5.1. Identify the following as specific ARIMA models. That is what are p, d, q and what are the values of the parameters (the ϕ's and θ's)?

(a) $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$

Solution: $Y_t = \underbrace{Y_{t-1} - 0.25Y_{t-2}}_{AR(2)} + \underbrace{e_t - 0.1e_{t-1}}_{MA(1)}$

AR(2) $\begin{cases} \phi_1 = 1 \\ \phi_2 = -0.25 \end{cases}$ since $\begin{cases} \phi_1 + \phi_2 = 1 - 0.25 = 0.75 < 1 \\ \phi_2 - \phi_1 = -0.25 - 1 = -1.25 < 1 \\ |\phi_2| = 0.25 < 1 \end{cases}$ The process is stationary

MA(1) $\theta_1 = 0.1$ $1 - 0.1x = 0$ $x = 10 > 1$ The process is invertible

So the process is a stationary and invertible ARMA(2,1) model with $\phi_1 = 1$, $\phi_2 = -0.25$ and $\theta_1 = 0.1$

(b) $Y_t = 2Y_{t-1} - Y_{t-2} + e_t$

Solution: $Y_t - 2Y_{t-1} + Y_{t-2} = e_t$ since $\nabla^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$.

so $\{Y_t\}$ is an IMA(2,0) model

(c) $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

Solution: $Y_t = \underbrace{0.5Y_{t-1} - 0.5Y_{t-2}}_{AR(2)} + \underbrace{e_t - 0.5e_{t-1} + 0.25e_{t-2}}_{MA(2)}$

AR(2) $\begin{cases} \phi_1 = 0.5 \\ \phi_2 = -0.5 \end{cases}$ since $\begin{cases} \phi_1 + \phi_2 = 0.5 - 0.5 = 0 < 1 \\ \phi_2 - \phi_1 = -0.5 - 0.5 = -1 < 1 \\ |\phi_2| = 0.5 < 1 \end{cases}$ The process is stationary

MA(2) $\begin{cases} \theta_1 = 0.5 \\ \theta_2 = -0.25 \end{cases}$ since $\begin{cases} \theta_1 + \theta_2 = 0.5 - 0.25 = 0.25 < 1 \\ \theta_2 - \theta_1 = -0.25 - 0.5 = -0.75 < 1 \\ |\theta_2| = 0.25 < 1 \end{cases}$ The process is invertible

So $\{Y_t\}$ is a stationary and invertible ARMA(2,2) model with $\phi_1 = 0.5$, $\phi_2 = -0.5$, $\theta_1 = 0.5$, and $\theta_2 = -0.25$

5.4 Suppose that $Y_t = A + Bt + X_t$, where $\{X_t\}$ is a random walk.

First suppose that A and B are constant.

(a) Is $\{Y_t\}$ stationary?

Solution: $E(Y_t) = E(A+Bt+X_t) = A + Bt$ since $E(X_t)=0$ $\{X_t\}$ is R.W.

since $E(Y_t)$ is not free of t we conclude $\{Y_t\}$ is not stationary

(b) Is $\{\nabla Y_t\}$ stationary?

Solution: $\nabla Y_t = Y_t - Y_{t-1} = A + Bt + X_t - (A + B(t-1) + X_{t-1})$

$= B + X_t - X_{t-1} = B + \nabla X_t$.

since $\{X_t\}$ is random walk, so ∇X_t is white noise. e_t

i. $E(\nabla Y_t) = E(B + \nabla X_t) = B$ free of t.

ii. $Var(\nabla Y_t) = Var(B + \nabla X_t) = Var(\nabla X_t) = \sigma_e^2$ free of t

$E(\nabla Y_t)^2 = Var(\nabla Y_t) + [E(\nabla Y_t)]^2 = \sigma_e^2 + B^2$ free of t

iii. $Cov(\nabla Y_t, \nabla Y_{t+k}) = Cov(B + \nabla X_t, B + \nabla X_{t+k}) = Cov(e_t, e_{t+k})$

$$= \begin{cases} \sigma_e^2 & k=0 \\ 0 & k \neq 0 \end{cases}$$ free of t

We conclude that $\{\nabla Y_t\}$ is stationary.

