CSC0056 Data Communication

The Aloha System

Instructor: Chao Wang

Networked Cyber-Physical Systems Laboratory
Department of Computer Science and Information Engineering
National Taiwan Normal University

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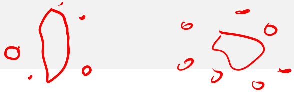
References

1. Harchol-Balter, Mor. Performance modeling and design of computer systems: queueing theory in action. Cambridge University Press, 2013. ISBN 9781107027503. (Section 10.2)
2. Bertsekas, Dimitri and Gallager, Robert. Data networks (2nd edition). Prentice Hall, 1992. ISBN 0132009161. (Section 4.1–4.2.2)

Outline

1. The Aloha system and its design
2. The Aloha system analysis
   - Worst-Case Analysis
   - Best-Case Analysis
3. Concluding remarks

The Aloha system (the Slotted Aloha)

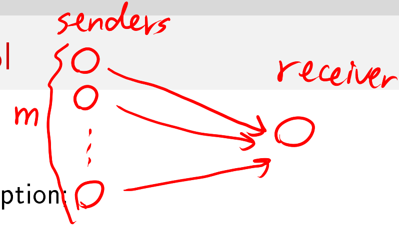


- The Aloha network: developed in 1970s at University of Hawaii, a precursor to the Ethernet protocol. e.g. LoRaWAN
- Definition of the Slotted Aloha protocol: Assume that time is divided into discrete time units called *time slots* (or simply, slots), and that each data item requires one slot for transmission. Given one receiver and m senders that share the communication channel, each of the m nodes may independently decide whether or not to send one data item at the beginning of a slot. gateway
  - If only one node transmits at a slot, the transmission is *successful.* backend
  - collision → If multiple nodes transmit at the same slot, all of them are *unsuccessful.* server
  - Each node having an unsuccessful transmission is called a *backlogged node,* and it will try to retransmit in some later slot.
- The initial design of the Aloha system suffered from occasional freezes, after which no transmission will be successful. In the following, we will use queueing analysis to make sense about it, and to guide us into some improved design :)

Design considerations for the Slotted Aloha system

1. Rate of transmission:
    1. From the receiver's view, what might be the needed transmission rate (for data application)?
    2. From the sender's view, what might be the achievable rate?
    3. How does the rate of transmission impact the choice of the rate of retransmission?
2. Rate of retransmission:
    1. What if every backlogged node makes retransmission at the next slot?
    2. If the rate of retransmission is too slow, would that cause the average delay to be longer than the system using some time-division transmission schedule?
    3. What to do if a backlogged node has some new data to transmit?
3. Number of senders:
    1. How does the number of senders impact the above two parameters?
    2. What might be the reasonable range of number of senders for an application?

Analyzing the slotted Aloha protocol



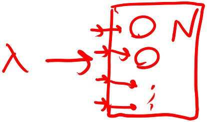
Assumption-1 Either collision or perfect reception;

Assumption-2 Immediate feedback;

Assumption-3a (best case) No buffering;

Assumption-3b (worst case) $m \to \infty$ (m = number of senders; see [2]).

- We may estimate the average waiting time for each data item in this way: First, build a Markov chain to analyze the average number of backlogged nodes at the end of a time slot, and denote this number by N. Then, along with a given rate of transmission (due to the arrivals of new data items), we may obtain the average waiting time by Little's Law :)



$N = \lambda \cdot T$

Initial modeling and the maximum throughput

(Assumption 3b.)

