STARS RATED FOR EXCELLENCE 2012

FPT Fpt University

TRƯỜNG ĐẠI HỌC FPT

DC programming and DCA for Secure Guarantee with Null Space Beamforming in Two-Way Relay Networks

PRESENTER:

LUONG THUY CHUNG

SUPERVISOR:

TRAN THI THUY

AUTHORS:

LUONG THUY CHUNG,

NGUYEN THE DUY

Outline

1. Introduction
2. Optimization of beamforming vector algorithm
3. Proposed algorithm
4. Experimental results
5. Conclusion and Future Works

01

Introduction

- The problem
- System model

Eavesdropper

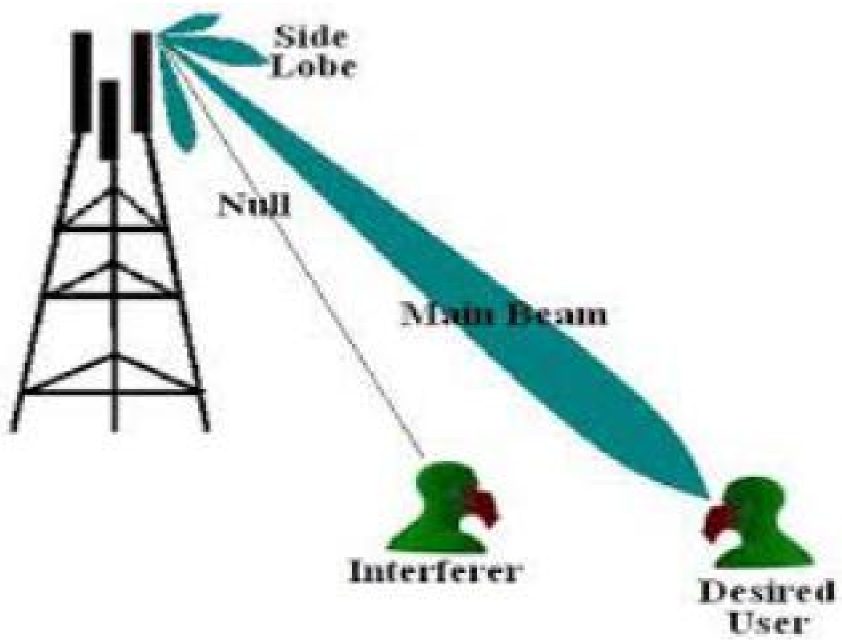
Wireless channel

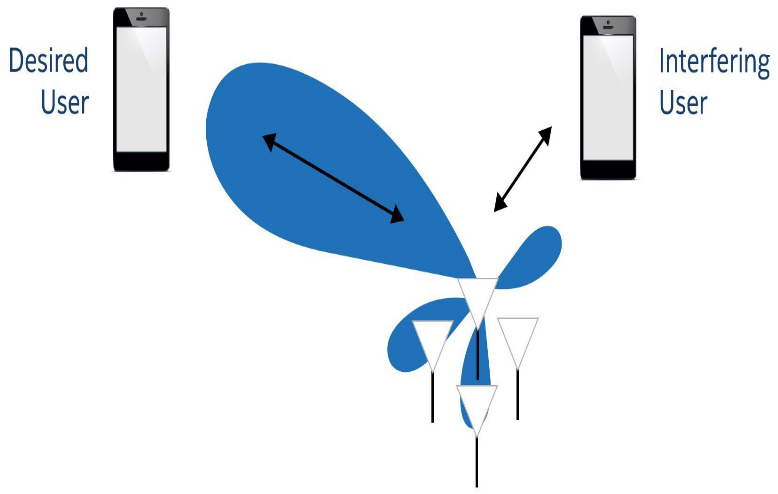
Create secure links relying on physical characteristic of wireless channel without relying on the privacy cryptograph

