Practice Problem Set

BUSS386 Futures and Options

1 Barrier Options

What is the price of a down-and-out put when the barrier is greater than the strike price?

2 Forward-Start Options

Suppose that $S = \$100$, $\sigma = 30\%$, and $r = 8\%$. Today you buy a contract which, 6 months from today, will give you one 3-month to expiration at-the-money call option. (This is called a forward start option.) Assume that r and σ are certain not to change in the next 6 months.

a Six months from today, what will be the value of the option if the stock price is \$100? \$50? \$200? (Use the Black-Scholes formula to compute the answer.) In each case, what fraction of the stock price does the option cost?

b What investment today would guarantee that you had the money in 6 months to buy an at-the-money option?

c What would you pay today for the forward start option in this example?

3 Monte Carlo Pricing of Asian Options

Estimate the value of a 6-month European-style (arithmetic) average price call option on a non-dividend-paying stock. The initial stock price is \$30, the strike price is \$30, the risk-free rate is 5%, and the stock price volatility is 30%.

a Run a Monte Carlo simulation to estimate the price of the option. Use 1,000 stochastic paths for the risk-neutral representation of the evolution of stock prices assuming lognormality, drawing innovations from a normal distribution, and with a time step h equal to 1 month.

b Repeat this exercise but now set the time step h equal to 1 week (treating 6 months as 26 weeks).

c Using the same Monte Carlo simulation of stock prices as in Part (b), what is the price of a knock-in call option with a strike price of \$30 and a barrier of \$35?

Hint: We can simulate the lognormal stock price process under the risk-neutral representation using the following algorithm:

$$S_{t+h} = S_t e^{(r-\sigma^2/2)h+\sigma\sqrt{h}\epsilon_t},$$

where the time step $h = \frac{1}{12}$ and $\epsilon_t \sim N(0,1)$.

After generating 1,000 paths for the stock price, we can calculate the payoff on each path i as:

$$V_i = \max\left(\frac{1}{6}\sum_{t=1}^{6} S_t^i - K, 0\right)$$

Finally, the value of the average price call is given by $c = e^{-0.05(0.5)}\frac{1}{1000}\sum_i V_i$.

4 Monte Carlo Valuation of Knock-In Option

The knock-in call option comes into existence when the stock price reaches \$35 before expiration. Using the same algorithm to simulate the lognormal stock price process as in Part (b) with $h = 1/52$, we can simulate the payoff of the knock-in call on a particular path as $\max(S_T - K, 0)$ if the stock price reaches \$35, and 0 otherwise.

Closed-form formula (Optional): The option's payoff depends on the arithmetic average of the price of the underlying stock during the life of the option. In particular, the payoff is $\max(0, A(T) - K)$, where $A(T)$ is the arithmetic average price of the stock. Under the assumption that $A(T)$ is lognormally distributed, the average price call can be valued using a similar formula to the one we have used to price a regular European call. Suppose M_1 and M_2 are the first two moments of $A(T)$. The value of the average price call is given by Black's model:

$$
\begin{aligned}
c &= e^{-rT}[F_0 N(d_1) - K N(d_2)] \\
d_1 &= \frac{\ln(F_0/K) + (\sigma^2/2)T}{\sigma_F \sqrt{T}} \\
d_2 &= d_1 - \sigma_F sqrtT,
\end{aligned}
$$

where $F_0 = M_1$ and $\sigma_F^2 = \frac{1}{T} \ln \frac{M_2}{M_1^2}$ Assuming that the average is calculated continuously,

$$
\begin{aligned}
M_1 &= \frac{e^{(r-q)T} - 1}{(r-q)T} S_0 \\
M_2 &= \frac{2e^{[2(r-q)+\sigma^2]TS_0^2}}{(r-q+\sigma^2)(2r-2q+\sigma^2)T^2} + \frac{2S_0^2}{(r-q)T^2}\left(\frac{1}{2(r-q)+\sigma^2} - \frac{e^{(r-q)T}}{r-q+\sigma^2}\right).
\end{aligned}
$$

Plugging in $r = 5\%, q = 0, \sigma = 30\%, T = 0.5, S_0 = 30, K = 30$ to the expressions for M_1 and M_2 above, we get that $M_1 = 30.378, M_2 = 936.9, \sigma_F^2 = 17.41$. Therefore,

$$
c = e^{-0.05(0.5)}[30.378N(0.163) - 30N(0.04)] = 1.64
$$