

Common Limites and Errors

Professor Ji-Woong Chung
Korea University

Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

Data Limitations

- ▶ The data we use is almost never perfect:
 - ▶ Variables are often reported with error.
 - ▶ Exit and entry into dataset typically not random.
 - ▶ Datasets only cover certain types of firms.

Measurement Error – Examples

- ▶ Measurement error occurs when observed values differ from the true values.
- ▶ Two main types:
 - ▶ **Random (innocent) errors:** Pure noise, not systematically related to other variables.
 - E.g. Survey respondents round or misremember income.
 - ▶ Leads to greater variance, but no systematic bias if uncorrelated with regressors.
 - ▶ **Systematic (nonrandom) errors:** Certain groups misreport in predictable ways.
 - E.g. High-GPA teenagers underreport marijuana use; firms understate liabilities.
 - ▶ Correlation with covariates \Rightarrow biased estimates.
- ▶ **Key question:** How do these errors affect causal inference and estimation?

Measurement Error – Why It Matters

- ▶ The impact depends on which variable is measured with error.
- ▶ **If the dependent variable (y) is mismeasured:**
 - ▶ Random noise: Increases residual variance \Rightarrow larger standard errors.
 - ▶ Systematic error: If correlated with x , coefficient estimates become biased.
E.g. Low-education respondents underreport income \Rightarrow downward bias on education effect.
- ▶ **If the independent variable (x) is mismeasured:**
 - ▶ Classical error (mean-zero, uncorrelated): Attenuation bias \Rightarrow slope biased toward 0.
 - ▶ Non-classical error (correlated): Bias in unpredictable directions; contaminates other coefficients.
E.g. Noisy education measure \Rightarrow underestimated returns to schooling.
- ▶ **Summary:** Random \Rightarrow inefficiency; Systematic \Rightarrow bias.

Measurement Error – Solutions

- ▶ Measurement error correction requires knowing its **source and structure**.
- ▶ **Common approaches:**
 - ▶ **Instrumental Variables (IV):** Find variable correlated with true x but not error.
E.g. Administrative wage data as instrument for self-reported income.
 - ▶ **Validation samples or repeated measures:** Estimate or correct error variance.
 - ▶ **Structural modeling:** Explicitly model the measurement process
- ▶ **Challenges:**
 - ▶ Hard to correct without auxiliary or validation data.
 - ▶ Unknown error patterns \Rightarrow unpredictable bias.
- ▶ Always scrutinize data accuracy—small errors can distort inference.

Survivorship Issues – Examples

- ▶ Observations may be missing or included for **systematic reasons**, not by chance.
- ▶ **Example 1:** IPO firms
 - ▶ Datasets of public firms exclude private firms.
 - ▶ Firms that go public may already differ (e.g., more profitable, faster-growing, or better governed).
- ▶ **Example 2:** Distressed or failed firms
 - ▶ Firms severely affected by a shock may disappear due to bankruptcy.
 - ▶ Remaining sample overrepresents “survivors.”
- ▶ **Question:** How do such missing or selective exits bias our estimates?

Survivorship Issues – Why It Matters

- ▶ **Selection bias** can lead to misleading conclusions.
- ▶ **Example 1: IPOs and growth**
 - ▶ High post-IPO growth may not be caused by going public.
 - ▶ Rather, firms that went public were already high-growth candidates.
- ▶ **Example 2: Negative events and exits**
 - ▶ If failing firms disappear after a shock, the observed average effect looks smaller (or even positive).
 - ▶ Survivors are systematically different from those that dropped out.
- ▶ **Result:** Bias in estimates, especially in causal or panel analyses.

Survivorship Issues – Solutions

- ▶ No perfect fix, but several **diagnostic checks** help:
- ▶ **1. Check for selective attrition:**
 - ▶ In DiD, test whether treatment status predicts dropping from the data.
 - ▶ If treatment increases exit probability, estimate may be biased.
- ▶ **2. Compare characteristics of dropouts vs. survivors:**
 - ▶ Are exiting observations systematically different in key covariates?
 - ▶ If yes, assess how their absence might affect estimates.
- ▶ **3. Sensitivity checks:**
 - ▶ Include censored or imputed outcomes where possible.
 - ▶ Use survival models (hazard or selection models) if dropout is endogenous.