Now suppose that A and B are random variables that are independent of random walk $\{X_t\}$

(c) Is $\{Y_t\}$ stationary?

Solution:

$E(Y_t) = E(A + Bt + X_t) = E(A) + t \cdot E(B) + 0$ depends on t.

We conclude that $\{Y_t\}$ is not stationary.

(d) Is $\{\nabla Y_t\}$ stationary?

Solution: $\nabla Y_t = B + \nabla X_t$

i $E(\nabla Y_t) = E(B + \nabla X_t) = E(B)$ free of t

ii. $Var(\nabla Y_t) = Var(B + \nabla X_t) = Var(B) + \sigma_e^2$ since B is independent of $\{X_t\}$

$E(\nabla Y_t)^2 = Var(\nabla Y_t) + [E(\nabla Y_t)]^2 = Var(B) + [E(B)]^2 + \sigma_e^2$ free of t

iii $Cov(\nabla Y_t, \nabla Y_{t-k}) = Cov(B + \nabla X_t, B + \nabla X_{t-k})$

$= Cov(B, B) + Cov(\nabla X_t, \nabla X_{t-k})$

$= \begin{cases} \sigma_B^2 + \sigma_e^2 & k = 0 \\ \sigma_B^2 & k \neq 0 \end{cases}$ free of t.

We conclude that $\{\nabla Y_t\}$ is stationary.

HW5 Chunhua Yu.

5.9 5.11 5.14
6.5 6.12 6.13

30/30

5.9 Verify Equation (5.1.10) $\rho_1 = -\{1/[2+(\sigma_\varepsilon^2/\sigma_e^2)]\}$

Solution:

5/5

$Y_t = M_t + e_t$. with $M_t = M_{t-1} + \varepsilon_t$.

$\nabla Y_t = \varepsilon_t + e_t + e_{t-1}$ where $\{e_t\}$ and $\{\varepsilon_t\}$ are independent W.N.

$Var(\nabla Y_t) = Var(\varepsilon_t + e_t - e_{t-1}) = Var\,\varepsilon_t + Var(e_t) + Var(e_{t-1})$

$= \sigma_\varepsilon^2 + 2\sigma_e^2$

$\gamma_1 = Cov(\nabla Y_t, \nabla Y_{t-1}) = Cov(\varepsilon_t + e_t - e_{t-1},\ \varepsilon_{t-1} + e_{t-1} - e_{t-2})$

$= Cov(\varepsilon_t, \varepsilon_{t-1}) + Cov(\varepsilon_t, e_{t-1}) - Cov(\varepsilon_t, e_{t-2})$

$+ Cov(e_t, \varepsilon_{t-1}) + Cov(e_t, e_{t-1}) - Cov(e_t, e_{t-2})$

$- Cov(e_{t-1}, \varepsilon_{t-1}) - Cov(e_{t-1}, e_{t-1}) + Cov(e_{t-1}, e_{t-2})$

$= -Cov(e_{t-1}, e_{t-1}) = -Var(e_{t-1}) = -\sigma_e^2$

$\rho_1 = \dfrac{\gamma_1}{\gamma_0} = \dfrac{-\sigma_e^2}{\sigma_\varepsilon^2 + 2\sigma_e^2} = -\{1/[2+(\sigma_\varepsilon^2/\sigma_e^2)]\}$ Verified.

