

## Evidence for the Wobbling Mode in Nuclei

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The nucleus  $^{163}\text{Lu}$  has been populated through the fusion-evaporation reaction  $^{139}\text{La}(^{29}\text{Si}, 5n)^{163}\text{Lu}$  with a beam energy of 152 MeV. The electromagnetic properties of several connecting transitions between two presumably triaxial, strongly deformed **TSD** bands have been studied. Evidence is presented for the **assignment** of the excited TSD band as a wobbling mode built on the yrast TSD band, based on comparisons to new calculations in which an **aligned** particle is coupled to a strongly deformed triaxial rotor. The wobbling mode is uniquely related to triaxiality in nuclei.

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The wobbling mode, of which the classical analog is the **spinning motion** of an asymmetric top [1], is a direct consequence of rotational motion of **a triaxial body** with moments of inertia  $J_x \gg J_y \neq J_z$ . In the high-spin limit, with most of the spin aligned along one of the principal axes, the wobbling degree of freedom introduces **sequences of bands** with an increasing number of wobbling quanta,  $n_w = 0, 1, 2, \dots$ . The wobbling phonon energy is  $\hbar\omega_w = \hbar\omega_{\text{rot}}\sqrt{(J_x - J_y)(J_x - J_z)/(J_y J_z)}$  with  $\hbar\omega_{\text{rot}} = I/J_x$  [2]. **A characteristic pattern** is expected for the **decay** between the bands in competition with the in-band decay. Although the wobbling mode was predicted more than 25 years ago, it was until now never realized in experimental high-spin spectra. Writing the quadrupole moment in the intrinsic coordinate system [2],  $Q_0 = \langle \sum_k (2z^2 - x^2 - y^2)_k \rangle$ ,  $Q_2 = \langle \sqrt{3/2} \sum_k (x^2 - y^2)_k \rangle$ , and  $\tan\gamma = \sqrt{2}(Q_2/Q_0)$ , triaxiality is parametrized by  $\gamma$  in the angular regime  $0^\circ \leq \gamma \leq 60^\circ$ . **When rotation sets in, the specification of the largest component of the angular momentum in the intrinsic coordinate system needs 3 times the range of  $\gamma$  values,  $-120^\circ \leq \gamma \leq 60^\circ$**  [3].

For nuclei with  $N \sim 94$  and  $Z \sim 71$ , calculations [4,5] predict **stable triaxial shapes** ( $\gamma \sim \pm 20^\circ$ ) with large quadrupole deformation ( $\varepsilon_2 \sim 0.38$ ) for all combinations of **parity,  $\pi$ , and signature,  $\alpha$** , where  $I = \alpha \bmod 2$ . The local minimum with  $\gamma > 0^\circ$  is generally lowest, and at both local minima the  $\pi i_{13/2}$  orbital is lowest in energy in the **proton system** [5] with the **preferred signature  $\alpha_f = +1/2$** .

Corresponding to the degree of **shell filling**, a particular triaxial shape (a particular value of  $\gamma$ ) is **preferred** by the fully aligned high- $j$  particle [6]. The preferred  $\gamma$  value for  $i_{13/2}$  protons in  $^{163}\text{Lu}$  is around  $+20^\circ$ . If a given triaxial shape is energetically **preferred** very much by the fully aligned  $i_{13/2}$  proton, the unfavored-signature state  $\alpha_u (= -1/2)$  which consists of the aligned  $i_{13/2}$  proton together with a wobbling motion of the rotational angular momentum of the core, may appear very low in energy. When the gain in the intrinsic energy of the  $i_{13/2}$  proton configuration in the wobbling mode wins against the loss in the collective rotational energy of the core, the wobbling mode becomes the lowest  $\alpha_u$  state. However, at very high spins the rotational energy dominates over the intrinsic energy and, thus, in the lowest  $\alpha_u$  state the wobbling mode will be replaced by the crankinglike mode. In either case, both  $M1$  and  $E2$  transition strength are expected in the  $\alpha_u \rightarrow \alpha_f$  decay.

