# Model Predictive Control Scheduling for the 24-24-24 Appalachian Trail Challenge

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***Abstract*—The 24-24-24 challenge of hiking 24 miles and drinking 24 beers in 24 hours presents an intriguing opportunity for control theory application. Modeling the challenge as a control problem allows for study of dynamic scheduling under safety-critical constraints. This paper develops a discrete-time state-space model to represent the hydration and blood alcohol concentration (BAC) of a hiker undertaking the challenge. A Model Predictive Control (MPC) framework is implemented to optimize the pace of hiking and beer consumption while adhering to hydration and BAC safety constraints. Simulation studies demonstrate that the MPC controller successfully schedules safe trajectories to complete the challenge that mitigate the risks associated with dehydration and intoxication. Robust MPC techniques further enhance safety by accounting for unmodeled disturbances to hydration. This work illustrates an application of control theory a complex scheduling problems under safety constraints in a domain not typically explored by control literature.**

***Index Terms*—Biological System Dynamics Modeling, Model Predictive Control, Safety-Constrained Optimization**

## I. Introduction

The 24-24-24 challenge is a folk challenge for through-hikers on the Appalachian trail. The goal is to hike 24 miles and drink 24 beers in 24 hours. While traveling by foot during binge-drinking is unsafe and inadvisable[1], the challenge presents an application for control theory: it is a scheduling problem for a hiker who must select their paces of traveling and drinking to meet the goal. In addition to the constraints of the challenge, the hiker must consider their own safety. For example, minimizing extreme exertion, dehydration, and intoxication. If measurements are not applied to close a feedback loop, the scheduling task can be solved with a quadratic programming formulation. However, any optimal plan the quadratic program provides is open loop and will not have any provision for unmodeled system disturbances. Model predictive control has been widely used for scheduling optimization under disturbances because it can apply online state measurements to a continuously updated mathematical model of a system.

[1]To reiterate, this challenge is hazardous, should not be attempted without further safety research, and was not performed in real life in the scope of this work.

## II. Literature Review

The academic literature lacks comprehensive study on applications of modern control techniques on wilderness drinking games. However, negative social effects of alcohol abuse such as drunk driving and domestic violence have demonstrated a legitimate need to build control-oriented models of alcohol consumption. A neural network is trained in [1] to represent an input-output system model of alcohol consumption and risk of drunk driving accidents. A discrete-time model to study the effects of alcohol rehabilitation services on population levels of heavy drinking is proposed in [2]. Using the same method to classify and predict population drinking habits, [3] considers the effectiveness of, among others, media and education as control inputs to the system. While these works identify control-oriented system models for drinking dynamics on a social epidemic scale, this project considers an individual drinker, not an aggregate. Therefore, this project will propose a control-oriented model of the blood alcohol and hydration states of a hiker, pulling parameters from literature in biological sciences and sports medicine.

Systems modeling for human-centric applications has been explored in the medical field, such as closed-loop insulin delivery systems [4], [5], or in the athletic field, such as pacing strategies in endurance sports [6], [7]. This work poses a novel consideration where the biochemistry of the hiker/athlete, i.e. their blood alcohol concentration, is also modeled. Due to the dangerous outcomes of reckless alcohol consumption combined with intense physical exertion, safety-focused modeling and control of the problem is critical.

## III. Problem and Model Formulation

### A. Problem Definition

The primary goal of the controller is to determine optimal inputs at each time step that will lead to completion of the challenge while maintaining hiker safety. If the challenge is successful, the hiker will have traveled the entire distance and consumed all the beer. For safety, key constraints on the states will be a maximum BAC to prevent extreme intoxication, and a minimum hydration level to prevent dehydration. For physical feasibility, the inputs will be constrained to reasonable levels.

TABLE I
SYSTEM STATES, INITIAL CONDITIONS, AND SET POINTS

| State | Initial condition | Set point | Units |
|---|---|---|---|
| Hiker Position | -8 | 0 | Miles |
| Hydration | 1808 | 1808 | oz. |
| Blood alcohol concentration | 0 | 0 | % |
| Pack weight | 192 | 0 | oz |
| Beer consumed | -96 | 0 | oz |

### B. Model Derivation

Mathematical relationships for the modeled system need to be developed in discrete-time state-space. First, we will generate governing equations for each of the states and formulate the system in state-space. Then, we will source values for model parameters from literature in the fields of pharmacokinetics and sports medicine.