Sample is Limited – Examples

- ▶ Many widely used datasets cover only a **subset of firms**.
- ▶ **Example 1: Compustat**
 - ▶ Focuses on large, listed U.S. firms.
 - ▶ Excludes small, private, and young firms.
- ▶ **Example 2: Execucomp**
 - ▶ Covers CEO pay and incentives only for S&P 1500 firms.
 - ▶ Omits privately held or smaller listed firms.
- ▶ **Question:** How could this limited coverage bias our findings?

Sample is Limited – Why It Matters

- ▶ Limited samples threaten **external validity**.
- ▶ **Example 1: Treatment effects in Compustat**
 - ▶ You may find no effect in large public firms.
 - ▶ But the same treatment could strongly affect unobserved small or private firms.
- ▶ **Example 2: CEO incentives in Execucomp**
 - ▶ Correlation between incentives and risk-taking may reflect large-firm governance structures.
 - ▶ May not generalize to smaller or family-controlled firms.
- ▶ Key issue: **Selection on observables and unobservables** into the dataset.

Sample is Limited – Solutions

- ▶ **1. Be explicit about scope:**
 - ▶ Avoid overgeneralization; limit conclusions to the covered population.
 - ▶ Emphasize that results apply to large public firms if using Compustat or Execucomp.
- ▶ **2. Argue representativeness:**
 - ▶ Show your sample captures an economically important segment.
 - ▶ E.g., S&P 1500 firms represent majority of U.S. market capitalization.
- ▶ **3. Extend the data:**
 - ▶ Hand-collect missing data or merge with private firm databases.
 - ▶ Building new datasets can yield high-impact, publishable research.

Example

- ▶ Ali, Klasa, and Yeung (RFS 2009) provide a striking case of data mismeasurement.
- ▶ Many finance theories emphasize **industry concentration** as a key variable:
 - ▶ E.g., competition, market power, R&D incentives, and financing constraints.
- ▶ Researchers typically measure concentration (Herfindahl index) using **Compustat**.
- ▶ **Question:** What's wrong with that approach?

Example [Part 1]

- ▶ **Systematic measurement error:**
 - ▶ Compustat **excludes** private firms ⇒ distorted Herfindahl index.
 - ▶ Ali, Klasa, and Yeung construct an alternative using **U.S. Census data** (which includes all firms).
 - ▶ Correlation between the Compustat and Census-based measures = only **13%**.
- ▶ The error is not random:
 - ▶ Bias is related to observable industry traits—e.g., turnover, entry, exit, and listing propensity.

Example [Part 2]

- ▶ Using the Census-based measure, Ali, Klasa, and Yeung (RFS 2009) show that:
 - ▶ The mismeasurement meaningfully changes empirical conclusions.
 - ▶ Four major published results are overturned.
- ▶ **Example:**
 - ▶ Previous studies (using Compustat) found a **negative** link between concentration and R&D.
 - ▶ With accurate Census data, the relationship becomes **positive**.
- ▶ **Lesson:** Measurement error in key variables can fundamentally alter conclusions.

Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

Hypothesis Testing Mistakes

- ▶ Researchers often compare treatment effects across groups by estimating separate DiDs.
- ▶ **Example:**
 - ▶ Estimate treatment effect for small firms.
 - ▶ Estimate treatment effect for large firms.
- ▶ Then they conclude: "The effect is stronger for large firms."

Example Inference from Analysis

	Small Firms	Large Firms	Low D/E Firms	High D/E Firms
Treatment × Post	0.031 (0.121)	0.104** (0.051)	0.056 (0.045)	0.081*** (0.032)
N	2,334	3,098	2,989	2,876
R ²	0.11	0.15	0.08	0.21
Year FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓

- ▶ Researchers often conclude:
 - ▶ “Treatment effect is larger for big firms.”
 - ▶ “High D/E firms respond more.”
 - ▶ *But are those differences statistically significant?*

Be Careful Making Such Claims

- ▶ **Problem:** Differences across subsamples may not be statistically significant.
- ▶ You can't tell by "eyeballing" coefficients.
 - ▶ Statistical significance depends on the **covariance** between estimates.
- ▶ **Proper test:** Include an interaction term (triple difference) in a single regression.

