Chunlei2_hw03

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Q1:

$h(x) = sin(x^4), \pi(x) = 1(0 \leqq x \leqq 1), E(sin(x^4)) = \int_0^1 sin(x^4) * 1dx$

I choose $g(x) \sim B(5,1), g(x) = 5 * x^4$ as the proposal distribution, as g(x) is close to $h(x) * \pi(x)$. So $E(sin(x^4)) = \int_0^1 \frac{sin(x^4)}{g(x)} * g(x)dx$.

1000 samples $x^{(1)}, x^{(2)}..., x^{(1000)}$ are chosen from the g(x). Calculate $\frac{h(x)\pi(x)}{g(x)}$ basen on X. Estimate μ by $\hat{\mu_n} = \frac{1}{1000} * [\frac{h(x^{(1)})\pi(x^{(1)})}{g(x^{(1)})} + ... + \frac{h(x^{(1000)})\pi(x^{(1000)})}{g(x^{(1000)})}]$. Estimate σ by $\sqrt{\frac{1}{1000}\hat{\sigma}}$

The mean estimate is 0.1874643, the standard error estimate is 0.0002984545.

```r
set.seed(9301)
target = function(x){
  sin(x^4)/(x^4*dbeta(1,5,1))
}
n = 1000
X = rbeta(n, 5, 1)
mean(target(X))
```

```
## [1] 0.1874643
```

```r
sqrt(1/1000*var(target(X)))
```

```
## [1] 0.0002984545
```

Q2:

In problem 2 of homework 2, I need 1922 samples to have 1000 accepted samples. So, the proposal funtion is the normal distribution with $\mu = 1.04, \sigma^2 = 0.2, g(\theta) = \frac{1}{\sqrt{2\pi * 0.2}} e^{-\frac{(x-1.04)^2}{2*0.2}}$.

$E(\theta) = \int_{-\infty}^{\infty} \theta\pi(\theta|x_1, x_2, x_3, x_4, x_5)d\theta$
$= \int_{-\infty}^{\infty} \frac{\theta\pi(\theta|x_1,x_2,x_3,x_4,x_5)}{g(\theta)} g(\theta)d\theta$

So draw 1922 samples $\theta^{(1)}...\theta^{(1922)}$ from the distribution $g(\theta)$ $W = \frac{\pi(\theta|x_1,x_2,x_3,x_4,x_5)}{g(\theta)}, h(\theta) = \theta, l(\theta) = \pi(\theta|x_1, x_2, x_3, x_4, x_5)$
$w^{(1)}...w^{(1922)}, h(\theta^{(1)})...h(\theta^{(1922)})$
$\tilde{\mu} = \frac{w^{(1)}h(\theta^{(1)})+...+w^{(1922)}h(\theta^{(1922)})}{w^{(1)}+...+w^{(1922)}}$

$Var(\tilde{\mu}) \approx \frac{var_g(\frac{h(\theta)l(\theta)}{g(\theta)}) + \tilde{\mu}^2 var_g(\frac{l(\theta)}{g(\theta)}) - 2\tilde{\mu}cov(\frac{h(\theta)l(\theta)}{g(\theta)}, \frac{l(\theta)}{g(\theta)})}{nE_g^2(\frac{l(\theta)}{g(\theta)})}$

The mean estimate is 0.8700508, the standard error estimate is 0.01056657. It is smaller than the one in the previous homework.

```r
set.seed(9301)
n = 1922
weight = function(x){
  ((1/(pi*(1+x^2))*(1/sqrt(2*pi))^5*exp(-(1.1-x)^2/2-(0.7-x)^2/2-(1.4-x)^2/2-(1.2-x)^2/2-(0.8-x)^2/2))),
}
target = function(x){
```

```r
  ((1/(pi*(1+x^2))*(1/sqrt(2*pi))^5*exp(-(1.1-x)^2/2-(0.7-x)^2/2-(1.4-x)^2/2-(1.2-x)^2/2-(0.8-x)^2/2))*
}
X = rnorm(n, 1.04, sd = sqrt(1/5))
sum(target(X))/sum(weight(X))
```

```
## [1] 0.8700508
```

```r
mu = sum(target(X))/sum(weight(X))
sqrt((var(target(X)) + mu^2*var(weight(X)) - 2*mu*cov(target(X), weight(X)))/(n*(mean(weight(X)))^2))
```

```
## [1] 0.01056657
```

Q3:

$\pi(x) \propto l(x) = e^{-\frac{|x|^3}{3}}, -\infty < x < \infty$ is hard to sample, $h(x) = x^2$. Find the proposal function $g(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ which is close to $l(x) = c\pi(x)$

So draw 1000 samples $x^{(1)}...x^{(1922)}$ from the distribution $g(x)$ $W = \frac{l(x)}{g(x)}, h(x) = x^2, l(x) = e^{-\frac{|x|^3}{3}}$
$w^{(1)}...w^{(1000)}, h(x^{(1)})...h(x^{(1000)})$
$\tilde{\mu} = \frac{w^{(1)}h(x^{(1)})+...+w^{(1000)}h(x^{(1000)})}{w^{(1)}+...+w^{(1000)}}$

$Var(\tilde{\mu}) \approx \frac{var_g(\frac{h(x)l(x)}{g(x)})+\tilde{\mu}^2 var_g(\frac{l(x)}{g(x)})-2\tilde{\mu}cov(\frac{h(x)l(x)}{g(x)}, \frac{l(x)}{g(x)})}{nE_g^2(\frac{l(x)}{g(x)})}$

The mean estimate is 0.7321103, the standard error estimate is 0.02478644.

```r
set.seed(9301)
n = 1000
weight = function(x){
  exp(-(abs(x)^3/3))/(1/(sqrt(2*pi*1))*exp(-x^2/(2*1)))
}
target = function(x){
  (x^2)*exp(-(abs(x)^3/3))/(1/(sqrt(2*pi*1))*exp(-x^2/(2*1)))
}
X = rnorm(n, 0, sd = sqrt(1))
sum(target(X))/sum(weight(X))
```

```
## [1] 0.7321103
```

```r
mu = sum(target(X))/sum(weight(X))
sqrt((var(target(X)) + mu^2*var(weight(X)) - 2*mu*cov(target(X), weight(X)))/(n*(mean(weight(X)))^2))
```

```
## [1] 0.02478644
```