Review Lecture 02 probability

# Lab 02 Stat 230 2022W2

**Theorem 2.2 (Fundamental Principle of Counting)** P17

*If a particular task may be accomplished $n_1$ ways and then a second task may be accomplished in $n_2$ ways, then the first task followed by the second task may the accomplished in $n_1 n_2$ different ways.*

**Definition 2.10** P32

Experimental probability refers to the probability of an event occurring when an experiment was conducted.

$$\text{Probability} = \frac{\text{Number of event occurrences}}{\text{Total number of trials}}$$

### Question 1

A group of 7 students consisting of 3 girls and 4 boys have agreed to form a study group to meet in the library.

(a) They have decide that one person should be in charge of organizing the meeting times, and a different person should be in charge of booking the table. In how many ways can these two people be chosen?

(b) Midway through studying, the group gets hungry and select two people to grab snacks. What is the probability that they select a pair of the opposite sex?

(a) $\boxed{7}\boxed{6}$ $\quad 3+4=7 \quad 7-1=6$

$7 \times 6 = 42$

(b) snack group

G + B

$\binom{3}{1} \times \binom{4}{1}$ like (a)

$$\text{prob} = \frac{\# \binom{3}{1} \times \binom{4}{1}}{\# \text{total} \binom{7}{2}} \approx 0.5714 \in [0,1]$$

→ choosing 2 people in any ways

thousands $\neq 0$ 4 tasks

### Question 2

Consider all four digit integers (i.e. all integers $\geq 1000$ and $\leq 9999$)

(a) How many 4-digit integers can be formed without repeating a digit? (eg. 2213 would not be allowed)

(b) How many of the numbers from part (a) are odd?

Counting Rule Extension (Thm 2.3) P18

$n_1 n_2 \cdots n_k$ k tasks

odd unique (no repeating)

(a) $\neq 0$ $\boxed{9}\boxed{9}\boxed{8}\boxed{7}$

1, 2 … 9 $\quad$ 0, 1, 9 $\quad n_3 \quad n_4 = 7$

$n_1 = 9 \quad 10-1=9 \quad 10-2=8$

$n_2$

$9 \times 9 \times 8 \times 7 = 4536$

(b) $\neq 0$ $\boxed{8}\boxed{8}\boxed{7}\boxed{5}$ ones

9-1 $\quad$ 0-9 $\quad$ 10-3 $\quad$ # 10-2

odd: 1, 3, 5, 7, 9

$8 \times 8 \times 7 \times 5 = 2240$.

### Question 3

Two cards are chosen at random from a well shuffled pack. What is the probability that:

(a) Both are red? $= \{$ the first card is red, the second -- red $\}$

(b) None are red? $= \{$ Both are black $\}$

(c) One is a club and one is a heart?

Multiplication Rule ("AND" Rule) P53

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{also} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B|A)\,P(A) = P(A|B)\,P(B)$$

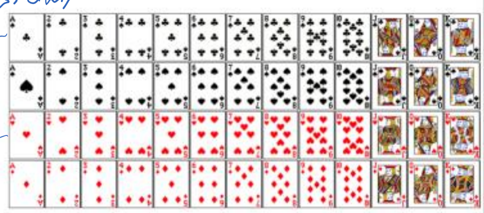
(a) $1R = \{$ the first card is red $\}$

$2R = \{$ the second card is red $\}$

1R → 2R

$$P(1R \cap 2R) = P(1R)\,P(2R|1R) = \frac{26}{52} \cdot \frac{25}{51} = \frac{25}{102}$$

club



$\to 13$, $\to 13$, $\to 13$, $\to 13$

heart

(b) $1R^c$ $2R^c$ $\quad P(1R^c \cap 2R^c) = P(1R^c)\,P(2R^c|1R^c) = \frac{26}{52} \cdot \frac{25}{51} = \frac{25}{102}$

(c) $= \{\underbrace{(1C, 2H), (1H, 2C)}_{2}\}$ disjoint events

$$P\{(1C,2H) \cup (1H,2C)\} = P(1C,2H) + P(1H,2C)$$
$$= \frac{13}{52} \cdot \frac{13}{51} + \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204} \times 2 = \frac{13}{102}$$

$$P(1C, 2H) = \frac{13}{52} \cdot \frac{13}{51}$$

### Question 4

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a non-accident-prone person. If we assume that 30 percent of the population is accident prone,

(a) what is the probability that $\{$ a new policyholder will have an accident within a year of purchasing a policy? $\} = A$

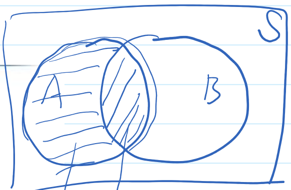
(b) suppose that a new policyholder has an accident within a year of purchasing a policy, What is the porbability that he or she is accident prone?

**Fact**

Let $A$ and $B$ be events. We may express $A$ as

$$A = \{A \cap B\} \cup \{A \cap \bar{B}\}$$

where $\bar{B}$ is the complement of event $B$



the population is accident prone,

(a) what is the probability that {a new policyholder will have an accident within a year of purchasing a policy?} = A

(b) suppose that a new policyholder has an accident within a year of purchasing a policy, What is the porbability that he or she is accident prone?

Or (Present the Tree Diagram of your answer!)

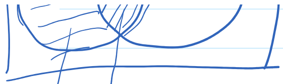
Let $A$ and $B$ be events. We may express $A$ as

$$A = \{A \cap B\} \cup \{A \cap \bar{B}\}$$

where $\bar{B}$ is the complement of event $B$.

- $\{A \cap B\}$ and $\{A \cap \bar{B}\}$ are disjoint events
- The probability of event $A$ is

$$P(A) = P(A \cap B) + P(A \cap \bar{B}) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$$ P68



$A \cap \bar{B}$ $A \cap B$

mutually exclusive

(a) $A = \{ \cdots \}$ $P(A) = ?$

$Ap = \{$ the policyholder is prone to have an accident $\}$

$P(Ap) = 0.3$, $P(\overline{Ap}) = 1 - 0.3 = 0.7$

$P(A|Ap) = 0.4$, $P(A|\overline{Ap}) = 0.2$

$\Rightarrow P(A) = P(A|Ap)P(Ap) + P(A|\overline{Ap})P(\overline{Ap})$

$= 0.4 \cdot 0.3 + 0.2 \cdot 0.7 = 0.26$

(b) $P(Ap|A) = \dfrac{P(A|Ap)P(Ap)}{P(A)} = \dfrac{0.4 \times 0.3}{0.26} = 0.4615$

tree diagram

$Ap$ — $P(Ap) = 0.3$
- $P(A|Ap) = 0.4$ — $P(A \cap Ap) = 0.3 \times 0.4 = 0.12$
- $P(\bar{A}|Ap) = 1 - 0.4 = 0.6$ — $P(\bar{A} \cap Ap) = 0.3 \times 0.6 = 0.18$

$\overline{Ap}$ — $P(\overline{Ap}) = 0.7$
- $P(A|\overline{Ap}) = 0.2$ — $P(A \cap \overline{Ap}) = 0.7 \times 0.2 = 0.14$
- $P(\bar{A}|\overline{Ap}) = 1 - 0.2 = 0.8$ — $P(\bar{A} \cap \overline{Ap}) = 0.7 \times 0.8 = 0.56$

$P(A) = P(A \cap Ap) + P(A \cap \overline{Ap}) = 0.12 + 0.14 = 0.26$

$P(\bar{A}) = 0.18 + 0.56 = 0.74$

$\Sigma = 1$