The states that will be modeled in this project are summarized in Table I. This state space representation models their dynamic relationships.

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

Where the state transition and input matrices $A$ and $B$ define the relationships among the states and inputs. The system parameters are recounted in Table II.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{2}$$

$$B = \begin{bmatrix} \Delta t/60 & 0 & 0 \\ \beta_1 & \Delta t\beta_2 & \Delta t \\ 0 & \Delta t\beta_2 & 0 \\ 0 & -\Delta t & -\Delta t \\ 0 & \Delta t & 0 \end{bmatrix}, \tag{3}$$

According to [8], a 175 lb. adult male with the healthy average of 25% body fat is approximately 65% water by weight, or 1808 oz. Daily water loss independently reported by [8], [9] is roughly 7%, or 125 oz lost daily to respiration, sweating, excreting, and other basic metabolic processes. These sources are used to determine the $\alpha_1$ time constant of free-response water loss.

The effect of exercise on water loss is parameterized by $\alpha_2$ and $\beta_1$. In [9], water loss during gentle walking was negligibly different from base water loss, but running at 6 mph led to increased water loss by 1 oz per minute. This is implemented in the model as the $\beta_1$ parameter. According to [8], vigorous exercise at maximum exertion depleted water at a faster rate than the jogging in [9]. The increased dehydration is modeled by increased exertion from pack weight in the $\alpha_2$ parameter.

The body's response to alcohol is characterized by the remaining $\alpha_3$, $\beta_2$, and $\beta_3$ parameters. While the kinetics of blood alcohol are widely agreed to be zero-order for BAC>0.10 [10], [11], the model will be built with a conservative approach using first-order kinetics to preserve model linearity. According to [11], the maximum rate of decrease is decay by 76% per hour. The $\alpha_3$ parameter represents this rate of exponential decay on the scale of the simulation step size interval. According to [12], consuming 4 oz of 50% ethanol, or 4 standard drinks, caused participants to urinate an additional 24 oz over 2 hours of the study. Since urination counts against the body's water budget, $\beta_2$ parameter represents the 0.5 units water loss per unit of beer consumed. According to [10] the BAC of a 175 lb. adult male increases by 0.034 per 12 oz drink, and this describes the $\beta_3$ parameter.

TABLE II
PARAMETERS OF STATE TRANSITION AND INPUT MATRICES

| Parameter | Physical Meaning | Value (Units) |
|---|---|---|
| $\alpha_1$ | Hydration decay | 0.99975 (oz/oz) |
| $\alpha_2$ | Effective water loss per pack weight | 0.0173 (oz/oz) |
| $\alpha_3$ | BAC decay | 0.9905 (%/%) |
| $\beta_1$ | Effective water loss per hiking pace | 0.833 (oz) |
| $\beta_2$ | Effective water loss per beer consumption | 0.5 (oz/oz) |
| $\beta_3$ | Blood alcohol increase per beer consumption | 0.00283 (%/oz) |
| $\Delta t$ | Time elapsed between steps | 5 (minutes) |

## IV. Controller Formulation

Complete controller formulation is presented, explained in detail, and mathematically correct. Formulation builds on the preliminary work from the previous phase to address shortcomings that were identified.

### A. MPC Control

The MPC Control law solves an online optimization problem that selects an input trajectory which minimizes an objective cost over a prediction horizon . This controller will use Linear-Quadratic MPC, which means the system state update is calculated as a linear combination of the states and input in 4b, and the objective cost is calculated as a quadratic combination of the states and inputs as in 4a. In addition to the state update constraint, the states and inputs are constrained to state and input sets according to 4c. The following optimization problem mathematically describes the controller:

$$u^*_{k+1} = \underset{u}{\operatorname{argmin}} \sum_{j=0}^{N-1} x_k^T Q x_k + u_k^T R u_k + x_k^T S u_k + x_N^T P x_N \tag{4a}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k \tag{4b}$$

