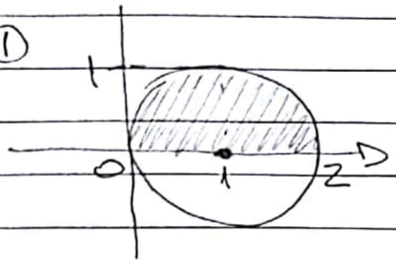
Gabarito PSub

①



$$\int_0^2\int_0^{\sqrt{2x-x^2}} x\,dy\,dx = \int_0^{\pi/2}\int_0^{2\cos\theta} r\cos\theta\, r\,dr\,d\theta$$

$$=\int_0^{\pi/2}\int_0^{2\cos\theta} r^2\cos\theta\,dr\,d\theta$$

$y=0$

$y=\sqrt{2x-x^2} \Rightarrow y^2=\sqrt{2x-x^2}$

$x^2-2x+y^2=0$

$x^2-2x+1+y^2=1$

$(x-1)^2+y^2=1$

$C(1,0),\ r=1$

$x^2+y^2=2x$

$r^2=2r\cos\theta$

$r(r-2\cos\theta)=0$

$r=0,\ 2\cos\theta$

$$=\int_0^{\pi/2}\cos\theta\,\frac{r^3}{3}\Big|_0^{2\cos\theta}d\theta$$

$$=\frac{1}{3}\int_0^{\pi/2}\cos\theta\ 8\cos^3\theta\,d\theta$$

$$=\frac{8}{3}\int_0^{\pi/2}\underbrace{\cos^4\theta}_{(\cos^2\theta)^2}\,d\theta$$

$$=\frac{8}{3}\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta=\frac{\cancel{8}^{2}}{3}\frac{1}{\cancel{4}}\int_0^{\pi/2}(1+\cos 2\theta)^2\,d\theta$$

$$=\frac{2}{3}\int_0^{\pi/2}(1+2\cos 2\theta+\cos^2 2\theta)\,d\theta=\frac{2}{3}\,\theta+2\frac{\operatorname{sen}2\theta}{2}+\int_0^{\pi/2}\cos^2\theta\,d\theta$$

$$=\frac{2}{3}\theta+\operatorname{sen}2\theta+\frac{1}{2}\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta$$

$$=\frac{2}{3}\theta+\operatorname{sen}2\theta+\frac{1}{2}\left(\theta+\frac{\operatorname{sen}2\theta}{2}\right)\Big|_0^{\pi/2}=$$

$$=\left[\frac{2}{3}\left(\frac{\pi}{2}\right)+\cancel{\operatorname{sen}\pi}^{\,0}+\frac{1}{2}\left(\frac{\pi}{2}+\frac{1}{2}\cancel{\operatorname{sen}\pi}^{\,0}\right)\right]-\left[0+\cancel{\operatorname{sen}0}^{\,0}+\underbrace{\frac{1}{2}\left(0+\frac{\operatorname{sen}\pi}{2}\right)}_{=0}\right]$$

$$=\left[\frac{\pi}{3}+0+\frac{1}{2}\left(\frac{\pi}{2}\right)\right]=\frac{\pi}{3}+\frac{\pi}{4}=\frac{7\pi}{12}$$

② Seja os caminhos $mx = y - 1$ que partam em $(0,1)$, e $m \in \mathbb{R}$.

$$\lim_{(x,y)\to(0,1)} \frac{x^3 \cdot mx - x(mx)^3}{x^4 + (mx)^4} = \lim_{(x,y)\to(0,1)} \frac{mx^4 - m^3x^4}{x^4 + m^4x^4}$$

$$= \lim_{(x,y)\to(0,1)} \frac{(m-m^3)\cancel{x^4}}{(1+m^4)\cancel{x^4}} = \frac{m-m^3}{1+m^4}.$$

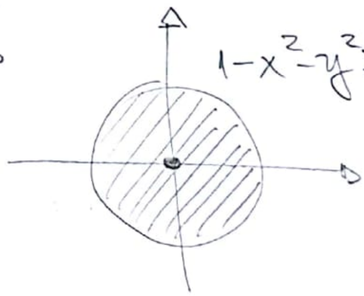
Se $m=0$, limite $\frac{m-m^3}{1+m^4} = 0$ L_1