Goals

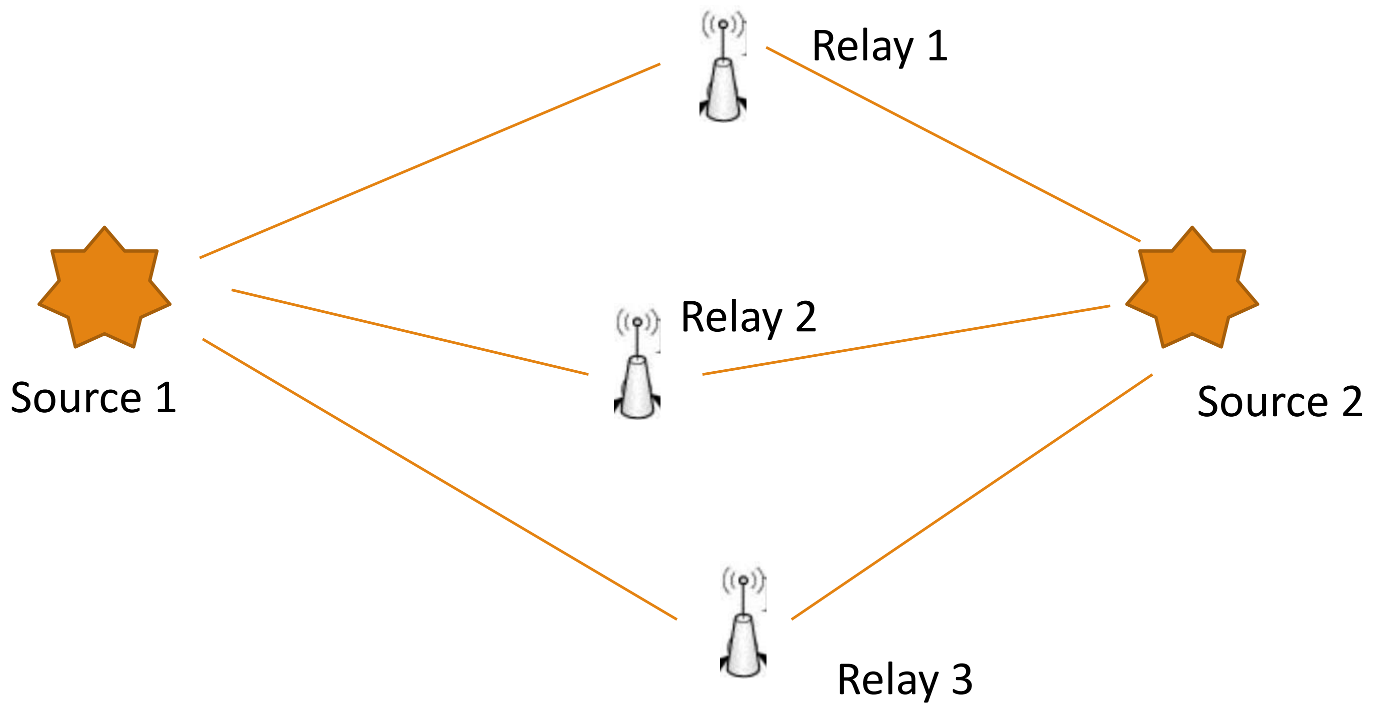
Find the maximum rate received at the legitimate receiver, while keeping the eavesdropper completely ignorant of the transmitted messages
=> Reduce the probability of interception

Null-Space Beamforming

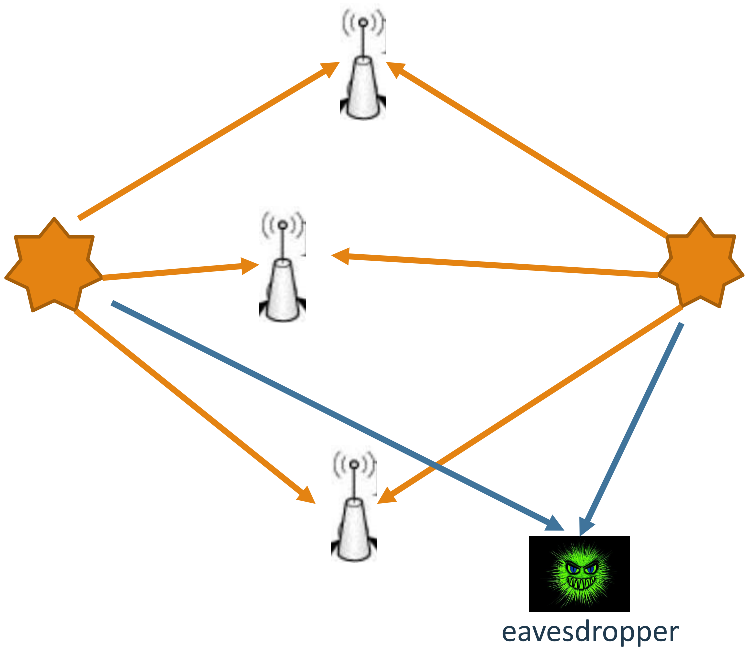




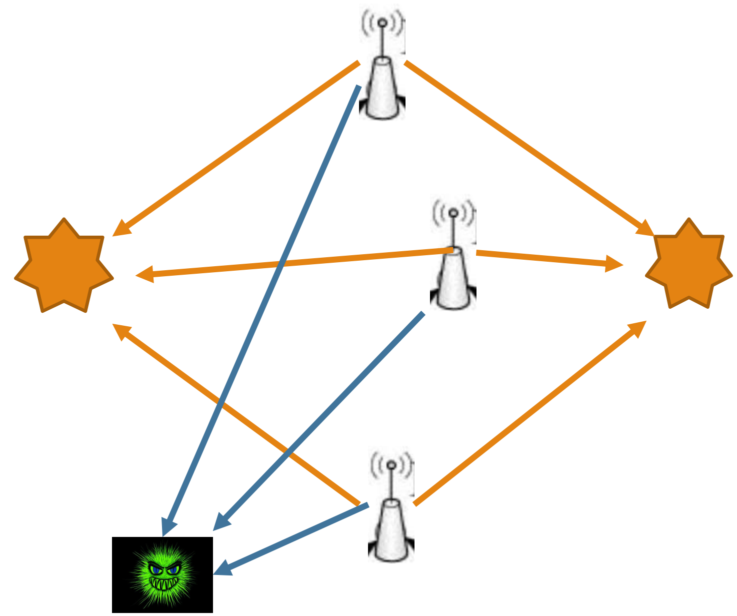
Amplify-and-forward (AF) relaying protocol



