Categorical Data Analysis $\boxed{\text{CH3}}$

#24.

NO, We obtain confidence interval for odds ratio θ by using $\log \hat{\theta}$

#27.

put $\hat{\theta}_1 = \hat{\pi}_1$. Recall Score interval for binomial parameter.

$$\hat{\pi}\left(\frac{n}{n+z_{\alpha/2}^2}\right) + \frac{1}{2}\cdot\frac{z_{\alpha/2}^2}{n+z_{\alpha/2}^2}$$

$$\pm z_{\alpha/2}\sqrt{\frac{1}{n+z_{\alpha/2}^2}\left[\hat{\pi}(1-\hat{\pi})\frac{n}{n+z_{\alpha/2}^2} + \frac{1}{2}\frac{1}{2}\cdot\frac{z_{\alpha/2}^2}{n+z_{\alpha/2}^2}\right.}$$

put α so that $z_{\alpha/2} = 1$.

$$\Rightarrow \hat{\pi}\cdot\frac{n}{n+1} + \frac{1}{2}\cdot\frac{1}{n+1} \pm \sqrt{\frac{1}{(n+1)}\left[\hat{\pi}(1-\hat{\pi})\frac{n}{n+1} + \frac{1}{4}\cdot\frac{1}{n+1}\right]}$$

#30

$$X^2 = \sum \frac{(n_{ij} - \mu_{ij})^2}{\mu_{ij}}$$

$$= \sum \frac{(n \cdot p_{ij} - n \cdot p_{i+}p_{+j})^2}{n \cdot p_{i+}p_{+j}}$$

$$= n \sum \frac{(p_{ij} - p_{i+}p_{+j})^2}{p_{i+}p_{+j}}$$

$$= n \sum p_{i+}p_{+j} \left(\frac{p_{ij}}{p_{i+}p_{+j}} - 1 \right)^2 \quad , \quad a_{ij} = \frac{p_{ij}}{p_{i+}p_{+j}}$$

#33.

(2.13) $\cdots$ $e_{ij} = \dfrac{n_{ij} - \hat{\mu}_{ij}}{\sqrt{\hat{\mu}_{ij}}}$, $X^2 = \sum e_{ij}^2$

$e_{11} = (y - n\pi_0) / \sqrt{n\pi_0}$.

$e_{12} = (n - y - (n - n\pi_0)) / \sqrt{n(1-\pi_0)}$

$= -(y - n\pi_0) / \sqrt{n(1-\pi_0)}$

$$r_{ij} = \frac{n_{ij} - \hat{\mu}_{ij}}{\sqrt{\hat{\mu}_{ij}(1-p_{i+})(1-p_{+j})}}$$

$r_{11} = (y - n\pi_0) / \sqrt{n\pi_0(1-\pi_0)}$, $r_{12} = -(y - n\pi_0) / \sqrt{n(1-\pi_0)(1-(1-\pi_0))}$

#36.

a) $Ep_{ij} = \frac{1}{n} E n_{ij} = \frac{1}{n} \cdot n\pi_{ij} = \pi_{ij}$. $E\hat{\pi}_{ij} = Ep_{i+}p_{+j} = Ep_{i+}Ep_{+j}$

$= \pi_{i+}\pi_{+j}$

b) $\mathrm{Var}\, p_{ij} = \frac{1}{n^2} \mathrm{Var}\, n_{ij}$

$= \frac{1}{n^2} \cdot n(\pi_{ij}(1-\pi_{ij})) = \pi_{i+}\pi_{+j}(1-\pi_{i+}\pi_{+j})/n.$

c) $\mathrm{Var}(\hat{\pi}_{ij}) = \mathrm{Var}(p_{i+}p_{+j})$

$= E(p_{i+}^2 p_{+j}^2) - E(p_{i+}p_{+j})^2$

$= Ep_{i+}^2 Ep_{+j}^2 - (Ep_{i+})^2 (Ep_{+j})^2$

$Ep_{i+}^2 = \mathrm{Var}(p_{i+}) + E(p_{i+})^2$

$= \frac{\pi_{i+}(1-\pi_{i+})}{n} + \pi_{i+}^2$

$\mathrm{Var}\, p_{i+} \mathrm{Var}\, p_{+j} + Ep_{i+}^2 \mathrm{Var}\, p_{+j} + Ep_{+j}^2 \mathrm{Var}\, p_{i+}$

$\Rightarrow \frac{\pi_{i+}\pi_{+j}(1-\pi_{i+})(1-\pi_{+j})}{n^2} + \pi_{+j}^2 \cdot \frac{\pi_{i+}(1-\pi_{i+})}{n} + \pi_{i+}^2 \frac{\pi_{+j}(1-\pi_{+j})}{n}$

d) $\mathrm{Var}(\sqrt{n}\,\hat{\pi}_{ij}) = \frac{\pi_{i+}\pi_{+j}(1-\pi_{i+})(1-\pi_{+j})}{n} + \pi_{i+}\pi_{+j}\left(\pi_{i+}(1-\pi_{+j}) + \pi_{+j}(1-\pi_{i+})\right)$

$\lim \mathrm{Var}(\sqrt{n}\,\hat{\pi}_{ij}) = \pi_{i+}\pi_{+j}(\pi_{i+}+\pi_{+j}-2\pi_{i+}\pi_{+j})$

$\leq \pi_{i+}\pi_{+j}(1-\pi_{i+}\pi_{+j}) = \lim \mathrm{Var}(\sqrt{n}\, p_{ij}).$

#42.

$$\overset{p_1}{\begin{array}{cc} 0 & 4 \\ 4 & 0 \end{array}} \qquad \overset{p_2}{\begin{array}{cc} 1 & 3 \\ 3 & 1 \end{array}} \qquad \overset{p_3}{\begin{array}{cc} 2 & 2 \\ 2 & 2 \end{array}} \qquad \overset{p_4}{\begin{array}{cc} 3 & 1 \\ 1 & 3 \end{array}} \qquad \overset{p_5}{\begin{array}{cc} 4 & 0 \\ 0 & 4 \end{array}}$$

$$\theta = 0 \qquad \frac{1}{9} \qquad 1 \qquad 9 \qquad \infty$$

$$\text{Pval} = \begin{cases} p_5 & : n_{11} = 4 \\ p_4 + p_5 & : n_{11} = 3 \\ p_3 + p_4 + p_5 & : n_{11} = 2 \\ p_2 + p_3 + p_4 + p_5 & : n_{11} = 1 \\ p_1 + \cdots + p_5 & : n_{11} = 0 \end{cases}$$

$$\text{Mid-Pval} = \begin{cases} \frac{1}{2} p_5 & : n_{11} = 4 \\ \frac{1}{2} p_4 + p_5 & : n_{11} = 3 \\ \frac{1}{2} p_3 + p_4 + p_5 & : n_{11} = 2 \\ \frac{1}{2} p_2 + p_3 + p_4 + p_5 & : n_{11} = 1 \\ \frac{1}{2} p_1 + \cdots + p_5 & : n_{11} = 0. \end{cases}$$