$\lambda = m \cdot \frac{\lambda}{m} \longrightarrow$ $m \begin{pmatrix} O \\ O \\ \vdots \\ O \end{pmatrix}$

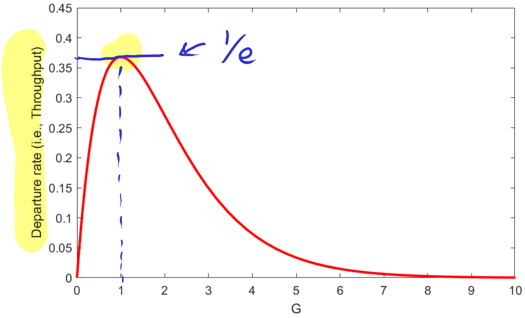
- Assume that for each of the m nodes, a new data item that needs transmission arrives according to a Poisson processes with arrival rate λ/m. Under this model, each sender will transmit with probability $p = 1 - e^{-\lambda/m}$, and the overall arrivals of those new data items will follow a Poisson process with parameter λ.
- Therefore, the probability of only one new arrival in the current slot is

$$P\{A(t+1) - A(t) = 1\} = \frac{\lambda^1}{1!}e^{-\lambda} = \lambda e^{-\lambda}.$$

- Take into account the retransmissions, then, approximately, the total number of transmissions in a slot is a Poisson random variable with parameter $G > \lambda$, and the probability of a *successful* transmission in a slot is Ge^{-G}.

Initial modeling and the maximum throughput (cont.)

- Now we may plot this probability of a successful transmission in a slot, Ge^{-G}, and get some initial idea of the dynamics of the system:



- The maximum possible throughput is $1/e$, which happens when $G = 1$ by considering $\frac{d}{dG}(Ge^{-G}) = 0$. $\frac{d}{dx}(xe^{-x}) = x\cdot(-e^{-x}) + 1\cdot e^{-x}$

Worst-case analysis (Assumption-3b) (Sec. 10.2.1 in [1])

First, assume $m \to \infty$ and that each sender makes a new transmission at each slot with probability p. Further, assume that each backlogged node will make a retransmission at the next slot with probability q, until the retransmission become successful.

Now, let p_k be the probability that at a certain time slot there are k new transmissions. Then we have

$$p_k = b(k; m, p) = \binom{m}{k} p^k (1-p)^{m-k}.$$

Similarly, let q_k^n be the probability that at a certain time slot, out of the n backlogged nodes there are k of them retransmit. Then we have

$$q_k^n = b(k; n, q) = \binom{n}{k} q^k (1-q)^{n-k}.$$

Worst-case analysis (cont.)

\# of backlogged nodes

Let $P_{i,j}$ be the probability of moving from state i to state j, and we have

no new transmission

just one transmission

$$P_{0,0} = (1-p)^m + mp(1-p)^{m-1};$$

$$P_{0,j} = \binom{m}{j} p^j (1-p)^{m-j} \quad \text{for } 2 \le j \le m;$$

$$P_{0,j} = 0 \quad \text{for } j = 1 \text{ or } j > m;$$

$$P_{n,j} = 0 \quad \text{for } j \le n-2 \text{ or } j > n+m;$$

no new transmission

just one transmission out of n backlogged nodes

$$P_{n,n-1} = (1-p)^m nq(1-q)^{n-1};$$

$$P_{n,n} = m(1-q)^n p(1-p)^{m-1} + (1-nq(1-q)^{n-1})(1-p)^m;$$

case ①: one new only

case ②: no one moves

case ③: no new & multiple re-trans

$$P_{n,n+1} = \binom{m}{1} p(1-p)^{m-1}(1-(1-q)^n);$$

one from new

at least one retransmission

$$P_{n,n+j} = P_{0,j} \quad \text{for } 2 \le j \le m.$$

Worst-case analysis (cont.)

Now, let $P_{\text{back}}^{(n)}$ be the probability of moving from state n to a lower numbered state. Then we can compute the limiting probabilities as follows:

$$
\begin{aligned}
\lim_{n\to\infty} P_{\text{back}}^{(n)} &= \lim_{n\to\infty} n(1-p)^m q(1-q)^{n-1} \\
&= q(1-p)^m \lim_{n\to\infty} \frac{n}{(1-q)^{1-n}} = q(1-p)^m \lim_{n\to\infty} \frac{\frac{\partial}{\partial n} n}{\frac{\partial}{\partial n}((1-q)^{1-n})} \\
&= q(1-p)^m \lim_{n\to\infty} \frac{1}{((1-q)^{1-n})(\ln(1-q))(-1)} \\
&= 0.
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty} P_{n,n} &= \lim_{n\to\infty} (m(1-q)^n p(1-p)^{m-1} + (1-nq(1-q)^{n-1})(1-p)^m) \\
&= (1-p)^m - \lim_{n\to\infty} P_{\text{back}}^{(n)} \\
&= (1-p)^m.
\end{aligned}
$$

Worst-case analysis (cont.)

