Problem Set

1 Delta

What does it mean to assert that the delta of a call option is 0.7? How can a short position in 1,000 options be made delta neutral when the delta of each option is 0.7?

2 Delta

Calculate the delta of an at-the-money six-month European call option on a non-dividend-paying stock when the risk-free interest rate is 10% per annum and the stock price volatility is 25% per annum. Use the BSM model.

3 Portfolio Insurance

Why did portfolio insurance not work well on October 19, 1987?

4 Delta of Futures Option

What is the delta of a short position in 1,000 European call options on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine-month futures price is $8 per ounce, the exercise price of the options is $8, the risk-free interest rate is 12% per annum, and the volatility of silver futures prices is 18% per annum. Hint: Use Black's model.

Extra: If you want to hedge the option position using the underlying asset (i.e., silver), how many units of silver do you need? (Assume no storage cost for silver)

5 Delta and Gamma

A company uses delta hedging to hedge a portfolio of long positions in put and call options on a currency. Which of the following would give the most favorable result?

1. A virtually constant spot rate
2. Wild movements in the spot rate

Explain your answer.

6 Gamma and Vega

Under what circumstances is it possible to make a European option on a stock index both gamma neutral and vega neutral by adding a position in one other European option with the same underlying asset but different K and T? Use the BSM model.

7 The Put-Call Parity and Greeks

Use the put-call parity relationship to derive, for a non-dividend-paying stock, the relationship between:

1. The delta of a European call and the delta of a European put.
2. The gamma of a European call and the gamma of a European put.
3. The vega of a European call and the vega of a European put.
4. The theta of a European call and the theta of a European put.

8 Hedging using Greeks

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of Option	Gamma of Option	Vega of Option
Call	$-1,000$	0.5	2.2	1.8
Call	-500	0.8	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

1. What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?
2. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

9 Portfolio Insurance

Suppose that \$70 billion of equity assets are the subject of portfolio insurance schemes. Assume that the schemes are designed to provide insurance against the value of the assets declining by more than 5% within one year. Suppose $S_0 = 70$, $K = 66.5$, $T = 1$. Other parameters are estimated as $r = 0.06$, $\sigma = 0.25$, and $q = 0.03$. Calculate the value of the stock or futures contracts that the administrators of the portfolio insurance schemes will attempt to sell if the market falls by 23% in a single day.

10 Hedging with Delta and Gamma

A bank's position in options on the dollar-euro exchange rate has a delta of 30,000 and a gamma of $-80,000$. Explain how these numbers can be interpreted. The exchange rate (dollars per euro) is 0.90. What position would you take to make the position delta neutral? After a short period of time, the exchange rate moves to 0.93. Estimate the new delta. What additional trade is necessary to keep the position delta neutral? Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange-rate movement?

11 Implied Volatility

A stock price is currently \$50 and the risk-free interest rate is 5%. Compute implied volatilities, assuming no dividends. Are the option prices consistent with the assumptions underlying Black-Scholes-Merton?

Stock Price	Maturity = 3 months	Maturity = 6 months	Maturity = 12 months
45	7.00	8.30	10.50
50	3.50	5.20	7.50
55	1.60	2.90	5.10

12 Implied Volatilities of Calls and Puts

A European call and put option have the same strike price and time to maturity. The call has an implied volatility of 30% and the put has an implied volatility of 25%. What trades would you do?

13 Implied Volatilities of Calls and Puts

The market price of a European call is \$3.00 and its price given by Black-Scholes-Merton model with a volatility of 30% is \$3.50. The price given by this Black-Scholes-Merton model for a European put option with the same strike price and time to maturity is \$1.00. What should the market price of the put option be? Explain the reasons for your answer.

14 OTM Options and Volatility

Option traders sometimes refer to deep-out-of-the-money options as being options on volatility. Why do you think they do this?

15 Practitioners' Approach

Using the table below, calculate the implied volatility a trader would use for an 8-month option with $K/S_0 = 1.04$.

	K/S_0				
	0.90	0.95	1.00	1.05	1.10
1 month	14.2	13.0	12.0	13.1	14.5
3 month	14.0	13.0	12.0	13.1	14.2
6 month	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0