$$x_k \in \mathscr{X}, u_k \in \mathscr{U} \tag{4c}$$

### B. Objectives

The objective function $Q$ and $R$ matrices are selected using Bryson's rule. Bryson's rule is a strategy for selecting the gains of a Linear Quadratic Regulator (LQR) controller:

$$Q_{ii} = \frac{1}{\tilde{x}_i^2}, R_{ii} = \frac{1}{\tilde{u}_i^2} \tag{5}$$

where $\tilde{x}_i$ and $\tilde{u}_i$ are the maximum acceptable values of $x_i$ and $u_i$. This selection for the LQR gains scales the state and input costs to unity, and weights them relative to their allowable ranges.

The chosen values for the simulation are as follows:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.003 & 0 & 0 & 0 \\ 0 & 0 & 44.4 & 0 & 0 \\ 0 & 0 & 0 & 10^{-4} & 0 \\ 0 & 0 & 0 & 0 & 4\times 10^{-4} \end{bmatrix}, \tag{6}$$

$$R = \begin{bmatrix} 2.78 & 0 & 0 \\ 0 & 69.4 & 0 \\ 0 & 0 & 69.4 \end{bmatrix}, \tag{7}$$

### C. Constraints

Applying state constraints to the optimization problem will ensure safe levels of hydration and intoxication.

$$x_2 \geq 1752 \tag{8a}$$
$$x_3 \leq 0.11 \tag{8b}$$

The minimum safe hydration level represented in 8a corresponds to water loss of no more than 56 oz from the starting amount. This value represents 2% of body weight, the amount of water loss that [8] reports can significantly impair cognitive abilities.

The maximum safe BAC level represented in 8b is 0.11% by volume. Above this level, [10] reports that cognitive function, motor coordination, and reaction time are all significantly impaired. Vomiting is also likely to occur above this level of intoxication, which would lead to further dehydration.

Applying input constraints to the optimization problem will ensure the control actions are reasonable.

$$0 \geq u_1 \geq 6 \tag{9a}$$
$$0 \geq u_2, u_3 \geq 12 \tag{9b}$$

The constraints on hiking pace in 9a ensure the hiking speed is in the forward direction and at 3 MPH or less, which a brisk walk, but slower than jogging. The constraints on drinking pace in 9b ensure the prescribed amount to drink is never more than one beer (12 oz) and an equal amount of water in a 5-minute time step.

These constraints are represented compactly as:

$$x_k \in \mathscr{X},\ \mathscr{X} = \{x_k \in \mathbb{R}^5 \,|\, A_x x_k \leq b_x\} \tag{10a}$$
$$u_k \in \mathscr{U},\ \mathscr{U} = \{u_k \in \mathbb{R}^3 \,|\, A_u u_k \leq b_u\} \tag{10b}$$
$$A_x = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix},\ b_x = \begin{bmatrix} 56 \\ 0.11 \\ 0 \end{bmatrix} \tag{10c}$$
$$A_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ b_u = \begin{bmatrix} 3 \\ 12/5 \\ 12/5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10d}$$

### D. Cross-term Considerations

In preliminary results where the controller was simulated as-is, the magnitude of the BAC state multiplied by the hiking pace input had large peaks. This means that the hiker is scheduled to hike fast while also being drunk. To mitigate this uncomfortable and potentially dangerous situation, the BAC and pace state-input cross term is represented the constraints and objectives. This is achieved as described in [13], where the weighted state-input cost

$$u_k^T S x_k \tag{11}$$

is added to each stage cost of the optimization problem. The cross-term cost is selected again according to Bryson's rule- the inverse square of the desired maximum error of that cross term.

$$S = \begin{bmatrix} 0 & 0 & 51.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{12}$$

A comfort constraint is also added to limit the pace * BAC product to .07, a five-fold reduction in the cross-term from the peaks seen in the preliminary simulations.

$$u_k x_k \leq 0.07 \tag{13}$$

### E. Time-varying Constraint Tightening

Some factors in the real physical system are not modeled, and the effect those factors have on the system states could lead to unsafe conditions if left unaddressed. A study on the effects of exercise on hydration found that an air temperature increase from 5°C to 32°C corresponded to a 30% increase in water loss due to sweating [14]. Another study found that an increase in trail grade from 3 (moderate steep sections) to 5.4 (steep section requiring handholds) likewise increased respiratory water losses by 20% [15]. Since the air temperature and trail steepness are not modeled in the system, the effects of these conditions acts as unknown disturbances to the hydration state of the system.

