exercise_4

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2020/3/21

```r
# load data
rm(list = ls())
data(iris)

# checking the data structure
iris
```

```
##    Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1           5.1         3.5          1.4         0.2  setosa
## 2           4.9         3.0          1.4         0.2  setosa
## 3           4.7         3.2          1.3         0.2  setosa
## 4           4.6         3.1          1.5         0.2  setosa
## 5           5.0         3.6          1.4         0.2  setosa
## 6           5.4         3.9          1.7         0.4  setosa
## 7           4.6         3.4          1.4         0.3  setosa
## 8           5.0         3.4          1.5         0.2  setosa
## 9           4.4         2.9          1.4         0.2  setosa
## 10          4.9         3.1          1.5         0.1  setosa
## 11          5.4         3.7          1.5         0.2  setosa
## 12          4.8         3.4          1.6         0.2  setosa
## 13          4.8         3.0          1.4         0.1  setosa
## 14          4.3         3.0          1.1         0.1  setosa
## 15          5.8         4.0          1.2         0.2  setosa
## 16          5.7         4.4          1.5         0.4  setosa
## 17          5.4         3.9          1.3         0.4  setosa
## 18          5.1         3.5          1.4         0.3  setosa
## 19          5.7         3.8          1.7         0.3  setosa
## 20          5.1         3.8          1.5         0.3  setosa
## 21          5.4         3.4          1.7         0.2  setosa
## 22          5.1         3.7          1.5         0.4  setosa
## 23          4.6         3.6          1.0         0.2  setosa
## 24          5.1         3.3          1.7         0.5  setosa
## 25          4.8         3.4          1.9         0.2  setosa
## 26          5.0         3.0          1.6         0.2  setosa
## 27          5.0         3.4          1.6         0.4  setosa
## 28          5.2         3.5          1.5         0.2  setosa
## 29          5.2         3.4          1.4         0.2  setosa
## 30          4.7         3.2          1.6         0.2  setosa
## 31          4.8         3.1          1.6         0.2  setosa
## 32          5.4         3.4          1.5         0.4  setosa
## 33          5.2         4.1          1.5         0.1  setosa
## 34          5.5         4.2          1.4         0.2  setosa
```

```
## 35          4.9          3.1          1.5          0.2        setosa
## 36          5.0          3.2          1.2          0.2        setosa
## 37          5.5          3.5          1.3          0.2        setosa
## 38          4.9          3.6          1.4          0.1        setosa
## 39          4.4          3.0          1.3          0.2        setosa
## 40          5.1          3.4          1.5          0.2        setosa
## 41          5.0          3.5          1.3          0.3        setosa
## 42          4.5          2.3          1.3          0.3        setosa
## 43          4.4          3.2          1.3          0.2        setosa
## 44          5.0          3.5          1.6          0.6        setosa
## 45          5.1          3.8          1.9          0.4        setosa
## 46          4.8          3.0          1.4          0.3        setosa
## 47          5.1          3.8          1.6          0.2        setosa
## 48          4.6          3.2          1.4          0.2        setosa
## 49          5.3          3.7          1.5          0.2        setosa
## 50          5.0          3.3          1.4          0.2        setosa
## 51          7.0          3.2          4.7          1.4 versicolor
## 52          6.4          3.2          4.5          1.5 