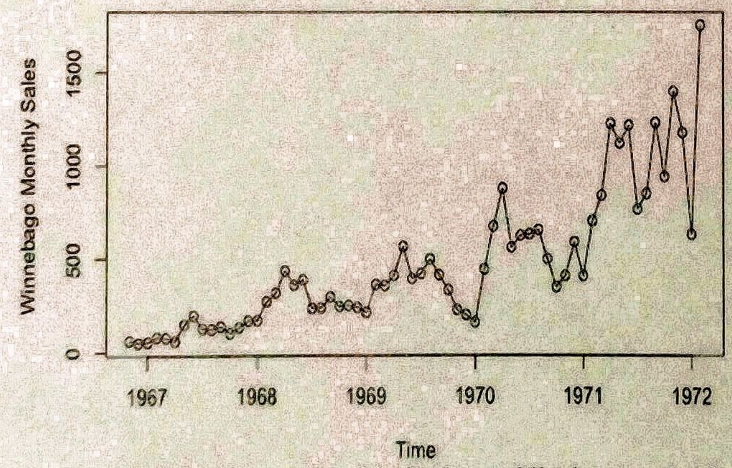
5.11

10/10

The data file winnebago contains monthly unit sales of recreational vehicles (RVs) from Winnebago, Inc., from November 1966 through February 1972.

(a) Display and interpret the time series plot for these data.

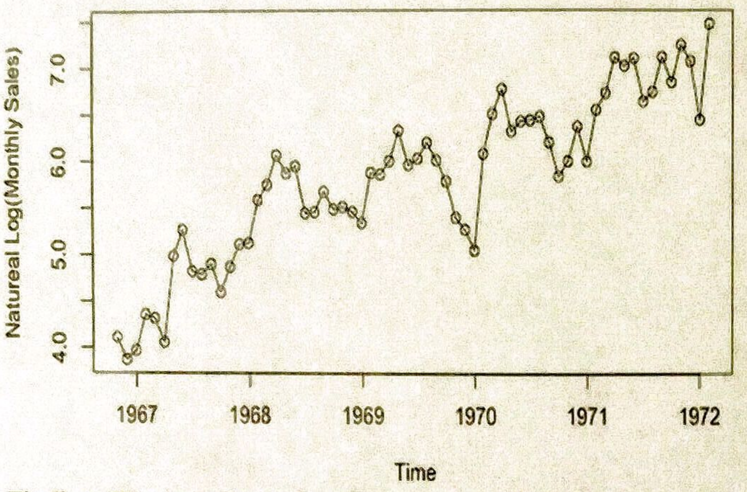
```
> library(TSA)
> data("winnebago")
> plot(winnebago,type='o',ylab='Winnebago Monthly Sales')
```



Finding: the series increases with the time, while the variation is larger and larger.

(b) Now take natural logarithms of the monthly sales figures and display the time series plot of the transformed values. Describe the effect of the logarithms on the behavior of the series.

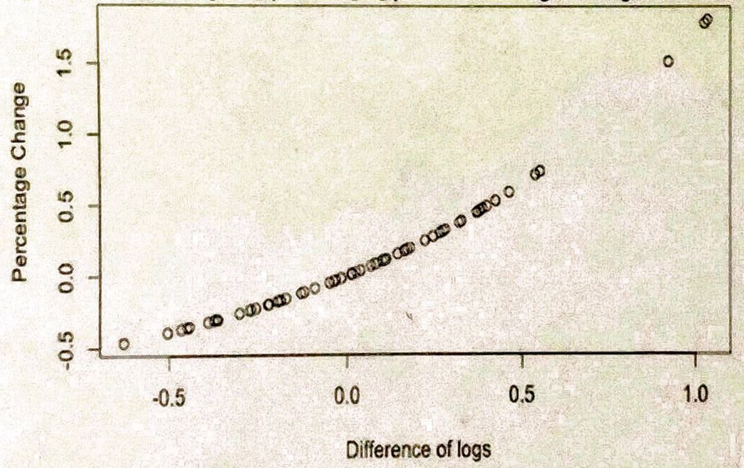
> win.log=log(winnebago)
> plot(win.log, type='o', ylab='Natureal Log(Monthly Sales)')



Finding: The transformed series increase with the time, the variation is approximately same.

(c) Calculate the fractional relative changes, $(Y_t - Y_{t-1})/Y_{t-1}$, and compare them with the differences of (natural) logarithms, $\nabla \log(Y_t) = \log(Y_t) - \log(Y_{t-1})$.

> win.f=na.omit((winnebago-zlag(winnebago))/zlag(winnebago))
> plot(x=diff(win.log)[-1],y=win.f[-1],ylab='Percentage Change',xlab='Difference of logs')



How do they compare for smaller values and for larger values?

