Introduction and Overview

BUSS386 Futures and Options

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- Overview of derivatives/markets
- Review: measures of return and risk
- Reading: Hull, Ch. 1.1–1.10 and 22.1–22.3

Overview

What are derivatives?

- A derivative is a financial security (i.e., instrument, contract, asset) whose value depends on other underlying **variables**.
- Example: A contract to buy 50,000 barrels of crude oil on September 16, 2017, for $50 per barrel.
- Example: An option contract that gives the holder the right, but not the obligation, to buy 100 shares of a company's stock at $100 per share within the next three months.

What are the underlying variables?

- Usually, the price of traded assets (e.g., equities, bonds, currencies, commodities)
- Some properties of asset prices (e.g., volatility)
- Certain events (e.g., default)
- Other factors like weather (e.g., temperature, rainfall), inflation, etc.

$\Rightarrow$ All variables should be measurable and observable.

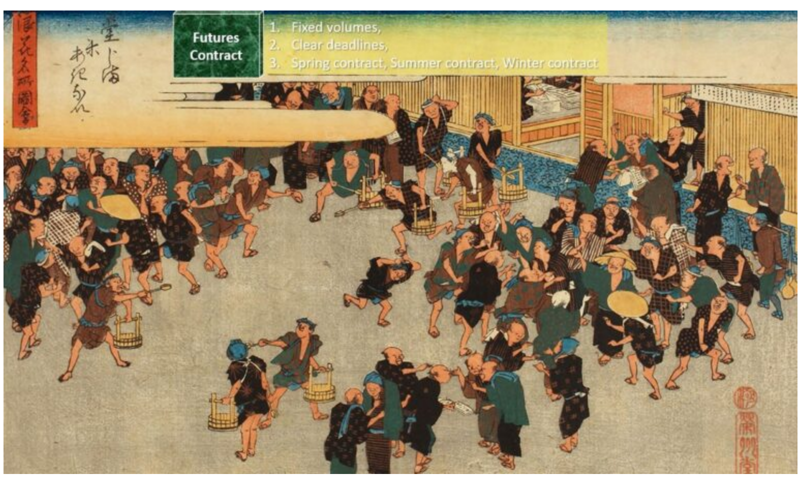
Type of derivatives

- **Contract Derivatives**
  - Examples: Futures, forwards, swaps, options, warrants, callable bonds (embedded), etc.
  - These contracts bind two counterparties to make a transaction at a future date. All profits and losses result from cash flows between the counterparties, making it a zero-sum game.
- **Securitized or Structured Products**

- Unlike contract derivatives, securitizations involve a pool or portfolio of underlying securities.
- Securitization creates new derivative securities that allocate the cash flows from the underlying pool to different classes of investors based on their risk tolerance.
- Examples: Collateralized mortgage obligations, asset-backed securities, etc.

- **Key Difference**
  - A contract derivative **transfers** risk from one counterparty to the other.
  - A securitized derivative **redistributes** the risk inherent in the underlying assets among different investors.

History of derivatives



Source: https://professornerdster.com/tag/forward-contracts/

- Established in the 1690s. Initially, rice exchange
- Futures trading began in 1730 for hedging.

History of derivatives

- Derivatives have been used by farmers and merchants for thousands of years:
  - Around 2000 B.C., derivatives were used in trade between India and the Arab Gulf.
  - In 300 B.C., olive growers in ancient Greece utilized derivatives.
- In the 12th century, European merchants used forward contracts for the future delivery of their goods.
- During Amsterdam's tulip mania in the 1630s, derivatives helped some merchants manage price swings.
- In the 17th century, Japan developed a forward market in rice.
- Modern developments:
  - The Chicago Board of Trade (CBOT) was established in 1848 to trade futures.
  - The Chicago Mercantile Exchange (CME) was founded in 1919. (CBOT and CME later merged to form the CME Group).
  - The Chicago Board Options Exchange (CBOE) introduced call options in 1973 and put options in 1977.
  - In Korea, forex derivatives began trading in 1968, and exchanges were established in 1996.

Where to trade derivatives

Derivatives can be traded in two main types of markets:

Exchange-Traded Market

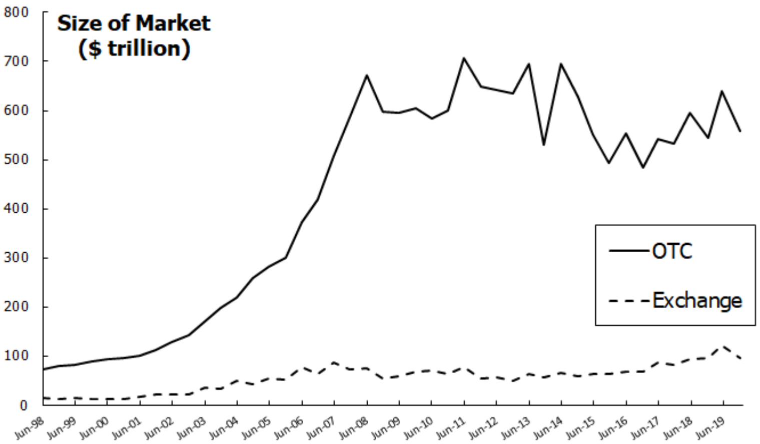
- **Centralized Trading**: All buy and sell orders are centralized in one place, either physically or electronically.
- **Standardized Contracts**: Contracts are standardized, ensuring uniformity and reducing the risk of counterparty default.
- **Types of Derivatives**: Futures and options are commonly traded.
- **Examples**: Chicago Mercantile Exchange (CME), Chicago Board Options Exchange (CBOE), Hong Kong Exchanges and Clearing (HKEX).
- **Liquidity**: High concentration of trades creates liquidity, which in turn attracts more liquidity.

