Gabarito P3 - AD

① Coord Esf:

$$\begin{cases} x = \rho \operatorname{sen}\phi \cos\theta \\ y = \rho \operatorname{sen}\phi \operatorname{sen}\theta \\ z = \rho\cos\phi \end{cases} \quad , \quad 1 \le \rho \le 2 \ , \ 0 \le \theta \le 2\pi \ , \ 0 \le \phi \le \pi/4$$

$$\int_0^{\pi/4}\int_0^{2\pi}\int_1^{2} \frac{\rho^2 \cos^2\phi}{(\rho^2)^{3/2}} \cdot \rho^2 \operatorname{sen}\phi \, d\rho\, d\theta\, d\phi =$$

$$= \int_0^{\pi/4}\int_0^{2\pi}\int_1^{2} \rho \operatorname{sen}\phi \cos^2\phi \, d\rho\, d\theta\, d\phi$$

$$= \int_1^2 \rho\, d\rho \int_0^{\pi/4} \cos^2\phi \operatorname{sen}\phi\, d\phi \int_0^{2\pi} d\theta = \frac{\rho^2}{2}\Big|_1^2 \cdot \int_1^{\sqrt{2}/2} u^2(-du) \cdot 2\pi$$

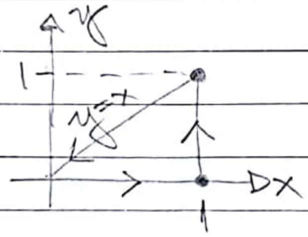
$u = \cos\phi$

$du = -\operatorname{sen}\phi\, d\phi$

$$= \frac{3}{2}\cdot 2\pi \int_{\sqrt{2}/2}^{1} u^2\, du = 3\pi \cdot \frac{u^3}{3}\Big|_{\sqrt{2}/2}^{1} = \pi\left(1 - \frac{2\sqrt{2}}{4\cdot 2}\right) = \left(1 - \frac{\sqrt{2}}{4}\right)\pi$$

② Teorema de Green: $\displaystyle\int_C F\cdot dr = \iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

$$F = 0\vec{i} + x^2 e^y \vec{j} \Rightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2xe^y - 0 = 2xe^y$$



Pelo Teorema de Green:

$$\oint_C x^2 e^y\, dy = \int_0^1\int_0^x 2xe^y\, dy\, dx$$

$$= 2\int_0^1 xe^y\Big|_0^x dx = 2\int_0^1 (xe^x - x)\, dx = 2\left[\int_0^1 xe^x\, dx - \int_0^1 x\, dx\right]$$

$$\int_0^1 xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x \Big|_0^1 = (e^1 - e^1) - (0-1)$$
$$= 1$$

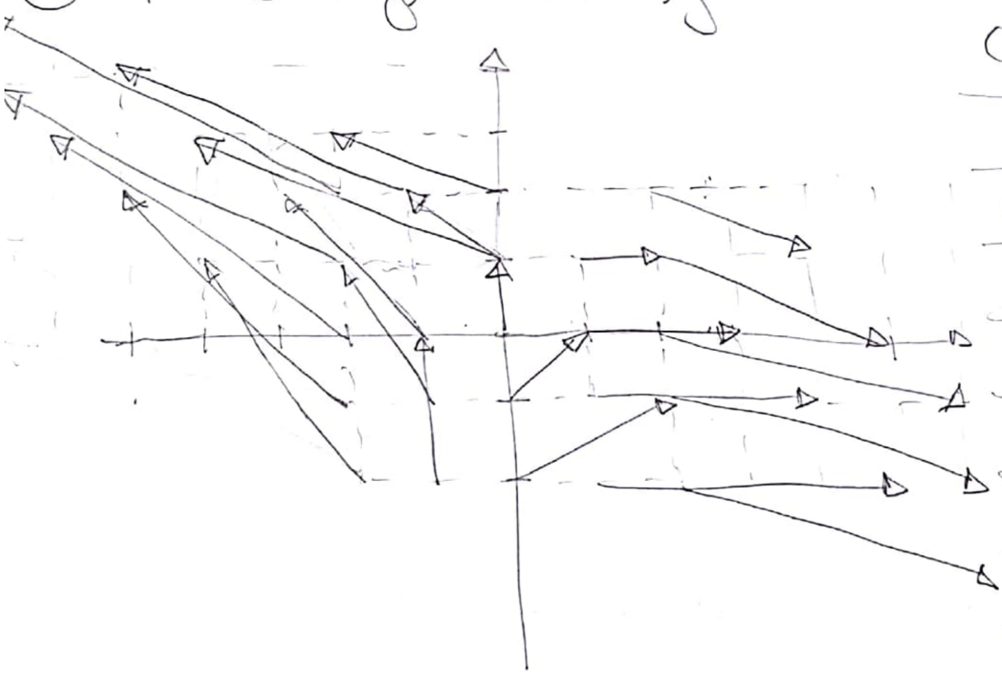
$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

