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## Resampling, Smoothing and Interest points of curves (via CSS) in OpenCV [w/ code]

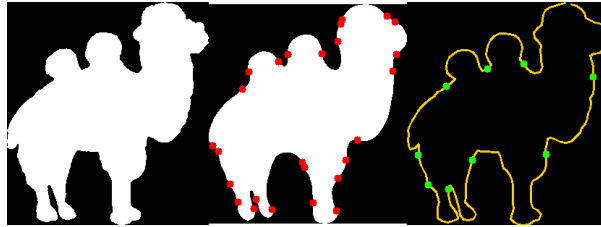
Utility functions for Resampling, Smoothing and finding Interest points of 2D curves in OpenCV.

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[December 7, 2012 <  
https://www.morethantechnical.com/2012/12/07/resampling-smoothing-  
and-interest-points-of-curves-via-css-in-opencv-w-code/>](https://www.morethantechnical.com/2012/12/07/resampling-smoothing-and-interest-points-of-curves-via-css-in-opencv-w-code/)

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smoothing-and-interest-points-of-curves-via-css-in-opencv-w-  
code/#disqus\\_thread>](https://www.morethantechnical.com/2012/12/07/resampling-smoothing-and-interest-points-of-curves-via-css-in-opencv-w-code/#disqus_thread)





I'm so glad to be back to work on a graphics project (of which you will probably hear later), because it takes me back to reading papers and implementing work by talented people. I want share a little bit of utilities I've developed for working with 2D curves in OpenCV.

## Smooth operator

So I've been implementing a paper called "Affine-invariant Shape Matching and Recognition Under Partial

Occlusion" by Mai, Chang and Hung from 2010 ([Here < http://hub.hku.hk/handle/10722/136450>](http://hub.hku.hk/handle/10722/136450)), that is using the CSS Image (Curvature Scale Space, rather an old concept, developed by Mukhtarian in the mid-1990s) to extract affine-invariant points on curves. The work is rather simple and the paper is short, but nevertheless powerful.

In the process of computing the CSS Image, once must smooth the curve with gaussian kernels of varying sizes, and this I'd like to show you.

The implementation is simple:

```

1  /* 1st and 2nd derivative of 1D gaussian
2  */
3  void getGaussianDerivs(double sigma, int M, vector<double>
4      int L = (M - 1) / 2;
5      double sigma_sq = sigma * sigma;
6      double sigma_quad = sigma_sq*sigma_sq;
7      dg.resize(M); d2g.resize(M); gaussian.resize(M);
8      Mat_<double> g = getGaussianKernel(M, sigma, CV_64
9      for (double i = -L; i < L+1.0; i += 1.0) {
10         int idx = (int)(i+L);
11         gaussian[idx] = g(idx);
12         // from http://www.cedar.buffalo.edu/ < http://
13         dg[idx] = (-i/sigma_sq) * g(idx);
14         d2g[idx] = (-sigma_sq + i*i)/sigma_quad * g(id
15     }
16 }
17 /* 1st and 2nd derivative of smoothed curve point */
18 void getdX(vector<double> x,
19     int n,
20     double sigma,
21     double& gx,
22     double& dgx,
23     double& d2gx,
24     vector<double> g,
25     vector<double> dg,
26     vector<double> d2g,
27     bool isOpen = false)
28 {
29     int L = (g.size() - 1) / 2;
30     gx = dgx = d2gx = 0.0;
31     // cout << "Point " << n << ": ";
32     for (int k = -L; k < L+1; k++) {
33         double x_n_k;
34         if (n-k < 0) {
35             if (isOpen) {
36                 //open curve - mirror values on border
37                 x_n_k = x[-(n-k)];
38             } else {
39                 //closed curve - take values from end
40                 x_n_k = x[x.size()+(n-k)];
41             }
42         } else if(n-k > x.size()-1) {
43             if (isOpen) {
44                 //mirror value on border
45                 x_n_k = x[n+k];
46             } else {
47                 x_n_k = x[(n-k)-(x.size())];
48             }
49         } else {
50             // cout << n-k;
51             x_n_k = x[n-k];

```

```

52     }
53     //     cout << "* g[" << g[k + L] << "], ";
54     gx += x_n_k * g[k + L]; //gaussians go [0 -> M
55     dgx += x_n_k * dg[k + L];
56     d2gx += x_n_k * d2g[k + L];
57 }
58 //     cout << endl;
59 }
60 /* 0th, 1st and 2nd derivatives of whole smoothed curve
61 void getdXcurve(vector<double> x,
62                double sigma,
63                vector<double>& gx,
64                vector<double>& dx,
65                vector<double>& d2x,
66                vector<double> g,
67                vector<double> dg,
68                vector<double> d2g,
69                bool isOpen = false)
70 {
71     gx.resize(x.size());
72     dx.resize(x.size());
73     d2x.resize(x.size());
74     for (int i=0; i<x.size(); i++) {
75         double gausx,dgx,d2gx; getdX(x,i,sigma,gausx,d
76         gx[i] = gausx;
77         dx[i] = dgx;
78         d2x[i] = d2gx;
79     }
80 }