versicolor
## 53          6.9          3.1          4.9          1.5 versicolor
## 54          5.5          2.3          4.0          1.3 versicolor
## 55          6.5          2.8          4.6          1.5 versicolor
## 56          5.7          2.8          4.5          1.3 versicolor
## 57          6.3          3.3          4.7          1.6 versicolor
## 58          4.9          2.4          3.3          1.0 versicolor
## 59          6.6          2.9          4.6          1.3 versicolor
## 60          5.2          2.7          3.9          1.4 versicolor
## 61          5.0          2.0          3.5          1.0 versicolor
## 62          5.9          3.0          4.2          1.5 versicolor
## 63          6.0          2.2          4.0          1.0 versicolor
## 64          6.1          2.9          4.7          1.4 versicolor
## 65          5.6          2.9          3.6          1.3 versicolor
## 66          6.7          3.1          4.4          1.4 versicolor
## 67          5.6          3.0          4.5          1.5 versicolor
## 68          5.8          2.7          4.1          1.0 versicolor
## 69          6.2          2.2          4.5          1.5 versicolor
## 70          5.6          2.5          3.9          1.1 versicolor
## 71          5.9          3.2          4.8          1.8 versicolor
## 72          6.1          2.8          4.0          1.3 versicolor
## 73          6.3          2.5          4.9          1.5 versicolor
## 74          6.1          2.8          4.7          1.2 versicolor
## 75          6.4          2.9          4.3          1.3 versicolor
## 76          6.6          3.0          4.4          1.4 versicolor
## 77          6.8          2.8          4.8          1.4 versicolor
## 78          6.7          3.0          5.0          1.7 versicolor
## 79          6.0          2.9          4.5          1.5 versicolor
## 80          5.7          2.6          3.5          1.0 versicolor
## 81          5.5          2.4          3.8          1.1 versicolor
## 82          5.5          2.4          3.7          1.0 versicolor
## 83          5.8          2.7          3.9          1.2 versicolor
## 84          6.0          2.7          5.1          1.6 versicolor
```

```
## 85          5.4          3.0          4.5          1.5 versicolor
## 86          6.0          3.4          4.5          1.6 versicolor
## 87          6.7          3.1          4.7          1.5 versicolor
## 88          6.3          2.3          4.4          1.3 versicolor
## 89          5.6          3.0          4.1          1.3 versicolor
## 90          5.5          2.5          4.0          1.3 versicolor
## 91          5.5          2.6          4.4          1.2 versicolor
## 92          6.1          3.0          4.6          1.4 versicolor
## 93          5.8          2.6          4.0          1.2 versicolor
## 94          5.0          2.3          3.3          1.0 versicolor
## 95          5.6          2.7          4.2          1.3 versicolor
## 96          5.7          3.0          4.2          1.2 versicolor
## 97          5.7          2.9          4.2          1.3 versicolor
## 98          6.2          2.9          4.3          1.3 versicolor
## 99          5.1          2.5          3.0          1.1 versicolor
## 100         5.7          2.8          4.1          1.3 versicolor
## 101         6.3          3.3          6.0          2.5  virginica
## 102         5.8          2.7          5.1          1.9  virginica
## 103         7.1          3.0          5.9          2.1  virginica
## 104         6.3          2.9          5.6          1.8  virginica
## 105         6.5          3.0          5.8          2.2  virginica
## 106         