Where to trade derivatives

Over-the-Counter (OTC) Market

- **Decentralized Trading**: There is no central place for collecting orders. Participants trade directly with each other or through a network of dealers.
- **Customizable Contracts**: Contracts are not standardized and can be tailored to meet the specific needs of the participants.
- **Main Participants**: Large institutions such as banks, hedge funds, and corporations.
- **Types of Derivatives**: Forwards, swaps, options, and other customized derivatives are traded.

Market Types and Trading Volume



Source: Bank for International Settlement

What contributed to rapid growth?

"Necessity is the mother of invention" - Plato

- Deregulation, Increased Volatility, and Technological Innovation
  - Key academic contributions: Black and Scholes (1972), Merton (1973)
  - Major events:
    * 1971: Currencies began to free float, leading to the introduction of currency futures in 1972.
    * 1973: The oil shock caused significant volatility in oil prices.
    * 1970s: Inflation and recessions resulted in volatile interest rates.
    * 1978: Deregulation of natural gas.
    * 1990s: Deregulation of electricity markets.

Why are derivatives useful?

- **Derivatives facilitate the transfer of risk** from those who are exposed to it to those more willing to bear it, making them a powerful tool for risk management.

- While risk management often aims to reduce risk, it can also involve **strategically assuming risks** that offer potential benefits.

- By effectively redistributing risk, derivatives **enable productive activities** that might otherwise be deemed too risky to pursue.

- However, derivatives **can be misused**, which is why regulations exist to mitigate potential abuses and ensure market stability.

Dangers of derivatives trading

- Without proper risk management, derivatives trading can lead to significant losses. Here are some notable examples:
  - **Société Générale (2008)**: Jérôme Kerviel lost over $7 billion by speculating on the future direction of equity indices. Source
  - **UBS (2011)**: Kweku Adoboli lost $2.3 billion by taking unauthorized speculative positions in stock market indices. Source
  - **Shell (1993)**: A single employee in the Japanese subsidiary of Shell lost $1 billion in unauthorized trading of currency futures. Source
  - **Barings Bank (1995)**: Nick Leeson lost £827 million, leading to the bank's collapse. Source
  - **Long-Term Capital Management (1998)**: The hedge fund lost $4.6 billion due to high-risk arbitrage trading strategies. Source

- **AIG (2008)**: AIG faced a liquidity crisis due to losses on credit default swaps, leading to a $182 billion government bailout. Source

- Effective risk management is crucial:
  - Define risk parameters and set risk limits.
  - Conduct various scenario analyses to anticipate potential outcomes and mitigate risks.

The OTC Market Prior to 2008

- The OTC market was largely unregulated.
- Banks acted as market makers, quoting bid and ask prices.
- Transactions between two parties were usually governed by master agreements provided by the International Swaps and Derivatives Association (ISDA).[1]
- Some transactions were cleared through central counterparties (CCPs), which act as intermediaries between the two sides of a transaction, similar to an exchange.

Changes Since 2008

- The OTC market has become more regulated with the following objectives:
  - Reduce systemic risk.
  - Increase transparency.
- In the U.S. and other countries, collateral and clearing of trades through a central clearing house (CCP) are required for all standard OTC contracts.

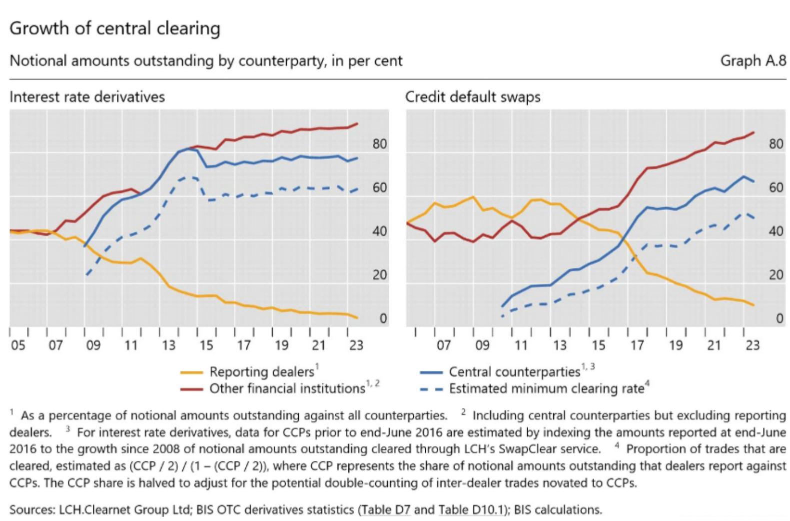
[1] The ISDA is a trade organization of participants in the market for over-the-counter derivatives. ISDA has created a standardized contract (the ISDA Master Agreement) to govern derivative transactions, which helps to reduce legal and credit risks.

