

1 (b)

$$H_0: \mu = 90 \quad (\mu = \mu_0 \text{ where } \mu_0 = 90)$$

$$H_a: \mu < 90$$

$$\bar{x} = 87 \quad \sigma = 1.5$$

Test statistic  $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{87 - 90}{1.5/\sqrt{49}} = \frac{-3}{1.5/7} = -14$

(iv)

If  $H_0$  is true,

$$P(\bar{X} > 87)$$

$$\bar{X} \sim N(\mu_0, \sigma)$$

that is

$$= P\left(\frac{\bar{X} - 90}{1.5} > \frac{87 - 90}{1.5}\right) = P(Z > -2) \quad N(90, 1.5)$$

$$= 1 - P(Z < -2)$$

$$= 1 - 0.0228 = 0.9772$$

### Question 1

25 hamburgers were randomly selected from the production process at a fast food restaurant and the mean sodium content was measured to be 400 mg. High levels of sodium are considered to be unhealthy if consumed too frequently. Assume that the sodium measurements are well approximated by a normal distribution with a standard deviation of 150 mg.

- Find a 99% confidence for the true mean sodium content.
- We are interested in showing that these hamburgers are healthier than a competitor who advertises that their hamburgers (which are similar in all other respects) have 480 mg of sodium. Write down the null and alternative hypotheses for the appropriate statistical test.
- (2 points) Calculate the test statistic for this test.
- (2 points) Calculate the  $p$ -value for this test, and interpret your result.

let  $\mu$  be the mean sodium content of our hamburgers.

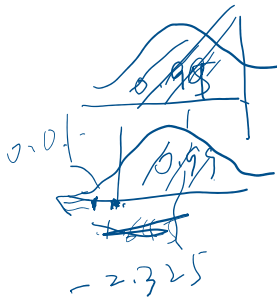
$$(b) H_0: \mu = 480$$

$$H_a: \mu < 480$$

$$(c) Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{400 - 480}{150/\sqrt{25}} = -2.67$$

(d)

$< -2.325$   
we reject  $H_0$ .



① critical Z-score

$$\alpha = 0.01$$

$$\alpha/2 \Rightarrow 1.645$$

$$-1.645$$

②  $p$ -value

$$P(Z < -2.67) = 0.0038 < 0.01 \quad (0.05)$$

2. (27)

$$\mu_0 \in (\bar{X} - z^* \cdot se, \bar{X} + z^* \cdot se)$$