To robustify the system against these disturbances, this work draws on time-varying constraint tightening methods as described in [16]. First, a candidate state-feedback control law is selected such that all the closed-loop system poles are at the origin.

$$u_k = -Kx_k,\ eig(A + BK) = \vec{0} \tag{14}$$

Placing the poles at the origin means that the state constraints will only shrink until the set is a nominal control invariant set. The state transition of the candidate nilpotent closed loop control law is therefore

$$L_{k+1} = (A + BK)L_k,\ L_0 = I \tag{15}$$

The tightened state constraint set at time step $k$, $\hat{\mathscr{X}}_k$, is then calculated by taking the Pontryagin difference of the constraint set $\mathscr{X}$ and the disturbance set $\mathscr{W}$ multiplied by the affine map of the nilpotent state transition matrix $L_k$:

$$\hat{\mathscr{X}}_k = \mathscr{X} \ominus L_k \mathscr{W} \tag{16}$$

This is a simplification from the method in [16], as the states are being constrained, not the system outputs. This is functionally the same, because the outputs are the system states. In other words, $C+DK = I$, since the output matrix $C$ is the identity matrix, and the feed-forward matrix $D$ is zero.

## V. Numerical Results

Numerical example or examples showcase(s) the key ideas of the project and MPC formulation. Simulation studies are described in detail and include well-formatted plots and other visualizations to effectively communicate the results

### A. Preliminary LQR study

. Using the system matrices $A$ and $B$ identified in Section III-B as the dynamic model, and the $Q$ and $R$ identified in Section IV-B as the gains for an LQR controller, the system was simulated under a reduced 8-mile, 8-beer, 8-hour challenge. The results of this initial simulation are depicted in Figures 1 and 2.

The controller successfully schedules a hike that covers 8 miles in 8 hours (plot $x_1$) while maintaining less than 40 oz of hydration loss (plot $x_2$). However, the maximum pace 5 MPH (plot $u_1$), which is higher than reasonable, and all 96 oz of beer are not consumed in the allotted time (plot $x_5$). Since an LQR controller cannot put constraints on states and inputs, there is no way to guarantee that a state will reach a desired value in finite time. The gains can be tuned by hand to produce the desired system results, but this process is time-consuming and not rigorous.

