chunlei2_hw02

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Q1:

Draw sample x from g(x) = 1, $0 \leqq x \leqq 1$, $g(x) \sim U(0,1)$

$\pi(x) = x^3 + \frac{5}{4}x^2 + \frac{1}{3}x + \frac{1}{6}, g(x) = 1, \pi(x) \leqq cg(x), 0 \leqq x \leqq 1$

$c \geqq \frac{\pi(x)}{g(x)} \geqq \pi(x), max(\pi(x)) = 1 + \frac{5}{4} + \frac{1}{3} + \frac{1}{6} = 2.75$. I choose c = 3.

Draw sample u from $U \sim U(0,1)$, accept x if $u \leqq \frac{\pi(x)}{3g(x)}$, otherwise reject x. Use this algorithm to generate 1000 samples.

The 1000 iid samples are $x^{(1)}, x^{(2)}, ..., x^{(1000)}$, E(X^2) can be estimated by $\frac{1}{1000}((x^{(1)})^2 + (x^{(2)})^2 + ... + (x^{(1000)})^2)$. Var(X^2) can be estimated by $\frac{1}{1000}\hat{\sigma}$

$E(X^2) = 0.5666202, SD(X^2) = 0.009202648$

```
set.seed(9301)
n = 1000 #the target number
k = 0 #the count number
X = numeric(n)
while (k < n) {
  u = runif(1)
  x = runif(1)
  if ((x^3 + 5/4*x^2 + 1/3*x + 1/6)/3 >= u) {
    k = k + 1
    X[k] = x
  }
}
mean(X^2)
```

```
## [1] 0.5666202
```

```
sqrt(1/1000*var(X^2))
```

```
## [1] 0.009202648
```

Q2:

$L(\theta|x_1 = 1.1, x_2 = 0.7, x_3 = 1.4, x_4 = .2, x_5 = 0.8) = \frac{1}{\pi(1+\theta^2)} \prod_{i=1}^{5} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_i-\theta)^2}{2}}$,

insert the x values, after simplification, $L(\theta|x_1 = 1.1, x_2 = 0.7, x_3 = 1.4, x_4 = .2, x_5 = 0.8) = \frac{1}{4\sqrt{5}\pi^3} e^{-0.166} \frac{1}{1+\theta^2} \frac{1}{\sqrt{2\pi\frac{1}{5}}} e^{-\frac{(\theta-1.04)^2}{0.4}}$

$\because \frac{1}{1+\theta^2} \leqq 1 \therefore L(\theta|x_1 = 1.1, x_2 = 0.7, x_3 = 1.4, x_4 = .2, x_5 = 0.8) \leqq \frac{1}{4\sqrt{5}\pi^3} e^{-0.166} \frac{1}{\sqrt{2\pi\frac{1}{5}}} e^{-\frac{(\theta-1.04)^2}{0.4}} = cg(\theta)$

$c = \frac{1}{4\sqrt{5}\pi^3} e^{-0.166}, g(\theta) = \frac{1}{\sqrt{2\pi\frac{1}{5}}} e^{-\frac{(\theta-1.04)^2}{0.4}}$

Draw sample θ from $\theta \sim N(1.04, \frac{1}{5})$

Draw sample u from $U \sim U(0,1)$, accept θ if $u \leqq \frac{L(\theta|x_1=1.1,x_2=0.7,x_3=1.4,x_4=.2,x_5=0.8)}{cg(\theta)}$, otherwise reject θ. Use this algorithm to generate 1000 samples.

My instrumental function is $g(\theta) = \frac{1}{\sqrt{2\pi\frac{1}{5}}}e^{-\frac{(\theta-1.04)^2}{0.4}}$.

I need 1922 samples to have 1000 accepted samples. The estimated acceptance rate is 0.5202914.

The 1000 iid samples are $\theta^{(1)}, \theta^{(2)}, ..., \theta^{(1000)}$, E($\theta$) can be estimated by $\frac{1}{1000}(\theta^{(1)} + \theta^{(2)} + ... + \theta^{(1000)})$. $Var(\theta)$ can be estimated by $\frac{1}{1000}\hat{\sigma}$.

The mean is 0.8765656. The standard error is 0.01327826.

```
set.seed(9301)
n = 1000
k = 0
N = 0
X = numeric(1000)
while (k < n) {
  N = N + 1
  u = runif(1)
  x = rnorm(1, 1.04, sd = sqrt(1/5))
  if ( (1/(pi*(1+x^2))*(1/sqrt(2*pi))^5*exp(-(1.1-x)^2/2-(0.7-x)^2/2-(1.4-x)^2/2-(1.2-x)^2/2-(0.8-x)^2/
    k = k + 1
    X[k] = x
  }
}
N
```

```
## [1] 1922
```

```
1000/N
```

```
## [1] 0.5202914
```

```
M = 1000/N*((1/(4*sqrt(5)*(pi^3))*exp(-0.166)))
mean(X)
```

```
## [1] 0.8765656
```

```
sqrt(1/1000*var(X))
```

```
## [1] 0.01327826
```

Q3:

(a)

$f(x) = \frac{4}{7}(\frac{x}{7})^3 e^{-(\frac{x}{7})^4}$. Instrumental function $g(x) = f(x)$.

$L(x) = f(x)1_{0<x<1} \leq 1g(x)$, draw sample x from g(x), draw sample u from $U \sim U(0,1)$, if $\frac{L(x)}{1g(x)} \geqq u$, accept x, otherwise reject x. Let's assume I = 1 if x is accepted, otherwise I = 0.

$P(I = 1) = \int_{-\infty}^{\infty} P(I = 1|X = x)g(x)dx$
$= \int_{-\infty}^{0} P(I = 1|X = x)g(x)dx + \int_{0}^{1} P(I = 1|X = x)g(x)dx + \int_{1}^{\infty} P(I = 1|X = x)g(x)dx$
$= \int_{0}^{1} P(I = 1|X = x)g(x)dx$
$= \int_{0}^{1} \frac{L(x)}{1g(x)}g(x)dx$
$= Pweibull(1, 4, 7) - Pweibull(0, 4, 7) = 0.0004164064$

So the acceptance rate is 0.0004164064.

(b)

I choose easy to sample distribution uniform distribution. So $g(x) = 1, 0 \leqq x \leqq 1$, $L(x) = \frac{f(x)}{F(1)-F(0)}, 0 \leqq x \leqq 1$, $L(X) \leqq cg(x)$

$c \geqq \frac{f(x)}{F(1)-F(0)}, max(f(x)) = 0.001665279, c_{min} = 0.001665279/0.0004164064 = 4$

Let's assume I = 1 if x is accepted, otherwise I = 0.

Draw sample x from g(x), draw sample u from $U \sim U(0,1)$, if $\frac{L(x)}{4g(x)} \geqq u$, accept x, otherwise reject x.

$P(I=1) = \int_0^1 P(I=1|X=x)g(x)dx$
$= \int_0^1 \frac{L(x)}{cg(x)} 1dx$
$= \frac{1}{c} \int_0^1 L(x)dx$
$= \frac{1}{c} = \frac{1}{4}$

So the acceptance rate is 0.25.

```r
4/7*(1/7)^3*exp(-(1/7)^4)
```

```
## [1] 0.001665279
```

```r
pweibull(1, 4, 7)
```

```
## [1] 0.0004164064
```

```r
pweibull(0, 4, 7)
```

```
## [1] 0
```