- CCPs must be used to clear standardized transactions between financial institutions in most countries.
- All trades must be reported to a central repository.

The Lehman Bankruptcy

- Lehman Brothers filed for bankruptcy on September 15, 2008, marking the largest bankruptcy in U.S. history.
- Lehman was heavily involved in the OTC derivatives markets and faced financial difficulties due to high-risk activities and an inability to roll over its short-term funding.
- The firm had hundreds of thousands of outstanding transactions with approximately 8,000 counterparties.
- The process of unwinding these transactions has been challenging for both Lehman's liquidators and their counterparties.

Central Clearing



Source: Bank for International Settlement

Who trades derivatives?

Derivatives are traded by various market participants:

- **Corporations**: Hedge future cash flows and manage risks (e.g., fuel futures for airlines).
- **Financial Institutions**: Manage risks and offer risk management solutions (e.g., interest rate swaps).
- **Hedge Funds**: Achieve higher returns through leverage and complex strategies.
- **Market Makers (Dealers)**: Provide liquidity and profit from bid-ask spreads.
- **Financial Engineers**: Design new derivative products to meet specific needs.

Each participant contributes to the market's depth and liquidity.

Measures of Return and Risk

Simple Return

Suppose a stock price evolves as follows:

```r
P0 <- 100
P1 <- 110
P2 <- 100

R_t <- P1 / P0
r_t <- (P1 - P0) / P0
log_return <- log(P1 / P0)

list(GrossReturn = R_t, NetReturn = r_t, LogReturn = log_return)
```

```
$GrossReturn
[1] 1.1

$NetReturn
[1] 0.1

$LogReturn
[1] 0.09531018
```

Compounded Return

If a bond pays 10% semiannually, its compounded return is:

$$\left(1+\frac{r}{k}\right)^{k}-1$$

Continuously compounded return:

$$\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{k}-1=e^{r}-1$$

In R:

```r
r <- 0.10
compounded_return <- (1 + r / 2)^2 - 1
continuously_compounded <- exp(r) - 1
list(CompoundedReturn = compounded_return, ContinuousReturn = continuously_compounded)
```

```
$CompoundedReturn
[1] 0.1025

$ContinuousReturn
[1] 0.1051709
```

What is the equivalent c.c. return of Bond XYZ?

$$e^r = \left(1 + \frac{10\%}{2}\right)^2$$

$$r = \ln\left(1 + \frac{10\%}{2}\right)^2 = \ln\frac{P_1}{P_0} = 9.758\%$$

Multi-Period Return

Again, suppose $P_0 = 100$, $P_1 = 110$, and $P_2 = 100$. 2-year gross return is

$$R(0,2) = \frac{P_2}{P_0}$$

2-year gross return using 1-year return:

$$R(0,2) = \frac{P_2}{P_0} = \frac{P_1}{P_0}\frac{P_2}{P_1} = R(0,1)R(1,2)$$

cf. Log returns

Annualization

Typically, returns are expressed as an annual return for comparison.

Monthly return 1% for 12 months:

$$r = (1 + 0.01)^{12} - 1$$

A two-year return 10%:

$$
\begin{aligned}
&(1+r_1)(1+r_2) = 1.1 \\
&\text{Set } r_1 = r_2 = r \\
&(1+r)^2 = 1.1 \Rightarrow r = (1.1)^{1/2} - 1 = 4.89\%
\end{aligned}
$$

In general, an annualized return $= (1+r_c)^{(365/Days)} - 1$, where r_c is the cumulative (holding-period) return, i.e., $P_t/P_0 - 1$.

Arithmetic vs. Geometric Average Return

- Arithmetic Mean Return:

$$r_{AM} = \frac{(r_1 + r_2 + \cdots + r_T)}{T}$$

- Geometric Mean Return:

$$r_{GM} = [(1+r_1)(1+r_2)\dots(1+r_T)]^{1/T} - 1$$

In R:

```r
returns <- c(0.05, 0.10, -0.02, 0.07)
mean_arith <- mean(returns)
mean_geom <- prod(1 + returns)^(1/length(returns)) - 1
list(ArithmeticMean = mean_arith, GeometricMean = mean_geom)
```

```
$ArithmeticMean
[1] 0.05

$GeometricMean
[1] 0.04905428
```

Arithmetic vs. Geometric Average Return

- Fact 1: $r_{AM} \geq r_{GM}$
- Fact 2: The greater the volatility of returns, the greater $r_{AM} - r_{GM}$
- Typically, use r_{AM} as a proxy for the expected return.

