Lab8

Question 1
Find a 95% confidence interval for the expected value of concentration measurements taken from a chemical process. Sample measurements are

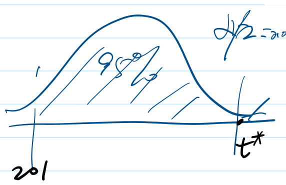
204	190	202	207
204	202	201	195

$n = 8 \quad df = n-1 = 7$

$\alpha = 0.05$

a 95% CI for $\bar{x}$ is

$$\bar{x} \pm t_{0.025,7} \cdot \frac{s}{\sqrt{n}}$$



sol)

mean) in R

$$\bar{x} = \frac{204 + 190 + \cdots + 195}{8} = 201$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = 5.5$$

$P(t < t^*) = 0.975$

$t_{0.025,7} = 2.365$ use t-table or $qt(0.975, 7) = t^*$

So $201 \pm 2.365 \times 5.5/\sqrt{8}$

i.e. 201 ± 4.6

$(196.4, 205.6)$

SE ✓
ME ✓
CL ✓
Lower ✓
upper ✓

Question 2
A chemical process has produced, on the average, 800 tons of chemical per day. The daily yields for the past week are 785, 805, 790, 793, and 802 tons.

$X \sim N(\mu, \sigma^2)$

(a) Do these data indicate that the average yield is less than 800 tons and hence that something is wrong with the process? Test at the 5% level of significance.

(b) What assumptions are required for the valid use of the procedure you used to analyze these data.

(a) $H_0: \mu = 800$ vs. $H_a: \mu < 800$ at $\alpha = 0.05$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \mu_0 = 800$$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

σ is unknown

s

$\frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t$

$df = n-1$

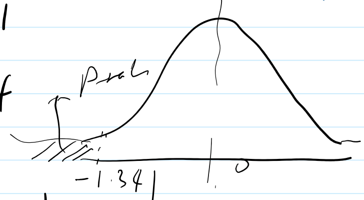
$\bar{x} = 795 \quad n = 5$

$s = 8.34$

$$t = \frac{795 - 800}{8.34/\sqrt{5}}$$

$= -1.341$

$df = 5 - 1 = 4$



$1.190 < |-1.341| < 1.533$

$P(-1.341, 4)$

$0.1 <$ p-value < 0.15

which is greater than 0.05

We fail to reject H_0.

Question 3
In what follows, $Z_1, Z_2, \ldots, Z_n$ is a sequence of independent standard normal random variables. Also, $X_1, X_2, \ldots, X_n$, $W_1, W_2, \ldots, W_m$ and $V_1, V_2, \ldots, V_k$ are independent sets of normal random variables with common variance σ^2 and respective means μ_X, μ_W and μ_V.
S_X^2, S_W^2 and S_V^2 denote the sample variances of the X's, W's and V's, respectively. Identify the distributions (specifying values of parameters wherever possible) of the following random variables:

(a).
$$Z_1^2 + Z_2^2 + Z_3^2 \sim \chi^2(3) \qquad (1)$$

(b).
$$\frac{1}{2\sigma^2}(X_1 - X_2)^2 \qquad X_1, X_2 \sim N(\mu_X, \sigma^2) \qquad (2)$$

(d).
$$\frac{Z_1^2}{Z_2^2} \qquad (5)$$

(e).
$$\frac{n(\bar{X} - \mu_X)^2}{\sigma^2} + \frac{m(\bar{W} - \mu_W)^2}{\sigma^2} + \frac{k(\bar{V} - \mu_V)^2}{\sigma^2} \qquad (7)$$

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Review. Standard normal dist. chi-square and F-dist (or F-ratio)

$Z_i \sim N(0,1) \quad i = 1, 2, \ldots$

$Z_i^2 \sim \chi^2(1)$

$\sum_{i=1}^{n} Z_i^2 \sim \chi^2(n)$

$$\frac{\sum_{i=1}^{n} Z_i^2 / n}{\sum_{i=1}^{m} Z_i^2 / m} \sim F(n, m)$$

(b) $E(X_1 - X_2) = \mu_x - \mu_x = 0$

$Var(X_1 - X_2) = Var X_1 + Var X_2 = 2\sigma^2$

$\therefore X_1 - X_2 \sim N(0, 2\sigma^2)$

$$\Rightarrow \frac{X_1 - X_2 - 0}{\sqrt{2}\sigma} \sim N(0,1)$$

$$\text{so} \left(\frac{X_1 - X_2}{\sqrt{2}\sigma}\right)^2 \sim \chi^2(1)$$

(c) $\sum_{i=1}^{n} Z_i^2 \sim \chi^2(n)$

(d)
$$\frac{Z_1^2/1}{Z_2^2/1} \sim F(1,1)$$

$Z_1^2 \sim \chi^2(1), \quad Z_2^2 \sim \chi^2(1)$

(e) $\bar{X} \sim N(\mu_X, \frac{\sigma^2}{n})$

$\bar{W} \sim N(\mu_W, \frac{\sigma^2}{m})$

$\bar{V} \sim N(\mu_V, \frac{\sigma^2}{k})$

$$\therefore \frac{\bar{X} - \mu_X}{\sigma/\sqrt{n}} \sim N(0,1)$$

(c) $\sum_{i=1}^{n} Z_i^2 \sim \chi^2(n)$

$$\left(\frac{\bar{X}-\mu_X}{\sigma/\sqrt{n}}\right)^2 + \left(\frac{\bar{W}-\mu_W}{\sigma/\sqrt{m}}\right)^2 + \left(\frac{\bar{V}-\mu_V}{\sigma/\sqrt{k}}\right)^2$$

Z_i

$\chi^2(1) \quad \chi^2(1) \quad \chi^2(1)$

$\because \frac{\bar{X}-\mu_X}{\sigma/\sqrt{n}} \sim N(0,1)$

$\frac{\bar{W}-\mu_W}{\sigma/\sqrt{m}} \sim N(0,1)$

$\frac{\bar{V}-\mu_V}{\sigma/\sqrt{k}} \sim N(0,1)$

$\sim \chi^2(3)$