hw07

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Q1:

The target distribution is $\pi(x) \propto exp(\mu \sum_{i=1}^{d-1} x_i x_{i+1}) = g(x)$,

at time step t, $x^{(t)} = (x_1^{(1)}, ..., x_d^{(d)})$,

at each direction i, draw $x_i^{(t+1)}$ from the conditional distribution $g(x_i^{(t+1)}|x_1^{(t+1)}, ..x_{i-1}^{(t+1)}, x_{i+1}^{(t)}, ..., x_d^{(t)}) = \frac{g(x_i^{(t+1)}, x_1^{(t+1)}, ..., x_{i-1}^{(t+1)}, x_{i+1}^{(t)}, ..., x_d^{(t)}}{g(x_1^{(t+1)}, ..., x_{i-1}^{(t+1)}, x_{i+1}^{(t)}, ..., x_d^{(t)})} = C_i * exp(\mu x_{i-1}^{(t+1)} x_i + \mu x_i x_{i+1}^{(t)}), C_i$ is a constant.

Becasue x_i only equals 1 or -1, $C_i * exp(\mu(x_{i-1}^{(t+1)} * 1 + 1 * x_{i+1}^{(t)})) + C_i * exp(\mu(x_{i-1}^{(t+1)} * -1 + -1 * x_{i+1}^{(t)})) = 1$
$C_i = \frac{1}{exp(\mu(x_{i-1}^{(t+1)} + x_{i+1}^{(t)})) + exp(\mu(-x_{i-1}^{(t+1)} - x_{i+1}^{(t)}))}$

There are two special cases,

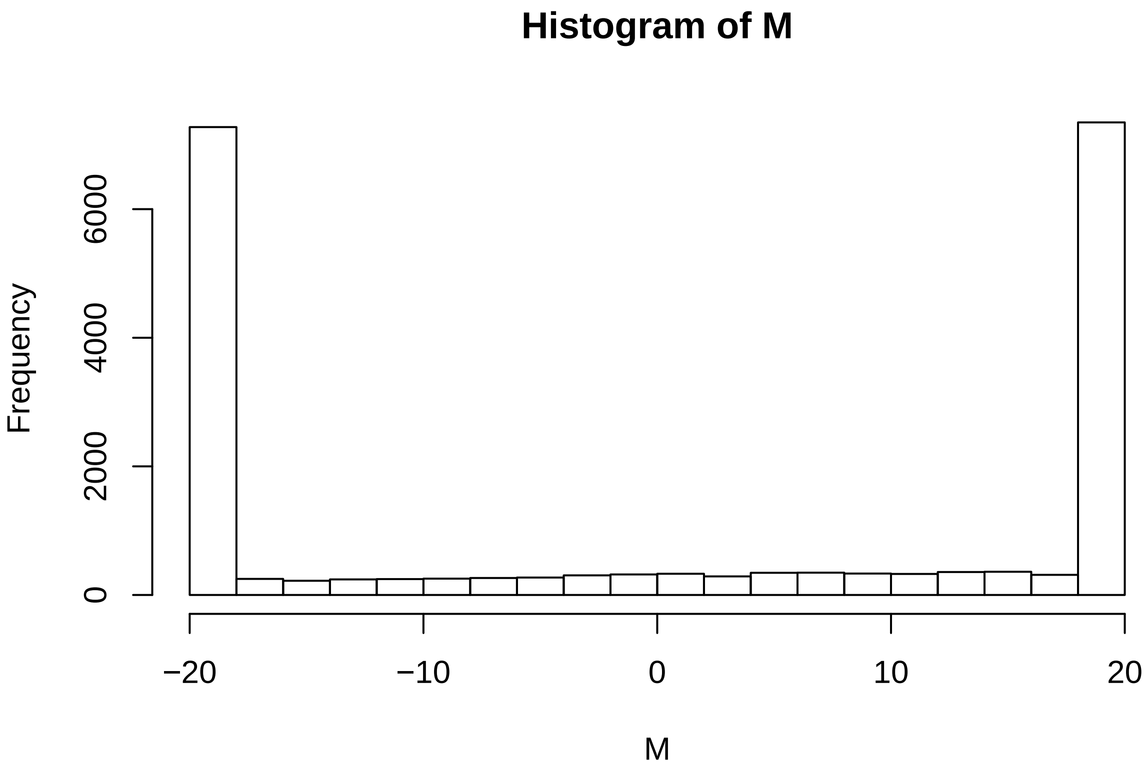
if i = 1, $g(x_1^{(t+1)}|x_{[-1]}^{(t)}) = C_1 * exp(\mu x_1 x_2^{(t)})$, same rule as before $C_1 = \frac{1}{exp(\mu x_2^{(t)}) + exp(-\mu x_2^{(t)})}$