7.6          3.0          6.6          2.1  virginica
## 107         4.9          2.5          4.5          1.7  virginica
## 108         7.3          2.9          6.3          1.8  virginica
## 109         6.7          2.5          5.8          1.8  virginica
## 110         7.2          3.6          6.1          2.5  virginica
## 111         6.5          3.2          5.1          2.0  virginica
## 112         6.4          2.7          5.3          1.9  virginica
## 113         6.8          3.0          5.5          2.1  virginica
## 114         5.7          2.5          5.0          2.0  virginica
## 115         5.8          2.8          5.1          2.4  virginica
## 116         6.4          3.2          5.3          2.3  virginica
## 117         6.5          3.0          5.5          1.8  virginica
## 118         7.7          3.8          6.7          2.2  virginica
## 119         7.7          2.6          6.9          2.3  virginica
## 120         6.0          2.2          5.0          1.5  virginica
## 121         6.9          3.2          5.7          2.3  virginica
## 122         5.6          2.8          4.9          2.0  virginica
## 123         7.7          2.8          6.7          2.0  virginica
## 124         6.3          2.7          4.9          1.8  virginica
## 125         6.7          3.3          5.7          2.1  virginica
## 126         7.2          3.2          6.0          1.8  virginica
## 127         6.2          2.8          4.8          1.8  virginica
## 128         6.1          3.0          4.9          1.8  virginica
## 129         6.4          2.8          5.6          2.1  virginica
## 130         7.2          3.0          5.8          1.6  virginica
## 131         7.4          2.8          6.1          1.9  virginica
## 132         7.9          3.8          6.4          2.0  virginica
## 133         6.4          2.8          5.6          2.2  virginica
## 134         6.3          2.8          5.1          1.5  virginica
```
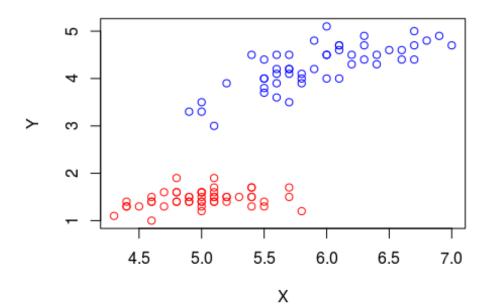
```
## 135            6.1          2.6          5.6          1.4  virginica
## 136            7.7          3.0          6.1          2.3  virginica
## 137            6.3          3.4          5.6          2.4  virginica
## 138            6.4          3.1          5.5          1.8  virginica
## 139            6.0          3.0          4.8          1.8  virginica
## 140            6.9          3.1          5.4          2.1  virginica
## 141            6.7          3.1          5.6          2.4  virginica
## 142            6.9          3.1          5.1          2.3  virginica
## 143            5.8          2.7          5.1          1.9  virginica
## 144            6.8          3.2          5.9          2.3  virginica
## 145            6.7          3.3          5.7          2.5  virginica
## 146            6.7          3.0          5.2          2.3  virginica
## 147            6.3          2.5          5.0          1.9  virginica
## 148            6.5          3.0          5.2          2.0  virginica
## 149            6.2          3.4          5.4          2.3  virginica
## 150            5.9          3.0          5.1          1.8  virginica
```

```r
# sepal and petal length
irissubdf <- iris[1:100, c(1, 3, 5)]
names(irissubdf) <- c("sepal", "petal", "species")
irissubdf
```

```
