Using the simplifications of numerator and denominator:

$$r^* = E(c^*(\alpha)|c(\alpha) = r)$$

$$= \frac{(r+1) E_{N+1}(N_{r+1})}{E_N(N_r)}$$

 $\simeq (r+1)\frac{N_{r+1}}{N_r}$