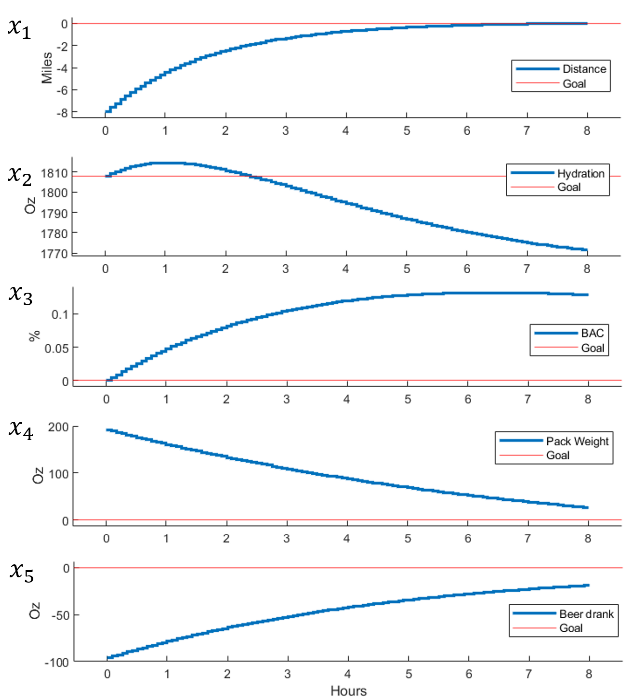


Fig. 1. System states for the LQR simulation

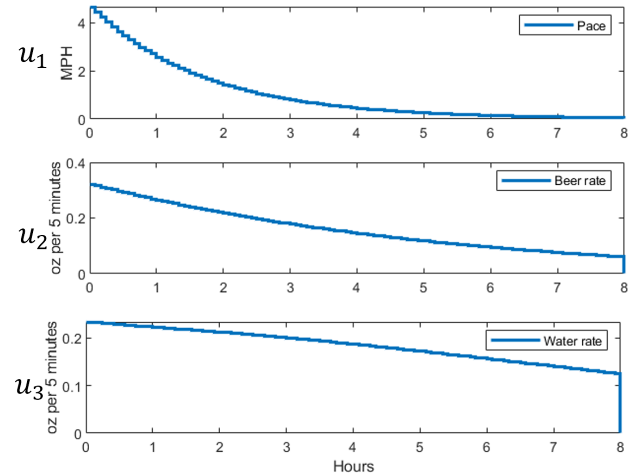


Fig. 2. System inputs for the LQR simulation

### B. Constrained MPC study

The MPC controller was simulated with a time step of 5 minutes and a prediction horizon of 96 steps. The simulation duration was expanded to the full 24-24-24 challenge.

In this simulation, the $A$ and $B$ matrices in the controller are unchanged from the LQR case. However, the dynamics for the simulated BAC have been changed for improved accuracy. Instead of a first order decay as modeled in the controller prediction, the simulated model decreases BAC by a zero-order decay rate equivalent to 1 drink per hour. Since the first-order decay of BAC in the MPC state dynamics is linearized around a slower rate, the controller will overestimate that state. This means the prediction for BAC will always be conservative and will not allow BAC to exceed the safety state constraint, despite the model mismatch.

Another adjustment to the simulated model that is not represented in the controller is water refilling. In the simulation, whenever the amount of water remaining in the pack drops to zero, 64 oz of water is added to the pack. This represents the hiker picking up previously cached water to carry in their pack.

The results of the preliminary simulation are reported in Figures 3 and 4. In the numerical simulation, the safety constraints are obeyed and the goal condition of the challenge is satisfied. The beer consumption and the hiking mileage are both completed in the given time.

A comparison of the nominal vs. cross-term weighted and constrained controllers is reported in Figure 5. As shown in the 'BAC', 'Beer Drank' states trajectories, $x_3$ and $x_5$ in Figure 3, and 'Beer rate' input trajectory, $u_2$ in 4, the addition of the cross-term constraint pushes the beer consumption to times in the challenge when the pace is slow. This reduces the peak pace by BAC product by over five-fold, as shown in Figure 5. Thus, the cross term theoretically will lead to a less intoxicated state during the faster beginning of the hike, and a gentler pace while becoming drunk at the end of the hike.

