Ministério da Educação
UNIVERSIDADE FEDERAL DO PARANÁ
Setor de Ciências Exatas

Nota: ______

Disciplina: ______

Curso: P1

Professor: ______

Aluno: ______

Turma: Gabarito - AN

Data: ___/___/___

1) a) $f(0,0)=4$; $f(0,-1)=6$

b) $\frac{\partial f}{\partial x}(0,0) = \frac{f(0+h,0)-f(0,0)}{h} = \frac{f(h,0)-f(0,0)}{h} = \frac{3-4}{0{,}4}$

$= \frac{-1}{0{,}4} = -1 \cdot \frac{10}{4} = -\frac{5}{2}$

c) $\frac{\partial f}{\partial y}(0,-1) = \frac{f(0,-1+h)-f(0,-1)}{h} = \frac{5-6}{-0{,}5} = \frac{-1}{-0{,}5} = 2$

2) a) 1º jeito

$-1 \leq \cos \frac{1}{x^2+y^2} \leq 1$

$-(x^2+y) \leq (x^2+y) \cos \frac{1}{x^2+y^2} \leq (x^2+y)$

$\lim_{(x,y)\to(0,0)} -(x^2+y) \leq \lim_{(x,y)\to(0,0)} (x^2+y) \cos \frac{1}{x^2+y^2} \leq \lim_{(x,y)\to(0,0)} (x^2+y)$

$0 \leq \lim_{(x,y)\to(0,0)} (x^2+y) \cos \frac{1}{x^2+y^2} \leq 0$

$\therefore \lim_{(x,y)\to(0,0)} (x^2+y) \cos \frac{1}{x^2+y^2} = 0$

2º jeito

$\lim_{(x,y)\to(0,0)} \underbrace{(x^2+y)}_{\text{tende a } 0} \underbrace{\cos \frac{1}{x^2+y^2}}_{\text{função limitada}} = 0$ pela 3ª técnica

b) $y-1=mx \Rightarrow \lim_{(x,y)\to(0,1)} \frac{x^3\cdot mx - x(mx)^3}{x^4+(mx)^4} =$

$= \lim_{(x,y)\to(0,1)} \frac{(m-m^3)x^4}{(1+m^4)x^4} = \frac{m-m^3}{1+m^4} = \frac{m(1-m^2)}{1+m^4}$

Se $m=0 \Rightarrow \lim_{(x,y)\to(0,1)} \frac{x^3(y-1)-x(y-1)^3}{x^4+(y-1)^4} = 0$

Se $m=1 \Rightarrow \lim_{(x,y)\to(0,1)} \frac{x^3(y-1)-x(y-1)^3}{x^4+(y-1)^4} = 0$

Se $m=2 \Rightarrow \lim_{(x,y)\to(0,1)} \frac{x^3(y-1)-x(y-1)^3}{x^4+(y-1)^4} = \frac{2(-3)}{1+16} = -\frac{6}{17}$

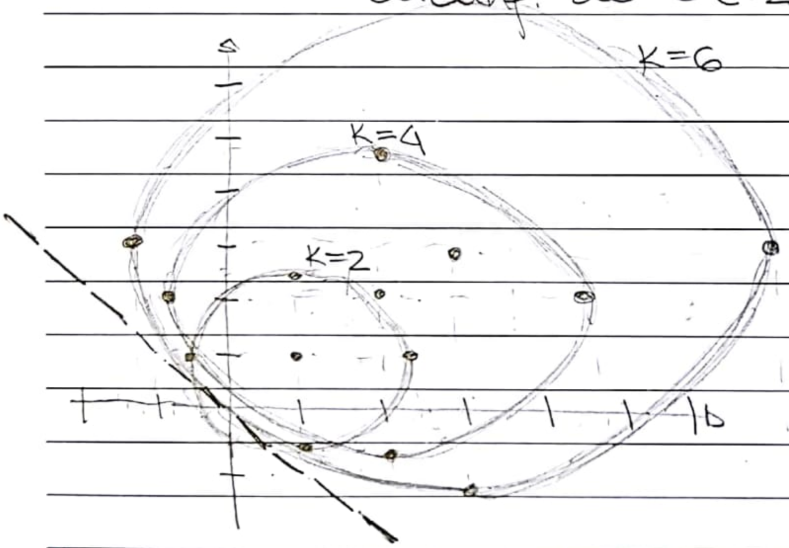
$\therefore$ o limite ñ existe

③ $f(x,y) = \frac{x^2+y^2}{x+y} \Rightarrow k = \frac{x^2+y^2}{x+y} \Rightarrow x^2-kx+y^2-ky=0$

$\Rightarrow x^2-kx+(k/2)^2+y^2-ky+(k/2)^2 = 2(k/2)^2 = 2\cdot k^2/4$

$\Rightarrow \underbrace{(x-k/2)^2+(y-k/2)^2 = k^2/2 = (k/\sqrt{2})^2}$

circunf. de $c(k/2, k/2)$ e $r = k/\sqrt{2} = \frac{k\sqrt{2}}{2}$



Dom: $\{x+y\neq 0 \Rightarrow y\neq -x\}$

④ $u(t,x) = \cos(x-\pi t) + 2\,\text{sen}(x+\pi t)$

$\frac{\partial u}{\partial t} = \pi\,\text{sen}(x-\pi t) + 2\pi\cos(x+\pi t)$

$\frac{\partial^2 u}{\partial t^2} = -\pi^2\cos(x-\pi t) - 2\pi^2\,\text{sen}(x+\pi t)$