Expected Return

- The probability weighted average return
- In population (when we know the probability function),
  - Discrete: $E(r) = \sum_{i=1}^{n} P(r_i) r_i$
  - Continuous: $E(r) = \int_{-\infty}^{+\infty} r f(r) dr$
    * $E(ar_1 + br_2) = aE(r_1) + bE(r_2)$
- In sample (when we only observe history),
  - $\overline{X} = \frac{\sum_{i=1}^{N} x_i}{N}$

Expected Return

- The expected return is the probability-weighted average of all possible returns.
- In practice, the true probability distribution of returns is often unknown (i.e., from the future). Therefore, we estimate it using historical data.
- Typically, we use the arithmetic average of historical returns to estimate the expected return.
  - Law of Large Numbers: If X_i's are independent and identically distributed (i.i.d) with mean μ, then $\frac{1}{N}\sum_{i=1}^{N} X_i \to \mu$ as N approaches infinity.

- The higher the risk, the greater the required rate of return. In equilibrium, the required rate of return should be equal to the expected return.

Risk: Variance/Standard Deviation

- Measures the degree of dispersion of return (around its mean)

In population

- $Var(r) = \sigma^2 = E[(r - E(r))^2] = E(r^2) - E(r)^2$
  - $\sigma(r) = \sqrt{var(r)}$
  - $Var(ar) = a^2 Var(r)$
  - $Var(ar_1 + br_2) = a^2 Var(r_1) + b^2 Var(r_2) + 2abCov(r_1, r_2)$

In sample

- $s^2 = \hat{\sigma}^2 = \frac{\sum_{i=1}^{N}(x_i - \overline{X})^2}{N-1}$

Note

- $Cov(X, Y) = E[(X - E(X))(Y - E(Y))] = E(XY) - E(X)E(Y)$
  - Correlation: $\rho = cov(X, Y)/\sigma(X)\sigma(Y)$, $-1 \leq \rho \leq +1$
  - $Cov(aX + b, eY + f) = aeCov(X, Y)$
  - $Cov(X + Y, Z) = Cov(X, Z) + Cov(Y, Z)$

The Reward-to-Volatility (Sharpe) Ratio

$$\frac{E(r) - r_f}{\sigma}$$

Example in R:

```
# Load necessary libraries
library(quantmod)
```

```
Loading required package: xts

Loading required package: zoo


Attaching package: 'zoo'

The following objects are masked from 'package:base':

    as.Date, as.Date.numeric

Loading required package: TTR

Registered S3 method overwritten by 'quantmod':
  method            from
  as.zoo.data.frame zoo
```

```
library(PerformanceAnalytics)
```

```
Attaching package: 'PerformanceAnalytics'

The following object is masked from 'package:graphics':

    legend
```

```r
# Load Tesla (TSLA) stock data from Yahoo Finance
getSymbols("TSLA", src = "yahoo", from = "2015-01-01", to = Sys.Date(), auto.assign = TRUE)
```

```
[1] "TSLA"
```

```r
# Compute daily returns and remove missing values
tsla_returns <- na.omit(dailyReturn(Cl(TSLA)))

# Compute annualized return and standard deviation
annualized_return <- Return.annualized(tsla_returns, geometric = TRUE)
annualized_std_dev <- sd(tsla_returns) * sqrt(252)  # Convert daily std to annualized

# Get the latest 3-month Treasury Bill (IRX) as risk-free rate
getSymbols("^IRX", src = "yahoo", from = Sys.Date() - 30, to = Sys.Date(), auto.assign = TRUE)
```

```
Warning: ^IRX contains missing values. Some functions will not work if objects
contain missing values in the middle of the series. Consider using na.omit(),
na.approx(), na.fill(), etc to remove or replace them.
```

```
[1] "IRX"
```

```r
risk_free_rate <- as.numeric(last(Cl(IRX))) / 100  # Convert from percentage

# Compute Sharpe Ratio (Annualized)
sharpe_ratio <- (annualized_return - risk_free_rate) / annualized_std_dev
sharpe_ratio
```

```
                  daily.returns
Annualized Return     0.5105525
```

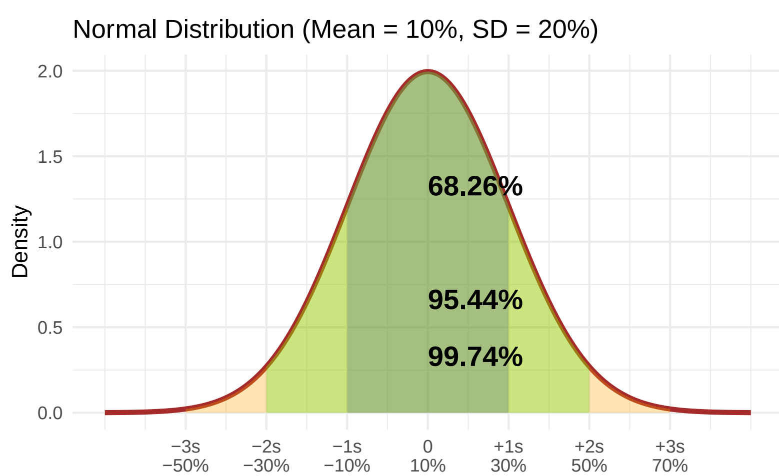
- Risk premium: $E(r) - r_f$ vs. Excess return: $r - r_f$.