Se $m=2$, limite $\frac{m-m^3}{1+m^4} = \frac{2-8}{1+16} = -\frac{6}{17}$ L_2

$\therefore \lim_{(x,y)\to(0,1)} \frac{x^3(y-1) - x(y-1)^3}{x^4 + (y-1)^4}$ não existe

③ Procurando ptos críticos no interior:

$f(x,y) = x^2 - y^2$



$$\begin{cases} f_x = 2x = 0 \\ f_y = -2y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

Como o pto crítico $(0,0)$ está no interior do disco $1-x^2-y^2 \geq 0$, então aceita-se o pto $(0,0)$

Procurando ptos críticos na fronteira:

$$\begin{cases} \text{max/min } f(x,y) = x^2 - y^2 \\ \text{s.a } x^2 + y^2 = 1 \end{cases}$$

Pelo Método de Lagrange temos

$$\mathcal{L}(x,y,\lambda) = \underbrace{f(x,y)}_{\text{f. objetivo}} - \lambda \underbrace{g(x,y)}_{\text{restrição}}$$

$$\mathcal{L}(x,y,\lambda) = x^2 - y^2 - \lambda(x^2+y^2-1)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2x - \lambda(2x) = 0 \quad \Rightarrow \quad 2x(1-\lambda)=0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = -2y - \lambda(2y) = 0 \quad \Rightarrow \quad -2y(1+\lambda)=0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Rightarrow \quad x^2+y^2=1 \quad (3)$$

De eq(1): $2x(1-\lambda)=0$

$\Rightarrow x=0$

$x^2+y^2=1 \Rightarrow y^2=1 \Rightarrow y=\pm 1$

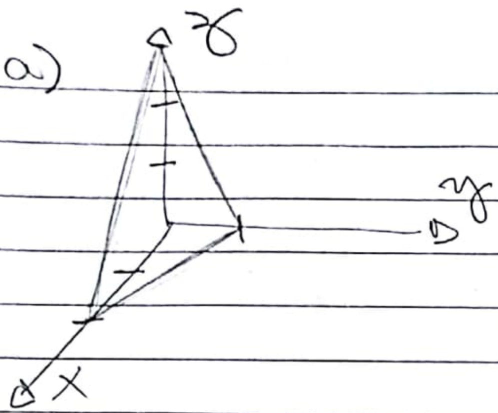
$\Rightarrow \lambda=1$

$-2y(1+\lambda)=0 \Rightarrow y=0 \Rightarrow x=\pm 1$

Ptos Críticos Encontrados:

	(x,y)	$f(x,y)$	
interior do disco	$(0,0)$	0	
fronteira do disco	$(0,1)$	-1	máximos
	$(0,-1)$	-1	
	$(1,0)$	1	mínimos
	$(-1,0)$	1	

④ a)



$\vec{BC} = (-2,1,0)$

$\vec{BD} = (-2,0,3)$

$\vec{n} = \vec{BC} \times \vec{BD}$

$$\vec{n} = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ -2 & 0 & 3 \end{vmatrix} = 3i+6j+2k$$

$3x+6y+2z+d=0$

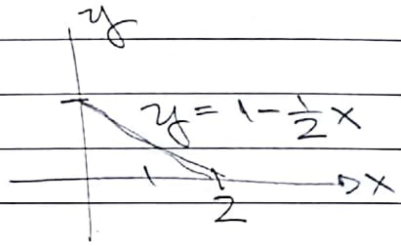
(A) $3\cdot 2+6\cdot 0+2\cdot 0+d=0$

$6+0+0+d=0 \rightarrow d=-6$

∴ eq. plano é $3x+6y+2z=6$

$$z = \frac{6-3x-6y}{2} = 3 - \frac{3x}{2} - 3y$$

b) $$\int_0^2 \int_0^{1-\frac{1}{2}x} \int_0^{\frac{6-3x-6y}{2}} 1 \, dz\,dy\,dx$$