Multiple Access Channel (MAC) and Broadcast Channel (BC)



Broadcast Channel



Multiple Access Channel

Model System

$$\max_{P_1,P_2,\mathbf{w}} \quad \frac{1}{2}\left[\log_2 \frac{\left(1+\frac{P_2\left|\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2\right|^2}{\sigma_R^2\|\mathbf{w}^H\mathbf{F}_1\|_2^2+\sigma_1^2}\right)\left(1+\frac{P_1\left|\mathbf{w}^H\mathbf{F}_2\mathbf{f}_1\right|^2}{\sigma_R^2\|\mathbf{w}^H\mathbf{F}_2\|_2^2+\sigma_2^2}\right)}{1+\frac{P_1|g_1|^2+P_2|g_2|^2}{\sigma_R^2}}\right]^+$$

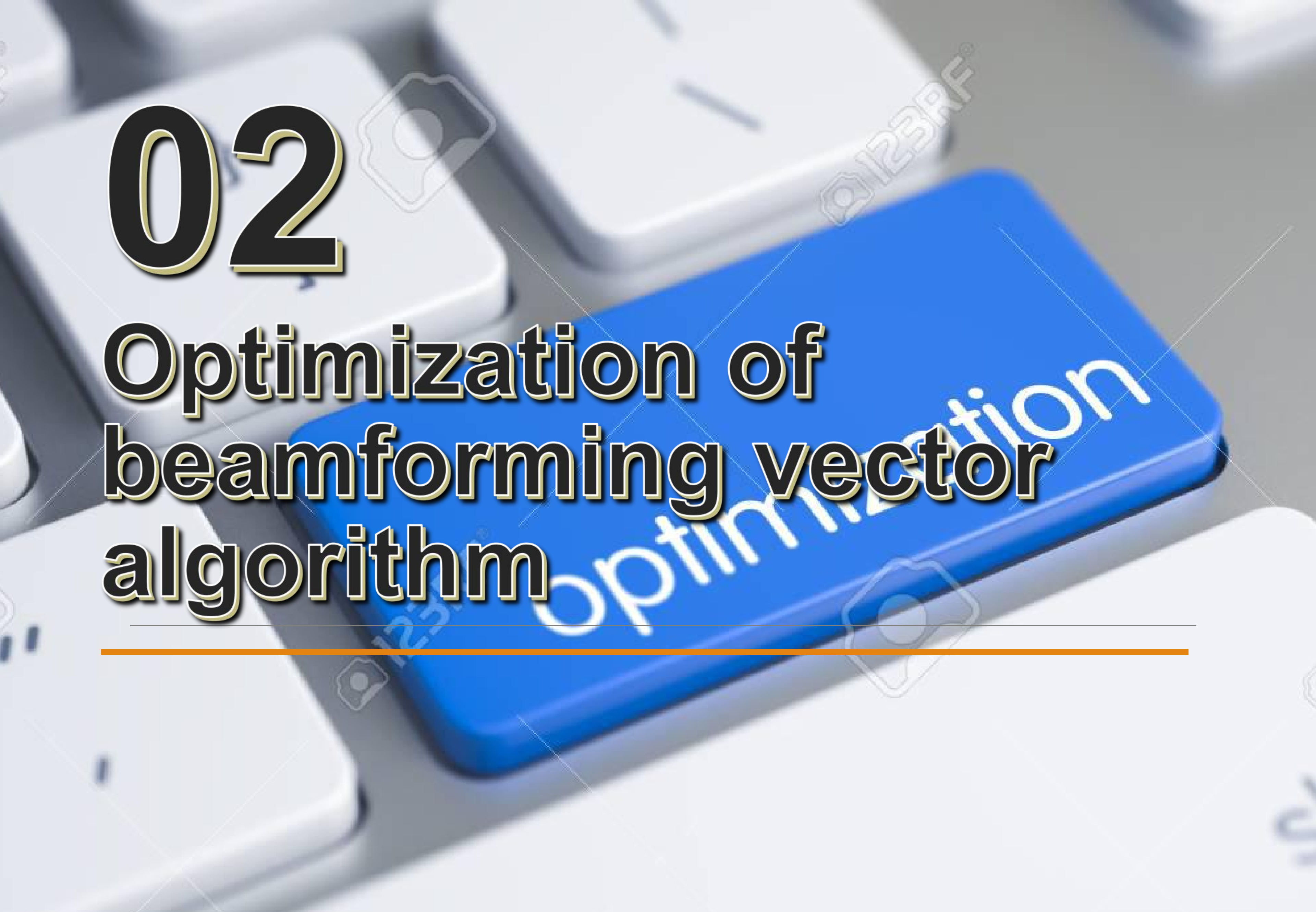
s.t $\quad P_1 + P_2 + P_1\mathbf{w}^H\mathbf{D}_1\mathbf{w} + P_2\mathbf{w}^H\mathbf{D}_2\mathbf{w} + \sigma^2\mathbf{w}^H\mathbf{w} \leq P_T$

$\mathbf{w}^H\mathbf{Z} = 0$

[2] Y. Yang, C. Sun, H. Zhao et al., "Algorithms for secrecy guarantee with null space beamforming in two-way relay networks", *IEEE Trans. Signal Process.*, vol. 62, no. 8, pp. 2111-2126, 2014.