if i = d, $g(x_d^{(t+1)}|x_{[-d]}^{(t+1)}) = C_d * exp(\mu x_{d-1}^{(t+1)} x_d)$, same rule as before $C_d = \frac{1}{exp(\mu x_{d-1}^{(t+1)}) + exp(-\mu x_{d-1}^{(t+1)})}$

```r
rm(list = ls())
set.seed(1)
d = 20
mu = 2
t = 50000
x_0 = sample(c(-1,1), d, replace = T)
x_matrix = matrix(0, t+1, d)
x_matrix[1, ] = x_0
gibbs = function(x, mu) {
  x_new = x
  for (i in 1:d) {
    if (i == 1) {
      c = 1/(exp(mu * x_new[i+1]) + exp(-mu * x_new[i+1]))
      prob = exp(mu * x_new[i+1])*c
      x_new[i] = rbinom(1,1,prob)*2-1
    }
    else if (i == d) {
      c = 1/(exp(mu * x_new[i-1]) + exp(-mu * x_new[i-1]))
      prob = exp(mu * x_new[i-1])*c
      x_new[i] = rbinom(1,1,prob)*2-1
    }
    else {
      c = 1/(exp(mu * (x_new[i-1] + x_new[i+1])) + exp(-mu * (x_new[i-1] + x_new[i+1])))
      prob = exp(mu * (x_new[i-1] + x_new[i+1]))*c
      x_new[i] = rbinom(1,1,prob)*2-1
    }
  }
  return(x_new)
}
```

```r
for (j in 1:t) {
x_matrix[j+1,] = gibbs(x_matrix[j,], mu)
}

trunc_sample = tail(x_matrix, 20000)
M = apply(trunc_sample, 1, sum)
hist(M)
```



Q2:

$\omega = 0.04, \alpha = 2, \beta = 0.5, n = 5$, The target distribution is $p(\mu, \tau | y_1, ..., y_n) \propto (2\pi)^{-\frac{n+1}{2}} \tau^{\frac{n}{2}} e^{-\frac{\tau}{2}\sum_{i=1}^{n}(y_i-\mu)^2} \omega^{\frac{1}{2}} e^{-\frac{\omega}{2}\mu^2} \frac{\tau^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} e^{-\frac{\tau}{\beta}}$

at time step t, we draw

$\mu^{(t+1)}$ from $p(\mu | \tau^{(t)}, y_1 ... y_n) \propto e^{-\frac{\tau}{2}\sum_{i=1}^{n}(y_i-\mu)^2} e^{-\frac{\omega}{2}\mu^2} \propto e^{-\frac{n\tau+\omega}{2}(\mu - \frac{n\tau\bar{y}}{n\tau+\omega})^2} \sim N(\frac{n\tau\bar{y}}{n\tau+\omega}, \frac{1}{n\tau+\omega})$,

$\tau^{(t+1)}$ from $p(\tau | \mu^{(t+1)}, y_1 ... y_n) \propto \tau^{\frac{n}{2}+\alpha-1} e^{-(\frac{\sum(y_i-\mu)^2}{2}+\frac{1}{\beta})\tau} \sim \Gamma(\frac{n}{2}+\alpha, \frac{\sum(y_i-\mu)^2}{2}+\frac{1}{\beta})$

The mean is (2.1130266, 0.8561928)

```r
rm(list = ls())
set.seed(1)
y = c(1.9, 3.4, 0.3, 2.5, 2.6)
n = 5
omega = 0.04
alpha = 2
beta = 0.5
y_mean = mean(y)
t = 20000
x_0 = c(0, 1)
x_matrix = matrix(0, t+1, 2)
```

```r
x_matrix[1, ] = x_0
gibbs = function(x){
  x_new = x
  x_new[1] = rnorm(1, (n*x_new[2]*y_mean)/(n*x_new[2]+omega), sqrt(1/(n*x_new[2]+omega)))
  x_new[2] = rgamma(1, shape = n/2+alpha, rate = (sum((y-x_new[1])^2)/2+1/beta))
  return(x_new)
}
for (i in 1:t) {
  x_matrix[i+1, ] = gibbs(x_matrix[i, ])
}
trunc_sample = tail(x_matrix, 10000)
M = apply(trunc_sample, 2, mean)
M
```

```
## [1] 2.1130266 0.8561928
```