

# **Practice Problem Set**

BUSS386 Futures and Options

## **1 Replicating Portfolio**

A stock price is currently \$40. It is known that at the end of one month it will be either \$42 or \$38. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-month European call option with a strike price of \$39?

## **2 Binomial Trees: One-Step**

A stock price is currently \$100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of \$100?

## **3 Binomial Trees: Call Option with Two-Step**

A stock price is currently \$50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of \$51?

## **4 Binomial Trees: Put Option with Two-Step**

For the situation considered in the previous problem, what is the value of a six-month European put option with a strike price of \$51? Verify that the European call and European put prices satisfy put-call parity. If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree?

## **5 Binomial Trees: Application**

A stock price is currently \$25. It is known that at the end of two months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose  $S_T$  is the stock price at the end of two months. What is the value of a derivative that pays off  $S_T^2$  at this time?

## 6 Finding $u$ and $d$ for Currency Option

Calculate  $u$ ,  $d$ , and  $p$  when a binomial tree is constructed to value an option on a foreign currency. The tree step size is one month, the domestic interest rate is 5% per annum, the foreign interest rate is 8% per annum, and the volatility is 12% per annum.

## 7 Finding $u$ and $d$ for Index Option

A stock index is currently 1,500. Its volatility is 18%. The risk-free rate is 4% per annum (continuously compounded) for all maturities and the dividend yield on the index is 2.5%. Calculate values for  $u$ ,  $d$ , and  $p$  when a six-month time step is used. What is the value of a 12-month American put option with a strike price of 1,480 given by a two-step binomial tree.

## 8 Binomial Trees: European vs. American

A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding.

- a What is the value of a six-month European put option with a strike price of \$42?
- b What is the value of a six-month American put option with a strike price of \$42?

## 9 Binomial Trees: Application

A stock price is currently \$30. During each two-month period for the next four months it is expected to increase by 8% or reduce by 10%. The risk-free interest rate is 5%. Use a two-step tree to calculate the value of a derivative that pays off  $[\max(30 - S_T, 0)]^2$  where  $S_T$  is the stock price in four months? If the derivative is American-style, should it be exercised early?