1.1 2.5
2.1 2.11
2.2 2.15

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Great Work!

Time Series HW1 Chunhua

1. If Z_t, $t=0, \pm1, \pm2, \cdots$, are independent random variables with mean 0 and variance σ^2 and $Y_t = \sum_{j=-q}^{q} a_j Z_{t-j}$. Find EY_t and $Var(Y_t)$

Solution: $E(Y_t) = E\left(\sum_{j=-q}^{q} a_j Z_{t-j}\right) = \sum_{j=-q}^{q} E(a_j Z_{t-j}) = \sum_{j=-q}^{q} a_j E(Z_{t-j}) = 0.$

$$Var(Y_t) = Var\left(\sum_{j=-q}^{q} a_j Z_{t-j}\right) = \sum_{j=-q}^{q} Var(a_j Z_{t-j}) = \sum_{j=-q}^{q} a_j^2 \, Var Z_{t-j} = \sum_{j=-q}^{q} a_j^2 \cdot \sigma^2$$

$$= \sigma^2 \cdot \sum_{j=-q}^{q} a_j^2$$

2.1 Suppose $E(X)=2$, $Var X = 9$, $EY=0$, $Var Y = 4$ and $Corr(X, Y) = 0.25$. Find:

5/5 (a) $Var(X+Y)$ (b) $Cov(X, X+Y)$ (c) $Corr(X+Y, X-Y)$

Solution: (a) $Var(X+Y) = Var X + Var Y + 2Cov(X, Y)$

since $Corr(X, Y) = \dfrac{Cov(X, Y)}{\sqrt{Var X \, Var Y}} = \dfrac{Cov(X, Y)}{\sqrt{4 \cdot 9}} = 0.25$

$Cov(X, Y) = 0.25 \times 2 \times 3 = 1.5$

$Var(X+Y) = 9 + 4 + 2 \times 1.5 = 16$ ✓

(b) $Cov(X, X+Y) = Cov(X, X) + Cov(X, Y) = Var X + Cov(X, Y)$

$= 9 + 1.5 = 10.5$ ✓

(c) $Corr(X+Y, X-Y) = \dfrac{Cov(X+Y, X-Y)}{\sqrt{Var(X+Y)\, Var(X-Y)}}$

since $Var(X-Y) = Var X + Var Y - 2Cov(X, Y) = 9 + 4 - 2 \times 1.5 = 10$

$Cov(X+Y, X-Y) = Cov(X, X) - Cov(X, Y) + Cov(Y, X) - Cov(Y, Y)$

$= Var X - Var Y = 9 - 4 = 5$

$Corr(X+Y, X-Y) = \dfrac{5}{\sqrt{16 \times 10}} = \dfrac{1}{4} \cdot \sqrt{\dfrac{5}{2}} \approx 0.395$ ✓

2.2 If X and Y are dependent but $Var(X) = Var Y$ find $Cov(X+Y, X-Y)$

5/5 Solution: $Cov(X+Y, X-Y) = Cov(X, X) - Cov(X, Y) + Cov(Y, X) - Cov(Y, Y)$

$= Var(X) - Var(Y) = 0$ ✓

2.5 Suppose $Y_t = 5+2t+X_t$, where $\{X_t\}$ is a zero-mean stationary series with autocovariance function γ_k.

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(a) Find the mean function for $\{Y_t\}$

(b) Find the autocovariance function for $\{Y_t\}$

(c) Is $\{Y_t\}$ stationary? Why or why not?

Solution: (a) $E\{Y_t\} = E(5+2t+X_t) = 5+2t+E(X_t) = 5+2t$ ✓

(b) $Cov(Y_t, Y_{t+k}) = Cov(5+2t+X_t,\ 5+2(t+k)+X_{t+k})$

$= Cov(X_t, X_{t+k}) = \gamma_k$ ✓

(c) since $E(Y_t) = 5+2t$ depends on t, $\{Y_t\}$ is not stationary.
✓

2.11 Suppose $Cov(X_t, X_{t-k}) = \gamma_k$ is free of t but that $E(X_t) = 3t$

(a) Is X_t stationary?

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(b) Let $Y_t = 7-3t+X_t$, Is $\{Y_t\}$ stationary?

Solution: (a) since $E(X_t) = 3t$ depends on t, X_t is not stationary.

(b) i. $E(Y_t) = E(7-3t+X_t) = 7-3t+E(X_t) = 7-3t+3t = 7$

ii. $E(Y_t^2) = E[(7-3t+X_t)^2] = E(49+9t^2+X_t^2-42t+14X_t-6tX_t)$

$= 49+9t^2-42t+E(X_t^2)+14EX_t-6tEX_t$

$= 49+9t^2-42t+VarX_t+(EX_t)^2+14\cdot 3t-6t\cdot 3t$

$= 49+\cancel{9t^2}-\cancel{42t}+\gamma_0+\cancel{9t^2}+\cancel{42t}-\cancel{18t^2}$ since $VarX_t = Cov(X_t, X_t) = \gamma_0$

$= 49+\gamma_0$

iii. $Cov(Y_t, Y_{t+k}) = Cov(7-3t+X_t,\ 7-3(t+k)+X_{t+k}) = Cov(X_t, X_{t+k}) = \gamma_k$

In summary $E(Y_t)$, $E(Y_t^2)$ and $Cov(Y_t, Y_{t+k})$ are all free of t we conclude that $\{Y_t\}$ is stationary. ✓

Chunhua Yu

2.15 Suppose that X is a random variable with zero mean, Define a time series by $Y_t = (-1)^t X$

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(a) Find the mean function for $\{Y_t\}$

(b) Find the covariance function for $\{Y_t\}$

(c) Is $\{Y_t\}$ stationary?

Solution: (a) $E\{Y_t\} = E((-1)^t X) = (-1)^t EX = 0$ ✓

(b) $Cov(Y_t, Y_{t+k}) = Cov((-1)^t X, (-1)^{t+k} X) = (-1)^{t+t+k} Cov(X, X)$

$= (-1)^{2t+k} \cdot \sigma_X^2 = (-1)^k \sigma_X^2$

(c) $E(Y_t^2) = Var Y_t + [E(Y_t)]^2 = (-1)^0 \sigma_X^2 + 0 = \sigma_X^2$

Since $E\{Y_t\}$, $E(Y_t^2)$ and $Cov(Y_t, Y_{t+k})$ are all free of t

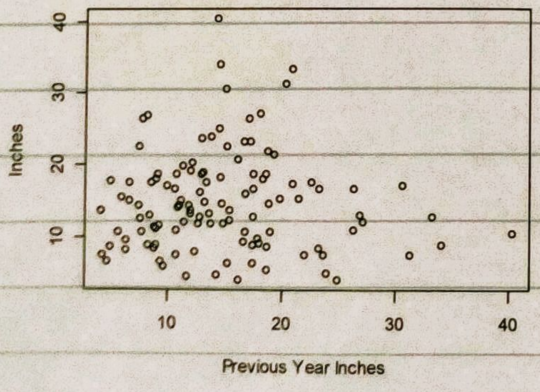
we conclude that $\{Y_t\}$ is stationary. ✓

1.1

5/5

```
> library(TSA)

载入程辑包：'TSA'
The following objects are masked from 'package:stats':    acf, arima
The following object is masked from 'package:utils':    tar
Warning message:
程辑包'TSA'是用R版本3.5.2 来建造的
> data(larain)
> win.graph(width = 3,height = 3,pointsize = 8)
> plot(y=larain,x=zlag(larain),ylab='Inches',xlab = 'Previous Year Inches')
```



✓