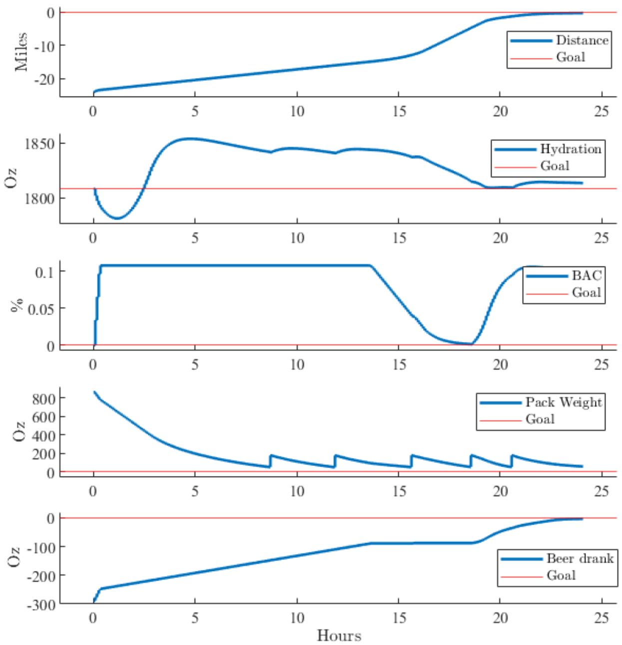


Fig. 3. System states in the MPC simulation

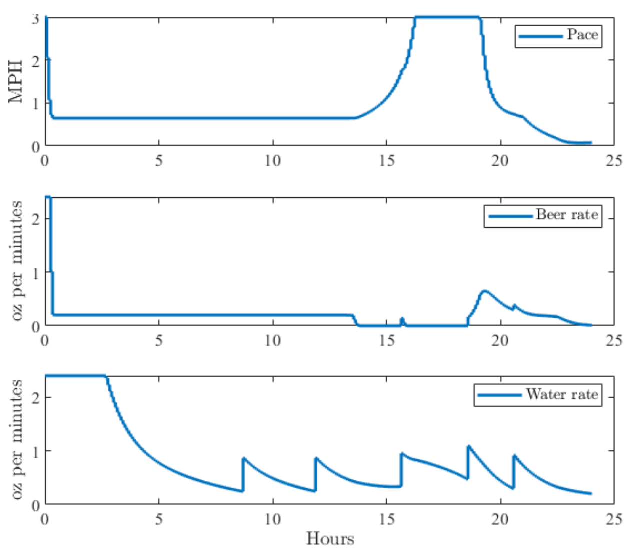


Fig. 4. System inputs in the MPC simulation

### C. Robust MPC study

According to the potential unmodeled water losses proposed in Section IV-E, another batch of 10 simulations was performed over an 8-hour segment of the challenge where every time step, the hydration state has a random chance of losing up to 6 oz. of water. The magnitude of this disturbance corresponds to the potential water losses discussed in [14] and [15], and it is equal to 50% of the available 'water rate' input control action to compensate. Both simulations began with the same RNG seed for the random disturbance application. The results of the 10 nominal and robust controller simulations are reported in Figures 6 and 7. As shown in Figure 6, 50%

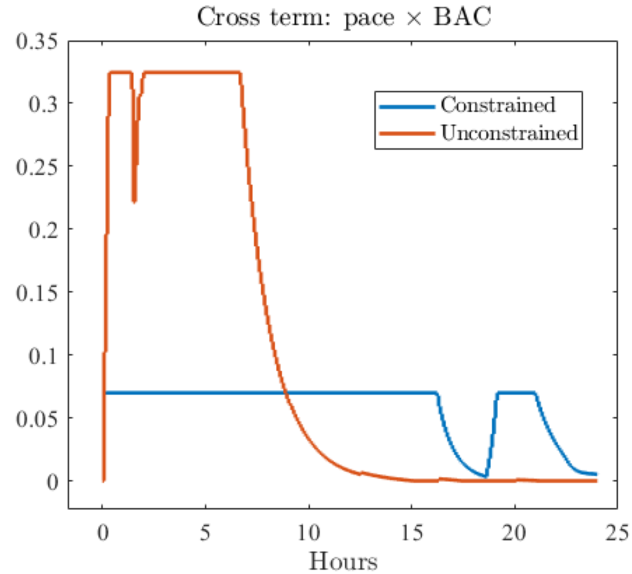


Fig. 5. Comparison of the speed and intoxication cross term time histories for the free and cross-term-constrained controllers

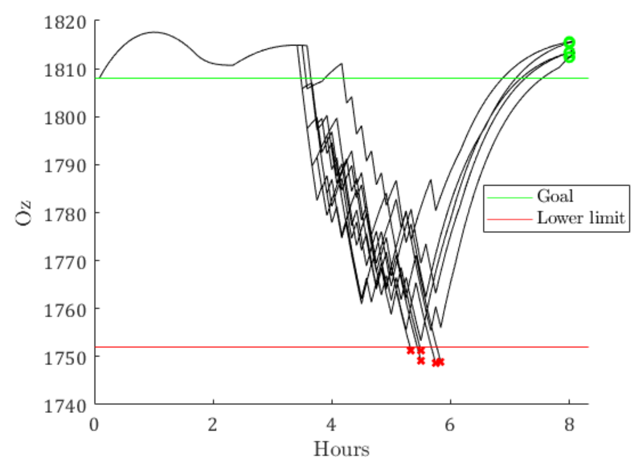


Fig. 6. Hydration state time history for 10 simulated nominal MPC attempts. The 5 trajectories ending in a red 'x' represent simulations where disturbances drove the hydration state below the minimum acceptable level.

of simulated challenge attempts result in the hydration state dropping below the safe limit when a hydration disturbance is applied to the simulation. On the other hand, Figure 7 shows that applying the time-varying constraint tightening method kept all of the hydration trajectories in the safe region.