02

Optimization of beamforming vector algorithm



Generalized Rayleigh quotient

- For a given pair $(\mathbf{A}, \mathbf{B})$ of matrices, and a given non-zero vector x, the generalized Rayleigh quotient is defined as:

$$\mathrm{R}(\mathbf{A}, \mathbf{B}, \mathbf{x}) = \frac{\mathbf{x}^{\mathbf{T}}\mathbf{A}\mathbf{x}}{\mathbf{x}^{\mathbf{T}}\mathbf{B}\mathbf{x}}$$

- For any vector $\mathbf{x}$, one has $\mathrm{R}(\mathbf{A}, \mathbf{B}, \mathbf{x}) \in [\lambda_{\mathrm{min}}, \lambda_{\mathrm{max}}]$ where $\lambda_{\mathrm{min}}, \lambda_{\mathrm{max}}$ are the smallest and largest eigenvalue of $\mathbf{B}^{-1}\mathbf{A}$, if $\mathbf{B}^{-1}\mathbf{A}$ is Hermitian matrix.

Generalized Rayleigh quotient

- Step 1 : Transform the model of problem into generalized Rayleigh quotient form.

$$\max_{x \in C^{2(N-2)\,x\,1}} \quad f(\mathbf{x}) = \frac{\mathbf{x}^{\mathbf{H}}(\widehat{\mathbf{P}}_{\mathbf{1}}^{\mathbf{T}} \otimes \widehat{\mathbf{P}}_{\mathbf{2}})\mathbf{x}}{\mathbf{x}^{\mathbf{H}}(\widehat{\mathbf{Q}}_{\mathbf{1}}^{\mathbf{T}} \otimes \widehat{\mathbf{Q}}_{\mathbf{2}})\mathbf{x}}$$

$$\text{s.t} \quad \mathbf{x}^{\mathbf{H}}\mathbf{x} = 1,$$

$$\mathrm{vec}^{-1}(\mathbf{x}) \succcurlyeq \mathbf{0}\,,\ \mathrm{rank}(\mathrm{vec}^{-1}(\mathbf{x})) = 1.$$

Generalized Rayleigh quotient

- Step 1 : Transform the model of problem into generalized Rayleigh quotient form.
- Step 2 : Check if $\mathbf{x}_{opt} = V_{max}((\widehat{\mathbf{Q}}_1^{\mathbf{T}} \otimes \widehat{\mathbf{Q}}_2)^{-1}(\widehat{\mathbf{P}}_1^{\mathbf{T}} \otimes \widehat{\mathbf{P}}_2))$ is vectorization of Hermitian matrix, then we can find the solution immediately.
- Step 3: If not, using randomization to find approximate solution.

Detail of the transformation in the paper·
$\mathrm{V}_{max}(\mathbf{X})$ is an unit-norm eigenvector of the largest eigenvalue of matrix X

03

Proposed algorithm

- Difference of Convex Program
- DC decomposition for Optimization of Relay Beamforming Vector

Difference of Convex Program
Difference of Convex Functions

In DC approach, the original non-convex function is decomposed to the difference of two convex functions.

A standard DC program is of the form as the following:

$$\alpha = \inf\{ f(x) \coloneqq g(x) - h(x) : x \in S \}$$

A general DC program usually has the form as the following:

$$\alpha = \inf\{ f(x) \coloneqq g(x) - h(x) : g_i(x) - h_i(x) \leq 0, i = 1, \ldots, m\}$$

With g, h being lower semi-continuous proper convex functions on R^n , S is a convex set.

Difference of Convex Program
Difference of Convex Functions Algorithm

Firstly we decompose the objective function f to a difference of two convex functions: g – h
Starting from a primal point x^0 in R^n.
With standard DCA, we repeat those steps, for k = 0,1,2,..:

- Compute $y^k \in \partial h(x^k)$
- Compute x^{k+1}

$$x^{k+1} \in \operatorname{argmin}\{g(x)-[h(x^k)+\langle y^k; x-x^k\rangle]\}$$

- Until either $\frac{\|x^{k-1}-x^k\|}{\|x^{k-1}\|+1} < \epsilon$ or $\frac{|f(x^{k-1})-f(x^k)|}{|f(x^{k-1})|+1} < \epsilon$

DC decomposition for Optimization of Relay Beamforming Vector (OBV)

