Ministério da Educação
UNIVERSIDADE FEDERAL DO PARANÁ
Setor de Ciências Exatas

Nota: ______

Disciplina: ______

Curso: Gabarito — Professor: ______

Aluno: P2 AD

Turma: ______ Data: ___ / ___ / ___

① $z=(xe^y)^7$ — e — $x=1$ $y=0$

$dx=0{,}01$ $dy=0{,}015$

$\frac{\partial z}{\partial x} = 7e^y(xe^y)^6 \Rightarrow \frac{\partial z}{\partial x}(1,0) = 7\cdot1(1\cdot1)^6 = 7$

$\frac{\partial z}{\partial y} = 7xe^y(xe^y)^6 \Rightarrow \frac{\partial z}{\partial y}(1,0) = 7(1\cdot1)(1\cdot1)^6 = 7\cdot1\cdot1 = 7$

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = 7\cdot0{,}01 + 7\cdot0{,}015 = 7\cdot0{,}025$

$dz = 0{,}175$

$\therefore (1{,}01e^{0{,}015})^7 \cong \underbrace{(1\cdot1)^7}_{z} + \underbrace{0{,}175}_{+dz} = 1{,}175$

② $f(x,y) = \frac{y^3}{3} - 2xy + x^2y.$

Teste 1ª deriv.

$\frac{\partial f}{\partial x} = -2y + 2xy = 0 \Rightarrow y(2x-2)=0 \Rightarrow \begin{cases} x=1 \\ y=0. \end{cases}$

$\frac{\partial f}{\partial y} = y^2 - 2x + x^2 = 0$

se $x=1$: $y^2-2+1=0$ — se $y=0$: $x^2-2x=0$

$y^2-1=0$ — $x(x-2)=0$

$y=\pm1$ — $x=0, 2.$

$(1,\pm1)$ — $(0,0)$ e $(2,0)$

Teste 2ª deriv.

$H = \begin{bmatrix} 2y & -2+2x \\ -2+2x & 2y \end{bmatrix}$

Ptos Críticos	Classif
$(1,1)$	$H=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow D=4>0 \Rightarrow$ Pto Mínimo
$(1,-1)$	$H=\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow D=4>0 \Rightarrow$ Pto Máximo
$(0,0)$	$H=\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} \Rightarrow D=-4<0 \Rightarrow$ Pto Sela
$(2,0)$	$H=\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \Rightarrow D=-4 \Rightarrow$ Pto Sela

③ $z=f(x,y)$ $\quad x=r\cos\theta$ $\quad y=r\,\text{sen}\,\theta$

$$\begin{cases} \dfrac{\partial x}{\partial r}=\cos\theta \\ \dfrac{\partial x}{\partial \theta}=-r\,\text{sen}\,\theta \end{cases} \qquad \begin{cases} \dfrac{\partial y}{\partial r}=\text{sen}\,\theta \\ \dfrac{\partial y}{\partial \theta}=r\cos\theta \end{cases}$$

$$\frac{\partial z}{\partial r}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial r}=\frac{\partial z}{\partial x}\cdot\cos\theta+\frac{\partial z}{\partial y}\,\text{sen}\,\theta$$

$$\left(\frac{\partial z}{\partial r}\right)^2=\left(\frac{\partial z}{\partial x}\cos\theta\right)^2+\left(\frac{\partial z}{\partial y}\,\text{sen}\,\theta\right)^2$$

$$=\left(\frac{\partial z}{\partial x}\right)^2\cos^2\theta+2\frac{\partial z}{\partial x}\cos\theta\,\frac{\partial z}{\partial y}\,\text{sen}\,\theta+\left(\frac{\partial z}{\partial y}\right)^2\text{sen}^2\theta$$

$$\frac{\partial z}{\partial \theta}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}=\frac{\partial z}{\partial x}(-r\,\text{sen}\,\theta)+\frac{\partial z}{\partial y}(r\cos\theta)$$

$$\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2 r^2\,\text{sen}^2\theta-2\frac{\partial z}{\partial x}r\,\text{sen}\,\theta\,\frac{\partial z}{\partial y}r\cos\theta+\left(\frac{\partial z}{\partial y}\right)^2 r^2\cos^2\theta$$

$$\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2\text{sen}^2\theta-2\frac{\partial z}{\partial x}\,\text{sen}\,\theta\,\frac{\partial z}{\partial y}\cos\theta+\left(\frac{\partial z}{\partial y}\right)^2\cos^2\theta$$

Assim

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2\cos^2\theta+2\frac{\partial z}{\partial x}\cos\theta\,\frac{\partial z}{\partial y}\,\text{sen}\,\theta+\left(\frac{\partial z}{\partial y}\right)^2\text{sen}^2\theta+$$
$$\left(\frac{\partial z}{\partial x}\right)^2\text{sen}^2\theta-2\frac{\partial z}{\partial x}\,\text{sen}\,\theta\,\frac{\partial z}{\partial y}\cos\theta+\left(\frac{\partial z}{\partial x}\right)^2\cos^2\theta$$

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2(\cos^2\theta+\text{sen}^2\theta)+\left(\frac{\partial z}{\partial y}\right)^2(\text{sen}^2\theta+\cos^2\theta)$$

$$=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2$$

como queríamos mostrar.