Example Triple Interaction Result

	All Firms
Treatment \times Post	0.031 (0.121)
Treatment \times Post \times Large	0.073 (0.065)
N	5,432
R^2	0.12
Year FE	✓
Firm FE	✓
Year \times Large FE	✓

- ▶ Difference between large and small firms is **not statistically significant**.
- ▶ Always include interaction with year dummies to match subgroup DiDs.

Practical Advice

- ▶ Don't make claims you haven't statistically tested.
- ▶ Always report the **p-value for the difference** across groups.
- ▶ If the difference isn't significant (e.g., $p = 0.15$), say so — triple differences are noisy.
- ▶ Be cautious with phrasing:
 - ▶ Instead of: "Large firms respond more,"
 - ▶ Say: "We find an effect for large firms but not for small firms."

Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

Unobserved Heterogeneity – Motivation

- ▶ Controlling for **unobserved heterogeneity** is a fundamental challenge in empirical finance.
- ▶ **Why?** Many important factors cannot be directly measured or included in data:
 - ▶ Managerial talent, corporate culture, or risk appetite.
 - ▶ Local demand or regulatory conditions.
 - ▶ Investor sentiment or regional economic trends.
- ▶ These unobservables can be **correlated** with key explanatory variables:
 - ▶ ⇒ Leads to **omitted variable bias**.
- ▶ Important sources of heterogeneity are often shared across **groups**:
 - ▶ Industry-level demand shocks.
 - ▶ Region-specific economic or policy environments.
 - ▶ Time-period shocks common to all firms.

Many Different Strategies Are Used

- ▶ As discussed earlier, **Fixed Effects (FE)** can control for unobserved heterogeneity and yield consistent estimates when the unobservables are time-invariant.
- ▶ But researchers use several alternative or complementary strategies to remove **group-level heterogeneity**:
 - ▶ **Adjusted-Y (AdjY)**: Demean the dependent variable within groups (e.g., subtract the industry-year average: “industry-adjusted” outcomes).
 - ▶ **Average Effects (AvgE)**: Include group-level averages of outcomes as controls (e.g., add the mean of y for a given state-year or industry-year).
- ▶ Each method aims to remove variation driven by shared shocks or persistent group differences.
 - ▶ FE fully removes group-level heterogeneity (e.g., via industry-year dummies).
 - ▶ AdjY and AvgE are simplified approximations useful in small samples, but only FE yields consistent estimates when unobservables correlate with regressors.

The Underlying Model [Part 1]

- ▶ Start with a simple data-generating process:

$$y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ i : Group index (e.g., industry, state, bank, or fund family)
- ▶ j : Observation within group (e.g., firm, branch, fund)
- ▶ Model components:
 - ▶ $y_{i,j}$: outcome (e.g., investment, leverage, return)
 - ▶ $X_{i,j}$: explanatory variable of interest (e.g., policy, treatment)
 - ▶ f_i : unobserved group-level factor (e.g., industry demand, regulation)
 - ▶ $\epsilon_{i,j}$: idiosyncratic error term
- ▶ The key question: what happens if we try to control for f_i without properly including a fixed effect?

The Underlying Model [Part 2]

- ▶ Standard assumptions about the data structure:
 - ▶ N : Number of groups is large; J : Observations per group is small.
 - ▶ $\text{Var}(f_i) = \sigma_f^2$, $\mathbb{E}[f_i] = 0$
 - ▶ $\text{Var}(X_{i,j}) = \sigma_X^2$, $\mathbb{E}[X_{i,j}] = 0$
 - ▶ $\text{Var}(\epsilon_{i,j}) = \sigma_\epsilon^2$, $\mathbb{E}[\epsilon_{i,j}] = 0$
- ▶ X and ϵ are i.i.d. across groups, but may be correlated **within** groups:
 - ▶ Within-group correlation \Rightarrow common shocks.
 - ▶ Across-group independence ensures valid asymptotics.