Assim, $2\left[\int_0^1 xe^x\,dx - \int_0^1 x\,dx\right] = 2\left[(1) - \frac{x^2}{2}\Big|_0^1\right] = 2\left[1 - \frac{1}{2}\right]$

$= 2 \cdot \frac{1}{2} = 1$ //

③ $F = (2x - y)\vec{i} + (1 - x)\vec{j}$



tabela

(x,y)	F
$(0,0)$	$\langle 0,1\rangle$
$(1,0)$	$\langle 2,0\rangle$
$(1,1)$	$\langle 1,0\rangle$
$(0,1)$	$\langle -1,1\rangle$
$(-1,1)$	$\langle -3,2\rangle$
$(-1,0)$	$\langle -2,2\rangle$
$(-1,-1)$	$\langle -1,2\rangle$
$(0,-1)$	$\langle 1,1\rangle$
$(1,-1)$	$\langle 3,0\rangle$
$(2,0)$	$\langle 4,-1\rangle$
$(2,1)$	$\langle 3,-1\rangle$
$(2,2)$	$\langle 2,-1\rangle$
$(-1,-2)$	$\langle 0,2\rangle$
$(0,-2)$	$\langle 2,1\rangle$

(x,y)	F
$(1,2)$	$\langle 0,0\rangle$
$(0,2)$	$\langle -2,1\rangle$
$(-1,2)$	$\langle -4,2\rangle$
$(-2,2)$	$\langle -6,3\rangle$
$(2,-1)$	$\langle 5,-1\rangle$

(x,y)	F
$(-2,1)$	$\langle -5,3\rangle$
$(-2,0)$	$\langle -4,3\rangle$
$(-2,-1)$	$\langle -3,3\rangle$
$(-2,-2)$	$\langle -2,3\rangle$
$(1,-2)$	$\langle 4,0\rangle$
$(2,-2)$	$\langle 6,-1\rangle$

a) $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -1-(-1) = 0 \Rightarrow$ F é conservativo

TFIL: $\int_C F \cdot dr = f\left(\begin{matrix}\text{pto}\\ \text{final}\end{matrix}\right) - f\left(\begin{matrix}\text{pto}\\ \text{inicial}\end{matrix}\right)$

$f_x = 2x - y \Rightarrow f = \int (2x-y)dx = x^2 - xy + g(y)$

$f_y = -x + g'(y) = 1 - x \Rightarrow g'(y) = 1 \Rightarrow g(y) = y + c$

$\therefore f = x^2 - xy + y + c$

pto final: $r(1) = \langle 1, e^1 \rangle \Rightarrow f\left(\begin{matrix}\text{pto}\\ \text{final}\end{matrix}\right) = 1 - \cancel{e} + \cancel{e} + c$
$t=1$

$= 1 + c$

pto inicial: $r(0) \langle 0, 1 \rangle \Rightarrow f\left(\begin{matrix}\text{pto}\\ \text{inicial}\end{matrix}\right) = 0 - 0 + 1 + c$
$t=0$

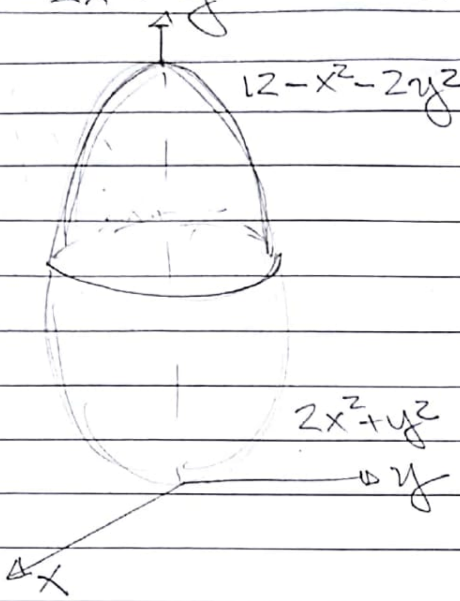
$\int_C F \cdot dr = f\left(\begin{matrix}\text{pto}\\ \text{final}\end{matrix}\right) - f\left(\begin{matrix}\text{pto}\\ \text{inicial}\end{matrix}\right) = 1+c-(1+c) = 0$