(continued from the previous page)
This implies that, as n increases, we will have

- the probability of moving to a lower numbered state tends to zero;
- the probability of staying at the current state tends to $(1 - p)^m$;
- the probability of moving to a higher numbered state tends to $1 - (1 - p)^m$.

This help explains why the initial design of the Aloha system may suffer from occasional freezes, after which no transmission will be successful.

A design idea based on the analytical result

- Key idea: we need to prevent the expected total number of transmissions at a state to be higher than 1. → prevent $mp+nq \geq 1$
- Given that the system is in state n, the expected total number of transmissions $E_n[\#]$ is

$$E_n[\#] = mp + nq$$

we want $mp+nq < 1$

and we see that, for example, whenever n gets higher than $\frac{1}{q}$ then unavoidably the system will have $E_n[\#] > 1$.

$\Rightarrow nq < 1 - mp$

$\Rightarrow q < \frac{1-mp}{n}$

- A plausible design: set $q < \frac{1-mp}{n}$.
- A downside of such a design: ______ waiting time

Graphical analysis (worst case)

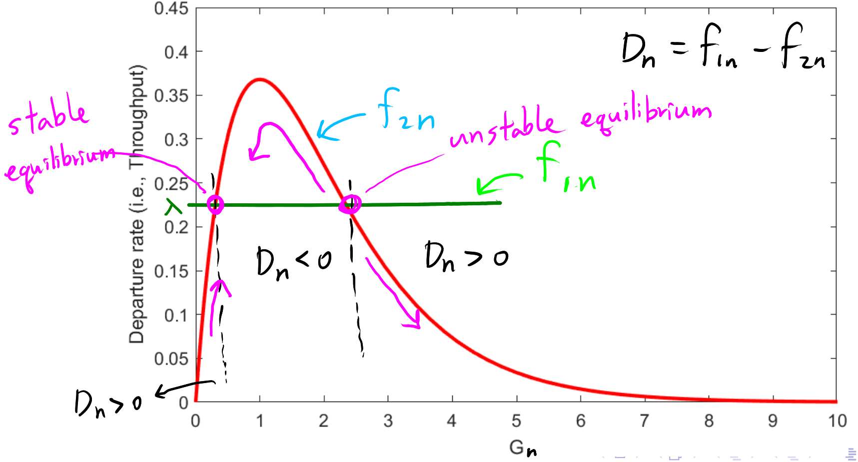
- To gain some more insight into the dynamics of the system, often we may resort to graphical analysis.
- Now, we define *the drift D_n* to be the expected change in the number of backlogged nodes over one slot time, starting in state n:

$$D_n = f_{1n} - f_{2n}$$

  where f_{1n} is the expected total number of new transmissions in a slot and f_{2n} the expected total number of successful transmissions therein.
    - $f_{1n} = \lambda$.
    - $f_{2n} = G_n e^{-G_n}$, where $G_n = \lambda + nq$ is the rate of transmissions in a slot.
- Graphically, D_n is equal to the difference between the curve of the arrival rate and that of the departure rate.

Graphical analysis (worst case) (cont.)

