Ministério da Educação
UNIVERSIDADE FEDERAL DO PARANÁ
Setor de Ciências Exatas

Nota: ______

Disciplina: ______

Curso: Mecânica - AD

Professor: P1

Aluno: GABARITO

Turma: ______ Data: ___ / ___ / ___

① $\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} \cong \frac{3,5-4}{0,25} \cong \frac{-0,5}{0,25} \cong -2$

$\frac{\partial f}{\partial y}(0,-1) = \lim_{h \to 0} \frac{f(0,-1+h) - f(0,-1)}{h} \cong \frac{5,75-6}{0,25} = \frac{-0,25}{0,25} = -1$

② a) $\lim_{(x,y) \to (0,0)} \ln(1+x^2+y) \cos\left(\frac{1}{x^2+y^2}\right)$

1º jeito: $-1 \leq \cos\left(\frac{1}{x^2+y^2}\right) \leq 1$

$-\ln(1+x^2+y) \leq \ln(1+x^2+y) \cos\left(\frac{1}{x^2+y^2}\right) \leq \ln(1+x^2+y)$

Aplicando limite fica

$\underbrace{0 \leq \lim_{(x,y) \to (0,0)} \ln(1+x^2+y) \cos\frac{1}{x^2+y^2} \leq 0}_{=0}$

2º jeito: $\lim_{(x,y) \to (0,0)} \underbrace{\ln(1+x^2+y)}_{\to 0} \underbrace{\cos\frac{1}{x^2+y^2}}_{\text{limitada}} = 0$ //

② b) $y = 2x+1 \Rightarrow$ $y-1 = 2x$ $\lim \frac{x^3(2x) - x(2x)^3}{x^4 + (2x)^4} = \frac{2x^4 - 8x^4}{x^4 + 16x^4} = \frac{-6x^4}{17x^4}$

$= \frac{-6}{17}$

$y = x+1 \Rightarrow \lim = 0$ //

③ $f(x,y) = \dfrac{x^2+y^2}{y^2+1} \Rightarrow k = \dfrac{x^2+y^2}{y^2+1}$

$ky^2+k = x^2+y^2 \Rightarrow x^2+(1-k)y^2 = k$

Se $k=1$:

$x^2=1 \Rightarrow x=\pm 1$

Se $k>1$:

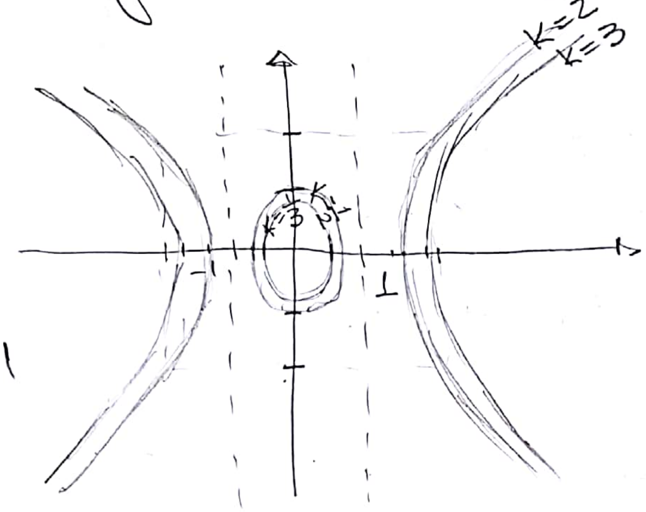
$\underbrace{\dfrac{x^2}{k}+\dfrac{(1-k)}{k}y^2=1}_{\text{hipérbole}}$

exemplo: $k=2$

$\dfrac{x^2}{2}-\dfrac{y^2}{2}=1$

$k=3$

$\dfrac{x^2}{3}-\dfrac{y^2}{\frac{3}{2}}=1$



Se $k<1$:

$\underbrace{\dfrac{x^2}{k}+\dfrac{(1-k)}{k}y^2=1}_{\text{elipse}}$

exemplo: $k=\frac{1}{2}$

$\dfrac{x^2}{\frac{1}{2}}+\dfrac{\frac{1}{2}}{\frac{1}{2}}y^2=1$

$k=\dfrac{1}{3}$

$\dfrac{x^2}{\frac{1}{3}}+\dfrac{\frac{2}{3}y^2}{\frac{1}{3}}=1$

$\dfrac{x^2}{\frac{1}{3}}+\dfrac{y^2}{\frac{1}{2}}=1$

④ $u(t,x) = \cos(x-\pi t)+2\,\mathrm{sen}(x+\pi t)$

$\dfrac{\partial^2 u}{\partial t^2} = c^2\dfrac{\partial^2 u}{\partial x^2}$

$\dfrac{\partial u}{\partial x} = -\mathrm{sen}\,(x-\pi t)+2\cos(x+\pi t)$

$\dfrac{\partial^2 u}{\partial x^2} = -\cos(x-\pi t)-2\,\mathrm{sen}(x+\pi t)$

$$\frac{\partial u}{\partial t} = -(-\pi)\,\text{sen}(x-\pi t) + 2\pi\cos(x+\pi t)$$

$$= \pi\,\text{sen}(x-\pi t) + 2\pi\cos(x+\pi t)$$

$$\frac{\partial^2 u}{\partial t^2} = -\pi^2\cos(x-\pi t) - 2\pi^2\,\text{sen}(x+\pi t)$$