Modify Model System

$$\min_{P_1,P_2,\mathbf{w}} \quad F(P_1,P_2,\mathbf{w}) = -\frac{1}{2\ln(2)}\left[\ln\frac{\dfrac{\sigma_1^2+\mathbf{w}^H\mathbf{A_1}\mathbf{w}}{\sigma_1^2+\mathbf{w}^H\mathbf{B_1}\mathbf{w}}\,\dfrac{\sigma_1^2+\mathbf{w}^H\mathbf{A_1}\mathbf{w}}{\sigma_1^2+\mathbf{w}^H\mathbf{B_1}\mathbf{w}}}{1+\dfrac{P_1|g_1|^2+P_2|g_2|^2}{\sigma_{E,1}^2}}\right]$$

$$\text{s.t} \quad P_1+P_2+P_1\mathbf{w}^H\mathbf{D}_1\mathbf{w}+P_2\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2\,\mathbf{w}^H\mathbf{w}\le P_T$$

$$\mathbf{w}^H\mathbf{Z}=0$$

DC decomposition for Optimization of Relay Beamforming Vector (OBV)

Real Form

$$\mathbf{x} = [\mathrm{Re}(\mathbf{w})^T \quad \mathrm{Im}(\mathbf{w})^T]^T$$

$$\bar{\mathbf{A}}_1 = \begin{bmatrix} \mathrm{Re}(\mathbf{A}_1) & \mathrm{Im}(\mathbf{A}_1) \\ -\mathrm{Im}(\mathbf{A}_1) & \mathrm{Re}(\mathbf{A}_1) \end{bmatrix}, \qquad \bar{\mathbf{A}}_2 = \begin{bmatrix} \mathrm{Re}(\boldsymbol{A}_2) & \mathrm{Im}(\mathbf{A}_2) \\ -\mathrm{Im}(\boldsymbol{A}_2) & \mathrm{Re}(\mathbf{A}_2) \end{bmatrix},$$

$$\bar{\mathbf{B}}_1 = \begin{bmatrix} \mathrm{Re}(\mathbf{B}_1) & \mathrm{Im}(\mathbf{B}_1) \\ -\mathrm{Im}(\mathbf{B}_1) & \mathrm{Re}(\mathbf{B}_1) \end{bmatrix}, \qquad \bar{\mathbf{B}}_2 = \begin{bmatrix} \mathrm{Re}(\mathbf{B}_2) & \mathrm{Im}(\mathbf{B}_2) \\ -\mathrm{Im}(\mathbf{B}_2) & \mathrm{Re}(\mathbf{B}_2) \end{bmatrix},$$

$$\bar{\mathbf{Z}} = \begin{bmatrix} \mathrm{Re}(\mathbf{Z}^{\mathbf{H}}) & \mathrm{Im}(\mathbf{Z}^{\mathbf{H}}) \\ -\mathrm{Im}(\mathbf{Z}^{\mathbf{H}}) & \mathrm{Re}(\mathbf{Z}^{\mathbf{H}}) \end{bmatrix},$$

$$\bar{\mathbf{D}}_1 = \begin{bmatrix} \mathrm{Re}(\mathbf{D}_1) & \mathrm{Im}(\mathbf{D}_1) \\ -\mathrm{Im}(\mathbf{D}_1) & \mathrm{Re}(\mathbf{D}_1) \end{bmatrix}, \qquad \bar{\mathbf{D}}_2 = \begin{bmatrix} \mathrm{Re}(\mathbf{D}_2) & \mathrm{Im}(\mathbf{D}_2) \\ -\mathrm{Im}(\mathbf{D}_2) & \mathrm{Re}(\mathbf{D}_2) \end{bmatrix}$$

DC decomposition for Optimization of Relay Beamforming Vector (OBV)

DC decomposition

$$F(\mathbf{x}) = \frac{1}{2\ln(2)}\left[G_2(\mathbf{x}) - H_2(\mathbf{x})\right]$$

$$G_2(\mathbf{x}) = \frac{1}{2}\rho\|\mathbf{x}\|^2 + \ln(C)\text{ , where } C = \frac{P_1|g_1|^2 + P_2|g_2|^2}{\sigma_{E,1}^2}$$

$$H_2(\mathbf{x}) = \frac{1}{2}\rho\|\mathbf{x}\|^2 + \ln(\sigma_1^2 + \mathbf{x}^{\mathbf{T}}\overline{\mathbf{A}}_{\mathbf{1}}\mathbf{x}) + \ln(\sigma_2^2 + \mathbf{x}^{\mathbf{T}}\overline{\mathbf{A}}_{\mathbf{2}}\mathbf{x}) - \ln(\sigma_1^2 + \mathbf{x}^{\mathbf{T}}\overline{\mathbf{B}}_{\mathbf{1}}\mathbf{x}) - \ln(\sigma_2^2 + \mathbf{x}^{\mathbf{T}}\overline{\mathbf{B}}_{\mathbf{2}}\mathbf{x})$$

Where ρ is chosen as the maximum eigenvalue of matrix $\left(\frac{\overline{\mathbf{A}}_{\mathbf{1}}}{2\sigma_1^2} + \frac{\overline{\mathbf{A}}_{\mathbf{2}}}{2\sigma_2^2} + \frac{2\overline{\mathbf{B}}_{\mathbf{1}}}{\sigma_1^2} + \frac{2\overline{\mathbf{B}}_{\mathbf{2}}}{\sigma_2^2}\right)$

DC decomposition for Optimization of Relay Beamforming Vector (OBV)