$\frac{\partial u}{\partial x} = -\text{sen}(x-\pi t) + 2\cos(x+\pi t)$

$\frac{\partial^2 u}{\partial x^2} = -\cos(x-\pi t) - 2\,\text{sen}(x+\pi t)$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \Rightarrow$$

$$-\pi^2 \cos(x-\pi t) - 2\pi^2 \operatorname{sen}(x+\pi t) = c^2\left(-\cos(x-\pi t) - 2\operatorname{sen}(x+\pi t)\right)$$

$$(-\pi^2 + c^2)\cos(x-\pi t) + (-2\pi^2 + 2c^2)\operatorname{sen}(x+\pi t) = 0$$

$$\begin{cases} c^2 - \pi^2 = 0 \Rightarrow c^2 = \pi^2 \Rightarrow c = \pm\pi \\ 2c^2 - 2\pi^2 = 0 \end{cases}$$

Como $c > 0$, então $\underline{c = \pi}$

b) $t = 0$, $x = \pi/4$ $\qquad u(0, \pi/4) = \cos \pi/4 + 2 \operatorname{sen} \pi/4$

$$= \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$$

$$\frac{\partial u}{\partial t}(0, \pi/4) = \pi \operatorname{sen}(\pi/4) + 2\pi \cos \pi/4$$

$$= \frac{\pi\sqrt{2}}{2} + 2\pi\frac{\sqrt{2}}{2} = \frac{3\pi\sqrt{2}}{2}$$

$$\frac{\partial u}{\partial x}(0, \pi/4) = -\operatorname{sen}(\pi/4) + 2\cos \pi/4$$

$$= -\frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \frac{\sqrt{2}}{2}.$$

∴ Plano Tangente é

$$u - u_0 = \frac{\partial u}{\partial t}(t_0, x_0)(t - t_0) + \frac{\partial u}{\partial x}(t_0, x_0)(x - x_0)$$

$$u - \frac{3\sqrt{2}}{2} = \frac{3\pi\sqrt{2}}{2}(t - 0) + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$$

$$u = \frac{3\sqrt{2}}{2} + \frac{3\pi\sqrt{2}}{2}t + \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}\pi}{8}$$

(5) $r(t) = \langle 2\operatorname{sen} t, 2\operatorname{sen} 2t, 2\operatorname{sen} 3t \rangle$

$r'(t) = \langle 2\cos t, 4\cos 2t, 6\cos 3t \rangle$

$t = ?$ tal que $r(t) = (1, \sqrt{3}, 2)$

$$\operatorname{sen} t = \frac{1}{2}, \quad \operatorname{sen} 2t = \frac{\sqrt{3}}{2}, \quad \operatorname{sen} 3t = \frac{2}{2} = 1$$

$$t = \frac{\pi}{6} \qquad 2t = \frac{\pi}{3}, \qquad 3t = \frac{\pi}{2}$$

$$t = \frac{\pi}{6} \qquad t = \frac{\pi}{6}.$$

$r'(\pi/6) = \langle 2\cos\pi/6, 4\cos\pi/3, 6\cos\pi/2 \rangle = \langle 2\sqrt{3}/2, 4\cdot 1/2, 6\cdot 0 \rangle$

$r'(\pi/6) = \langle \sqrt{3}, 2, 0 \rangle$

$$\text{rta}: \begin{cases} x = 1 + \sqrt{3}t \\ y = \sqrt{3} + 2t \\ z = 2 \end{cases}$$

⑥ $r(t) = \langle t\cos(\ln t), t\,\text{sen}(\ln t) \rangle$

$r'(t) = \langle \cos(\ln t) - \cancel{t}\,\text{sen}(\ln t)\frac{1}{\cancel{t}}, \text{sen}(\ln t) + \cancel{t}\cos(\ln t)\frac{1}{\cancel{t}} \rangle$

$r'(t) = \langle \cos(\ln t) - \text{sen}(\ln t), \text{sen}(\ln t) + \cos(\ln t) \rangle$

$\|r'(t)\| = \sqrt{\cos^2(\ln t) - \cancel{2\cos(\ln t)\text{sen}(\ln t)} + \text{sen}^2(\ln t) + \text{sen}^2(\ln t) + \cancel{2\cos(\ln t)\text{sen}(\ln t)} + \cos^2(\ln t)}$

$\|r'(t)\| = \sqrt{2}$

$$L = \int_1^e \sqrt{2}\,dt = \sqrt{2}\,t\Big|_1^e = \sqrt{2}(e-1)$$

⑦ $x^2 + y^2 - 4 > 0 \Rightarrow x^2 + y^2 > 4$

$xy - 1 > 0 \Rightarrow xy > 1 \Rightarrow y > \frac{1}{x}$