```

The smoothing runs on each of the curves dimensions (i.e. x and y) separately, so I create little helper functions:

```

1  template<typename T, typename V>
2  void PolyLineSplit(const vector<Point_<T> >& pl,vector
3      contourx.resize(pl.size());
4      contoury.resize(pl.size());
5      for (int j=0; j<pl.size(); j++)
6      {
7          contourx[j] = (V)(pl[j].x);
8          contoury[j] = (V)(pl[j].y);
9      }
10 }
11 template<typename T, typename V>
12 void PolyLineMerge(vector<Point_<T> >& pl, const vecto
13     assert(contourx.size()==contoury.size());
14     pl.resize(contourx.size());
15     for (int j=0; j<contourx.size(); j++) {
16         pl[j].x = (T)(contourx[j]);
17         pl[j].y = (T)(contoury[j]);
18     }
19 }

```

They also convert to different types if needed.

Smoothing is easy:

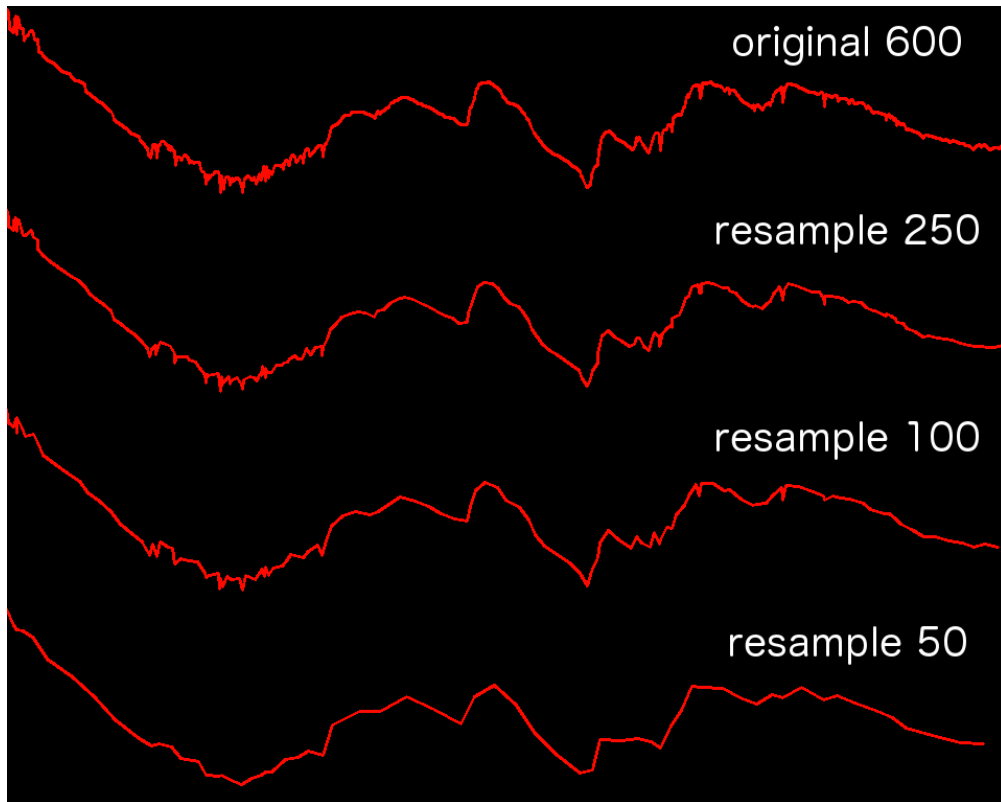
```

1  vector<Point> curve = ...;
2  double sigma = 3.0;
3  int M = round((10.0*sigma+1.0) / 2.0) * 2 - 1;
4  assert(M % 2 == 1); //M is an odd number
5  //create kernels
6  vector<double> g,dg,d2g; getGaussianDerivs(sigma,M,g,d
7  vector<double> curvex,curvey,smoothx,smoothy;
8  PolyLineSplit(curve,curvex,curvey);
9  vector<double> X,XX,Y,YY;
10 getdXcurve(curvex,sigma,smoothx,X,XX,g,dg,d2g,isOpen);
11 getdXcurve(curvey,sigma,smoothy,Y,YY,g,dg,d2g,isOpen);
12 PolyLineMerge(curve,smoothx,smoothy);

```

## Resampling

I implemented my own resampling code, where I sample points on the curve keeping the same regular distance between them. To get the regular distance I simply take the complete length of the original curve and divide by the number of desired points.