##      sepal petal    species
## 1      5.1   1.4     setosa
## 2      4.9   1.4     setosa
## 3      4.7   1.3     setosa
## 4      4.6   1.5     setosa
## 5      5.0   1.4     setosa
## 6      5.4   1.7     setosa
## 7      4.6   1.4     setosa
## 8      5.0   1.5     setosa
## 9      4.4   1.4     setosa
## 10     4.9   1.5     setosa
## 11     5.4   1.5     setosa
## 12     4.8   1.6     setosa
## 13     4.8   1.4     setosa
## 14     4.3   1.1     setosa
## 15     5.8   1.2     setosa
## 16     5.7   1.5     setosa
## 17     5.4   1.3     setosa
## 18     5.1   1.4     setosa
## 19     5.7   1.7     setosa
## 20     5.1   1.5     setosa
## 21     5.4   1.7     setosa
## 22     5.1   1.5     setosa
## 23     4.6   1.0     setosa
## 24     5.1   1.7     setosa
## 25     4.8   1.9     setosa
## 26     5.0   1.6     setosa
## 27     5.0   1.6     setosa
```

```
## 28      5.2    1.5      setosa
## 29      5.2    1.4      setosa
## 30      4.7    1.6      setosa
## 31      4.8    1.6      setosa
## 32      5.4    1.5      setosa
## 33      5.2    1.5      setosa
## 34      5.5    1.4      setosa
## 35      4.9    1.5      setosa
## 36      5.0    1.2      setosa
## 37      5.5    1.3      setosa
## 38      4.9    1.4      setosa
## 39      4.4    1.3      setosa
## 40      5.1    1.5      setosa
## 41      5.0    1.3      setosa
## 42      4.5    1.3      setosa
## 43      4.4    1.3      setosa
## 44      5.0    1.6      setosa
## 45      5.1    1.9      setosa
## 46      4.8    1.4      setosa
## 47      5.1    1.6      setosa
## 48      4.6    1.4      setosa
## 49      5.3    1.5      setosa
## 50      5.0    1.4      setosa
## 51      7.0    4.7 versicolor
## 52      6.4    4.5 versicolor
## 53      6.9    4.9 versicolor
## 54      5.5    4.0 versicolor
## 55      6.5    4.6 versicolor
## 56      5.7    4.5 versicolor
## 57      6.3    4.7 versicolor
## 58      4.9    3.3 versicolor
## 59      6.6    4.6 versicolor
## 60      5.2    3.9 versicolor
## 61      5.0    3.5 versicolor
## 62      5.9    4.2 versicolor
## 63      6.0    4.0 versicolor
## 64      6.1    4.7 versicolor
## 65      5.6    3.6 versicolor
## 66      6.7    4.4 versicolor
## 67      5.6    4.5 versicolor
## 68      5.8    4.1 versicolor
## 69      6.2    4.5 versicolor
## 70      5.6    3.9 versicolor
## 71      5.9    4.8 versicolor
## 72      6.1    4.0 versicolor
## 73      6.3    4.9 versicolor
## 74      6.1    4.7 versicolor
## 75      6.4    4.3 versicolor
## 76      6.6    4.4 versicolor
## 77      6.8    4.8 versicolor
```

```
## 78     6.7    5.0 versicolor
## 79     6.0    4.5 versicolor
## 80     5.7    3.5 versicolor
## 81     5.5    3.8 versicolor
## 82     5.5    3.7 versicolor
## 83     5.8    3.9 versicolor
## 84     6.0    5.1 versicolor
## 85     5.4    4.5 versicolor
## 86     6.0    4.5 versicolor
## 87     6.7    4.7 versicolor
## 88     6.3    4.4 versicolor
## 89     5.6    4.1 versicolor
## 90     5.5    4.0 versicolor
## 91     5.5    4.4 versicolor
## 92     6.1    4.6 versicolor
## 93     5.8    4.0 versicolor
## 94     5.0    3.3 versicolor
## 95     5.6    4.2 versicolor
## 96     5.7    4.2 versicolor
## 97     5.7    4.2 versicolor
## 98     6.2    4.3 versicolor
## 99     5.1    3.0 versicolor
## 100    5.7    4.1 versicolor
```

```r
# setosa and versicolor data
g1_x = irissubdf[1:50,1]
g1_y = irissubdf[1:50,2]
g2_x = irissubdf[51:100,1]
g2_y = irissubdf[51:100,2]
g_x = c(g1_x, g2_x)
g_y = c(g1_y, g2_y)
N = 50
group = c(rep(-1,N), rep(1,N))

# scatter plot
print(g_x)
```

```
##   [1] 5.1 4.9 4.7 4.6 5.0 5.4 4.6 5.0 4.4 4.9 5.4 4.8 4.8 4.3 5.8 5.
## 7 5.4 5.1
##  [19] 5.7 5.1 5.4 5.1 4.6 5.1 4.8 5.0 5.0 5.2 5.2 4.7 4.8 5.4 5.2 5.
## 5 4.9 5.0
##  [37] 5.5 4.9 4.4 5.1 5.0 4.5 4.4 5.0 5.1 4.8 5.1 4.6 5.3 5.0 7.0 6.
## 4 6.9 5.5
##  [55] 6.5 5.7 6.3 4.9 6.6 5.2 5.0 5.9 6.0 6.1 5.6 6.7 5.6 5.8 6.2 5.
## 6 5.9 6.1
##  [73] 6.3 6.1 6.4 6.6 6.8 6.7 6.0 5.7 5.5 5.5 5.8 6.0 5.4 6.0 6.7 6.
## 3 5.6 5.5
##  [91] 5.5 6.1 5.8 5.0 5.6 5.7 5.7 6.2 5.1 5.7
```

```r
print(g_y)
```