> cor(diff(win.log)[-1],win.f[-1])
[1] 0.9646886

Finding: If they have a perfect relationship, the plot should be a straight line. Apparently, the relationship is quite good but not perfect. The correlation coefficient in this plot is 0.965 so the agreement is quite good. Since the model is fitted by using the last element of each year, there is seasonality in this series that has not involved in the model.

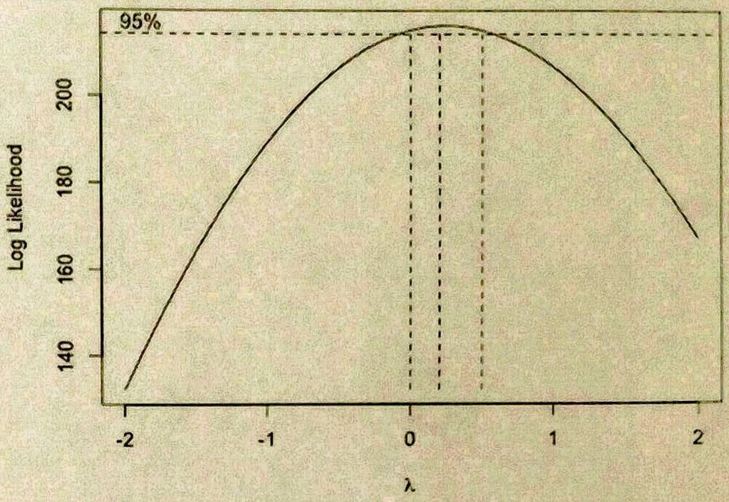
10/10

5.14

Consider the annual rainfall data for Los Angeles shown in Exhibit 1.1, on page 2. The quantile-quantile normal plot of these data, shown in Exhibit 3.17, on page 50, convinced us that the data were not normal. The data are in the file larain.

(a) Use software to produce a plot similar to Exhibit 5.11, on page 102, and deter- mine the "best" value of λ for a power transformation of the data.

Finding: From the R output,
95% CI for λ is (-0.086, 0.5782)
The estimate of λ is 0.25

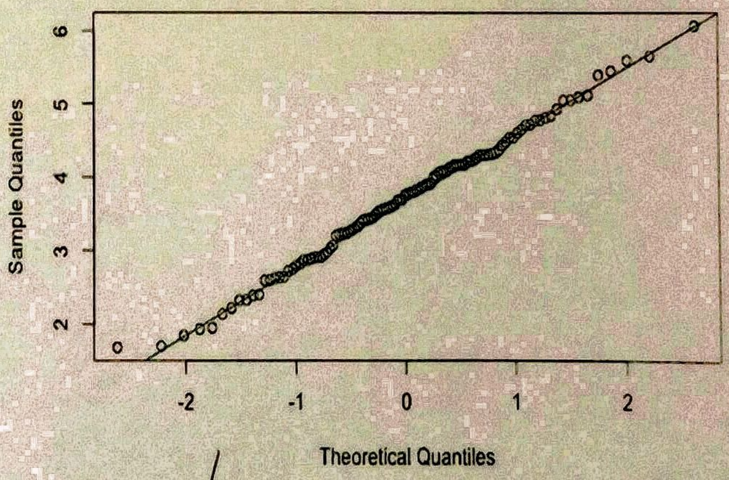



```
> data(larain)
> y=BoxCox.ar(larain,method='ols')
> y

$mle
[1] 0.25

$ci
[1] -0.08  0.57
```

✓

(b) Display a quantile-quantile plot of the transformed data. Are they more nor- mal?

```
> qqnorm((larain)^0.25,main='')
>  la.tp=(larain^0.25-1)/0.25
> qqnorm(la.tp,main = '')
> qqline(la.tp)
> qqline(la.tp)
```



