P2 Gabarito - AN

① $z = \sqrt{x^2+y^2}$ onde $x = 4$, $dx = 0{,}03$

$z = (x^2+y^2)^{1/2}$ $y = 3$, $dy = -0{,}1$

$z = 5$

$$\frac{\partial z}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}}$$

Então $\sqrt{(4{,}03)^2+(2{,}9)^2} = z + \underbrace{dz}_{\text{diferencial}}$

$$dz = \frac{\partial z}{\partial x}(x,y)\,dx + \frac{\partial z}{\partial y}(x,y)\,dy$$

$$dz = \frac{x}{\sqrt{x^2+y^2}}\,dx + \frac{y}{\sqrt{x^2+y^2}}\,dy = \overbrace{\frac{4}{5}\cdot 0{,}03}^{0{,}12} - \overbrace{\frac{3\cdot 0{,}1}{5}}^{0{,}30} = -\frac{0{,}18}{5}$$

$$\sqrt{(4{,}03)^2+(2{,}9)^2} = 5 - \frac{0{,}18}{5} = 5 - 0{,}036 = 4{,}964$$

② $f(x,y) = x^3 - 3x + y^3 - 12y$

Teste 1ª Deriv.

$f_x = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$ $\Rightarrow$ Ptos Críticos:

$f_y = 3y^2 - 12 = 0 \Rightarrow y = \pm 2$ $(1,2), (-1,2), (-1,2), (-1,-2)$

Teste 2ª Deriv.

$$H = \begin{bmatrix} 6x & 0 \\ 0 & 6y \end{bmatrix}$$

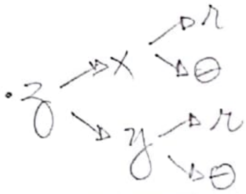
Ptos Críticos	Hessiana H	Classificação
$(1,2)$	$H = \begin{bmatrix} 6 & 0 \\ 0 & 12 \end{bmatrix} \Rightarrow D > 0$ com $f_{xx} > 0$	Pto Mínimo
$(-1,2)$	$H = \begin{bmatrix} -6 & 0 \\ 0 & 12 \end{bmatrix} \Rightarrow D < 0$	Pto Sela
$(1,-2)$	$H = \begin{bmatrix} 6 & 0 \\ 0 & -12 \end{bmatrix} \Rightarrow D < 0$	Pto Sela
$(-1,-2)$	$H = \begin{bmatrix} -6 & 0 \\ 0 & -12 \end{bmatrix} \Rightarrow D > 0$ com $f_{xx} < 0$	Pto Máximo

③ $F(y,z,x) = e^{z}x - e^{x}yz - x^2 + y^2$ com $f(\underset{y}{1},\underset{z}{1}) = \underset{x}{1}$

$$\frac{\partial x}{\partial y} = -\frac{F_y}{F_x} = -\frac{-e^{x}z + 2y}{e^{z} - e^{x}yz - 2x} \Rightarrow \frac{\partial x}{\partial y}(1,1) = -\frac{-e+2}{e-e-2} = \frac{2-e}{2}$$

$$\frac{\partial x}{\partial z} = -\frac{F_z}{F_x} = -\frac{e^{z}x - e^{x}y}{e^{z} - e^{x}yz - 2x} = -\frac{e-e}{e-e-2} = -\frac{0}{-2} = 0$$

④ $z = f(x,y)$, $x = r\cos\theta \Rightarrow \frac{\partial x}{\partial r} = \cos\theta$; $\frac{\partial x}{\partial \theta} = -r\,\text{sen}\,\theta$



$y = r\,\text{sen}\,\theta \Rightarrow \frac{\partial y}{\partial r} = \text{sen}\,\theta$; $\frac{\partial y}{\partial \theta} = r\cos\theta$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\,\text{sen}\,\theta$$

$$\left(\frac{\partial z}{\partial r}\right)^2 = \left(\frac{\partial z}{\partial x}\cos\theta - \frac{\partial z}{\partial y}\,\text{sen}\,\theta\right)^2$$

$$= \left(\frac{\partial z}{\partial x}\right)^2\cos^2\theta - 2\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\cos\theta\,\text{sen}\,\theta + \left(\frac{\partial z}{\partial y}\right)^2\text{sen}^2\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial z}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial z}{\partial y}r\cos\theta$$

$$\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \frac{1}{r^2}\left(\frac{\partial z}{\partial x}\right)^2 r^2\,\text{sen}^2\theta - \frac{1}{r^2}\cdot 2\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) r^2\,\text{sen}\,\theta\cos\theta + \frac{1}{r^2}\left(\frac{\partial z}{\partial y}\right)^2 r^2\cos^2\theta$$

$$= \left(\frac{\partial z}{\partial x}\right)^2\text{sen}^2\theta - 2\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\cos\theta\,\text{sen}\,\theta + \left(\frac{\partial z}{\partial y}\right)^2\cos^2\theta$$