The Underlying Model [Part 3]

- ▶ Additional assumptions:
 - ▶ $\text{Cov}(f_i, \epsilon_{i,j}) = 0$ — group factors are uncorrelated with idiosyncratic errors.
 - ▶ $\text{Cov}(X_{i,j}, \epsilon_{i,j}) = \text{Cov}(X_{i,j}, \epsilon_{i,-j}) = 0$ — exogeneity of X .
 - ▶ $X_{i,j}$ is exogenous with respect to both its own error term and the error terms of other group members, enabling unbiased and consistent estimation of β in the fixed effects model.
 - ▶ $\text{Cov}(X_{i,j}, f_i) = \sigma_{Xf} \neq 0$ — regressor correlated with group unobservables.
- ▶ Implication:
 - ▶ If we omit f_i , OLS suffers from classic **omitted variable bias**.
 - ▶ FE removes f_i through within-group demeaning. AdjY and AvgE only partially do so, leaving residual correlation with f_i . This incomplete adjustment can amplify—or even reverse—the bias.

We Already Know OLS Is Biased

True model: $y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$

But OLS estimates: $y_{i,j} = \hat{\beta}_{OLS} X_{i,j} + u_{i,j}^{OLS}$

- ▶ By omitting the group effect f_i , OLS suffers from standard omitted variable bias:

$$\hat{\beta}_{OLS} = \beta + \frac{\sigma_{Xf}}{\sigma_X^2}$$

- ▶ Direction and size of bias depend on the covariance between X and f_i .

Adjusted-Y ($AdjY$) Estimation

- **Idea:** Remove unobserved group effects by demeaning the dependent variable within groups:

$$y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$$

- Group mean:

$$\bar{y}_i = \frac{1}{J} \sum_{k \in i} y_{i,k} = \frac{1}{J} \sum_{k \in i} (\beta X_{i,k} + f_i + \epsilon_{i,k})$$

$$\Rightarrow \bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

- Some researchers exclude the observation itself or use medians, but bias remains.

Example: AdjY Estimation in Practice

- ▶ Example regression:

$$Q_{i,j,t} - \bar{Q}_{i,t} = \alpha + \beta X_{i,j,t} + \epsilon_{i,j,t}$$

- ▶ Variables:

- ▶ $Q_{i,j,t}$: Tobin's Q for firm j in industry i , year t
- ▶ $\bar{Q}_{i,t}$: industry-year mean Q ("industry-adjusted Q")
- ▶ $X_{i,j,t}$: explanatory variables (e.g., governance, leverage)
- ▶ Often combined with firm or year fixed effects

- ▶ **Question:** Why is AdjY still inconsistent?

Why $\text{Adj}Y$ Is Inconsistent

- ▶ Substitute the group mean:

$$\bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

$$y_{i,j} - \bar{y}_i = (\beta X_{i,j} + f_i + \epsilon_{i,j}) - (\beta \bar{X}_i + f_i + \bar{\epsilon}_i)$$

- ▶ The transformation removes f_i in the mean but not in the regressor.
- ▶ When we regress $(y_{i,j} - \bar{y}_i)$ on $X_{i,j}$ instead of $(X_{i,j} - \bar{X}_i)$, the omitted \bar{X}_i induces bias.
- ▶ Hence, $\text{Adj}Y$ omits a relevant group-level term.

AdjY and Omitted Variable Bias

- ▶ The true transformed model is:

$$y_{i,j} - \bar{y}_i = \beta(X_{i,j} - \bar{X}_i) + (\epsilon_{i,j} - \bar{\epsilon}_i)$$

- ▶ But *AdjY* estimates:

$$y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$$

- ▶ Because it omits \bar{X}_i , the estimator is biased:

$$\hat{\beta}_{AdjY} = \beta - \frac{\sigma_{X\bar{X}}^2}{\sigma_X^2}$$

- ▶ With positive $Cov(X, \bar{X})$ —common under shared industry shocks—bias is typically negative.

Adding a Second Variable, Z

- ▶ Suppose the true model has two regressors:

$$y_{i,j} = \beta X_{i,j} + \gamma Z_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ Maintain previous assumptions and add:
 - ▶ $\text{Cov}(Z_{i,j}, \epsilon_{i,j}) = 0, \text{Var}(Z) = \sigma_Z^2$
 - ▶ $\text{Cov}(X, Z) = \sigma_{xz}$
 - ▶ $\text{Cov}(Z, f_i) = \sigma_{zf}$
- ▶ AdjY still omits group-level means (\bar{X}_i, \bar{Z}_i) , creating intertwined biases.