Normal Distribution

- We commonly assume that returns are normally distributed, $N(\mu, \sigma^2)$

```
Warning: package 'ggplot2' was built under R version 4.4.3
```

```
Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
i Please use `linewidth` instead.
```



Normal Distribution

- Probability distribution function, (N(, ^2))

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

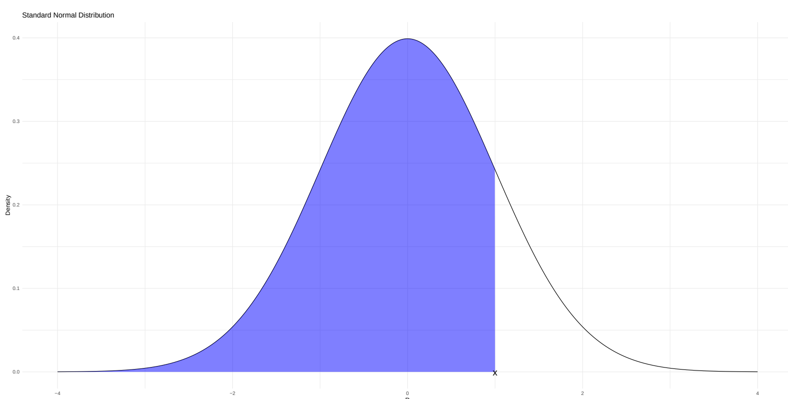
- Standard normal, (N(0,1))

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{r^2}{2}}$$

- If $Z \sim \phi(0,1)$, then $X = \mu + \sigma Z \sim N(\mu, \sigma^2)$
- If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$, then $X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2 + 2cov(X,Y))$

Standard Normal Random Variables

- Consider a normal random variable R with $\mu = 0$ and $\sigma = 1$. In other words, $R \sim N(0,1)$. We call it a standard normal random variable.
- Suppose that we want to find the probability that R is lower than x. Graphically, this probability is the shadowed area in the figure below:



Standard Normal Random Variables

- To find this probability, we calculate

$$\text{Prob}(R \leq x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{r^2}{2}} dr \equiv \Phi(x).$$

  $\Phi(x)$ is called the cumulative probability distribution function for a standard normal random variable.

- For any x, the value of $\Phi(x)$ can be found using the excel function, *norm.s.dist*(x, TRUE).

Standard Normal Random Variables

- ***Ex.1*** Suppose that $R_1 \sim \phi(0, 1)$. What is the probability that R_1 is larger than 1?

```
prob_R1 <- 1 - pnorm(1, mean = 0, sd = 1)
cat("P(R1 > 1) =", prob_R1, "\n")
```

```
P(R1 > 1) = 0.1586553
```

- ***Ex.2*** Suppose that $R_2 \sim \phi(0.1, 0.2)$. What is the probability that R_2 is equal to or smaller than 0.5?

```
prob_R2 <- pnorm(0.5, mean = 0.1, sd = 0.2)
cat("P(R2   0.5) =", prob_R2, "\n")
```

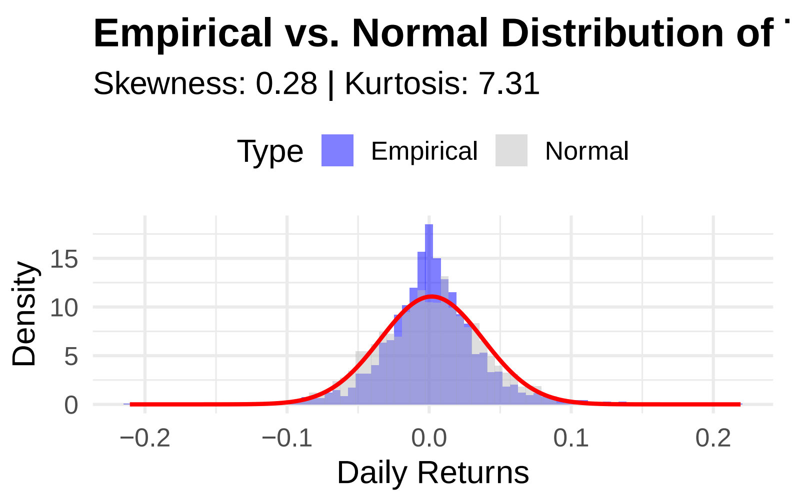
```
P(R2   0.5) = 0.9772499
```

Historic vs. Normal Distribution

- Historical returns often deviate from the normal distribution.
- Empirical distributions can exhibit skewness and kurtosis (fat tails).

```
Attaching package: 'moments'

The following objects are masked from 'package:PerformanceAnalytics':

    kurtosis, skewness
```

```
[1] "TSLA"
```



Log-Normal Distribution

- $X \sim LN(\mu, \sigma^2)$ if $\ln(X) \sim N(\mu, \sigma^2)$
- If a rate of return is normally distributed, security prices follow lognormal distribution.
  - log-return $= \ln R_t = \ln \frac{P_t}{P_{t-1}} = \ln P_t - \ln P_{t-1} \sim N(\cdot)$

- By the way,
  - $E(X) = e^{\mu+\sigma^2/2}$. In fact $E(X^n) = e^{n\mu+n^2\sigma^2/2}$
  - $Var(X) = E(X^2) - E(X)^2 = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$

Risk Measures

- Companies must assess and manage risks to avoid business failures.
- To understand the risk level of a project or business, we can analyze the probability distribution of possible outcomes.