c) $$\int_0^2 \int_0^{1-\frac{1}{2}x} \left(3 - \frac{3x}{2} - 3y\right) dy\,dx$$

$$= 3\int_0^2 \int_0^{1-\frac{1}{2}x} \left(1 - y - \frac{x}{2}\right) dy\,dx = 3\int_0^2 y - \frac{y^2}{2} - \frac{xy}{2} \Bigg|_0^{1-\frac{1}{2}x} dx$$

$$= 3\int_0^2 y\left(1 - \frac{x}{2} - \frac{y}{2}\right)\Bigg|_0^{1-\frac{1}{2}x} dx = 3\int_0^2 \left(1-\frac{x}{2}\right)\left(1 - \frac{x}{2} - \frac{1}{2} + \frac{x}{4}\right) dx$$

$$= 3\int_0^2 \left(1-\frac{x}{2}\right)\left(\frac{1}{2} - \frac{x}{2} + \frac{x}{4}\right) dx = 3\int_0^2 \left(1-\frac{x}{2}\right)\left(\frac{1}{2} - \frac{x}{4}\right) dx$$

$$= 3\int_0^2 \left(\frac{1}{2} - \frac{x}{4} - \frac{x}{4} + \frac{x^2}{8}\right) dx = 3\int_0^2 \left(\frac{1}{2} - \frac{2x}{4} + \frac{x^2}{8}\right) dx$$

$$= 3\left(\frac{1}{2}x - \frac{1}{2}\frac{x^2}{2} + \frac{1}{8}\frac{x^3}{3}\right)\Bigg|_0^2 = 3\left(1 - \frac{1}{2}\frac{4}{2} + \frac{1}{8}\frac{8}{3}\right) = 3\left(1-1+\frac{1}{3}\right)$$

$= 1$

$144 = 12 \cdot 12$
$= 3 \cdot 4 \cdot 12$
$= 3 \cdot 12 \cdot 4$

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a) $r'(t) = 12\vec{i} + \cancel{8}\frac{3}{\cancel{2}}t^{1/2}\vec{j} + 6t\vec{k}$

$$L = \int_0^1 \sqrt{144 + (12t^{1/2})^2 + 36t^2}\, dt = \int_0^1 \sqrt{(144 + 144t + 36t^2)}\, dt$$

$$L = \int_0^1 \sqrt{36(4 + 4t + t^2)}\, dt = \int_0^1 6\sqrt{(t+2)^2}\, dt = \int_0^1 6(t+2)\, dt$$

$$L = 6\int_0^1 (t+2)\, dt = 6\left(\frac{t^2}{2} + 2t\right)\Big|_0^1 = 6\left(\frac{1}{2} + 2\right) = \cancel{6}^{3} \cdot \frac{5}{\cancel{2}} = 15$$

b)
$$\text{rot } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 12x & y & z + e^{z} \end{vmatrix} = 0\vec{i} + 0\vec{j} + 0\vec{k} = 0$$

$\therefore F = 12x\vec{i} + y\vec{j} + (z + e^{z})\vec{k}$ é conservativo

Vou usar TFIL p/ calcular $\int_C F \cdot dr$.

$f_x = 12x \Rightarrow f = \int 12x\, dx = 12\frac{x^2}{2} = 6x^2 + g(y,z)$

$f_y = g_y(y,z) = y \Rightarrow g(y,z) = \int y\, dy = \frac{y^2}{2} + h(z)$

$\Rightarrow f = 6x^2 + \frac{y^2}{2} + h(z)$

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$$f_z = h'(z) = z + e^z \Rightarrow h(z) = \frac{z^2}{2} + e^z + C$$

$$\therefore f = 6x^2 + \frac{y^2}{2} + \frac{z^2}{2} + e^z + C$$

TFIL:

$$\int_C F \cdot dr = f\left(\begin{matrix}\text{pto}\\\text{final}\end{matrix}\right) - f\left(\begin{matrix}\text{pto}\\\text{inicial}\end{matrix}\right)$$