AdjY Estimates with Two Variables

- ▶ The biases are now complex:

$$\hat{\beta}_{AdjY} = \beta + \Delta, \quad \hat{\gamma}_{AdjY} = \gamma + \diamond$$

- ▶ Biases depend on correlations among X, Z, f_i .
- ▶ As Gormley and Matsa (2014) show:
 - ▶ Both coefficients can move in unpredictable directions.
 - ▶ Even sign reversals are possible.

Average Effects ($\text{Avg}E$) — Idea

- ▶ Researchers often want to control for unobserved group-level factors (f_i) when fixed effects are unavailable or costly.
- ▶ **Idea:** Include the group mean of the dependent variable as a proxy for f_i :

$$y_{i,j} = \beta^{\text{Avg}E} X_{i,j} + \gamma^{\text{Avg}E} \bar{y}_i + u_{i,j}^{\text{Avg}E}$$

- ▶ Example – Firm profitability regression:

$$\text{ROA}_{i,s,t} = \alpha + \beta X_{i,s,t} + \gamma \overline{\text{ROA}}_{s,t} + u_{i,s,t}$$

- ▶ $\overline{\text{ROA}}_{s,t}$: Average ROA among firms in state s , year t
- ▶ $X_{i,s,t}$: Firm-level controls (e.g., leverage, size, market share)
- ▶ **Goal:** Use \bar{y}_i to soak up unobserved shocks (f_i) that affect all group members.

Why $\text{Avg}E$ Is Problematic

- ▶ The true model:

$$y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ $\text{Avg}E$ substitutes a proxy for f_i :

$$\bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

- ▶ Substituting this into the regression gives:

$$y_{i,j} = \beta X_{i,j} + \gamma(\beta \bar{X}_i + f_i + \bar{\epsilon}_i) + u_{i,j}$$

- ▶ Two problems arise:

1. **Measurement error:** \bar{y}_i is an imperfect proxy for f_i — it includes $\beta \bar{X}_i + \bar{\epsilon}_i$.
2. **Endogeneity:** controlling for \bar{y}_i removes only the fraction of f_i ; The leftover f_i in the error is still correlated with $X_{i,j}$.

Measurement Error Bias

- ▶ Since

$$\bar{y}_i = f_i + \underbrace{(\beta \bar{X}_i + \bar{\epsilon}_i)}_{\text{measurement error}} ,$$

\bar{y}_i measures f_i with noise.

- ▶ This creates **measurement error bias**:
 - ▶ As is well known, even classical measurement error causes all estimated coefficients to be inconsistent
- ▶ Bias here is complicated because error can be correlated with both mismeasured variable, f_i , and with $X_{i,j}$.

Summary of OLS, $AdjY$, and $AvgE$

True model: $y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$

True model: $y_{i,j} - \bar{y}_i = \beta(X_{i,j} - \bar{X}_i) + \epsilon_{i,j} - \bar{\epsilon}_i$

$AdjY$ estimates: $y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$

$AvgE$ estimates: $y_{i,j} = \beta^{AvgE} X_{i,j} + \gamma^{AvgE} \bar{y}_i + u_{i,j}^{AvgE}$

- ▶ All three estimators are inconsistent in the presence of unobserved group heterogeneity.
- ▶ $AdjY$ and $AvgE$ are not necessarily an improvement over OLS.
- ▶ $AdjY$ and $AvgE$ can yield estimates with opposite signs to the true coefficient.

The Differences Will Matter! Example 1 — Capital Structure

- ▶ Regression model:

$$(D/A)_{i,t} = \alpha + \beta X_{i,t} + f_i + \epsilon_{i,t}$$

- ▶ $(D/A)_{i,t}$: Book leverage for firm i , year t
- ▶ $X_{i,t}$: Variables affecting leverage (e.g., tangibility, size, profitability)
- ▶ f_i : Firm fixed effect capturing unobserved, time-invariant factors
- ▶ Data: U.S. firms, 1950–2010, winsorized at 1% tails
- ▶ Goal: Compare how different estimators handle unobserved heterogeneity (f_i)

