(c). we have $R_{ij} = \begin{cases} +1, & w.p\ \frac{1}{2} \\ -1, & w.p\ \frac{1}{2} \end{cases}$

so $E[R_{ij}^2] = \frac{1}{2}(1) + \frac{1}{2}(1) = 1$

Want to show that $E[\|v\|^2] = \|u\|^2$

<u>Proof</u>: we have $E[\|v\|^2] = E\left[\sum_{i=1}^{k} v_i^2\right]$

$$= E\left[\sum_{i=1}^{k}\left(\frac{1}{\sqrt{k}}\sum_{j=1}^{d} R_{ij} u_j\right)^2\right]$$

$$= E\left[\frac{1}{k}\sum_{i=1}^{k}\left(\sum_{j=1}^{d} R_{ij} u_j\right)^2\right]$$

$$= \frac{1}{k}\sum_{i=1}^{k} E\left[\left(\sum_{j=1}^{d} R_{ij} u_j\right)^2\right]$$

$$= \frac{1}{k}\sum_{i=1}^{k}\sum_{j=1}^{d}\left(E[R_{ij}^2]\, u_j^2\right) \quad \left[\text{assume independent } R_{ij}\right]$$

$$= \frac{1}{k}\sum_{i=1}^{k}\sum_{j=1}^{d} u_j^2$$

$$= \frac{1}{k}\cdot k\cdot \sum_{j=1}^{d} u_j^2$$

$$= \sum_{j=1}^{d} u_j^2$$

$$= \|u\|^2 \quad \blacksquare$$