Rotational bands based on the **deformation driving  $\pi i_{13/2}$  intruder configuration** have been observed in the even- $N$  Lu isotopes  $^{163-167}\text{Lu}$  [5,7,8], and in odd-odd  $^{164}\text{Lu}$  [9] where the  $i_{13/2}$  proton is coupled to different neutron orbitals. The quadrupole moment is measured only in  $^{163}\text{Lu}$  ( $Q_2 \approx 10.7$  b) [7]. **However, the dynamic moments of inertia are very similar in all these bands, and larger than those of the bands built on the normal deformed (ND) structures.** It should be noted that very recently three similar bands were found in  $^{168}\text{Hf}$  and lifetime measurements resulted in a transition quadrupole moment of  $Q_2 \approx 11.4$  b [10]. So far, no direct experimental evidence for the triaxiality of the observed

triaxial, strongly deformed (TSD) bands has been given, but one possible and unique consequence of a rotating nucleus with a triaxial shape is the existence of “wobbling bands” [2].

In an investigation of the isotopes  $^{163,164}\text{Lu}$  with the Euroball III array [11], a second band (TSD2) with similar properties as the previously known  $i_{13/2}$  band (TSD1) has been observed in  $^{163}\text{Lu}$  [12]. This second band was found to decay to TSD1, **but no connections could be established.** The new band was considered [12] a candidate for a wobbling excitation. The present work firmly establishes the band as a wobbling excitation built on TSD1.

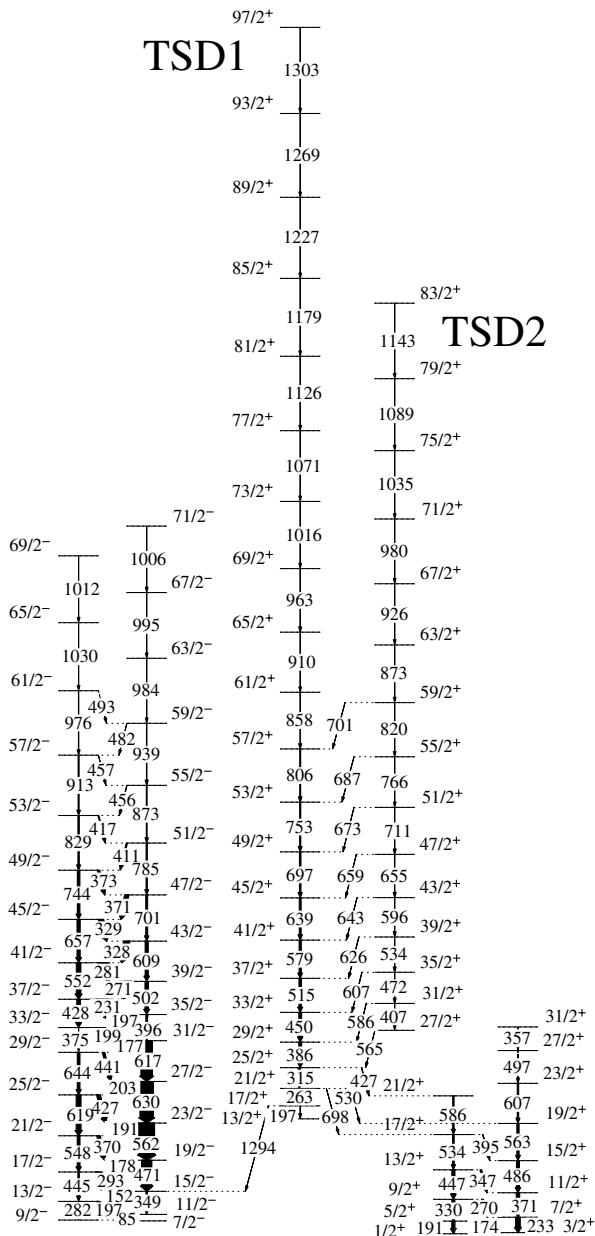


FIG. 1. Partial level scheme of  $^{163}\text{Lu}$  showing the two TSD bands together with the connecting transitions and the ND structures to which the TSD states decay.

To find and investigate the nature of the connecting transitions between TSD2 and TSD1, an experiment was performed with Euroball IV [11] in Strasbourg equipped with the BGO inner ball. With the  $^{139}\text{La}(^{29}\text{Si}, 5n)^{163}\text{Lu}$  reaction and a beam energy of 152 MeV, approximately  $2.4 \times 10^9$  events with 3 or more Compton suppressed  $\gamma$  rays in the Ge detectors and 8 or more  $\gamma$  rays detected in the BGO inner ball were collected and used in 3D and 4D coincidence analyses.

The band TSD2 could be extended to both lower ( $6\hbar$ ) and higher ( $4\hbar$ ) spins, and 9 connecting transitions to TSD1 were established; see Fig. 1. Furthermore, TSD1 has been extended  $10\hbar$  higher in spin. Gated spectra illustrating the connecting transitions and their angular dependence, as well as in-band transitions in TSD1 and TSD2 in the same energy range, are shown in Fig. 2. **The population** of TSD1 and TSD2 relative to yrast are  $\sim 10\%$  and  $\sim 2.5\%$ , respectively.