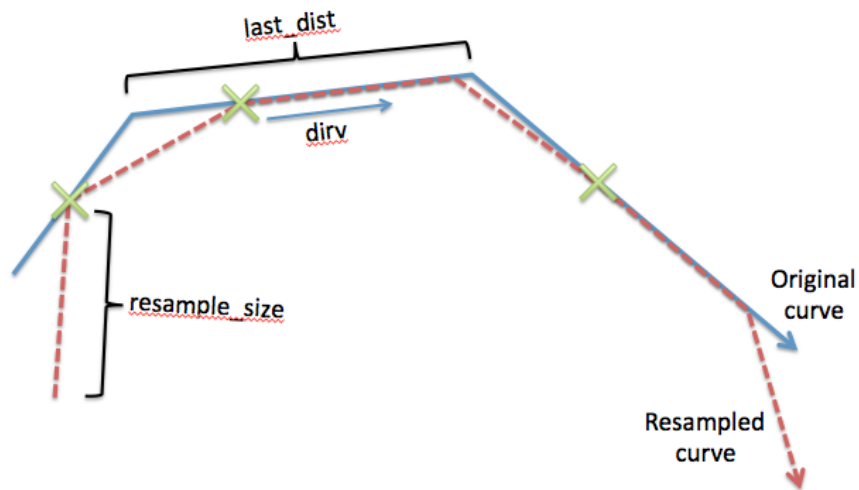
The implementation is again quite simple:

```

1 void ResampleCurve(const vector<double>& curvex, const
2                   vector<double>& resampleX, vector<d
3                   int N,
4                   bool isOpen
5                   ) {
6     assert(curvex.size()>0 && curvey.size()>0 && curve
7     vector<Point2d> resamplepl(N); resamplepl[0].x = c
8     vector<Point2i> pl; PolyLineMerge(pl,curvex,curvey
9     double pl_length = arcLength(pl, false);
10    double resample_size = pl_length / (double)N;
11    int curr = 0;
12    double dist = 0.0;
13    for (int i=1; i<N; ) {
14        assert(curr < pl.size() - 1);
15        double last_dist = norm(pl[curr] - pl[curr+1])
16        dist += last_dist;
17        // cout << curr << " and " << curr+1 << "\t\t" <<
18        if (dist >= resample_size) {
19            //put a point on line
20            double _d = last_dist - (dist-resample_siz
21            Point2d cp(pl[curr].x,pl[curr].y),cp1(pl[c
22            Point2d dirv = cp1-cp; dirv = dirv * (1.0
23            // cout << "point " << i << " between " << cu
24            assert(i < resamplepl.size());
25            resamplepl[i] = cp + dirv * _d;
26            i++;
27            dist = last_dist - _d; //remaining dist
28            //if remaining dist to next point needs mo
29            while (dist - resample_size > 1e-3) {
30                // cout << "point " << i << " between " <
31                assert(i < resamplepl.size());
32                resamplepl[i] = resamplepl[i-1] + dirv
33                dist -= resample_size;
34                i++;
35            }
36        }
37        curr++;
38    }
39    PolyLineSplit(resamplepl,resampleX,resampleY);
40 }

```

Essentially I just traverse the original curve, advancing in a regular pace (resample\_size) and adding new vertices as long as there is space to so on the current edge. If space to add more vertices on the edge – I move to the next edge and add more vertices there.