b) Como F é conservativo e C é uma curva fechada, então

$\oint F \cdot dr = 0$ pelo TFIL

Scanned by CamScanner

④ $2x^2+y^2 = 12-x^2-2y^2 \Rightarrow 3x^2+3y^2=12 \Rightarrow x^2+y^2=4$



$$V=\iint\int_{2x^2+y^2}^{12-x^2-2y^2} 1\,dz\,dA = \iint$$

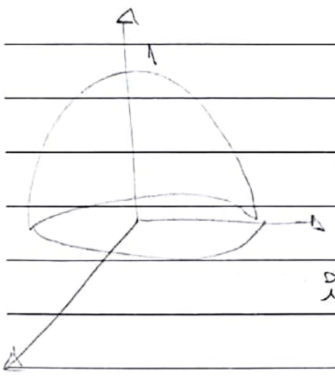
$$V=\iint (12-x^2-2y^2-2x^2-y^2)\,dA$$

$$V=\iint (12-3x^2-3y^2)\,dA$$

$$V=3\iint 4-(x^2+y^2)\,dA = 3\int_0^{2\pi}\int_0^{2}(4-r^2)\,r\,dr\,d\theta =$$

$$=3\int_0^{2\pi}d\theta\int_0^{2}(4r-r^3)\,dr = 3\cdot 2\pi\left(2r^2-\frac{r^4}{4}\right)\Bigg|_0^2 = 6\pi\left(8-\frac{\cancel{16}^{\,4}}{\cancel{4}}\right)=24\pi$$

⑤ Teorema de Stokes:



$$\int_C F\cdot dr = \iint_S \text{rot } F\cdot dS = \iint_S \text{rot } F\cdot(r_u\times r_v)\,dA$$

Temos 2 formas de resolver:

i) $\text{rot } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 z & y^2 & xy \end{vmatrix} = (x-0)\vec{i} - (y-x^2)\vec{j} + 0\vec{k}$

$= x\vec{i} - (y-x^2)\vec{j}$

$r(x,y) = \begin{cases} x=x \\ y=y \\ z=1-x^2-y^2 \end{cases}$ $\Rightarrow r_x=\langle 1,0,-2x\rangle$, $\Rightarrow r_y=\langle 0,1,-2y\rangle$

$r_x\times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & -2y \end{vmatrix} = 2x\vec{i}+2y\vec{j}+\vec{k}$

Ministério da Educação
UNIVERSIDADE FEDERAL DO PARANÁ
Setor de Ciências Exatas

Nota: ____________

Disciplina: ____________

Curso: ____________ Professor: ____________

Aluno: ____________

Turma: ____________ Data: ___ / ___ / ___

$\text{rot } F \cdot (r_x \times r_y) = 2x^2 - 2y(y - x^2)$

$$\iint_S \text{rot} F (r_x \times r_y)\, dA = \iint_S (2x^2 - 2y^2 + 2x^2 y)\, dA$$

$$= \int_0^{2\pi}\int_0^1 (2r^2\cos^2\theta - 2r^2 \text{sen}^2\theta + 2r^3\cos^2\theta\,\text{sen}\,\theta)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 (2r^3\cos 2\theta + 2r^4\cos^2\theta\,\text{sen}\,\theta)\, dr\, d\theta$$

$$= 2\int_0^1 r^3\, dr \int_0^{2\pi} \cos 2\theta\, d\theta + 2\int_0^1 r^4\, dr \int_0^{2\pi} \underbrace{\cos^2\theta}_{u^2}\, \underbrace{\text{sen}\,\theta\, d\theta}_{-du}$$

$$= 2 \left.\frac{r^4}{4}\right|_0^1 \left.\frac{\text{sen}\, 2\theta}{2}\right|_0^{2\pi} + 2 \left.\frac{r^5}{5}\right|_0^1 \int_1^1 u^2(-du)$$

$$= 2 \cdot \frac{1}{4} \cdot 0 + 2 \cdot \frac{1}{5} \cdot 0 = 0$$

ii) $$\int_C F \cdot dr = \int_C \langle \cos^2 t \cdot 0,\ \text{sen}^2 t,\ \cos t\,\text{sen}\,t \rangle \langle -\text{sen}\,t,\ \cos t,\ 0 \rangle\, dt$$

$x^2 + y^2 = 1$

$r(t) \begin{cases} x = \cos t \\ y = \text{sen}\, t \\ z = 0 \end{cases} \Rightarrow r'(t) = \begin{cases} dx/dt = -\text{sen}\, t \\ dy/dt = \cos t \\ dz/dt = 0 \end{cases}$

$$= \int_0^{2\pi} \text{sen}^2 t \cos t\, dt$$

$$= \int_0^0 u^2\, du = 0$$

⑥ $F = xz\vec{i} + yz\vec{j} + 2018z\vec{k}$

$r(\theta,z) = \begin{cases} x = 2\cos\theta \\ y = \operatorname{sen}\theta \\ z = z \end{cases}$ $\quad r_\theta = \langle -2\operatorname{sen}\theta, \cos\theta, 0 \rangle$