A determination of the multipolarity of the connecting transitions is crucial. The directional correlation of  $\gamma$  rays from the oriented states (DCO ratios) [13] were obtained for the strongest connecting transitions using “25°” and “90°” data. In addition, angular distribution ratios were produced from the same data. Linear polarization measurements were also attempted using the two “90°” rings of Clover detectors [11]. In all cases the data were selected by clean gates in TSD1 in any angle in the spin range  $21/2 - 45/2\hbar$ . The spin alignment, parametrized as  $\sigma/I$  for a Gaussian distribution of the  $m$ -substate population,  $P_m(I) \propto \exp(-\frac{m^2}{2\sigma(I)^2})$  [14], was determined for a number of stretched electric quadrupole ( $E2$ ) transitions in the same spin region as the connecting transitions. There was no detectable spin dependence. An average value is  $\sigma/I = 0.25 \pm 0.02$ . Both the angular correlation and angular distribution data are consistent with mixed  $M1/E2$  multipolarity for the connecting transitions. Within

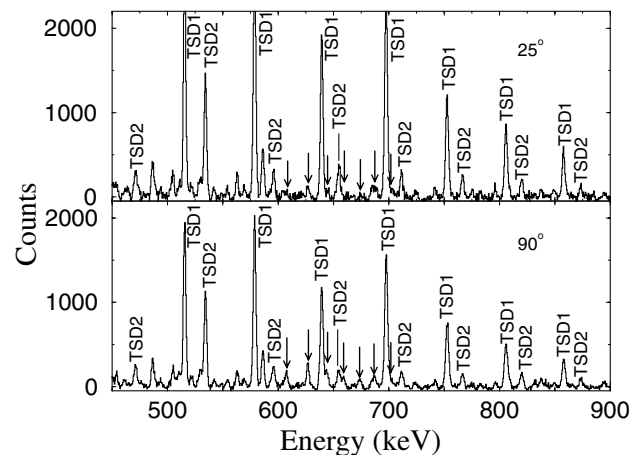


FIG. 2. Spectra from the angular distribution matrices gated on the 450 keV transition in TSD1. Connecting transitions are marked by arrows. Most other unmarked transitions belong to the decay of TSD1.

errors we found no spin dependence in the mixing ratio,  $\delta$ , and have therefore combined the values for the different connecting transitions and also averaged the results from both methods to a final value  $\delta = -3.10^{+0.36}_{-0.44}$  or  $-0.22^{+0.05}_{-0.03}$ . The latter, numerically smallest value of  $\delta$ , is rejected based on the polarization results yielding definite electric character. Our final result of  $\delta$  corresponds to  $(90.6 \pm 1.3)\%$   $E2$  and  $(9.4 \pm 1.3)\%$   $M1$  in the connecting transitions. The alternative solution with  $E1/M2$  mixing would result in unexpectedly large matrix elements for both  $M2$  and  $E1$  transitions, and is therefore disregarded.

With the present results the band TSD2 has firm parity and signature assignments,  $(\pi, \alpha) = (+, -1/2)$ . The angular distribution and DCO analysis also confirm the stretched quadrupole character of the in-band transitions in both TSD1 and TSD2, including their extensions and the decay-out transitions from TSD1. The small difference but overall similarity between TSD1 and TSD2 is illustrated in the plots of alignment and dynamic moment of inertia vs frequency in Fig. 3. The excitation energy of TSD2 is only 250–300 keV relative to TSD1, and decreases as the spin increases; see also Fig. 1. From the measured branching ratios,  $\lambda = T_{\gamma, \text{out}}(M1 + E2)/T_{\gamma, \text{in}}(E2)$  and mixing ratio  $\delta$ , the experimental reduced transition probabilities  $B(M1)$  and  $B(E2)_{\text{out}}$  can be determined relative to  $B(E2)_{\text{in}}$ , and compared to theoretical expectations. A few different possibilities for the configuration of TSD2 exist and must be considered along with the more exotic wobbling mode.

First, cranking calculations with the “ultimate cranker” (UC) [15,16], based on a modified harmonic oscillator potential, predict a large signature splitting of the  $\pi i_{13/2}$  orbital ( $>1$  MeV). The local minimum associated with the unfavored signature in the proton system is found within the spin range of interest at a smaller quadrupole deformation,  $\varepsilon_2 \sim 0.32$ , and a larger triaxiality,  $\gamma \sim 40^\circ$ .