## Curvature Scale Space

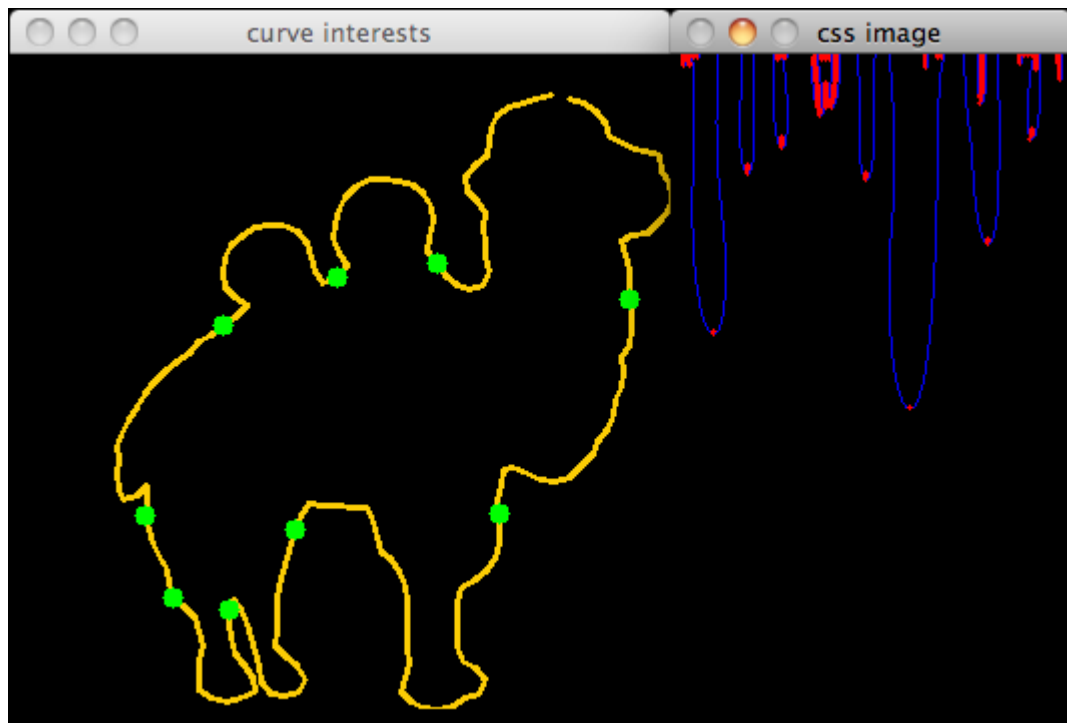
The core idea of the CSS image (as shown perhaps best by Mokhtarian and Abbassi in their [2002 paper <http://www.sciencedirect.com/science/article/pii/S0031320301000401>](http://www.sciencedirect.com/science/article/pii/S0031320301000401)) is to smooth the curve by ever increasing gaussian kernels and looking at zero crossings of the 2nd derivative. When you're smoothing the curve you will lose the noise, and be left with only the biggest changes in curvature, and a zero-crossing on the 2nd derivative will indicate a big change in curvature. The invariant points across the different sizes of the gaussian kernel – are the interest points of the curve (sometimes referred to as “affine invariant”).



```

1  /* compute curvature of curve after gaussian smoothing
2   from "Shape similarity retrieval under affine transfo
3   curvex - x position of points
4   curvey - y position of points
5   kappa - curvature coeff for each point
6   sigma - gaussian sigma
7   */
8  void ComputeCurveCSS(const vector<double>& curvex,
9                      const vector<double>& curvey,
10                      vector<double>& kappa,
11                      vector<double>& smoothX, vector<d
12                      double sigma,
13                      bool isOpen
14                      )
15  {
16      int M = round((10.0*sigma+1.0) / 2.0) * 2 - 1;
17      assert(M % 2 == 1); //M is an odd number
18      vector<double> g,dg,d2g; getGaussianDerivs(sigma,M
19      vector<double> X,XX,Y,YY;
20      getdXcurve(curvex,sigma,smoothX,X,XX,g,dg,d2g,isOp
21      getdXcurve(curvey,sigma,smoothY,Y,YY,g,dg,d2g,isOp
22      kappa.resize(curvex.size());
23      for (int i=0; i<curvex.size(); i++) {
24          // Mokhtarian 02' eqn (4)
25          kappa[i] = (X[i]*YY[i] - XX[i]*Y[i]) / pow(X[i
26      }
27  }
28  /* find zero crossings on curvature */
29  vector<int> FindCSSInterestPoints(const vector<double>
30      vector<int> crossings;
31      for (int i=0; i<kappa.size()-1; i++) {
32          if ((kappa[i] < 0 && kappa[i+1] > 0) || kappa[
33              crossings.push_back(i);
34          }
35      }
36      return crossings;
37  }

```



The evolution of the curve (iterating through many levels of the gaussian kernel) looks like the following:



**Code**

Grab the code: <http://web.media.mit.edu/~roys/src/CurveCSS.zip> <  
<http://web.media.mit.edu/~roys/src/CurveCSS.zip>>

(if the hosting dies – let me know)

Thanks for joining in

Roy.

⇐ [How I fell for QGLViewer for my Qt/OpenGL projects \[w/ code\]](https://www.morethantechnical.com/2012/11/04/how-i-fell-for-qglviewer-for-my-Qt/OpenGL-projects-w-code/) ≤  
[https://www.morethantechnical.com/2012/11/04/how-i-fell-for-qglviewer-for-my-Qt/OpenGL projects \[w/ code\]](https://www.morethantechnical.com/2012/11/04/how-i-fell-for-qglviewer-for-my-Qt/OpenGL-projects-w-code/) <  
⇒ [2D curve matching in OpenCV \[w/ code\]](https://www.morethantechnical.com/2012/12/27/2d-curve-matching-in-opencv-w-code/) ≤  
<https://www.morethantechnical.com/2012/12/27/2d-curve-matching-in-opencv-w-code/>>