HW3 4.21(a) 4.5(a)

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4.21(a) Sketch the autocorrelation functions for the following MA(2) model with parameters (a) $\theta_1 = 0.5$ and $\theta_2 = 0.4$

Solution: Using Equation (4.2.3)

$$\rho_1 = \frac{-\theta_1 + \theta_1\theta_2}{1+\theta_1^2+\theta_2^2} = \frac{-0.5+0.5\cdot 0.4}{1+0.5^2+0.4^2} = \frac{-0.3}{1.41} = -0.213.$$

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$$\rho_2 = \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} = \frac{-0.4}{1+0.5^2+0.4^2} = \frac{-0.4}{1.41} = -0.284.$$

✓

$\rho_k = 0$ for $k = 3, 4 \cdots$

sketch ? (-1)

4.4 Show that when θ is replaced by $1/\theta$, the autocorrelation function for an MA(1) process does not change.

5/5 Solution: Using Equation (4.2.2)

$$\rho_1 = \frac{-\theta}{1+\theta^2}$$ ✓

replacing θ by $1/\theta$

$$\rho_1 = \frac{-1/\theta}{1+(1/\theta)^2} = -\frac{1}{\theta} / (1+\frac{1}{\theta^2}) = \frac{-\theta}{1+\theta^2}$$

Verified.

4.10 Sketch the autocorrelation functions for ARMA(1, 1) $\phi = 0.7$ and $\theta = 0.4$

5/5 Solution: Using Equation (4.4.5)

$$\rho_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1} = \frac{(1-0.7\times0.4)(0.7-0.4)}{1-2\times0.7\times0.4+0.4^2} \times 0.7^{k-1} = 0.36\cdot 0.7^{k-1} \quad \text{for } k \geq 1$$

✓

eg $\rho_1 = 0.36$ and so on

$\rho_2 = 0.252$

$\rho_3 = 0.1764$

4.18 Consider a process that satisfies the zero-mean, "stationary" AR(1) equation

10/10 $Y_t = \phi Y_{t-1} + e_t$ with $-1 < \phi < +1$. Let c be any nonzero constant, and define $W_t = Y_t + c\phi^t$

(a) Show that $E(W_t) = c\phi^t$

Solution: $E(W_t) = E(Y_t + c\phi^t) = E(Y_t) + c\phi^t = c\phi^t$ since $E(Y_t) = 0$

(b) Show that $\{W_t\}$ satisfies the stationary AR(1) equation $W_t = \phi W_{t-1} + e_t$

$$\begin{cases} W_{t-1} = Y_{t-1} + c\phi^{t-1} \quad \& \quad Y_t = \phi Y_{t-1} + e_t \\ W_t = Y_t + c\phi^t = \phi Y_{t-1} + e_t + c\phi^t = \phi[Y_{t-1} + c\phi^{t-1}] + e_t = \phi \cdot W_{t-1} + e_t \end{cases}$$

Verified

(c) Is $\{W_t\}$ stationary?

since $E(W_t) = c\phi^t$ depends on t, we conclude $\{W_t\}$ is not stationary.

4.5 (a) Calculate and sketch the autocorrelation functions for AR(1) with $\phi_1 = 0.6$

5/5 Plot for sufficient lags that the autocorrelation function has nearly dies out.

Solution:

Using Equation (4.3.6)

$$\rho_k = \phi^k = 0.6^k \qquad \text{for } k = 1, 2, 3 \cdots$$

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> Acf=ARMAacf(ar=0.6,lag.max = 9)
> plot(y=Acf[-1],x=1:9,xlab='Lag',ylab='ACF',type = 'h')
> abline(h=0)
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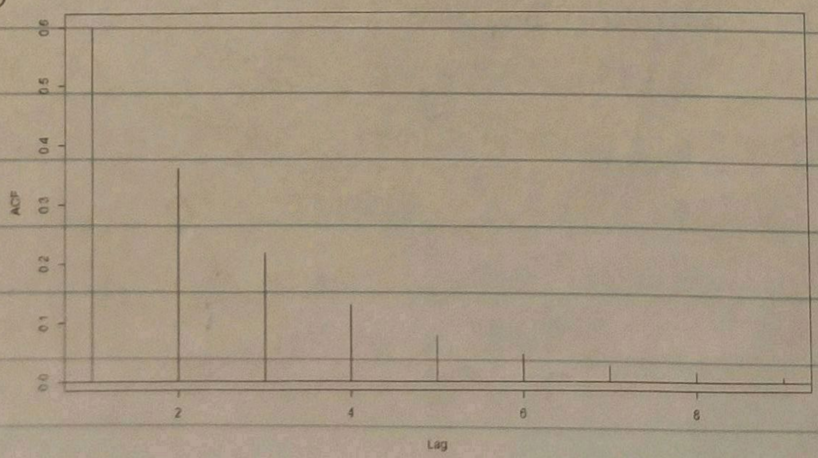

HW4

4.21
5.1
5.4

4.21 Consider the model $Y_t = e_{t-1} - e_{t-2} + 0.5e_{t-3}$

(a) Find the autocovariance function for the process.

$Var(Y_t) = (1+1+0.25)\sigma_e^2 = 2.25\sigma_e^2$

$Cov(Y_t, Y_{t-1}) = Cov(e_{t-1} - e_{t-2} + 0.5e_{t-3}, e_{t-2} - e_{t-3} + 0.5e_{t-4})$

$= Cov(-e_{t-2}, e_{t-2}) + Cov(0.5e_{t-3}, -e_{t-3})$

$= [-1 + 0.5\times(-1)]\sigma_e^2$

$= -1.5\sigma_e^2$

$Cov(Y_t, Y_{t-2}) = Cov(e_{t-1} - e_{t-2} + 0.5e_{t-3}, e_{t-3} - e_{t-4} + 0.5e_{t-5})$

$= Cov(0.5e_{t-3}, e_{t-3})$

$= 0.5\sigma_e^2$

In summary $\gamma(k) = \begin{cases} 2.25\sigma_e^2 & k=0 \\ -1.5\sigma_e^2 & k=\pm1 \\ 0.5\sigma_e^2 & k=\pm2 \\ 0 & |k|>2 \end{cases}$ $\rho_k = \begin{cases} -\frac{1.5}{2.25} = -\frac{2}{3} & k=\pm1 \\ \frac{0.5}{2.25} = \frac{2}{9} & k=\pm2 \\ 0 & |k|>2 \end{cases}$

(b) Show that this is a certain ARMA(p,q) process in disguise. That is identify values for p and q and for the θ's and φ's such that the ARMA(p, q) process has the same stationary properties as $\{Y_t\}$.

This is really just the MA(2) process $Y_t = e_t - e_{t-1} + 0.5e_{t-2}$ in disguise since we do not observe the error terms, there is no way to tell the difference between the two sequences defined as e_t and $e_t' = e_{t-1}$