$\boxed{3.6.1.}$ (a) Let F be a (d_1, d_2, p_1, p_2)-LSH family.

then, after a 2-way AND, we will get $F' : (d_1, d_2, p_1^2, p_2^2)$-LSH

applying 3-way OR on F', we get $F'' : (d_1, d_2, 1-(1-p_1^2)^3, 1-(1-p_2^2)^3)$-LSH

Hence we get a $(d_1, d_2, 1-(1-p_1^2)^3, 1-(1-p_2^2)^3)$-LSH.

(b). Let F be a (d_1, d_2, p_1, p_2)-LSH family

applying a 3-OR, we get $F' : (d_1, d_2, 1-(1-p_1)^3, (1-(1-p_2)^3))$-LSH

applying a 2-AND on F', we get $F'' : (d_1, d_2, [1-(1-p_1)^3]^2, [1-(1-p_2)^3]^2)$-LSH

Hence we get a $(d_1, d_2, [1-(1-p_1)^3]^2, [1-(1-p_2)^3]^2)$-LSH.

$\boxed{4.3.3}$ let false prob. rate be $f(k) = (1-e^{-\frac{m}{n}k})^k$

taking log of both sides, $\ln f(k) = k \ln(1-e^{-\frac{m}{n}k})$

$$\frac{d}{dk}[\ln f(k)] = \frac{d}{dk}\left[k \ln(1-e^{-\frac{m}{n}k})\right]$$

$$\frac{f'(k)}{f(k)} = k\left[\frac{1}{1-e^{-\frac{m}{n}k}} \cdot \frac{m}{n} e^{-\frac{m}{n}k}\right] + \ln(1-e^{-\frac{m}{n}k})$$

~~so $f'(k) =$~~

set $f'(k) = 0$, so that

$$① \longrightarrow 0 = \frac{km}{n} e^{-\frac{m}{n}k} + (1-e^{-\frac{m}{n}k}) \cdot \ln(1-e^{-\frac{m}{n}k})$$

let $b = e^{\frac{-m}{n}k}$,

then $\ln b = \frac{-m}{n} k$

$k = \frac{-n}{m} \ln b$ —— ②

Substituting ② into ①, $0 = -\ln b \cdot b + (1-b)\ln(1-b)$

$b \ln b = (1-b)\ln(1-b)$

So $b = 1-b$

$2b = 1$

$b = \frac{1}{2}$

Hence $k = \frac{-n}{m} \ln \frac{1}{2}$

$\boxed{k = \frac{n}{m} \ln 2}$ is the value that minimises $f(k)$, or the false prob. rate.