

#2.2. Sensitivity = specificity = 0.8

	Pos	Neg
True	0.8	0.2
False	0.2	0.8

$$\therefore \text{Odds ratio} = \frac{0.8 \times 0.8}{0.2 \times 0.2}$$

$$= \frac{64}{4} = 16 \quad \square$$

#2.5

	Gun-Death	O:W
U.S	14.24	
Can	4.31	
Aus	2.65	
Ger	1.24	
Eng	0.41	

$$RR_{\text{Can/USA}} = \frac{4.31}{14.24}$$

$$RR_{\text{Aus/USA}} = \frac{2.65}{14.24}$$

$$RR_{\text{Ger/USA}} = \frac{1.24}{14.24}$$

$$RR_{\text{Eng/USA}} = \frac{0.41}{14.24}$$

→ Interp: All countries have low RR for gun-death compared to USA.

ratio

#2.8. (a) $\text{Odds} = 11.4 \neq RR$

Such interpretation is for RR. ($\pi_1 = 11.4\pi_2$)

Correct interpretation is just female has 11.4 times "odds" for survival.

(b) Odds of survival for female = 2.9

$$\frac{\pi_{\text{surv}}}{1 - \pi_{\text{surv}}}$$

$$\therefore \pi_{\text{surv}} = \frac{2.9}{3.9} = 0.74 \text{ (female)}$$

$$\frac{1}{11.4} = \text{Odds ratio} = \frac{2.9}{\pi_{\text{surv}}} \cdot \frac{3.9}{1 - \pi_{\text{surv}}} = \frac{2.9\pi_{\text{surv}}}{2.9 - 2.9\pi_{\text{surv}}}$$

$$\therefore 2.9 - 2.9\pi_{\text{surv}} = 11.4 \times 2.9\pi_{\text{surv}}$$

$$(11.4 - 1)\pi_{\text{surv}} =$$

$$\frac{2.9}{11.4 \times 2.9 + 2.9}$$



No.
year month day ()

#2.11.	Smokers	Non-smokers
Lung Cancer	0.0014	0.0010
Coronary	0.00669	0.00413

(a)

Condition →	L	C
S	$\frac{140}{809}$	$\frac{669}{929}$
N-S	$\frac{100}{513}$	$\frac{413}{513}$

$$\Rightarrow \text{Difference of prop} = \frac{140}{809} - \frac{100}{513}$$

$$RR = (140/809) / (100/513)$$

$$\text{Odds ratio} = \frac{100 \times 140}{140 \times 413}$$

(b) Odds ratio가 3.2

(1.222 S와 N-S 대비 1.222배 더 높음) (3.2배 더 높음) (c) 안이 0.222배 (2)

#2.14.

		Victim	D.W.	
(a) Male	White	0.0049	$1 - \pi_1$	$\text{Odds}_1 = 0.0138$
	Non-white	0.0263	$1 - \pi_2$	$\text{Odds}_2 = 0.3179$
Female	White	0.0023	$1 - \pi_3$	Homogeneous X.
	Non-white	0.0072	$1 - \pi_4$	

(b) (Same #)

White	$\frac{(49+23)}{10000}$
Non-white	$\frac{(263+72)}{10000}$

$$\text{Odds} = 3.8$$

#2.17.

Yes

No

 $S < 20$

S

 π_1 $1 - \pi_1$

Odds ratio = 11.7

N-S

 π_2 $1 - \pi_2$ $S > 20$

S

 π_3 $1 - \pi_3$

" = 26.1

N-S

 π_2 $1 - \pi_2$

$$\frac{26.1}{11.7} = 2.2$$

N-S = 203
$S < 20 = y_{03}$
$S > 20 = z_{03}$

$$1 - \left(\frac{y\pi_1 + z\pi_3}{y+z} \right)$$

$$= \left(\frac{y\pi_1 + z\pi_3}{y+z} \cdot (1 - \pi_2) \right)$$

$$\pi_2 \cdot \frac{y+z - y\pi_1 - z\pi_3}{y+z}$$

$$= \frac{(1 - \pi_2)}{\pi_2}$$

$$= \frac{(1 - \pi_2)}{\pi_2}$$

$$\frac{(y\pi_1 + z\pi_3)}{(y+z) - y\pi_1 - z\pi_3}$$

$$\frac{y\pi_1 + z\pi_3}{y(1 - \pi_1) + z(1 - \pi_3)}$$

#2.20.

Victim

D

Y

N

W

W

19

132

0.68

< 1

B

B

11

52

0

< 1

B

W

0

9

Total

W

19

141

1.18

> 1

B

17

149

Simpson's paradox

#223.

