Problem Set

1 Option Greeks

A financial institution has just sold 1,000 7-month European call options on the Japanese yen. Suppose that the spot exchange rate is 0.80 cent per yen, the exercise price is 0.81 cent per yen, the risk-free interest rate in the United States is 8% per annum, the risk-free interest rate in Japan is 5% per annum, and the volatility of the yen is 15% per annum. Calculate the delta, gamma, vega, theta, and rho of the financial institution's position. Interpret each number.

2 Portfolio Insurance

A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth $360 million. The value of the S&P 500 is 1,200, and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next 6 months. The risk-free interest rate is 6% per annum. The dividend yield on both the portfolio and the S&P 500 is 3%, and the volatility of the index is 30% per annum.

1. If the fund manager buys traded European put options, how much would the insurance cost?
2. Create alternative strategies involving traded European call options, and show that they lead to the same result.

3 Risk Management

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of option	Gamma of option	Vega of option
Call	$-1,000$	0.50	2.2	1.8
Call	-500	0.80	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

1. What position in the traded option and in sterling (the underlying asset) would make the portfolio both gamma neutral and delta neutral?
2. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

4 The Black's Model

Consider a futures. Its current price is \$70.00 and its volatility is 16.70% per annum. Suppose the risk-free interest rate is 5.00% per annum (continuous compounding). Use the Black's model to calculate the value of a five-month European put on the futures with a strike price of \$65.00.

5 Implied Volatility

On November 4, 2021, the following options data (379 days until the maturity) was available on the S&P500. Assume that the index is at \$4670.00, that the risk-free rate is 0.25% (continuous compounding), and that the dividend yield is 1.17% (continuous compounding). (Base your calculations on the last trade prices of the put or call).

No.	Strike	Call				Put			
		Last	Net	Bid	Ask	Last	Net	Bid	Ask
1	SPX$4600.00	$382.60	+$12.60	$379.5	$382.8	$344.03	+$1.08	$343.3	$346.2
2	SPX$4625.00	$365.98	+$11.88	$363.4	$367.1	$348.29	−$3.66	$352.3	$355.2
3	SPX$4650.00	$321.40	+$00.00	$347.7	$351.4	$387.59	+$0.00	$361.4	$364.4
4	SPX$4675.00	$334.81	+$11.71	$332.3	$336.0	$366.81	−$3.94	$370.9	$373.8
5	SPX$4700.00	$294.80	+$00.00	$317.2	$320.8	$395.38	+$0.00	$380.6	$383.5
6	SPX$4725.00	$266.17	+$00.00	$302.4	$305.9	$425.23	+$0.00	$390.6	$393.5

1. Calculate the implied volatility for each of the call and put options listed. Use Excel's Goal-Seek function using the BSM formula.
2. For the call and put options with a strike price of $4675.00, estimate the option values when volatility decreases to 0.8 times the implied volatility in part (a), and when it increases to 1.2 times the implied volatility in part (a).

6 Implied Volatility

A European call option on a certain stock has a strike price of $30, a time to maturity of one year, and an implied volatility of 30%. A European put option on the same stock has a strike price of $30, a time to maturity of one year, and an implied volatility of 33%. What is the arbitrage opportunity open to a trader? Does the arbitrage work only when the lognormal assumption underlying Black-Scholes-Merton holds?

7 Option Greeks and Binomial Tree

According to the Black-Scholes-Merton (BSM) model, the delta of a European option on a non-dividend-paying stock has the analytical expressions. We can also compute the delta of an option without invoking the BSM model using the binomial tree model. By varying the price of the underlying stock and recalculating the option price, we can numerically approximate the derivative dC/dS using the formula:

$$\frac{dC}{dS} \approx \frac{\text{New Option Price} - \text{Original Option Price}}{\text{New Stock Price} - \text{Original Stock Price}}$$

Create a binomial tree using the following information: $S_0 = 100$, $K = 100$, $n = 10$, $T = 0.1$, $\sigma = 0.3$, and $r = 0.02$, where n is the number of steps, and find the price of the European call option. Vary the stock price and numerically find delta.

We can numerically approximate gamma as well.

$$\frac{d^2C}{dS^2} \approx \frac{\text{New Delta} - \text{Original Delta}}{\text{New Stock Price} - \text{Original Stock Price}}$$

Find gamma of the option.

8 Binomial Tree

Bank XYZ offers an equity option. The expiration date of the option is two periods from now. The option is not initially specified to be a put or a call. Instead, the owner makes this choice after one period. Once the choice is made, the option can be exercised at any time. For example, if after one period the owner chooses for the new exotic option to be a put, it would at that time become identical to an ordinary American put with one period remaining until expiration. A customer has asked for a quote for an option of this type on Stock ABC with a strike price of 100. The current price of Stock ABC is 100 per share, and over each period the stock price evolves as shown on the tree diagram below. The risk-neutral probability of an "up" move is 0.5. The stock does not pay dividends, and the interest rate is 10% per period. What is the lowest price the firm could charge and still break even?

$$\begin{tikzcd}
 & & \begin{matrix}S_0uu\\256\end{matrix} \\
 & \begin{matrix}S_0u\\160\end{matrix} \arrow[ur, "u"] \arrow[dr, "d"] & \\
\begin{matrix}S_0\\100\end{matrix} \arrow[ur, "u"] \arrow[dr, "d"'] & & \begin{matrix}S_0ud\\96\end{matrix} \\
 & \begin{matrix}S_0d\\60\end{matrix} \arrow[ur, "u"] \arrow[dr, "d"] & \\
 & & \begin{matrix}S_0dd\\36\end{matrix}
\end{tikzcd}$$

9 Option Greeks

All assumptions of the Black-Scholes-Merton option pricing model hold. Stock XYZ is priced at \$30. It has volatility 25% per year. The annualized continuously-compounded risk-free interest rate is 3.0%.

1. Compute the price of a European call option with strike price \$31, which matures in 6 months.
2. Compute the option Delta at time 0.
3. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$34. Compute the resulting change in the option price.
4. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$34. Compute the resulting change in the value of the replicating portfolio for this option.
5. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$26. Compute the resulting change in the option price.
6. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$26. Compute the resulting change in the value of the replicating portfolio for this option.
7. Does the change in the option price exceed the change in the value of its replicating portfolio? If so, why?