$r_z = \langle 0, 0, 1 \rangle$

$0 \le \theta \le 2\pi$

$0 \le z \le 2$

$$r_\theta \times r_z = \begin{vmatrix} i & j & k \\ -2\operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta\, i + 2\operatorname{sen}\theta\,\vec{j} = \langle \cos\theta, 2\operatorname{sen}\theta, 0 \rangle$$

$$F\cdot(r_\theta \times r_z) = 2\cdot\cos\theta\cdot z\cdot\cos\theta + \operatorname{sen}\theta\cdot z\cdot 2\operatorname{sen}\theta + 0$$
$$= 2z(\cos^2\theta + \operatorname{sen}^2\theta) = 2z$$

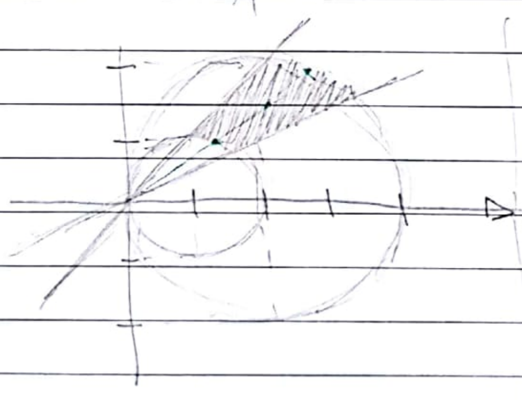
$$\int_0^2\int_0^{2\pi} 2z\, d\theta\, dz = 2\int_0^{2\pi} d\theta \int_0^2 z\, dz = 2\cdot 2\pi\cdot \frac{z^2}{2}\Big|_0^2 = 4\pi\cdot\frac{4}{2} = 8\pi$$

⑦ $x^2+y^2 = 2x$ $\qquad$ $x^2+y^2 = 4x$

$x^2-2x+1+y^2 = 1$ $\qquad$ $x^2-4x+4+y^2 = 4$

$(x-1)^2+y^2 = 1$ $\qquad$ $(x-2)^2+y^2 = 4$

$C(1,0),\ r=1$ $\qquad$ $C(2,0),\ r=2$



$$\int_{\pi/6}^{\pi/4}\int_{2\cos\theta}^{4\cos\theta} r\cdot r\, dr\, d\theta = \int_{\pi/6}^{\pi/4}\int_{2\cos\theta}^{4\cos\theta} r^2\, dr\, d\theta$$

$$= \int_{\pi/6}^{\pi/4} \frac{r^3}{3}\Big|_{2\cos\theta}^{4\cos\theta} d\theta = \frac{1}{3}\int_{\pi/6}^{\pi/4} (64\cos^3\theta - 8\cos^3\theta)\, d\theta$$

$x^2+y^2 = 2x$ $\qquad$ $x^2+y^2=4x$

$r^2 = 2r\cos\theta$ $\qquad$ $r^2 = 4r\cos$

$r(r-2\cos\theta)=0$ $\qquad$ $r(r-4\cos\theta)=0$

$\begin{cases} r=0 \\ r = 2\cos\theta \end{cases}$ $\qquad$ $\begin{cases} r=0 \\ r=4\cos\theta \end{cases}$

$$= \frac{1}{3}\, 56 \int_{\pi/6}^{\pi/4} \cos^3\theta\, d\theta =$$

$$= \frac{56}{3}\int_{\pi/6}^{\pi/4} (1-\operatorname{sen}^2\theta)\cos\theta\, d\theta$$

$y = x$ $\qquad$ $y = \sqrt{3}x/3$

$\Rightarrow \operatorname{tg}\theta = 1$ $\qquad$ $\Rightarrow \operatorname{tg}\theta = \frac{\sqrt{3}}{3}$

$\theta = \pi/4$ $\qquad$ $\theta = \pi/6$

$$= \frac{56}{3}\int_{1/2}^{\sqrt{2}/2} (1-u^2)\, du$$

Scanned by CamScanner

$$= \frac{56}{3}\left(u - \frac{u^3}{3}\right)\Bigg|_{1/2}^{\sqrt{2}/2} = \frac{56}{3}\left[\left(\frac{\sqrt{2}}{2} - \frac{1}{3}\frac{\cancel{2}}{4}\frac{\sqrt{2}}{\cancel{2}}\right) - \left(\frac{1}{2} - \frac{1}{3}\frac{1}{8}\right)\right]$$

$$= \frac{56}{3}\left[\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{12}\right) - \left(\frac{1}{2} - \frac{1}{24}\right)\right] = \frac{56}{3}\left(\frac{5\sqrt{2}}{12} - \frac{11}{24}\right) =$$

$$= \frac{\overset{7}{56}}{3}\,\frac{10\sqrt{2} - 11}{\underset{3}{24}} = \frac{7(10\sqrt{2} - 11)}{9}$$