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2(\operatorname{sen}^2\theta+\cos^2\theta)+\left(\frac{\partial z}{\partial y}\right)^2(\operatorname{sen}^2\theta+\cos^2\theta)$$

$$=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2 \quad \text{cqd}$$

⑤ a) $D_u f(x,y)=\underbrace{\nabla f(x,y)}_{\langle a,b\rangle}\cdot \vec{u}$ com $\|\vec{u}\|=1$

$$\begin{cases}\langle a,b\rangle\langle 3/5, 4/5\rangle = 2/5 \Rightarrow \\ \langle a,b\rangle\langle 4/5, -3/5\rangle = 11/5 \Rightarrow\end{cases} \begin{cases}3a+4b=2\\ 4a-3b=11\end{cases} \Rightarrow \begin{array}{l} 9a+12b=6\\ 16a-12b=44 \\ \hline 25a=50 \Rightarrow \begin{cases}a=2\\ b=-1\end{cases}\end{array}$$

$\therefore\ \nabla f(x,y)=\langle a,b\rangle=\langle 2,-1\rangle$

b) $\vec{w}=\vec{i}+k\vec{j}$ onde $k=d_8+1$

$\|\vec{w}\|=\sqrt{1+k^2} \Rightarrow \dfrac{\vec{w}}{\|\vec{w}\|}=\left\langle \dfrac{1}{\sqrt{1+k^2}}, \dfrac{k}{\sqrt{1+k^2}}\right\rangle$

$\therefore\ D_u f(0,\pi/2)=\langle 2,-1\rangle\left\langle \dfrac{1}{\sqrt{1+k^2}}, \dfrac{k}{\sqrt{1+k^2}}\right\rangle = \dfrac{2-k}{\sqrt{1+k^2}}$

⑥ $L(x,y)=x^2y-\lambda(x^2+2y^2-6)$

$$\begin{cases}L_x=0 \Rightarrow \\ L_y=0 \Rightarrow \\ L_\lambda=0 \Rightarrow\end{cases}\begin{cases}2xy-\lambda(2x)=0 \Rightarrow 2x(y-\lambda)=0 \Rightarrow \begin{cases}x=0\\ y=\lambda\end{cases}\\ x^2-\lambda(4y)=0\\ x^2+2y^2=6\end{cases}$$

Se $x=0 \Rightarrow 0^2+2y^2=6$

$y=\pm\sqrt{3}$

Ptos Críticos: $(0,\sqrt{3}),\ (0,-\sqrt{3})$

Se $y=\lambda \Rightarrow x^2-y(4y)=0$

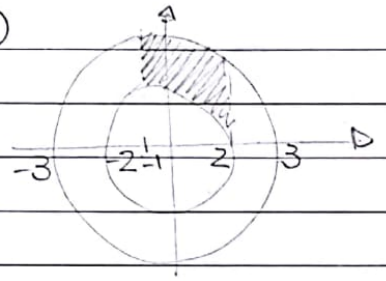
$x^2=4y^2 \Rightarrow x=\pm 2y$

$4y^2+2y^2=6 \Rightarrow y=\pm 1$

Ptos Críticos: $(2,1);\ (-2,1);\ (-2,-1);\ (2,-1)$

Ptos críticos	$f(x,y)=x^2y$	classif
$(0,\sqrt{3})$	0	
$(0,-\sqrt{3})$	0	
$(2,1)$	4	máximo global
$(-2,1)$	4	máximo global
$(-2,-1)$	-4	mínimo global
$(2,-1)$	-4	mínimo global

⑦ a)



b) $\displaystyle\int_{-1}^{2}\int_{\sqrt{4-x^2}}^{\sqrt{9-x^2}} x\,dy\,dx = \int_{-1}^{2} x\left(\sqrt{9-x^2}-\sqrt{4-x^2}\right)dx$

$$= \int_{-1}^{2} x\sqrt{9-x^2}\,dx - \int_{-1}^{2} x\sqrt{4-x^2}\,dx = -\frac{1}{2}$$

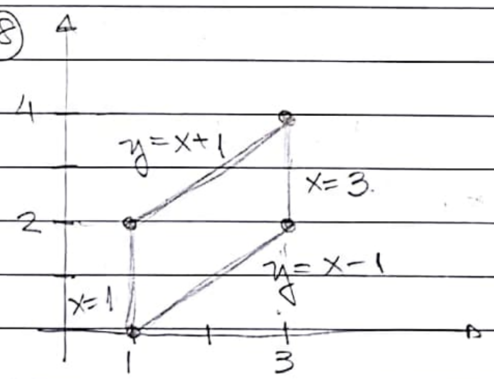
$u=9-x^2$ $\qquad$ $w=4-x^2$

$du=-2x\,dx$ $\qquad$ $dw=-2x\,dx$

$$= \int_{8}^{5} -\frac{1}{2}u^{1/2}du - \int_{3}^{0} -\frac{1}{2}w^{1/2}dw = -\frac{1}{2}\frac{u^{3/2}}{3/2}\Big|_{8}^{5} + \frac{1}{2}\frac{w^{3/2}}{3/2}\Big|_{3}^{0}$$

$$= -\frac{1}{3}\left(5^{3/2}-8^{3/2}\right) + \frac{1}{3}\left(0-3^{3/2}\right) = \frac{8^{3/2}-5^{3/2}-3^{3/2}}{3}$$

⑧



Tipo I:

$$\int_{1}^{3}\int_{x-1}^{x+1} f(x,y)\,dy\,dx$$

Tipo II:

$$\int_{0}^{2}\int_{1}^{y+1} f(x,y)\,dx\,dy + \int_{2}^{4}\int_{y-1}^{3} f(x,y)\,dx\,dy$$