Problem Set: Answer Key

1   Replicating Portfolio

A stock price is currently $40. It is known that at the end of one month it will be either $42 or $38. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-month European call option with a strike price of $39?

2   Binomial Trees: One-Step

A stock price is currently $100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of $100?

3   Binomial Trees: Call Option with Two-Step

A stock price is currently $50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of $51?

4   Binomial Trees: Put Option with Two-Step

For the situation considered in the previous problem, what is the value of a six-month European put option with a strike price of $51? Verify that the European call and European put prices satisfy put-call parity. If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree?

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5 Binomial Trees: Application

A stock price is currently $25. It is known that at the end of two months it will be either $23 or $27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of two months. What is the value of a derivative that pays off S_T^2 at this time?

6 Finding u and d for Currency Option

Calculate u, d, and p when a binomial tree is constructed to value an option on a foreign currency. The tree step size is one month, the domestic interest rate is 5% per annum, the foreign interest rate is 8% per annum, and the volatility is 12% per annum.

7 Finding u and d for Index Option

A stock index is currently 1,500. Its volatility is 18%. The risk-free rate is 4% per annum (continuously compounded) for all maturities and the dividend yield on the index is 2.5%. Calculate values for u, d, and p when a six-month time step is used. What is the value of a 12-month American put option with a strike price of 1,480 given by a two-step binomial tree.

8 Binomial Trees: European vs. American

A stock price is currently $40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding.

a What is the value of a six-month European put option with a strike price of $42?

b What is the value of a six-month American put option with a strike price of $42?

9 Binomial Trees: Application

A stock price is currently \$30. During each two-month period for the next four months it is expected to increase by 8% or reduce by 10%. The risk-free interest rate is 5%. Use a two-step tree to calculate the value of a derivative that pays off $[\max(30 - S_T, 0)]^2$ where S_T is the stock price in four months? If the derivative is American-style, should it be exercised early?

10 Binomial Trees: American on Foreign Currency

Calculate the value of a nine-month American call option on a foreign currency using a three-step binomial tree. The current exchange rate is 0.79 and the strike price is 0.80 (both expressed as dollars per unit of the foreign currency). The volatility of the exchange rate is 12% per annum. The domestic and foreign risk-free rates are 2% and 5%, respectively. Suppose a company has bought options on 1 million units of the foreign currency. What position in the foreign currency is initially necessary to hedge its risk?