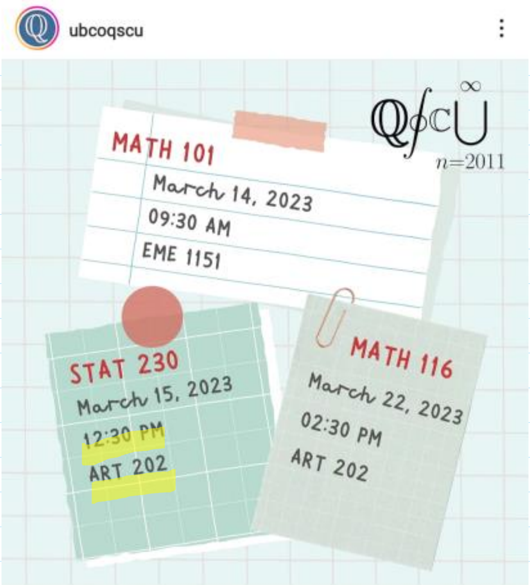
Review

March 11, 2023 10:10 AM



Sampling distribution

— Sampling from a Normal dist.

$X \sim N(\mu, \sigma^2)$ $\quad N(\mu, \sigma)$

$Z = \boxed{\frac{X-\mu}{\sigma}} \sim N(0,1)$ $\quad$ prob. . . $\quad$ Z-table $N(0,1)$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ $\triangle$ $\quad$ prove $E(\ )$ $Var(\ )$

random sampling $\quad$ $X \sim N(\mu, \sigma^2)$ population

X_1
X_2
$\vdots$
X_n

$\bar{X} = \frac{1}{n}\sum X_i$

— Sampling from any distribution with sufficiently large sample size n

Central limit Thm (CLT), $\quad \bar{X} \overset{appr.}{\sim} N(\mu, \frac{\sigma^2}{n})$ $\triangle$

proof: use MGF (moment generating function)

Q1

Question 1

West Indian manatees travel at a speed that is approximately normally distributed with a mean of 3.8 miles per hour and a standard deviation of 1.34 miles per hour.

(a) What proportion of West Indian manatees travel at speeds between 3 and 4.7 miles per hour?

(b) How slow do the slowest 45% of West Indian manatees travel?

(c) You sample 50 West Indian manatees. What is the sampling distribution of their average speed, $\bar{X}$?

(d) What is the probability that the mean speed of your sample is between 3 and 4.7 miles per hour?

W

$n=50$

suppose w be West Indian manatees' speed.

$W \sim N(3.8, 1.34)$ then $\frac{W-3.8}{1.34} \sim N(0,1)$

(a) $P(3 < W < 4.7) = P\left(\frac{3-3.8}{1.34} < \frac{W-3.8}{1.34} < \frac{4.7-3.8}{1.34}\right)$

Z

$= P(-0.6 < Z < 0.67)$

$= P(Z < 0.67) - P(Z < -0.6)$ pnorm( )

$= 0.7486 - 0.2743$

$= 0.4743$

(b) let w be the 45th percentile of the speed.

that is $P(W < w) = 45\%$

$P(W < w) = P\left(\frac{W-3.8}{1.34} < \frac{w-3.8}{1.34}\right) = P\left(Z < \frac{w-3.8}{1.34}\right) = 0.45$

(c) $\bar{W} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ or $N\left(\mu, \frac{\sigma^2}{n}\right)$

$\frac{\sigma}{\sqrt{n}} = \frac{1.34}{\sqrt{50}} = 0.19$ so. $\bar{W} \sim N(1.34, 0.19)$

(d) $P(3 < \bar{W} < 4.7)$

$= P\left(\frac{3-1.34}{0.19} < \frac{\bar{W}-1.34}{0.19} < \frac{4.7-1.34}{0.19}\right)$

$= P(-4.21 < Z < 4.74)$

$= P(Z < 4.74) - P(-4.21)$

$> P(Z < 3.49) - P(Z < -3.49) = 0.9998 - 0.0002$

> 0.9996 ✓

≈ 1

Q2

samplemean of this 25 hamburgers

sample

Question 2

25 hamburgers were randomly selected from the production process at a fast food restaurant and the mean sodium content was measured to be 400 mg. High levels of sodium are considered to be unhealthy if consumed too frequently. Assume that the sodium measurements are well approximated by a normal distribution with a standard deviation of 150 mg.

(a) Find a 99% confidence interval for the true mean sodium content.

$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

CI

99% CI for μ is $\bar{x} \pm \overset{2.58}{z_{0.005}} \underbrace{\frac{\sigma}{\sqrt{n}}}_{\text{S.E.}}$

$\bar{x} = 400$

99.5 percentile of the st. Normal dist.

that is $P(Z < z_{0.005}) = 99.5\% = 0.995$

$z_{0.005} = 2.58$

$\frac{\sigma}{\sqrt{n}} = \frac{150}{\sqrt{25}} = 30$

99% CI for the .... is

$400 \pm 2.58 \times 30$

$(322.6, \ 477.4)$

Q3

Question 3

The weight of a bag of potato chips is stated as 300 g. The amount that the packaging machine puts in each bag is normally distributed with mean 306 g and standard deviation 3.6 g.

(a) What is the proportion of all bags sold that are underweight? < 300

(b) A "bargain pack" contains two bags. Find the probability that both are underweight.

(c) Find the probability that the average weight of the bags in a bargain pack is below 300 g.

(d) Find the probability that the mean weight per bag in a 24-bag carton is below 300 g.

Let X be the weight of a bag of potato chips. $X \sim N(306, 3.6)$

(a) $P(X<300) = P\left(\boxed{\frac{X-306}{3.6}} < \frac{300-306}{3.6}\right) = P(Z<-1.67) = 0.04746$

(b) $P(\text{both underweight}) = P(X<300)P(X<300) = (0.04746)^2 = 0.002225$

(c) $\bar{X} \sim N(306, 3.6/\sqrt{2})$

$n=2$

$P(\bar{X}<300) = P\left(\frac{\bar{X}-306}{3.6/\sqrt{2}} < \frac{300-306}{3.6/\sqrt{2}}\right) = P(Z<-2.357)$

$= 0.0092$

(d) $n=24$

$\bar{X} \sim N(306, 3.6/\sqrt{24})$

$P(\bar{X}<300) = P\left(\frac{\bar{X}-306}{3.6/\sqrt{24}} < \boxed{\frac{300-306}{3.6/\sqrt{24}}}\right) = P(Z<-8.16)$?

$<<< P(Z<-3.49)$

0.0002

≈ 0