DC decomposition

Note that the linear approximation of $\mathrm{H}_2(\mathbf{x})$ at the point $\mathbf{x}^\mathbf{k}$ is given by:

$$\overline{\mathrm{H}}_2\ (\mathbf{x};\ \mathbf{x}^k) = \mathrm{H}_2(\mathbf{x}^k)\ \ + \langle \mathbf{y}^k;\ \mathbf{x} - \mathbf{x}^k \rangle$$

Where

$$\mathrm{y}^\mathrm{k} = \nabla \mathrm{H}_2(\mathbf{x}^\mathbf{k}) = \rho \mathbf{x}^\mathbf{k} + \frac{2\overline{\mathbf{A}}_1\mathbf{x}^\mathbf{k}}{\sigma_1^2 + (\mathbf{x}^\mathbf{k})^\mathbf{T}\overline{\mathbf{A}}_1\mathbf{x}^\mathbf{k}} + \frac{2\overline{\mathbf{A}}_2\mathbf{x}^\mathbf{k}}{\sigma_2^2 + (\mathbf{x}^\mathbf{k})^\mathbf{T}\overline{\mathbf{A}}_2\mathbf{x}^\mathbf{k}} - \frac{2\overline{\mathbf{B}}_1\mathbf{x}^\mathbf{k}}{\sigma_1^2 + (\mathbf{x}^\mathbf{k})^\mathbf{T}\overline{\mathbf{B}}_1\mathbf{x}^\mathbf{k}} - \frac{2\overline{\mathbf{B}}_2\mathbf{x}^\mathbf{k}}{\sigma_2^2 + (\mathbf{x}^\mathbf{k})^\mathbf{T}\overline{\mathbf{B}}_2\mathbf{x}^\mathbf{k}}$$

DC decomposition for Optimization of Relay Beamforming Vector

DCA Scheme

1.Initialization: start with any $\mathbf{x}^{\mathbf{0}} \in \mathbb{R}^{\mathbf{2}N}$, k = 0

2.Repeat:

Compute $\mathbf{x}^{\mathbf{k+1}}$ by solving the following problem

$$\min \quad G_2(\mathbf{x}) - \overline{\mathbf{H}}_{\mathbf{2}}(x; x^{\mathbf{k}})$$

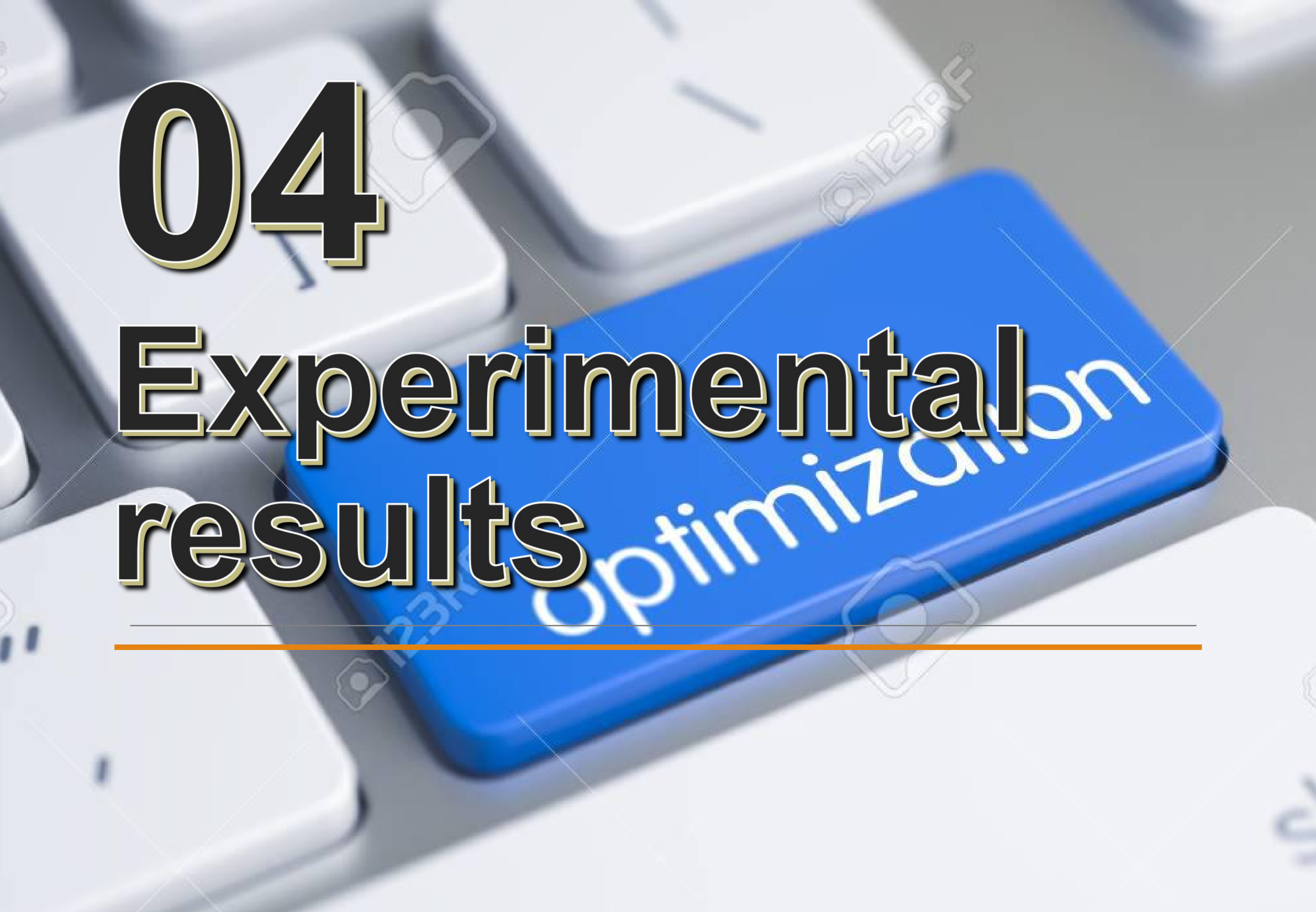
$$\textbf{s.t} \quad P_1 + P_2 + P_1\mathbf{x}^{\mathbf{T}}\overline{\mathbf{D}}_{\mathbf{1}}\mathbf{x} + P_2\mathbf{x}^{\mathbf{T}}\overline{\mathbf{D}}_{\mathbf{2}}\mathbf{x} + \sigma_R^2\mathbf{x}^{\mathbf{T}}\mathbf{x} \le P_T,$$