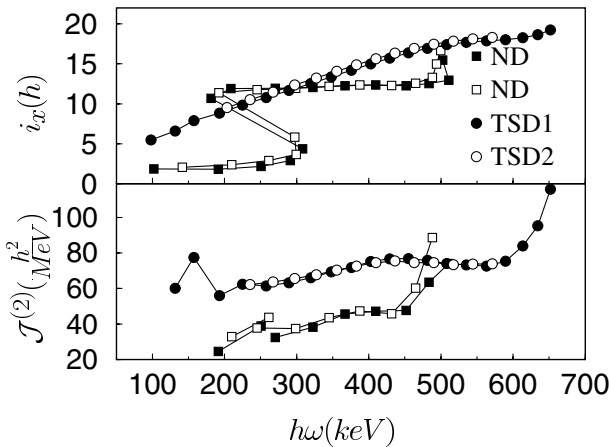


FIG. 3. Alignment  $i_x = I - I_{\text{ref}}$ , where  $I_{\text{ref}} = 25\hbar^2 \text{MeV}^{-1} \omega + 45\hbar^4 \text{MeV}^{-3} \omega^3$ , and dynamic moment of inertia vs rotational frequency for the bands TSD1 and TSD2 as well as the ND band in  $^{163}\text{Lu}$  shown leftmost in Fig. 1.

This highly excited signature partner therefore has features qualitatively different from TSD1, unlike those of TSD2.

Second, a configuration, in which the  $\alpha = -1/2$  signature is composed of  $\alpha = +1/2$  in the proton system, like TSD1, and a two-quasineutron excitation with  $\alpha = 1$ , is predicted by the UC calculations. This configuration has a local minimum identical to that of TSD1, but the excitation energy is approximately the same as that of the signature partner, and therefore 3–4 times higher than found experimentally for TSD2. Furthermore, an expected additional alignment of  $\sim 2\hbar$  relative to TSD1 is not compatible with the data for TSD2; cf. Fig. 3.

Finally, based on particle-rotor model calculations [17], the most interesting possibility that TSD2 is a wobbling excitation with  $n_w = 1$  built on the aligned  $i_{13/2}$  proton configuration TSD1 (with  $n_w = 0$ ) is suggested. Around the relevant angular-momentum region, a wobbling mode,  $\alpha = -1/2$ , based on TSD1 can appear energetically lower than the  $\alpha_u = -1/2$  signature partner of the  $i_{13/2}$  proton orbital, if appropriate values of  $\gamma$  and moments of inertia are chosen. Then, the estimated splitting of TSD2 and TSD1 may be a few hundred keV only. In the wobbling mode for  $\alpha_u$  the alignment of the  $i_{13/2}$  proton must be nearly equal to that of the yrast  $\alpha_f$  state. Nevertheless, the signature splitting of the energy has the same sign as in the cranking mode. Furthermore, in the presence of the aligned  $i_{13/2}$  proton, the  $\Delta I = 1$  electromagnetic transition matrix elements from the wobbling mode to the yrast state have the following characteristic features: (a) The transition is dominated by  $E2$  and not by  $M1$ ; (b)  $B(E2)$  values are proportional to  $1/I$  in the limit of high  $I$  values, in contrast to the cranking mode, in which they are

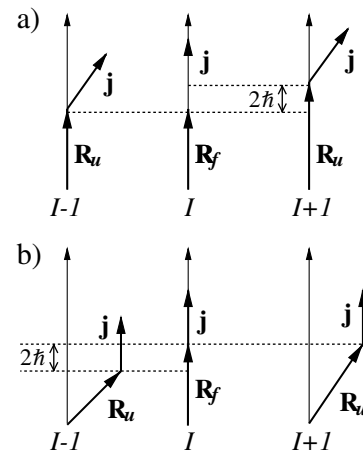


FIG. 4. Schematic coupling scheme of the particle and core angular momenta in the favored ( $I$ ) and unfavored ( $I \pm 1$ ) states for (a) the cranking regime and (b) the wobbling mode ( $n_w = 1$ ). The total angular momentum is  $\mathbf{I} = \mathbf{R} + \mathbf{j}$ , where the angular momentum of the collective rotation of the core is expressed by  $\mathbf{R}$ . The vertical axis shown ( $x$  axis) is the axis of the largest moment of inertia of the core, about which collective rotation is energetically cheapest. For  $n_w > 1$  the angle between the  $x$  axis and  $\mathbf{R}$  gets larger.