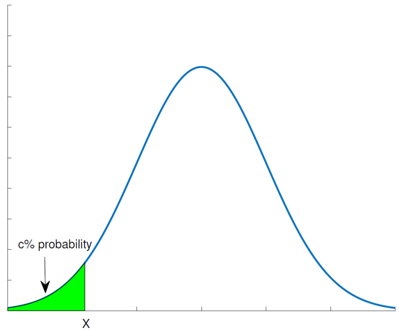
- Several risk measures are commonly used:
  - Standard Deviation
  - Value at Risk (VaR)
  - Expected Shortfall
  - ...
- Each measure focuses on different aspects of the distribution.

Standard Deviation

- Standard deviation measures the level of uncertainty about the outcomes, or the dispersion of probability distribution.
- The larger standard deviation is, the riskier a project.
- A disadvantage of the standard deviation is that it cannot distinguish between upside and downside movement.

Value at Risk

- Value at Risk (VaR) is intended to focus on downside risk of the distribution.
- VaR is the estimate of the losses that occur with a given probability.
  - ***e.g.*** How much loss we might have on our portfolio such that there is only a 5% chance that we will do worse?



- That is, we want to find X such that

$$\text{Prob}\,(R \leq X) = 0.05$$

Value at Risk

- How can we find X satisfying $\Pr\,(R \leq X) = 0.05$, i.e., 95% VaR?
- In a special case when $R \sim \phi(\mu, \sigma)$, we can find X using the Excel function `norm.inv()`.
  - For given $1-p$, `norm.inv(1-p, mean, sigma)` is X that satisfies $\text{Prob}\,(R \leq X) = 1 - p$.
    - VaR at 5% = `norm.inv(0.05,0,1) = -1.645`

    (R: `qnorm(0.05, mean = 0, sd = 1)`)
    - VaR at 10% = `norm.inv(0.1,0,1) = -1.282`

    (R: `qnorm(0.10, mean = 0, sd = 1)`)

Value at Risk - Example I

- Suppose we own a stock whose return is normally distributed with a mean of 15% and a standard deviation of 30%. What is the 5% Value at Risk (VaR) for this stock?

Answer: Let X denote the 5% VaR. Then, $\Pr(R \leq X) =$ $\texttt{norm.inv}(0.05, 0.15, 0.30) = -34.3\%$

Alternatively,

$$\text{Prob}\,(R \leq X) = \text{Prob}\left(\frac{R - 0.15}{0.3} \leq \frac{X - 0.15}{0.3}\right) = 0.05$$

Note that $\frac{R-0.15}{0.3} \sim \phi(0,1)$, so we can write

$$\frac{X - 0.15}{0.3} = \texttt{norm.s.inv}(0.05) = -1.645.$$

Thus, $X = -34.3\%$.

Value at Risk - Example II

- ***Q.*** A portfolio worth \$10 million has a 1-day standard deviation of \$200,000 and an approximate mean of zero. Assume that the change is normally distributed. What is the 1-day 99% VaR for our portfolio consisting of a \$10 million position in Microsoft? What is the 10-day 99% VaR?

  Answer: `norm.s.inv(0.01)` $= -2.326$, meaning that there is a 1% probability that a normally distributed variable will decrease in value by more than 2.326 standard deviations.

  Hence, the 1-day 99% VaR is $2.326 \times \$200,000 = \$465,300$.

  The 10-day 99% VaR is $2.326 \times (\$200,000 \times \sqrt{10}) = \$1,471,300$.

Value at Risk - Multiple Stocks

- Consider a portfolio consisting of $\boldsymbol{n}$ different stocks.
- The return on the portfolio is

$$R_p = \sum_{i=1}^{n} w_i R_i$$

where w_i is the fraction of wealth invested in stock i.
- If each stock return is normally distributed, then the portfolio return is also normally distributed.

Value at Risk - Multiple Stocks

- ***Ex.*** Consider a portfolio consisting of stock A and stock B. In the portfolio, \$5 million are invested in each of stock A and stock B. The return on each stock is normally distributed. Stock A has an expected return of 15% and a standard deviation of 30%. Stock B has an expected return of 18% and a standard deviation of 45%. The correlation between stock A and stock B is 0.4. What is the 10% VaR for the portfolio?
  - **Note:** When $X \sim \phi(\mu_x, \sigma_x^2)$ and $Y \sim \phi(\mu_y, \sigma_y^2)$, then $X + Y \sim \phi(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2 + 2\rho\sigma_x\sigma_y)$