$$\bar{\mathbf{Z}}\mathbf{x} = 0$$

3.Until stopping condition is satisfied, i.e

$$\frac{\|\mathbf{x}^{\mathrm{k+1}} - \mathbf{x}^{\mathrm{k}}\|}{\|\mathbf{x}^{\mathrm{k}}\|+1} < \epsilon \text{ or } \frac{|F(\mathbf{x}^{k+1}) - F(\mathbf{x}^{k})|}{|F(\mathbf{x}^{k})|+1} < \epsilon$$

04 Experimental results

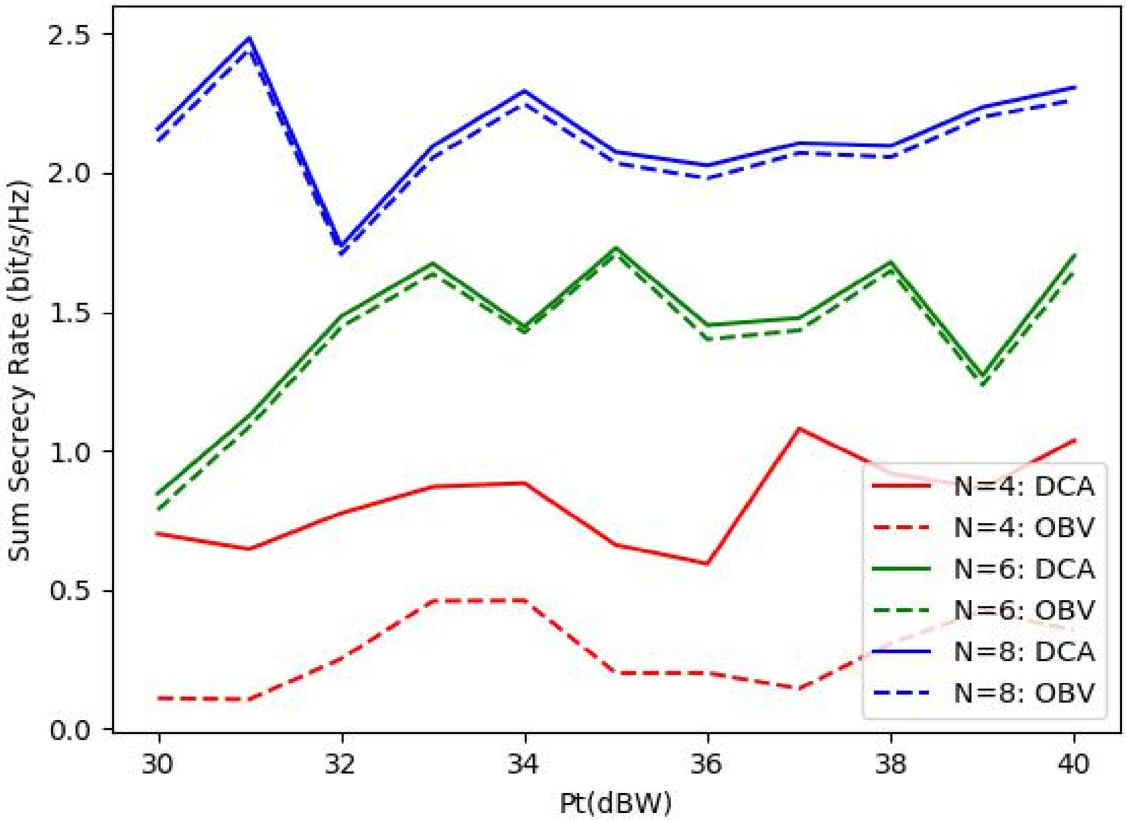


Experimental results

The analysis part of the comparison **only** focused on the objective value of each algorithm.

