Operations Research Case 3 Assignment

1. (40 points) Please formulate an integer program that may find an optimal allocation of CSRs. As always, please formulate a mixed linear integer program. Except having integer variables, do not have nonlinear constraints.
For this problem, please write down a compact mathematical formulation. Do not submit a computer program.

Step 1: Define Variables and Parameters

Define Variables

$$\mathrm{N_{jk}} = \text{numbers of CSRs working in shift k in date j}$$

$$\mathrm{N} = \mathrm{N_{31\times14}}$$

$$\mathrm{x_{ijk}} = \begin{cases} 1, & \text{if CSR i is assigned to shift k in day j} \\ 0, & \text{otherwise} \end{cases}$$

Define Parameters

Shift arrangement

$$\mathrm{A_{kp}} = \text{Working status at period p in shift s}$$

$$\mathrm{A} = \mathrm{A_{14\times24}}$$

Shift Category	Shift	09:00 09:30	09:30 10:00	10:00 10:30	10:30 11:00	11:00 11:30	11:30 12:00	12:00 12:30	12:30 13:00	13:00 13:30	13:30 14:00	14:00 14:30	14:30 15:00	15:00 15:30	15:30 16:00	16:00 16:30	16:30 17:00	17:00 17:30	17:30 18:00	18:00 18:30	18:30 19:00	19:00 19:30	19:30 20:00	20:00 20:30	20:30 21:00
Morning	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
	2	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
	3	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
	4	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
	5	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
Afternoon	7	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
	8	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
	9	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
	10	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0
Night	11	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
	12	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1
	13	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
Leave	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Shift working in every period

Demands estimation

$$\mathrm{D_{jp}} = \text{demand of CSRs at period p at date d}$$

$$\mathrm{D} = \mathrm{D_{31\times24}}$$

Time	1	2	3	4	5	6	7	8	9	10
09:00~09:30	29	20	21	29	20	21	17	29	20	21
09:30~10:00	24	20	18	24	20	18	18	24	20	18
10:00~10:30	25	21	20	25	21	20	17	25	21	20
10:30~11:00	24	22	20	24	22	20	18	24	22	20
11:00~11:30	24	19	22	24	19	22	20	24	19	22
11:30~12:00	24	18	19	24	18	19	17	24	18	19
12:00~12:30	17	12	13	17	12	13	12	17	12	13
12:30~13:00	13	14	11	13	14	11	12	13	14	11
13:00~13:30	17	14	14	17	14	14	11	17	14	14
13:30~14:00	23	18	17	23	18	17	15	23	18	17
14:00~14:30	21	20	20	21	20	20	18	21	20	20
14:30~15:00	23	20	21	23	20	21	17	23	20	21
15:00~15:30	21	18	18	21	18	18	16	21	18	18
15:30~16:00	18	20	18	18	20	18	17	18	20	18
16:00~16:30	20	17	17	20	17	17	14	20	17	17
16:30~17:00	20	17	15	20	17	15	15	20	17	15
17:00~17:30	16	11	15	16	11	15	13	16	11	15
17:30~18:00	11	11	13	11	11	13	11	11	11	13
18:00~18:30	10	8	8	10	8	8	9	10	8	8
18:30~19:00	8	8	6	8	8	6	7	8	8	6
19:00~19:30	8	9	6	8	9	6	8	8	9	6
19:30~20:00	8	8	8	8	8	8	7	8	8	8
20:00~20:30	9	8	7	9	8	7	7	9	8	7
20:30~21:00	8	7	6	8	7	6	6	8	7	6

Demand in day 1 to day 7 in the following month

CSRs parameters

$$Y_i = \text{work experience in years of CSR i}$$

$$P_i = \text{Position fo CSR i} = \begin{cases} 2, & \text{Manager} \\ 1, & \text{Assistant Manerger} \\ 0, & \text{otherwise} \end{cases}$$

Step 2: Define Objective Function

The main idea of this task is to minimize the "Lack" of CSRs' supply, which "lack" means we care only about the insufficient of supplements instead of over supplements. Therefore, objective function has to detect those "lack" periods that lack numbers' summation. So that objective function may be designed as follows.

⇨ Assume S_{dp} as lack of supplements in dated at period p

$$S_{jp} = D_{jp} - (NA)_{jp}$$

However, we only care the "lack" of supplements, so we only need positive S_{dp}.

$$\text{Positive}(x) = \{x \text{ , if } x \geq 0;\ 0 \text{ , if } x < 0\ \}$$

$$z = \sum_{d=1}^{31} \sum_{p=1}^{24} \text{Positive}(S_{jp})$$

To Linearize the problem, we have to add additional constraints.

⇨ Assume $\omega_{jp} = \text{Positive}(S_{jp})$

⇨ $\omega_{jp} \geq 0, \omega_{jp} \geq S_{jp}$

So the objective function becomes as followed

$$z = \sum_{d=1}^{31} \sum_{p=1}^{24} \omega_{jp}$$

$$\omega_{jp} \geq 0$$

$$\omega_{jp} \geq S_{jp}$$

$$\forall j = 1,2,\dots,31 \ \ \forall p = 1,2,\dots,24$$

Step 3: Define CSRs allocation's constraint

(1) Number of shifts summation

$$N_{jk} = \sum_{i=1}^{40} x_{ijk}$$

$$\forall j = 1,2,\dots,31 \ \ \forall k = 0,1,2,\dots,13$$

(2) Everyone should be assign to one shift each day.