D: Having certain disease

AR: proportion of disease attributable to that exposure

E: " exposure to a factor

(a) ^{Show} $P(\bar{E}) = 1 - P(E)$
 $AR = 1 - \frac{P(D|\bar{E})}{P(D)}$

P(D) 중에 E 인데 걸린 사람

(b) D N-D

E

N-E

$$\frac{P(D|E)}{P(E)}$$

$$\frac{P(D|\bar{E})}{P(\bar{E})}$$

$$RR = \frac{P(D|E)}{P(D|\bar{E})} = \frac{P(E)}{P(\bar{E})}$$

Given $AR + P(E)(RR-1) \cdot AR = P(E) \cdot (RR-1)$

$$\Leftrightarrow P(E)(RR-1)(1-AR) = AR$$

$$\Leftrightarrow RR-1 = \frac{AR}{1-AR} \cdot \frac{1}{P(E)}$$

$$\Leftrightarrow AR = 1 + \left(1 - \frac{P(D|\bar{E})}{P(D)}\right) \cdot \frac{1}{P(E)} \cdot \frac{P(D)}{P(D|\bar{E})}$$

$$RR = \frac{P(D|E)}{P(D|\bar{E})}$$

$$P(E)(RR-1) = \frac{P(D|E)}{P(D|\bar{E})} - 1$$

$$= \frac{P(D|E) - P(D|\bar{E})}{P(D|\bar{E})} \cdot P(E)$$

RHS =

$$1 + \frac{P(D) - P(D|\bar{E})}{P(D|\bar{E})} \cdot P(E)$$

$$= \frac{P(D|\bar{E}) - P(D|\bar{E})}{P(D|\bar{E}) + P(D|\bar{E})P(E)}$$

$$= \frac{P(D) \cdot (1 - P(\bar{E}|D))}{P(E) \cdot (1 - P(\bar{E}|D))}$$

#2.26.

	E_1	\bar{E}_1
E_2	—	—
\bar{E}_2	—	—

Think conditional RR.

& Marginal RR

→ Simpson's paradox.



#2.29

 $n_1 \quad n_2 \quad \dots \quad n_k$
 $m_1 \quad m_2 \quad \dots \quad m_k$
 $\pi_1 \dots \pi_k$ $p_1 \dots p_k$ $\pi_1 > p_1$ $\pi_k > p_k$

$$\Rightarrow \frac{n_1 \pi_1 + \dots + n_k \pi_k}{n_1 + \dots + n_k}$$

$$\frac{m_1 p_1 + \dots + m_k p_k}{m_1 + \dots + m_k}$$

$$< \frac{m_1 \pi_1 + \dots + m_k \pi_k}{m_1 + \dots + m_k}$$

		Hit	Not
year ₁	S	1	99
	J	0	2
year ₂	S	1	2
	J	49	51
year _k	S	2	101
	J	49	53

Simpson's paradox

$$\Rightarrow \frac{2}{101}$$

$$\Rightarrow \frac{49}{102}$$



#2.32.

$$\frac{\pi_{ij} \pi_{jj}}{\pi_{i+} \pi_{+j}} = \alpha_{ij}$$

$$\textcircled{1} \text{ indep} \Rightarrow \pi_{ij} = \pi_{i+} \cdot \pi_{+j}$$

$$\Rightarrow \alpha_{ij} = \frac{\pi_{i+} \cdot \pi_{+j} \cdot \pi_{i+} \pi_{+j}}{\pi_{i+} \pi_{+j} \cdot \pi_{i+} \pi_{+j}} = 1$$

$$\textcircled{2} \quad 1 = \frac{\pi_{ji} \pi_{ji}}{\pi_{ji} \pi_{ji}}$$

#2.35. Poisson multi (n = \sum n_i) 일 때 (2)의 식을 보.

#2.38. $V(Y) = \sum \pi_{tj} (1 - \pi_{tj}) = 1 - \sum \pi_{tj}^2$

(a)

N_1	\dots	N_k
Y_1	π_{11}	π_{1k}
\vdots	\vdots	\vdots
Y_m	π_{m1}	π_{mk}
	π_{t1}	π_{tj}

$= 1$

$\pi_{tj} = 1$ for some $j \Rightarrow$
 \Rightarrow other $\pi_{tj} = 0$

$(\because \sum_j \pi_{tj} = 1) \therefore V(Y) = 0$

Cauchy 보이기

$\Rightarrow \pi_{tj} = \frac{1}{J}$ 같은 값이면 Max

$\Rightarrow J \cdot \frac{1}{J} \cdot (1 - \frac{1}{J}) = \frac{J-1}{J} = V(Y) \quad \square$

★

(b) $E(V(Y|X)) = \sum_i \pi_{it} V(Y|t)$

$= \sum_i (\pi_{it} \sum_j \pi_{ij} (1 - \pi_{ij}))$

$= \sum_i \pi_{it} (\sum_j \pi_{ij} - \sum_j \pi_{ij}^2)$

$= \sum_i \pi_{it} (\pi_{it} - \sum_j \pi_{ij}^2)$

$= \sum_i \pi_{it}^2 - \pi_{it} \cdot \sum_j \pi_{ij}^2$

$= \sum_i \pi_{it} (\pi_{it} - \sum_j \pi_{ij}^2)$