Capital Structure Regression Results

Dependent Variable: Book Leverage

Variable	OLS	AdjY	AvgE	FE
Fixed Assets / Total Assets	0.270*** (0.008)	0.066*** (0.004)	0.103*** (0.004)	0.248*** (0.014)
Ln(Sales)	0.011*** (0.001)	0.011*** (0.000)	0.011*** (0.000)	0.017*** (0.001)
Return on Assets	-0.015*** (0.005)	0.051*** (0.004)	0.039*** (0.004)	-0.028*** (0.005)
Z-score	-0.017*** (0.000)	-0.010*** (0.000)	-0.011*** (0.000)	-0.017*** (0.001)
Market-to-Book Ratio	-0.006*** (0.000)	-0.004*** (0.000)	-0.004*** (0.000)	-0.003*** (0.000)
Observations	166,974	166,974	166,974	166,974
R ²	0.29	0.14	0.56	0.66

- ▶ Notice how *AdjY* and *AvgE* estimates differ sharply from both OLS and FE.
- ▶ For example, the profitability (ROA) coefficient flips sign under *AdjY/AvgE*.
- ▶ Partial corrections for heterogeneity distort inference — bias can even reverse direction.

The Differences Will Matter! Example 2 — Firm Value

- ▶ Regression model:

$$Q_{i,j,t} = \alpha + \beta X_{i,j,t} + f_{j,t} + \epsilon_{i,j,t}$$

- ▶ $Q_{i,j,t}$: Tobin's Q for firm i , industry j , year t
- ▶ $X_{i,j,t}$: Firm-level determinants of value (e.g., size, R&D, profitability)
- ▶ $f_{j,t}$: Industry-year fixed effect (controls for sectoral conditions)
- ▶ Data: U.S. manufacturing firms
- ▶ Question: Do OLS, AdjY, AvgE, and FE produce consistent results?

Firm Value Regression Results

Dependent Variable: Tobin's Q

Variable	OLS	AdjY	AvgE	FE
Delaware Incorporation	0.100*** (0.036)	0.019 (0.032)	0.040 (0.032)	0.086** (0.039)
Ln(Sales)	-0.125*** (0.009)	-0.054*** (0.008)	-0.072*** (0.008)	-0.131*** (0.011)
R&D Expenses / Assets	6.724*** (0.260)	3.022*** (0.242)	3.968*** (0.256)	5.541*** (0.318)
Return on Assets	-0.559*** (0.108)	-0.526*** (0.095)	-0.535*** (0.097)	-0.436*** (0.117)
Observations	55,792	55,792	55,792	55,792
R^2	0.22	0.08	0.34	0.37

- ▶ $AdjY$ and $AvgE$ substantially underestimate the R&D coefficient compared to FE (3.0 vs 5.5).
- ▶ Their partial corrections remove part of the true within-group variation.
- ▶ Overall fit (R^2) confirms this — FE explains far more variation, capturing persistent unobserved factors.

General Implications of the Framework

- ▶ The same logic applies well beyond firm-level panel regressions:
 - ▶ Many “adjusted” estimators implicitly assume the group mean or median removes unobserved heterogeneity.
- ▶ However, any estimator that subtracts off a noisy or endogenous benchmark still suffers from omitted-variable or measurement-error bias.
- ▶ **Examples of biased AdjY-type estimators:**
 - ▶ Subtracting the group **median** or **value-weighted mean** instead of the unobserved fixed effect.
 - ▶ Subtracting the mean outcome of a **matched control sample** (as in diversification-discount studies).
 - ▶ Comparing “adjusted” outcomes before vs. after an event (as in M&A announcement studies).
 - ▶ Using **characteristically adjusted returns** in asset pricing.
- ▶ These adjustments remove some noise but not the unobserved heterogeneity that actually drives bias.

AdjY-Type Estimators in Asset Pricing

- ▶ In empirical asset pricing, researchers often compare returns across portfolios sorted by firm characteristics.
- ▶ Returns are typically “**characteristically adjusted**”:
 - ▶ Subtract the mean return of a benchmark portfolio with similar size, book-to-market, or R&D intensity.
 - ▶ $r_{i,t} - \bar{r}_{\text{benchmark},t}$ is regressed on firm characteristics.
- ▶ **Problem:** This is mathematically equivalent to *AdjY*.
 - ▶ The benchmark mean ($\bar{r}_{\text{benchmark},t}$) is a noisy, endogenous proxy for the unobserved common component (e.g., systematic factor, industry effect).
 - ▶ It does not hold constant the variation in the independent variable across benchmark portfolios.
- ▶ Hence, the adjustment does not eliminate unobserved co-movement — it may even exaggerate it.