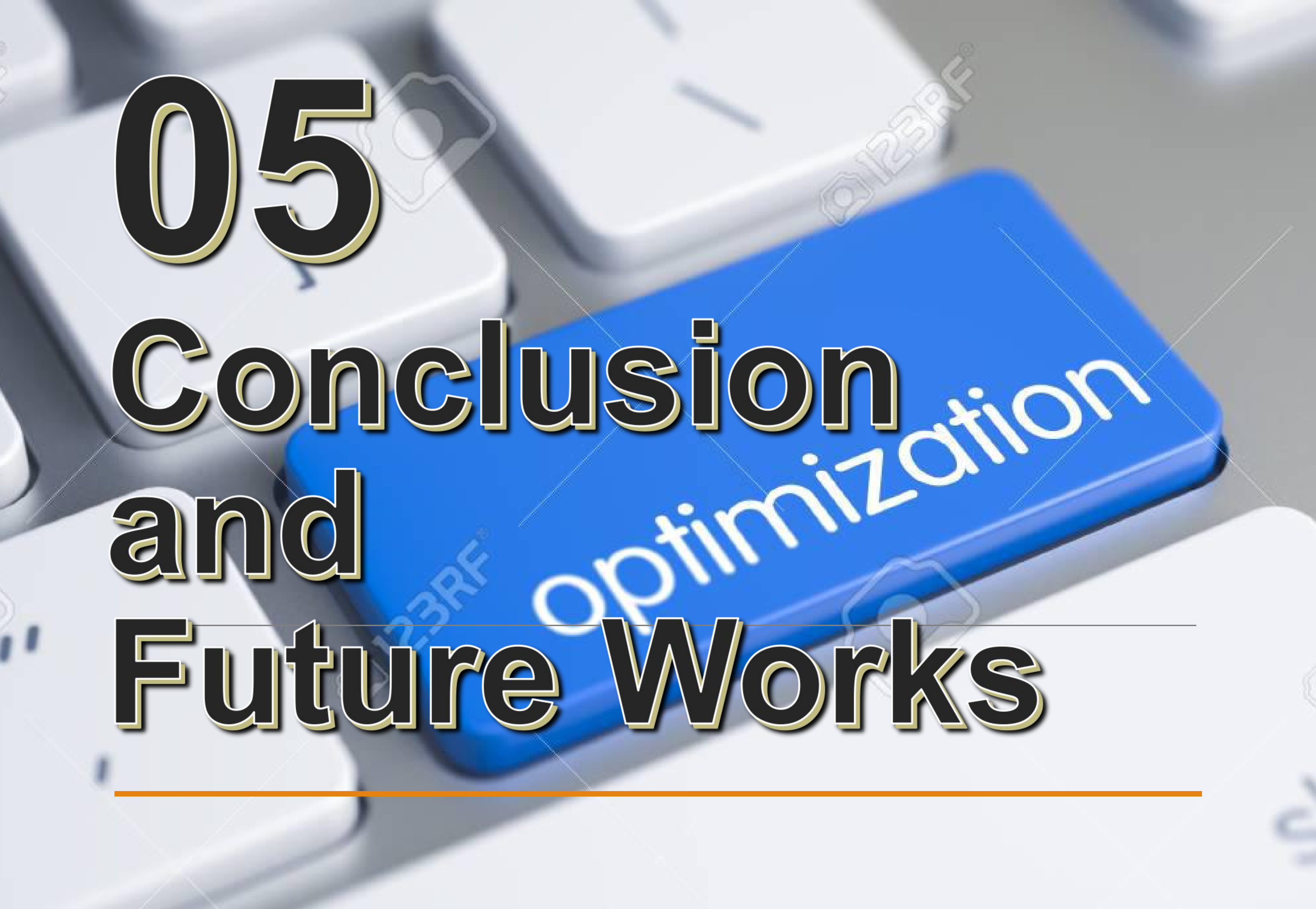
- Set of power $P_t = [30 : 1 : 40]$ (dBW), the number of relays $N \in \{4, 6, 8\}$, $P_1 = P_2 = \frac{\mathbf{P_T}}{\mathbf{4}}$ (dBW), $\sigma_1^2 = \sigma_2^2 = 1$ dB.
- Each pair of P_t and N we use 10 sets of randomized data, then getting the average.
- Run on an Intel i7 7500U 2.70 GHz of 8 GB.
- Solver CVX is used for DC algorithm.

Experimental results



- **There are a lot of test sets that OBV gives solution that is ZERO while DC algorithm can give much better results**

05 Conclusion and Future Works



Conclusion

- We proposed an algorithm base on DCA in order to solve a problem which is finding maximal secrecy sum rate of a two-way relay network system and compare it with an existed algorithm.
- Optimization of Beamforming Vector (OBV) can find quite good solutions in a short amount of time.
- However, DC algorithm even do the job better in a reasonable amount of time.
- While DC algorithm guarantees convergence, OBV cannot due to the fact that it has a randomization step.

Future Works

- The DC algorithm can be used to improve the objective value of this model in order to get better results for further research.

Thank you
for
your attention

Q&A