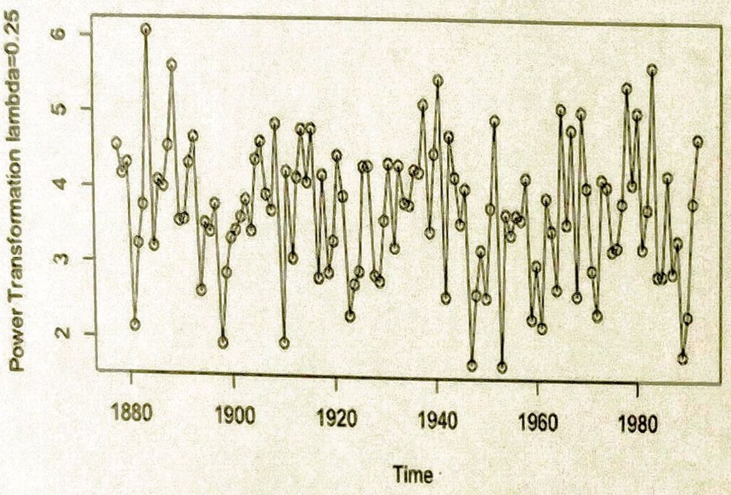

```
> shapiro.test(la.tp)

        Shapiro-Wilk normality test

data:  la.tp
W = 0.99408, p-value = 0.9096
```

✓

Finding: From the output of Shapiro test, the p value is 0.9096, we conclude that there is significant evidence that the transformed data are normally distributed.

(c) Produce a time series plot of the transformed values.

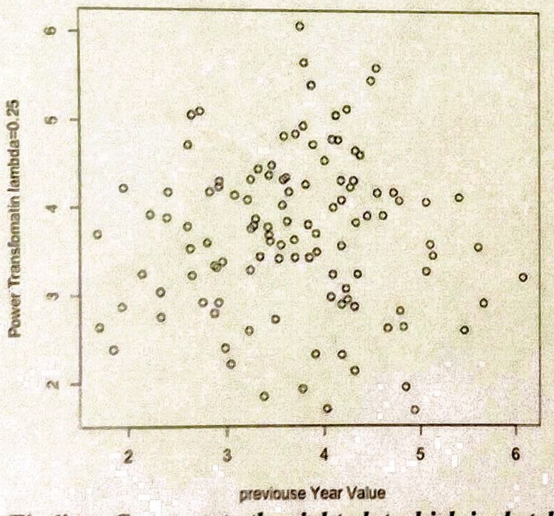
```
> plot(la.tp,type='o',ylab='Power Transformation lambda=0.25')
```

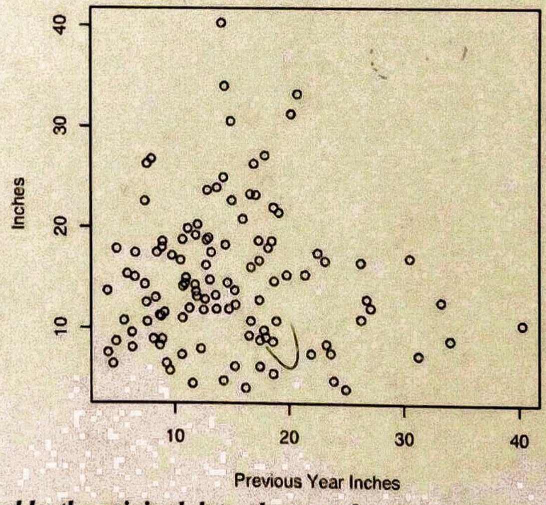


Finding: this transformed series could now be considered as normal white noise with a nonzero mean.

(d) Use the transformed values to display a plot of Y_t versus Y_{t-1} as in Exhibit 1.2, on page 2. Should we expect the transformation to change the dependence or lack of dependence in the series?

```
>plot(la.tp,x=zlag(la.tp),ylab = 'Power Transfomatin lambda=0.25',xlab='previouse Year Value')
```





Finding: Compare to the right plot which is sketched by the original data, the transformation shows the same relationship between year values, that is the lack of correlation or any other kind of dependency between year values. Also clearly from the plots, the transformation did not make any change the dependence or lack of dependence in the series.