Asset Pricing *AdjY* Example — R&D and Stock Returns

- ▶ Example: Firms sorted into quintiles by R&D intensity ($R&D/MVE$).
- ▶ Researchers compute “characteristically adjusted” yearly returns by subtracting industry-size benchmark means (i.e., an *AdjY* transformation).

Missing	Q1	Q2	Q3	Q4	Q5
-0.012*** (0.003)	-0.033*** (0.009)	-0.023*** (0.008)	-0.002 (0.007)	0.008 (0.013)	0.020*** (0.006)

- ▶ Benchmark portfolios: industry-size matched means of returns.
- ▶ Difference between Q5 and Q1 = 5.3 percentage points.
- ▶ Appears to suggest “high R&D firms outperform.”
- ▶ But since benchmark returns correlate with firm characteristics and unobserved shocks, this inference may be spurious.

Regression Comparison: *AdjY* vs Fixed Effects

Dependent Variable: Yearly Stock Return

R&D Quintile	<i>AdjY</i> Estimate	FE Estimate
Missing	0.021** (0.009)	0.030*** (0.010)
Quintile 2	0.010 (0.013)	0.019 (0.019)
Quintile 3	0.032*** (0.012)	0.051*** (0.014)
Quintile 4	0.041*** (0.015)	0.068*** (0.018)
Quintile 5	0.053*** (0.011)	0.094*** (0.020)
Observations	144,592	144,592
R^2	0.00	0.40

- ▶ Regression equivalent of the previous “sorts” result.
- ▶ The FE version applies benchmark-period fixed effects to both returns and R&D — conceptually a cleaner “within” estimator.
- ▶ *AdjY* coefficients are consistently smaller in magnitude.
- ▶ R^2 is near zero under *AdjY* but large under FE — indicating that benchmark adjustment misses systematic variation.

What If $AdjY$ or $AvgE$ Is the True Model?

- ▶ Suppose the data truly follow the $AvgE$ structure—where the **group mean outcome** directly affects each member's outcome:

$$y_{i,j} = \beta X_{i,j} + \gamma \bar{y}_i + u_{i,j}$$

- ▶ Then \bar{y}_i itself depends on \bar{u}_i , which includes $u_{i,j}$.
- ▶ This creates a simultaneous relationship: individuals affect the group mean and the group mean affects individuals.
- ▶ This is the classic **reflection problem** (Manski, 1993) — identifying peer or group effects becomes impossible without extra structure or instruments.
- ▶ In this case, **none** of the estimators (OLS, $AdjY$, $AvgE$, or FE) recover the true β . [See Leary and Roberts (2010) for a finance application.]

What If $AdjY$ or $AvgE$ Is the True Model?

- ▶ Even if the researcher is interested in deviations from the group mean: $(y_{i,j} - \bar{y}_i)$, the $AdjY$ estimator is only consistent if $X_{i,j}$ has no effect on $y_{i,j}$.
 - ▶ If $X_{i,j}$ influences $y_{i,j}$, then it must also influence others in the same group $(y_{i,-j})$ through correlated behavior or shared shocks.
 - ▶ Therefore, \bar{X}_i also affects $(y_{i,j} - \bar{y}_i)$, implying:

$$\text{Cov}(X_{i,j}, (y_{i,j} - \bar{y}_i)) \neq 0.$$

- ▶ In short, it is impossible for $X_{i,j}$ to affect $y_{i,j}$ but not the group deviation $(y_{i,j} - \bar{y}_i)$.
- ▶ When true interdependence exists among group members, simple “adjusted” or “demeaned” models confound cause and reflection. Identifying the effect of $X_{i,j}$ requires either instrumental variables or structural modeling of peer interactions.

Summary of Today [Part 1]

- ▶ Our data isn't perfect:
 - ▶ Watch for measurement error.
 - ▶ Watch for survivorship bias.
 - ▶ Be careful about external validity claims.
- ▶ Test that estimates across subsamples are statistically different (if you plan to claim differences).

Summary of Today [Part 2]

- ▶ Don't use AdjY or AvgE !
- ▶ Do use fixed effects:
 - ▶ Use benchmark portfolio-period FE in asset pricing rather than characteristically adjusted returns.