hw05_chunlei2

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Q1:

The state equation is the proposal distribution.

T from 1 to 10, $\mu_0 = 0$, every time $\mu_t = \mu_{t-1}$ with probability 0.9 or $\mu_t = Z_t, Z_t \sim N(0,1)$ with probability 0.1.

$W_0 = 1, W_t = W_{t-1}g(y_t|\mu_t)$. $g(y_t|\mu_t)$ is the density of y_t given μ_t, given $\mu_t, \varepsilon_t \sim N(0,1)$, $y_t \sim N(\mu_t, 1)$

The final estimation is $\frac{\sum_{j=1}^{m} h(\mu_t^{(j)})w_t^{(j)}}{\sum_{j=1}^{m} w_t^{(j)}}$.

```
set.seed(10)
m = 10000
t = 10
mu_matrix = matrix(0, m, t)
weight_matrix = matrix(0, m, t)
y_obs = c(0.2, -0.1, -0.4, -1.3, 1.1, -1.1, 1.1, -0.1, -2.4, 0.9)
for (i in 1:m) {
  mu = 0
  weight = 1
  for (j in 1:t){
    p = rbinom(1,1,0.9)
    mu = p*mu + (1-p)*rnorm(1)
    mu_matrix[i, j] = mu
    weight = weight*dnorm(y_obs[j], mean = mu, sd = 1)
    weight_matrix[i, j] = weight
  }

}
expect = rep(0, 10)
for (k in 1:10) {
  expect[k] = sum(weight_matrix[, k] * mu_matrix[, k])/sum(weight_matrix[, k])
}
expect
```

```
##  [1]  0.006168962 -0.003208903 -0.030774839 -0.155845971  0.036031282
##  [6] -0.107504385  0.070610249  0.024223908 -0.434505828 -0.051386380
```

Q2:

Naive Monte Carlo:

The naive Monte Carlo is generating 1000 iid samples from uniform distribution and insert into $\frac{1}{1+u}$, calculate the average.

The mean is 0.6910957. The variance is 1.969141e-05.

```
set.seed(10)
n = 1000
X = rep(0, n)
for (i in 1:n){
```

```
  u = runif(1)
  X[i] = 1/(1 + u)
}
mu = mean(X)
var_mu = 1/1000*var(X)
mu
```

```
## [1] 0.6910957
```

```
var_mu
```

```
## [1] 1.969141e-05
```

Control Variates:

Using control variates method. Calculate the optimal b. Calculate the mean and variance of new variable $X(b)$.

The mean is 0.6935891, the variance is 6.172178e-07. The variance is smaller than the result obtained with naive monte carlo method.

```
set.seed(10)
n = 1000
X = rep(0, n)
for (i in 1:n){
  u = runif(1)
  X[i] = u
}
b = cov(1/(1+X), (1+X))/var(1 + X)
mu = mean(1/(1+X) - b*((1 + X) - 1.5))
var_mu = 1/1000*var(1/(1+X) - b*((1 + X)) - 1.5)
mu
```

```
## [1] 0.6935891
```

```
var_mu
```

```
## [1] 6.172178e-07
```

Q3:

Naive Monte Carlo in HW1 Q2:

The mean is 2.711023, the variance is 0.0005780832.

```
set.seed(10)
#Q2
##generate 1000 samples from uniform distribution
set.seed(9301)
U2 = runif(1000, 0, 1)
##calculate the inversed cdf
X2 = 3*(-log(1-U2))^(1/4)
mu = mean(X2)
var = 1/1000*var(X2)
mu
```

```
## [1] 2.711023
```

```
var
```

```
## [1] 0.0005780832
```

Antithetic variates:

The pdf is $f(x) = \frac{4}{3}(\frac{x}{3})^3 e^{-(\frac{x}{3})^4}, 0 < x < \infty$.

Then the cdf is $F(x) = \int_0^x \frac{4}{3}(\frac{a}{3})^3 e^{-(\frac{a}{3})^4} da$
$= -\int_0^x e^{-(\frac{a}{3})^4} d - (\frac{a}{3})^4$
$= -e^{-(\frac{a}{3})^4}|_0^x$
$= -(e^{-(\frac{x}{3})^4} - 1) = U, U \sim U[0, 1]$.

Because the cdf in range(0,1), so the uniform distribution can be used to simulate. $U = 1 - e^{-(\frac{x}{3})^4}$, after calculation we can get $(\frac{x}{3})^4 = -log(1 - U)$.

$\because x > 0, \therefore x = 3 * \sqrt[4]{-log(1 - U)}, x^* = 3 * \sqrt[4]{-log(U)}$. X and x^* follow the same distribution.

Draw iid samples $u^{(1)}, u^{(2)}, ..., u^{(1000)}$ from $U[0, 1]$, at the same time we get $1 - u^{(1)}, 1 - u^{(2)}, ..., 1 - u^{(1000)}$.

The mean will be calculated by $\tilde{\mu} = \frac{1}{1000}(\sum_{i=1}^{1000} \frac{x+x^*}{2})$.

The variance will be calculated by $\frac{1}{1000}\sigma^2$.

The mean is 2.718838, the variance is 3.587899e-07. The variance is smaller than the naive monte carlo method.

```
set.seed(10)
U = runif(1000, 0, 1)
N_U = 1 - U
X = 3*(-log(1-U))^(1/4)
N_X = 3*(-log(1-N_U))^(1/4)
mu = mean((X+N_X)/2)
var = 1/1000*var((X+N_X)/2)
mu
```

```
## [1] 2.718838
```

```
var
```

```
## [1] 3.587899e-07
```