④ a) Seja $\nabla f(0,\pi/2)=\langle a,b\rangle$

$D_u f(0,\pi/2)=\nabla f(0,\pi/2)\cdot\langle 3/5,4/5\rangle=\langle a,b\rangle\langle 3/5,4/5\rangle=2/5$

$D_u f(0,\pi/2)=\nabla f(0,\pi/2)\cdot\langle 4/5,-3/5\rangle=\langle a,b\rangle\langle 4/5,-3/5\rangle=11/5$

$$\begin{cases}3/5a+4/5b=2/5\\4/5a-3/5b=11/5\end{cases}\Rightarrow\begin{cases}3a+4b=2\\4a-3b=11\end{cases}\Rightarrow\begin{cases}9a+12b=6\\16a-12b=44\end{cases}$$

$25a=50$

$\nabla f(0,\pi/2)=\langle 2,-1\rangle \Leftarrow \begin{cases}a=2\\b=-1\end{cases}$

b) $D_u f(0,\pi/2)=\nabla f(0,\pi/2)\cdot\langle 1/\sqrt{2},1/\sqrt{2}\rangle=\langle 2,-1\rangle\langle 1/\sqrt{2},1/\sqrt{2}\rangle$

$=2/\sqrt{2}-1/\sqrt{2}=\underline{1/\sqrt{2}}$

⑤ $L(x,y,\lambda)=xy^2-\lambda(2x^2+y^2-24)$

$\frac{\partial L}{\partial x}=y^2-\lambda(4x)=0.\Rightarrow$

$\frac{\partial L}{\partial y}=2xy-\lambda(2y)=0\Rightarrow 2y(x-\lambda)=0\Rightarrow\begin{cases}y=0\\\lambda=x\end{cases}$

$\frac{\partial L}{\partial \lambda}=0\Rightarrow 2x^2+y^2=24$

Se $y=0\Rightarrow 2x^2+y^2=24$

$2x^2=24$

$x=\pm\sqrt{12}$

Se $\lambda=x\Rightarrow y^2=4x^2$

$y=\pm 2x$

$2x^2+y^2=24$

$2x^2+4x^2=24$

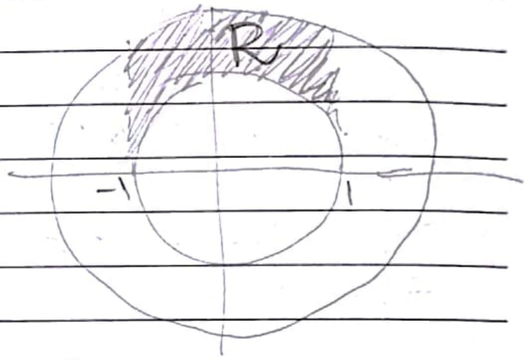
$x^2=4$

$x=\pm 2\Rightarrow y=\pm 4$

Ptos Críticos	$f(x,y)$	
$(\sqrt{12},0)$	0	
$(-\sqrt{12},0)$	0	
$(2,4)$	$2\cdot 4^2>0$	máximos globais
$(2,-4)$	$2\cdot(-4)^2>0$	
$(-2,4)$	$-2(4)^2<0$	mínimos globais
$(-2,-4)$	$-2(-4)^2<0$	

6) $\displaystyle\int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x\,dy\,dx$

a)



b) $\displaystyle\int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x\,dy\,dx = \int_{-1}^{1} x\left(\sqrt{4-x^2}-\sqrt{1-x^2}\right)dx =$

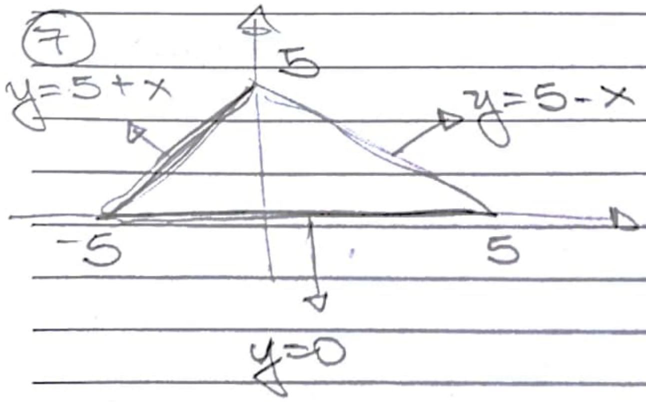
$\displaystyle= \int_{-1}^{1}\sqrt{4-x^2}\,x\,dx - \int_{-1}^{1}\sqrt{1-x^2}\,x\,dx =$

$u=4-x^2$ $\qquad$ $w=1-x^2$

$du=-2x\,dx$ $\qquad$ $dw=-2x\,dx$

$\displaystyle= \int_{3}^{3}\sqrt{u}\left(-\frac{1}{2}du\right) - \int_{0}^{0}\sqrt{w}\left(-\frac{1}{2}dw\right) = 0$

7)



Tipo I:

$\displaystyle\int_{-5}^{0}\int_{0}^{5+x} f(x,y)\,dy\,dx + \int_{0}^{5}\int_{0}^{5-x} f(x,y)\,dy\,dx$

Tipo II:

$\displaystyle\int_{0}^{5}\int_{y-5}^{5-y} f(x,y)\,dx\,dy$