```
##   [1] 1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 1.5 1.6 1.4 1.1 1.2 1.
5 1.3 1.4
##  [19] 1.7 1.5 1.7 1.5 1.0 1.7 1.9 1.6 1.6 1.5 1.4 1.6 1.6 1.5 1.5 1.
4 1.5 1.2
##  [37] 1.3 1.4 1.3 1.5 1.3 1.3 1.3 1.6 1.9 1.4 1.6 1.4 1.5 1.4 4.7 4.
5 4.9 4.0
##  [55] 4.6 4.5 4.7 3.3 4.6 3.9 3.5 4.2 4.0 4.7 3.6 4.4 4.5 4.1 4.5 3.
9 4.8 4.0
##  [73] 4.9 4.7 4.3 4.4 4.8 5.0 4.5 3.5 3.8 3.7 3.9 5.1 4.5 4.5 4.7 4.
4 4.1 4.0
##  [91] 4.4 4.6 4.0 3.3 4.2 4.2 4.2 4.3 3.0 4.1
```

```r
print(group)
```

```
##   [1] -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
 -1 -1 -1 -1
##  [26] -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
 -1 -1 -1 -1
##  [51]  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
  1  1  1  1
##  [76]  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
  1  1  1  1
```

```r
plot(g_x, g_y, type='n', xlab='X', ylab='Y')
points(g1_x, g1_y, col='red')
points(g2_x, g2_y, col='blue')
```

```r
# parameter initialization
θ0 = 0.1 # initial weitht
θ1 = 0.2 # initial weight
θ2 = 0.3 # initial weitht

M = 15            # number of epochs to run
eta = 0.005       # learning rate
th = 0.9          # threshold to stop
verbose = F    # whether detailed weight update info is printed

for (i in 1:M){
  print(paste('Epoch starts: ', i))

  ## We reshuffle the order of the datapoint for each epoch.
  index = 1:(2*N)
  index = sample(index)

  for (j in index){
    y_j = θ0 + θ1*g_x[j] + θ2*g_y[j]
    if (y_j >= 0){
      pred_j = 1
    }else{
      pred_j = -1}

    θ0 = θ0 + eta*(group[j] - pred_j)*1.0
    θ1 = θ1 + eta*(group[j] - pred_j)*g_x[j]
    θ2 = θ2 + eta*(group[j] - pred_j)*g_y[j]
    if (verbose == T){
      print(paste('  -> updating data point ', j, ' : '))
      print(paste('     -> θ0: ' ,θ0))
      print(paste('     -> θ0: ' ,θ1))
      print(paste('     -> θ0: ' ,θ2))
    }
  }
  y_all = θ0 + θ1*g_x + θ2*g_y
  y_pred = y_all
  y_pred[y_all >= 0] = 1
  y_pred[y_all< 0] = -1

  acc = sum(y_pred == group)/length(group)
  print(paste('Epoch ends: ', i, ' WITH accuracy: ', acc))
  if (acc >= th){
    break
  }
}

## [1] "Epoch starts:  1"
## [1] "Epoch ends:  1  WITH accuracy:  1"
```

```r
y_all = 00 + 01*g_x + 02*g_y
print(y_all)
```

```
##   [1] -0.2187 -0.1973 -0.1964 -0.1447 -0.2080 -0.1893 -0.1652 -0.187
5 -0.1438
##  [10] -0.1768 -0.2303 -0.1456 -0.1866 -0.1946 -0.3346 -0.2624 -0.271
3 -0.2187
##  [19] -0.2214 -0.1982 -0.1893 -0.1982 -0.2472 -0.1572 -0.0841 -0.167
0 -0.1670
##  [28] -0.2089 -0.2294 -0.1349 -0.1456 -0.2303 -0.2089 -0.2615 -0.176
8 -0.2490
##  [37] -0.2820 -0.1973 -0.1643 -0.1982 -0.2285 -0.1750 -0.1643 -0.167
0 -0.1162
##  [46] -0.1866 -0.1777 -0.1652 -0.2196 -0.2080  0.2545  0.2777  0.306
2  0.2715
##  [55]  0.2875  0.3526  0.3294  0.1922  0.2768  0.2831  0.2225  0.269
7  0.2180
##  [64]  0.3508  0.1788  0.2251  0.3633  0.2599  0.2991  0.2403  0.392
7  0.2073
##  [73]  0.3704  0.3508  0.2367  0.2358  0.2964  0.3481  0.3205  0.147
6  0.2305
##  [82]  0.2100  0.2189  0.4435  0.3847  0.3205  0.2866  0.2679  0.281
3  0.2715
##  [91]  0.3535  0.3303  0.2394  0.1815  0.3018  0.2911  0.2911  0.258
1  0.1093
## [100]  0.2706
```

```r
# prediction
y_pred = y_all
y_pred[y_all >= 0] = 1
y_pred[y_all< 0] = -1

print(y_pred)
```

```
##   [1] -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
 -1 -1 -1 -1
##  [26] -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
 -1 -1 -1 -1
##  [51]  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
  1  1  1  1
##  [76]  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
  1  1  1  1
```

```r
# accuracy and dividing line
acc = sum(y_pred == group)/length(group)
print(acc)
```

```
## [1] 1
```

```r
plot(g_x, g_y, type='n', xlab='X', ylab='Y')
points(g1_x, g1_y, col='red')
```

```
points(g2_x, g2_y, col='blue')
abline(a = -1.0*θ0/θ2, b = -1.0*θ1/θ2, col='dark green', lwd=3, lty=2)
```