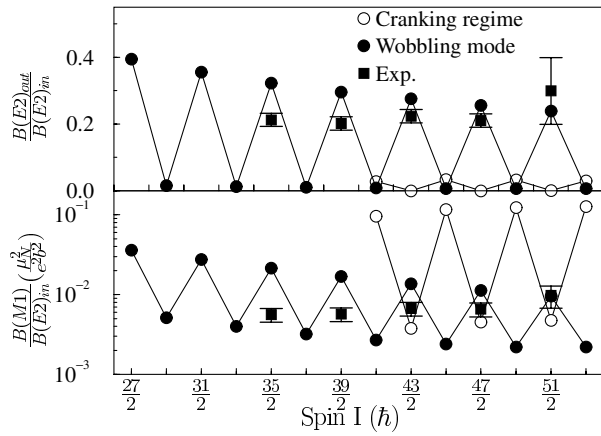


FIG. 5. Experimental and calculated electromagnetic properties of the connecting transitions.

proportional to  $1/I^2$ ; (c) the zigzag pattern of both  $B(E2)$  and  $B(M1)$  values is out of phase compared with the case for the cranking mode. The zigzag pattern can be understood by examining Fig. 4, obtained by analyzing the wave functions in the particle-rotor model. In the cranking regime both  $E2$  and  $M1$  ( $\alpha_u, I+1 \rightarrow \alpha_f, I$ ) transitions are strongly reduced because  $\Delta R_x \approx 2\hbar$  and, simultaneously,  $\Delta j_x \approx 1\hbar$ . In contrast, for the wobbling regime, in the ( $\alpha_f, I \rightarrow \alpha_u, I-1$ ) transitions, the  $M1$  strength is strongly reduced due to  $\Delta R_x \approx 2\hbar$ , while the  $E2$  strength is reduced, because the contributions from  $Q_0$  and  $Q_2$  almost cancel for  $\gamma = +20^\circ$ .

Calculated values of  $B(E2)_{\text{out}}/B(E2)_{\text{in}}$  and  $B(M1)/B(E2)_{\text{in}}$  are shown together with the experimental values for the connecting transitions of mixed  $E2/M1$  nature in Fig. 5. Transitions with alternate spin values are suppressed due to very low transition energies, and no meaningful upper limits could be obtained from the data. The agreement of the present data with the results calculated for the wobbling mode appears quite satisfactory from Fig. 5 and Table I, in view of the schematic character of the particle-rotor calculations including a single proton  $i_{13/2}$  subshell. The failure of a crankinglike solution is particularly obvious from the  $E2$  strength, and the extracted properties summarized in the table. The observed gradual increase of  $B(M1)$  values may come from the gradual increase of neutron alignment in the core, which is seen in the observed alignment  $i_x$  but not included in the calculation of  $B(M1)$  values. The very similar dynamic moment of inertia for the two TSD bands indicates their similar structure. In the case suggested of TSD2 as a one-phonon wobbling, the ratio  $\hbar\omega_w/\hbar\omega_{\text{rot}}$  varies from 1.5 to 0.5 with increasing spin, indicating a gradual change in the three moments of inertia.

In summary, the candidate for a wobbling excitation in  $^{163}\text{Lu}$ , TSD2, has been connected to TSD1 by nine linking transitions. The electromagnetic properties are in agreement with the assignment of TSD2 as a wobbling

TABLE I. Experimental and calculated values of mixing ratio  $\delta$ , branching ratio  $\lambda$ , and electromagnetic nature of the connecting transition for  $I = 43/2\hbar \rightarrow I = 41/2\hbar$ .

	$\delta$	$\lambda$	$E/M$
Expt.	$-3.10^{+0.36}_{-0.44}$	$0.36 \pm 0.04$	$E$
Wobbling	$-2.4^a$	$0.48^a$	$E$
Crankinglike	$\pm 0.15^{a,b}$	$0.02^a$	$M$

<sup>a</sup>Based on calculated matrix elements and experimental  $\gamma$ -ray energies.

<sup>b</sup>Both  $E2$  and  $M1$  transitions are approximately forbidden. Thus, the sign of the mixing ratio can be either plus or minus.

excitation in the presence of an aligned particle, built on TSD1. Alternative interpretations as a signature partner or a three-quasiparticle excitation could be rejected. For the first time, the wobbling mode which is uniquely related to nuclear triaxiality is established experimentally.

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