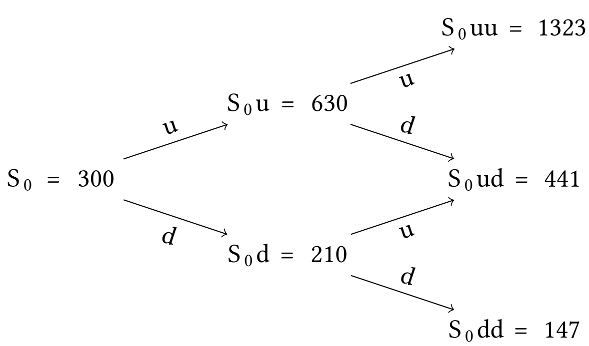
8. Consider a stock paying no dividends. Price movements follow the binomial model with the following tree. Interest rate is 5% per period.



a. What is the risk-neutral probability of the up-move at the time-0 node?

b. Using risk-neutral probabilities, compute the value of an option, which pays 1 at the end of the second period if the stock price at that time exceeds the initial price at time 0, and nothing otherwise.

Solution:

a. u=2.1, d=0.7, and p=0.25. f(uu)=f(ud)=1 and f(dd)=0. Hence, the option price = 3.97.

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9. All assumptions of the Black-Scholes-Merton option pricing model hold. Stock XYZ is priced at 50. It has volatility 30 per year. The annualized continuously-compounded risk-free interest rate is 1.6.

   a. Compute the price of a European call option with strike price 51, which matures in 3 months.

   b. Compute the price of a European put option with the same strike price and the same maturity date using the put-call parity.

Solution:

a. d1=-0.0303, d2=-0.1803, N(d1)=0.4878, and N(d2)=0.4284. Hence, c=2.631

b. Using c + K*exp(-rT) = p + S0, p=3.427

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10. All Black-Scholes-Merton assumptions hold. Stock XYZ is priced at 50. It has volatility 25 per year. The annualized continuously-compounded risk-free interest rate is 2.5 and will remain constant for the next 6 months. Consider two European options: a call with the strike price of 55, which matures in 3 months; and a put with the strike price of 50, which matures in 6 months. Which of the following statements is true?

    a. The call has higher implied volatility than the put.
    b. The put has higher implied volatility than the call.
    c. The call has the same implied volatility as the put.

Solution:

Under the BSM model, implied volatility of the option equals the physical volatility of the stock return. In our case, this implies that both options have the same implied volatility, equal to 25%