pto final: $r(1) = 12\vec{i} + 8\vec{j} + 3\vec{k}$

$$f(r(1)) = 6 \cdot 12^2 + 8^2 + \frac{9^2}{2} + e^3 + C$$

pto inicial: $r(0) = 0\vec{i} + 0\vec{j} + 0\vec{k}$

$$f(r(0)) = 0 + 0 + 0 + 1 + C$$

$$\therefore \int_C F \cdot dr = 6 \cdot 12^2 + 8^2 + \frac{9^2}{2} + e^3 - 1$$

⑥ Como o prisma é sólido fechado, então posso usar Teor. Divergente:

$$\iint_S F \cdot dS = \iiint_R \text{div } F \cdot dV$$

$$= \int_1^e \int_0^1 \int_1^x \left(\frac{\ln y}{y} + \frac{1}{x} \cancel{-2xz} \cancel{+2xz} \right) dy\, dz\, dx$$

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$$= \int_1^e \int_0^1 \int_1^x \frac{\ln y}{y} \frac{1}{x} \, dy \, dz \, dx = \int_1^e \int_0^1 \int_1^x \frac{1}{x} \underbrace{\ln y}_{u} \underbrace{\frac{1}{y} dy}_{du} \, dz \, dx$$

$$= \int_1^e \int_0^1 \frac{1}{x} \int_0^{\ln x} u \, du \, dz \, dx = \int_1^e \int_0^1 \frac{1}{x} \left. \frac{u^2}{2} \right|_0^{\ln x} dz \, dx$$

$$= \int_1^e \int_0^1 \frac{1}{x} \frac{1}{2} \ln^2 x \, dx \, dz = \frac{1}{2} \int_0^1 \int_1^e (\ln x)^2 \frac{1}{x} \, dx \, dz$$

$$= \frac{1}{2} \int_0^1 \int_0^1 u^2 \, du \, dz = \frac{1}{2} \int_0^1 \left. \frac{u^3}{3} \right|_0^1 dz = \frac{1}{2} \int_0^1 \frac{1}{3} \, dz$$

$$= \frac{1}{2} \frac{1}{3} \cdot 1 = \frac{1}{6}$$

7 $x y^3 z^5 = e^{x^2+y^2+z^2-3}$

$$\ln x y^3 z^5 = \ln e^{x^2+y^2+z^2-3}$$

$$\ln x + 3 \ln y + 5 \ln z = x^2 + y^2 + z^2 - 3$$

$$F(x,y,z) = \ln x - x^2 + 3 \ln y - y^2 + 5 \ln z - z^2 + 3$$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = -\frac{1/x - 2x}{3/y - 2y}$$

$\underline{y(1,1) = 1}$
$x = 1$
$z = 1$
$y = 1$

$$\frac{\partial y}{\partial x}(\underbrace{1}_{x}, \underbrace{1}_{z}) = -\frac{1-2}{3-2} = \frac{-(-1)}{1} = 1$$

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$$\frac{\partial y}{\partial z}(1,1) = -\frac{F_z}{F_y} = -\frac{\left(\frac{5}{z} - 2z\right)}{\frac{3}{y} - 2y} = -\frac{5-2}{3-2} = -\frac{3}{1} = -3$$

2º jeito de resolver:

$$F = e^{x^2+y^2+z^2-3} - xy^3z^5 = 0$$

$$F_x = 2x\left(e^{x^2+y^2+z^2-3}\right) - y^3z^5 \qquad F_x(1,1) = 2\cdot 1\, e^{0} - 1\cdot 1 = 1$$

$$F_y = 2y\left(e^{x^2+y^2+z^2-3}\right) - 3xy^2z^5 \qquad F_y(1,1) = 2\cdot 1\cdot e^0 - 3\cdot 1\cdot 1\cdot 1 = -1$$

$$F_z = 2z\left(e^{x^2+y^2+z^2-3}\right) - 5xy^3z^4 \qquad F_z(1,1) = 2\cdot 1\cdot e^0 - 5\cdot 1\cdot 1\cdot 1 = -3$$

$$\frac{\partial y}{\partial x}(1,1) = -\frac{F_x}{F_y} = -\frac{1}{-1} = 1$$

$$\frac{\partial y}{\partial z}(1,1) = -\frac{F_z}{F_y} = \frac{-(-3)}{-1} = -3$$