$$\sum_{k=0}^{13} x_{ijk} = 1, \ \ i = 1,2,\dots,40, \ \ j = 1,2,\dots,31$$

(3) At least 8 day-off per month

$$\sum_{j=1}^{31} x_{ij0} \geq 8 \quad \forall i = 1,2,\dots,40$$

(4) At most one night-shift and two afternoon shifts per week.
For night shift

$$\sum_{j=d}^{d+6}\sum_{k=11}^{13} x_{ijk} \le 1 \sum_{j=d}^{d+6}\sum_{k=7}^{10} x_{ijk} \le 2$$
$$\forall d = 1,2,\dots,25, \forall i = 1,2,\dots,40$$

(5) At least one day off in seven consecutive days.

$$\sum_{j=d}^{d+6} x_{ij0} \ge 1, \forall d = 1,2,\dots,25, \forall i = 1,2,\dots,40$$

(6) Leave and shift requests

If CSR i request for shift k in date j

⇨ $x_{ijk} = 1$

If CSR i request leave in date j_1 to j_2

⇨ $x_{ij0} = 1, \ \forall j = j_1, j_1 + 1, \dots, j_2$

(7) Manager limits

If date j night shift has manager limit, and require q CSRs who are Assistant Managers or above.

$$3M_i \ge P_i, \qquad M_i = \begin{cases} 1, & \text{CSR i is above Assistant Manager} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{k=11}^{13}\sum_{i=1}^{40} M_i x_{ijk} \ge q$$

If date j afternoon shift has manager limit, and require q CSRs who are Managers.

$$3M_i \ge P_i - 1, \qquad M_i = \begin{cases} 1, & \text{CSR i is above Manager} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{k=7}^{10}\sum_{i=1}^{40} M_i x_{ijk} \ge q$$

(8) Senior limits

Assume that we know that date I = 1 's weekday.

We define the following table:

weekday	Mon	Tue	Wed	Thu	Fri	Sat	Sun
index	0	1	2	3	4	5	6

If during daytime D in period p_1 to p_2, there is a senior limit of working experience of y years that is above a ratio of r.

Define L as a huge number $L = \sum_{i=1}^{40} Y_i$

$$LO_i \ge Y_i - y + 1 \ , O_i = \begin{cases} 1, & \text{CSR i is senior enough} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{40} O_i x_{ijk} \ge r \sum_{i=1}^{40} x_{ijk}, \qquad J \in j\%7 = D, K \in \text{shift that contain } p_1 \text{ to } p_2$$

2. (30 points) Submit a (set of) computer program that may solve the integer program you formulated in Problem 1. You may submit an AMPL model file and an AMPL data file. Alternatively, you may submit a (set of) Python or C++ programs that invoke gurobi to solve this problem.

(1) Please put “solution.py” and “OR108-2_case00_data.xlsx” in the same folder
(2) When running the program, please do not open “OR108-2_case00_data.xlsx”.
(3) This program takes around 5 sec, after that you can open “OR108-2_case00_data.xlsx”, there you can see three additional data sheet.

a. CSRs test place

It has each CSR’s allocation data in the hole month, and there are summations of day-offs, night, afternoons shifts and the end of each CSR’s line. Also you can see that different shifts have different colors, green is for day-offs, yellow is for morning shifts, red is for afternoon shifts and blue is for night shifts.

	A	B	C	D	E	F	G	
1	Name\Date	ID	1	2	3	4	5	6
2	Georgina Stevens	12	3	0	0	5	0	
3	Cory Grant	19	0	0	0	0	13	
4	Sasha Blouse	20	2	1	0	12	6	
5	Sam Burns	23	11	10	0	2	5	
6	Finley Hamilton	30	0	13	2	0	1	
7	Lynn Harris	36	8	2	10	13	6	
8	Avery Mejia	37	2	12	0	0	8	
9	Rowan Bailey	40	2	0	1	7	8	
10	Breanna Mckay	45	10	0	11	0	4	
11	Jay Gardner	49	11	3	0	10	1	
12	Paloma Blackwell	69	3	11	5	2	0	
13	Willy Shaw	70	0	0	9	13	6	

b. Output Data

It’s about the total shifts number in each day.

A	B	C	D	E	F
Date\shift	1	2	3	4	5
1	3	7	3	1	0
2	4	5	3	0	0
3	4	1	0	2	5
4	2	7	1	3	3
5	4	1	2	5	3
6	6	2	2	2	2
7	3	2	5	0	0

c. Output Demand

It’s about the total demand minus supply in each day, so that positive means the lack of supply, negative means over supply.

A	B	C	D	E	F
Date\period	1	2	3	4	5
1	-1	-6	-5	-6	-6
2	-2	-2	-1	0	-3
3	-5	-8	-6	-6	-4
4	0	-5	-4	-5	-5
5	-2	-2	-1	0	-3
6	-3	-6	-4	-4	-2
7	-4	-3	-4	-3	-1

3. (10 points) Please summarize the optimal solution you obtain with your computer programs submitted in Problem 2. You should at least summarize the allocation of CSRs and the total lack amount returned by your computer programs. If you believe that there is other information that will be useful for executive/managers, you want provide them.

4. (20 points) This problem is designed to give you some free points. Please write down your opinions and thoughts for the three case assignments. You may say that you learned a lot, you learned nothing, you think the case assignments help you get a feeling about real-world applications of Operations Research, you think the whole approach is useless, you think it is more interesting than homework, you think it is less interesting than homework, etc. As long as you write down something, regardless of the length and content, you get 20 points