## VI. Conclusion

Due to the assumptions taken in identifying the dynamic system in Section III, future work is needed to validate the appropriateness of the linear model in the optimization problem and the mixed linear/zero-order model used in the simulation. A possible method for validating the model derivation would be using data-driven methods for system identification. For example, instrumentation such as GPS to determine position, a scale to measure pack weight and water loss (offset from starting body weight), and a handheld breathalyzer to measure

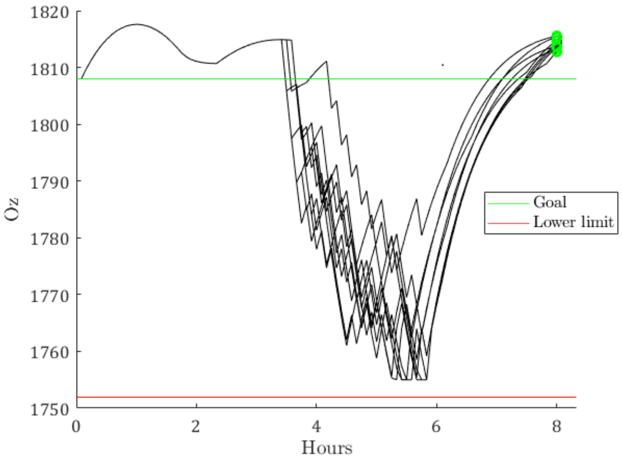


Fig. 7. Hydration state time history for 10 simulated robust MPC attempts. All trajectories, represented by a green 'o' endpoint, remained in the safe region.

BAC could be used to monitor the system responses to step inputs. This time-series data could then be used in time-domain system identification methods such as maximum likelihood estimation, which would generate a data-based model to compare to the derived physics-based system model.

Despite uncertainty in the model derivation, this project demonstrated a successful application of Model Predictive Control to a recreational challenge. A physics-based, state-space model was developed to capture the interconnected dynamics of alcohol metabolism, hydration, and physical exertion. This model was then used to design a multi-objective cost function and optimization constraints that consider the trade-offs between competing objectives, such as minimizing BAC and dehydration hydration while completing hiking and drinking goals. The robust control technique of time-varying constraint tightening was also implemented to increase the controller's robustness to unmodeled disturbances. Simulation studies illustrated the effectiveness of MPC generating optimal pacing strategies under constraints.

Advanced control methods like MPC provide actionable strategies for balancing competing objectives in complex dynamic systems, even in nontraditional domains, as demonstrated in this paper. MPC's ability to anticipate future states and adjust accordingly is a powerful feature for managing systems with coupled dynamics. The results show that MPC could extend to other domains where safety and efficiency are critical, such as health monitoring of athletes, or resource management in isolated environments.

## REFERENCES


[1] A. Saad and H. Mohammed, "Developing a New Optimization Algorithm to Predict the Risk of Car Accidents Due to Drinking Alcoholic Drinks by Using Feed-Forward Artificial Neural Networks," *Journal of Multidisciplinary Modeling and Optimization*, vol. 4, no. 1, pp. 11–18, Jan. 2022, number: 1 Publisher: Ahmet ŞAHİNER. [Online]. Available: https://dergipark.org.tr/en/pub/jmmo/issue/67903/979274

[2] B. Khajji, A. Labzai, A. Kouidere, O. Balatif, and M. Rachik, "A Discrete Mathematical Modeling of the Influence of Alcohol Treatment Centers on the Drinking Dynamics Using Optimal Control," *Journal of Applied Mathematics*, vol. 2020, no. 1, p. 9284698, 2020, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1155/2020/9284698. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1155/2020/9284698

