- Numerator:

$$\begin{aligned}
\sum_{i=1}^{s} N\ p_i\ p(c(\alpha_i) = r) &= \sum_{i=1}^{s} N\ p_i \binom{N}{r} p_i^r (1-p_i)^{N-r} \\
&= N\ \frac{N!}{N-r!r!} p_i^{r+1} (1-p_i)^{N-r} \\
&= N\ \frac{(r+1)}{N+1} \frac{N+1!}{N-r!r+1!} p_i^{r+1} (1-p_i)^{N-r} \\
&= (r+1)\ \frac{N}{N+1} E_{N+1}(N_{r+1}) \\
&\simeq (r+1)\ E_{N+1}(N_{r+1})
\end{aligned}$$