$$-\pi^2\cos(x-\pi t) - 2\pi^2\,\text{sen}(x+\pi t) =$$

$$= -c^2\cos(x-\pi t) - 2c^2\,\text{sen}(x+\pi t)$$

$$\begin{cases} \pi^2 = c^2 \\ \pi^2 = c^2 \end{cases} \Rightarrow c = \pm\pi \Rightarrow \text{Como } c > 0,$$

$$c = \pi$$

b) $\frac{\partial u}{\partial x}(0, \pi/4) = -\,\text{sen}(x-\pi t) + 2\cos(x+\pi t)$

$$= -\,\text{sen}\,\frac{\pi}{4} + 2\cos\frac{\pi}{4}$$

$$= -\frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$\frac{\partial u}{\partial y}(0, \pi/4) = \pi\,\text{sen}(x-\pi t) + 2\pi\cos(x+\pi t)$$

$$= \pi\,\text{sen}\,\frac{\pi}{4} + 2\pi\cos\frac{\pi}{4} = \pi\frac{\sqrt{2}}{2} + 2\pi\frac{\sqrt{2}}{2}$$

$$= \frac{3\pi\sqrt{2}}{2}$$

$$z - \tilde{z}_0 = f_x(0, \pi/4)\left(x - \frac{\pi}{4}\right) + f_t(0, \pi/4)(t-0)$$

$$u(0, \pi/4) = \cos\frac{\pi}{4} + 2\,\text{sen}\,\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$$

$$z - \frac{3\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) + \frac{3\pi\sqrt{2}}{2}(t)$$

(5) $r(t) = \langle 2\,\text{sen}\, t, 2\,\text{sen}\, 2t, 2\,\text{sen}\, 3t \rangle$

$r'(t) = \langle 2\cos t, 4\cos 2t, 6\cos 3t \rangle$

$(1, \sqrt{3}, 2) \Rightarrow 2\,\text{sen}\, t = 1 \Rightarrow \text{sen}\, t = 1/2 \Rightarrow t = \pi/6$

$2\,\text{sen}\, 2t = \sqrt{3} \Rightarrow \text{sen}\, 2t = \frac{\sqrt{3}}{2} \overset{?}{\Rightarrow} \text{sen}\, \pi/3 = \frac{\sqrt{3}}{2}$ OK!

$2\,\text{sen}\, 3t = 2 \Rightarrow \text{sen}\, 3t = 1 \overset{?}{\Rightarrow} \text{sen}\, \frac{\pi}{2} = 0$ OK!

pto inicial P_0 x: $2\cos \pi/6 = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

y: $4\cos 2t = 4\cos 2\frac{\pi}{6} = 4\cos\frac{\pi}{3} = 4\cdot\frac{1}{2} = 2$

z: $6\cos 3t = 4\cos 3\frac{\pi}{6} = 4\cos\frac{\pi}{2} = 4(0) = 0$

Assim a reta tangente é $\begin{cases} 1 + \sqrt{3}t \\ \sqrt{3} + 2t \\ 2 \end{cases}$

(6) $r(t) = \langle t\cos(\ln t), t\,\text{sen}(\ln t) \rangle$

$r'(t) = \langle \cos(\ln t) - \cancel{t}\,\text{sen}(\ln t)\frac{1}{\cancel{t}}, \text{sen}(\ln t) + t\cos(\ln t)\frac{1}{t} \rangle$

$r'(t) = \langle \cos(\ln t) - \text{sen}(\ln t), \text{sen}(\ln t) + \cos(\ln t) \rangle$

$|r'(t)| = \sqrt{(\cos(\ln t) - \text{sen}(\ln t))^2 + (\cos(\ln t) + \text{sen}(\ln t))^2}$

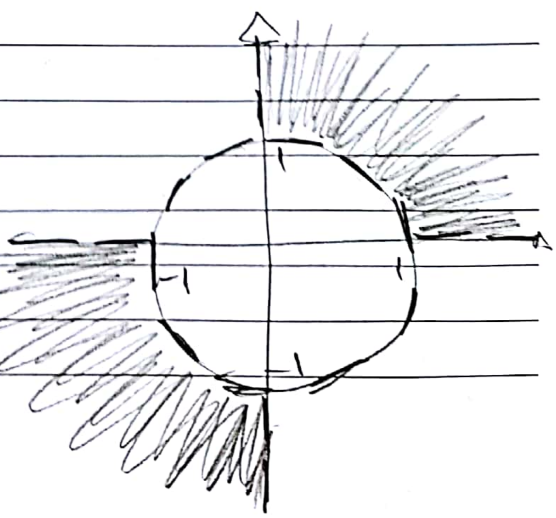
$|r'(t)| = \sqrt{\cos^2(\ln t) \cancel{- 2\cos(\ln t)\,\text{sen}(\ln t)} + \text{sen}^2(\ln t) + \cos^2(\ln t) \cancel{+ 2\cos(\ln t)\,\text{sen}(\ln t)} + \text{sen}^2(\ln t)}$

$|r'(t)| = \sqrt{1+1} = \sqrt{2}$

$L = \int_1^e \sqrt{2}\,dt = \sqrt{2}\,t \Big|_1^e = \sqrt{2}(e-1)$ //

(7) $x^2 + y^2 - 1 > 0 \Rightarrow x^2 + y^2 > 1$ //

$xy > 0 \Rightarrow \begin{cases} x > 0 \text{ e } y > 0 \\ x < 0 \text{ e } y < 0. \end{cases}$



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