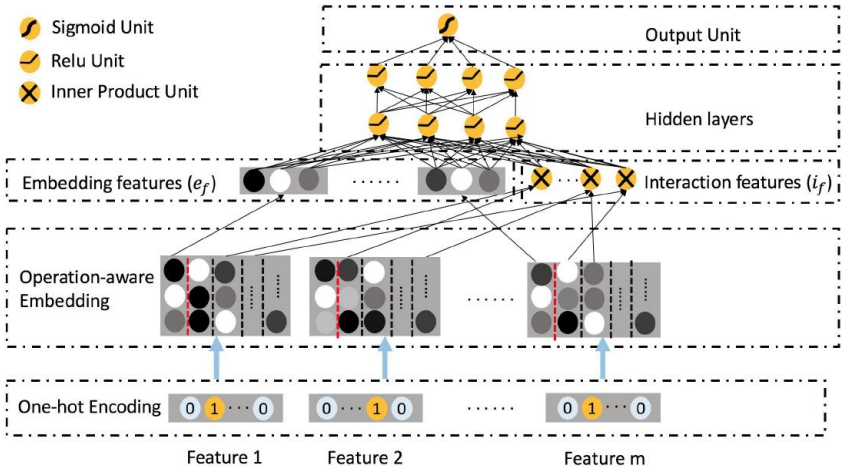
2016
PNN

2019
ONN

2020
LorentzFM

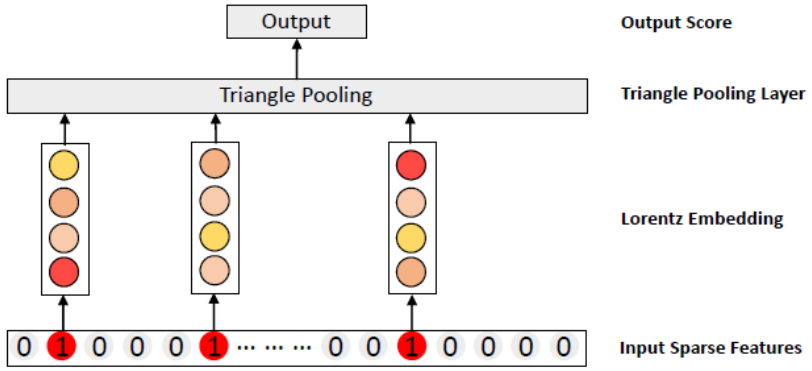
2019
HFM

Field-aware Embedding Layer → Operation-aware Embedding



Operation-aware Neural Networks (ONN)

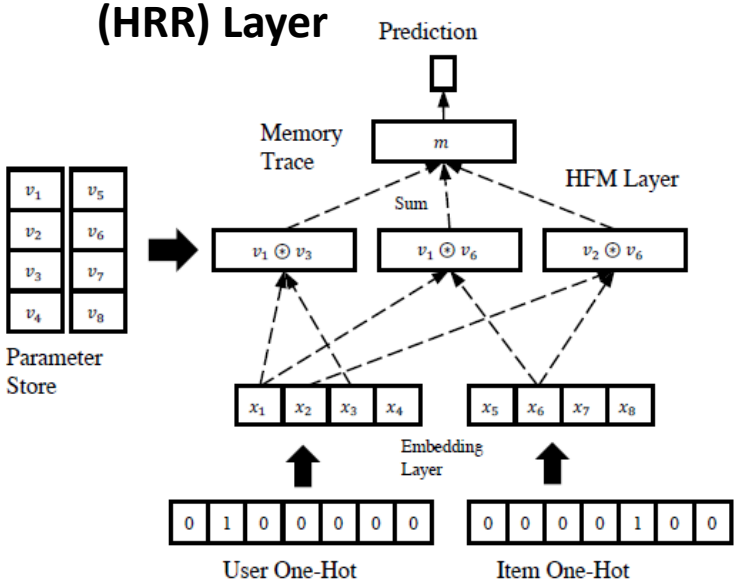
Field-aware Embedding + Production Layer→ Lorentz embedding + Triangle Pooling



$$\mathcal{T}(\mathbf{u},\mathbf{v}) = \frac{d^2_{\mathcal{L}}(\mathbf{u},\mathbf{v}) - d^2_{\mathcal{L}}(\mathbf{0},\mathbf{u}) - d^2_{\mathcal{L}}(\mathbf{0},\mathbf{v})}{2\langle \mathbf{0},\mathbf{u}\rangle_{\mathcal{L}}\langle \mathbf{0},\mathbf{v}\rangle_{\mathcal{L}}} = \frac{1 - \langle \mathbf{u},\mathbf{v}\rangle_{\mathcal{L}} - u_0 - v_0}{u_0 v_0} = \underbrace{\frac{1 - \langle \mathbf{u},\mathbf{v}\rangle_{\mathcal{L}}}{u_0 v_0}}_{\text{interaction term}} - \underbrace{\left(\frac{1}{u_0} + \frac{1}{v_0}\right)}_{\text{linear term}}.$$

Lorentz embedding

Production Layer → Holographic reduced representations (HRR) Layer



Holographic Factorization Machines (HFM)