  Answer:
  - The expected return of the portfolio is:

    $$\mu_p = 0.5 \times 0.15 + 0.5 \times 0.18 = 0.165 \text{ or } 16.5\%$$
  - The standard deviation of the portfolio is:

    $$\sigma_p = \sqrt{(0.5 \times 0.30)^2 + (0.5 \times 0.45)^2 + 2 \times 0.5 \times 0.5 \times 0.4 \times 0.30 \times 0.45} = 0.315 \text{ or } 31.5\%$$
  - The 10% VaR for the portfolio is:

    $$\text{VaR}_{10\%} = \mu_p + \sigma_p \times \texttt{norm.s.inv}(0.10) = 0.165 + 0.315 \times (-1.282) = -0.239 \text{ or } -23.9\%$$
  - Therefore, the 10% VaR for the \$10 million portfolio is:

    $$10,000,000 \times 0.239 = \$2,390,000$$

Value at Risk - Historical Data

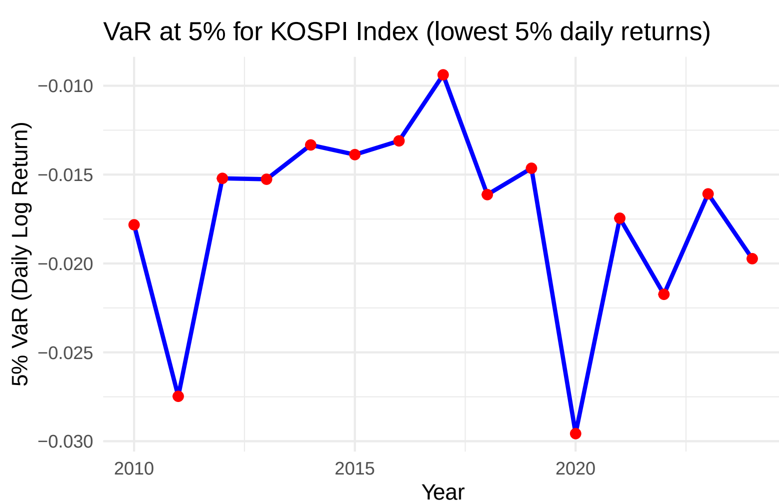
- We can also calculate the VaR using historical data without assuming a specific distribution.
- For example, let's consider 1-year-long historical data of daily returns for a stock price index.
- We aim to estimate the 5% VaR for the next day's return.
- To do this, we assume that the next day's return will be similar to one of the past year's returns.
- The 5% VaR is then the 5th percentile of these historical returns.

Value at Risk - Some Issues I

- VaR estimation is based on the assumption that the distribution of future return is the same as (at least similar to) the distribution of past return.
- This assumption may not hold in the real world.

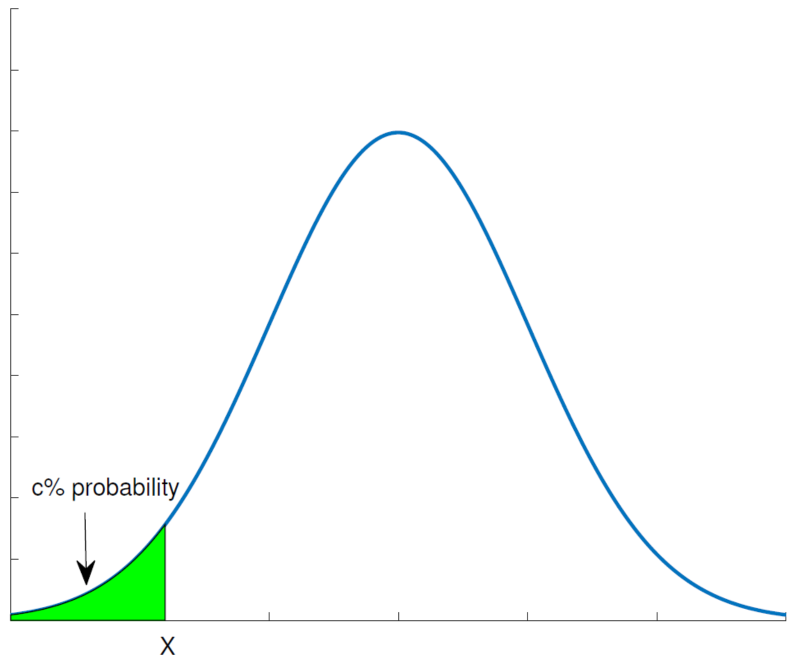
```
######################### Warning from 'xts' package ##########################
#                                                                              #
# The dplyr lag() function breaks how base R's lag() function is supposed to  #
# work, which breaks lag(my_xts). Calls to lag(my_xts) that you type or       #
# source() into this session won't work correctly.                            #
#                                                                              #
# Use stats::lag() to make sure you're not using dplyr::lag(), or you can add #
# conflictRules('dplyr', exclude = 'lag') to your .Rprofile to stop           #
# dplyr from breaking base R's lag() function.                                #
#                                                                              #
# Code in packages is not affected. It's protected by R's namespace mechanism #
# Set `options(xts.warn_dplyr_breaks_lag = FALSE)` to suppress this warning.  #
#                                                                              #
###############################################################################


Attaching package: 'dplyr'
```