[3] L. El Youssoufi, B. Khajji, O. Balatif, and M. Rachik, "A discrete mathematical modeling for drinking alcohol model resulting in road accidents and violence: an optimal control approach," *Commun. Math. Biol. Neurosci.*, vol. 2021, no. 0, p. Article ID 88, Nov. 2021. [Online]. Available: https://scik.org/index.php/cmbn/article/view/6578

[4] R. Hovorka, "Closed-loop insulin delivery: from bench to clinical practice," *Nature Reviews Endocrinology*, vol. 7, no. 7, pp. 385–395, Jul. 2011. [Online]. Available: https://www.nature.com/articles/nrendo.2011.32

[5] G. Steil, "Closed-loop insulin delivery—the path to physiological glucose control," *Advanced Drug Delivery Reviews*, vol. 56, no. 2, pp. 125–144, Feb. 2004. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0169409X0300190X

[6] C. R. Abbiss and P. B. Laursen, "Describing and Understanding Pacing Strategies during Athletic Competition:," *Sports Medicine*, vol. 38, no. 3, pp. 239–252, 2008. [Online]. Available: http://link.springer.com/10.2165/00007256-200838030-00004

[7] C. Thiel, K. Pfeifer, and G. Sudeck, "Pacing and perceived exertion in endurance performance in exercise therapy and health sports: A competence-oriented view," *German Journal of Exercise and Sport Research*, vol. 48, no. 1, pp. 136–144, Mar. 2018. [Online]. Available: http://link.springer.com/10.1007/s12662-017-0489-5

[8] S. K. Riebl and B. M. Davy, "The Hydration Equation: Update on Water Balance and Cognitive Performance," *ACSM's health & fitness journal*, vol. 17, no. 6, pp. 21–28, 2013. [Online]. Available: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4207053/

[9] S. M. Shirreffs, "Markers of hydration status," *European Journal of Clinical Nutrition*, vol. 57, no. S2, pp. S6–S9, Dec. 2003. [Online]. Available: https://www.nature.com/articles/1601895

[10] D. M. Cowan, J. R. Maskrey, E. S. Fung, T. A. Woods, L. M. Stabryla, P. K. Scott, and B. L. Finley, "Best-practices approach to determination of blood alcohol concentration (BAC) at specific time points: Combination of ante-mortem alcohol pharmacokinetic modeling and post-mortem alcohol generation and transport considerations," *Regulatory Toxicology and Pharmacology*, vol. 78, pp. 24–36, Jul. 2016. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0273230016300642

[11] J. Lee, B.-w. Nam, J.-s. Seo, J.-c. Seo, H.-j. Bang, M.-c. Ahn, B.-s. Hong, and S.-h. Lee, "Re-Analyses and Predictive Models on Blood Alcohol Elimination Rate and Associated Factors in Korean Adults," *Korean Academy of Addiction Psychiatry*, vol. 23, no. 2, pp. 82–93, Sep. 2019. [Online]. Available: https://scholar.kyobobook.co.kr/article/detail/4010027414726

[12] K. E. Roberts, "Mechanism of Dehydration Following Alcohol Ingestion," *Archives of Internal Medicine*, vol. 112, no. 2, p. 154, Aug. 1963. [Online]. Available: http://archinte.jamanetwork.com/article.aspx?doi=10.1001/archinte.1963.038600200520

[13] S. Cho, M. Otsuki, and T. Kubota, "Model Predictive Control with Cross-Product Term between State and Input," in *2018 18th International Conference on Control, Automation and Systems (ICCAS)*, Oct. 2018, pp. 1366–1369. [Online]. Available: https://ieeexplore.ieee.org/document/8571990/?arnumber=8571990

[14] R. J. Maughan and N. L. Meyer, "Hydration during intense exercise training," *Nestle Nutrition Institute Workshop Series*, vol. 76, pp. 25–37, 2013.

[15] B. Coetzee, D. Coetzee, and R. Schall, "Fitness tests as predictors of physical exertion on graded hiking trails," *Journal of Outdoor Recreation and Tourism*, vol. 46, p. 100760, Jun. 2024. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2213078024000288

[16] A. G. Richards, "Robust Constrained Model Predictive Control."