- By looking at the plot of our function of interest, we may gain great insights into the dynamics of a system.
- The plot for $G_n e^{-G_n}$, the arrival rate λ, and the drift:



Best-case analysis (Assumption-3a) (see page 6)

- The assumption of *no buffering*: All the backlogged nodes will discard any new data items and will not transmit any of them.
- Compared with the use of Assumption-3b ($m \to \infty$), now the system will have a lower probability to make k transmissions at a time slot (cf. page 9). So p_k in this case is smaller, while q_k^n remaining the same:

$$p_k = b(k; m-n, p) = \binom{m-n}{k} p^k (1-p)^{m-n-k};$$

$$q_k^n = b(k; n, q) = \binom{n}{k} q^k (1-q)^{n-k}.$$

- Also, in applying the DTMC, the transition probabilities $P_{i,j}$ will be different from that on page 10. See Equation (4.3) on page 279 in the second textbook.

Graphical analysis (best case)

- Following the same definition for *the drift* D_n:

$$D_n = f_{1n} - f_{2n}.$$

And now with the Assumption-3a, we have

- $f_{1n} = (m - n)p.$
- $f_{2n} = G_n e^{-G_n}$, where now $G_n = (m - n)p + nq$

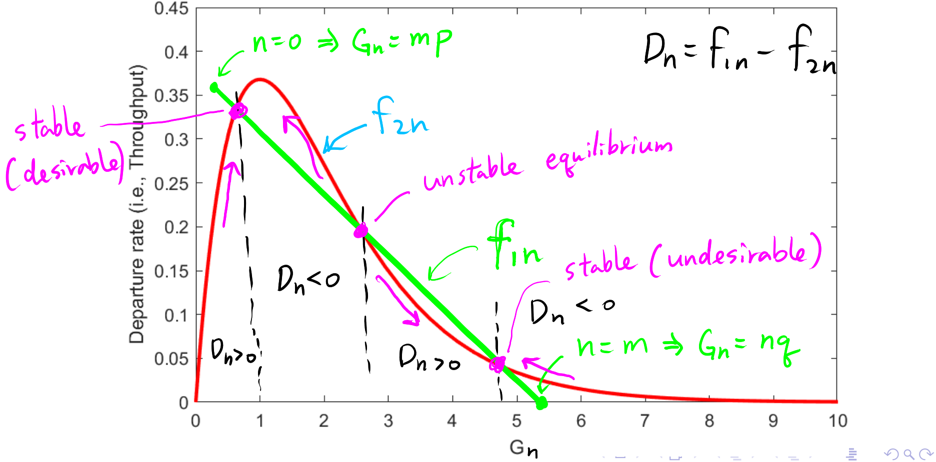
Graphical analysis (best case) (cont.)

$G_n = (m-n)p + nq$

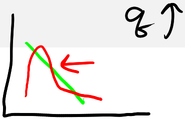
$f_{1n} = (m-n)p$

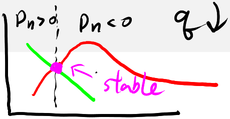
- Again, by looking at the plot, we see that there could be *stable equilibrium* and *unstable equilibrium*.
- The plot of $G_n e^{-G_n}$, the arrival rate λ, and the drift:

assumption: $p<q$



Insights learned from the graphical analysis, and a design stemmed from them





- Stable/unstable equilibriums.
- The impact of arrival rate λ.
- The impact of retransmission probability q.
- A design for the choice of q: exponential backoff [1]
  - Each sender waits for some random time after each collision before another retransmission. Set the mean waiting time to be an exponentially increasing function of the number of collisions the sender experienced so far.

[1] To learn more on the design of the Ethernet protocol, see: *Kurose, Jim and Ross, Keith. Computer Networking: A Top-Down Approach. Pearson, 2010. ISBN 0136079679.*

Concluding remarks

- Think about how the queueing analysis is used here. This is a concrete example of using queueing analysis.
- Design and performance of the Slotted Aloha protocol
  - Worst-case analysis, best-case analysis, graphical analysis
- Optional futher study: see the second textbook for further development on multiaccess communication along this line:
  - Stablizing Slotted Aloha (Section 4.2.3)
  - Unslotted Aloha (Section 4.2.4)
  - Improving throughput beyond $1/e$ (Section 4.3)
  - Carrier sensing (Section 4.4)