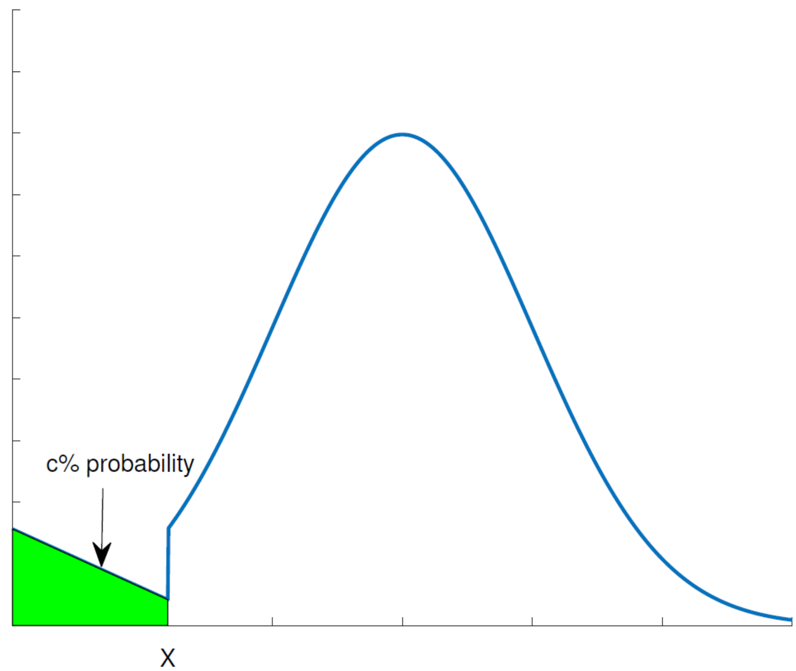
```
The following objects are masked from 'package:xts':

    first, last


The following objects are masked from 'package:stats':

    filter, lag


The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union


Warning: ^KS200 contains missing values. Some functions will not work if
objects contain missing values in the middle of the series. Consider using
na.omit(), na.approx(), na.fill(), etc to remove or replace them.


[1] "KS200"


Warning in to_period(xx, period = on.opts[[period]], ...): missing values
removed from data
```



Value at Risk - Some Issues II

- VaR specifies the ***minimum*** loss that will occur with a given probability.
- VaR tells nothing about the expected magnitude of the loss.
- Which is the better between the following two?





Expected Shortfall

- ***Expected Shortfall*** is another measure to address the shortcoming of VaR.
  - It asks ***"If things get bad, what is the expected loss?"***
- Suppose that we focus on the loss that will happen with 5% probability. Let $\boldsymbol{V}$ denote the 5% loss (VaR). Then,

$$\text{Expected shortfall} = E\left(R | R \leq V\right)$$

- Also known as Conditional Value at Risk (CVaR)

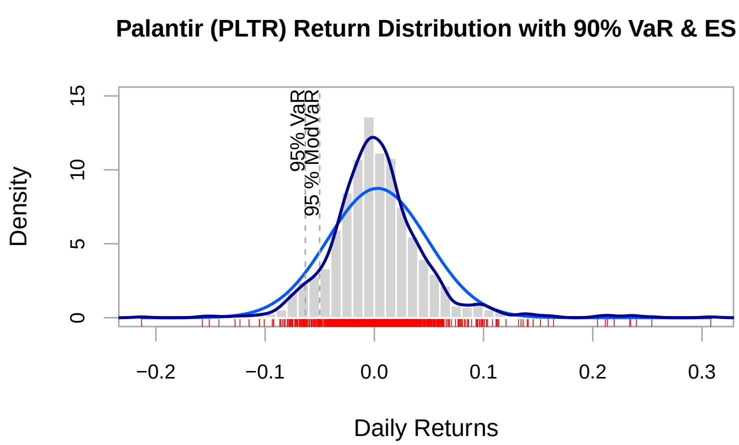
- Under normal distribution: $\text{ES} = \mu - \sigma \frac{\phi((V-\mu)/\sigma)}{\Phi((V-\mu)/\sigma)}$

Expected Shortfall

- Once historical data are given, we can compute the expected shortfall.
  - In Excel, use ***"averageif()"***.
- ***Ex.*** Let's use the 1-year-long data of daily returns on a stock index.
  - ***Q1.*** What is the expected shortfall with 5% probability?
  - ***Q2.*** What is the expected shortfall with 10% probability?

Value at Risk and Expected Shortfall

```
[1] "PLTR"

90% Historical VaR: -0.044821

90% Expected Shortfall: -0.069202
```



Application: Bank Regulation

- VaR and ES are widely used in the financial industry to measure and manage risk.
- The Basel Committee on Banking Supervision (BCBS) provides global banking regulations.
- Key frameworks include:
  - **1996 Amendment**: Required capital = ***k* × *VaR (1%, 10 days)***, where ***k 3***.
  - **Basel II (2007)**: Suggested ***VaR(0.1%, 1-year)*** for risk assessment.
  - **Basel IV (2021)**: Recommended 97.5% expected shortfall (ES) for a comprehensive risk view.