The differential cross-section for coherent vector meson production:

$$\frac{\mathrm{d}\sigma^{\gamma^*+A\to V+A}}{\mathrm{d}|t|} = \frac{1}{16\pi}\left|\langle\mathcal{A}\rangle_\Omega\right|^2 \tag{1}$$

Note

$$\mathrm{d}|t| = \mathrm{d}|\boldsymbol{\Delta}_\perp|^2 = 2|\boldsymbol{\Delta}_\perp|\mathrm{d}|\boldsymbol{\Delta}_\perp| = \frac{1}{\pi}\mathrm{d}^2\boldsymbol{\Delta}_\perp \tag{2}$$

Then we can write the differential cross-section as

$$\frac{\mathrm{d}\sigma^{\gamma^*+A\to V+A}}{\mathrm{d}^2\boldsymbol{\Delta}_\perp} = \frac{1}{16\pi^2}\left|\langle\mathcal{A}\rangle_\Omega\right|^2 \tag{3}$$

$$\sigma^{\gamma^*+A\to V+A} = \frac{1}{16\pi^2}\int \mathrm{d}^2\boldsymbol{\Delta}_\perp \left|\langle\mathcal{A}\rangle_\Omega\right|^2 \tag{4}$$

Recall

$$\mathcal{A} = 2i\int \mathrm{d}^2\boldsymbol{r}_\perp \mathrm{d}^2\boldsymbol{b}_\perp \frac{\mathrm{d}z}{4\pi} e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}[\Psi_V^*\Psi_\gamma](Q^2,\boldsymbol{r}_\perp,z)N_\Omega(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp,z,x) \tag{5}$$

where[1]

$$N_\Omega(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp,z,x) = 1 - \frac{1}{N_c}\mathrm{Tr}\left[V(\boldsymbol{b}_\perp + (1-z)\boldsymbol{r}_\perp)V^\dagger(\boldsymbol{b}_\perp - z\boldsymbol{r}_\perp)\right] \tag{6}$$

Let us define

$$F(\boldsymbol{b}_\perp,x) = \int \mathrm{d}^2\boldsymbol{r}_\perp \mathrm{d}z[\Psi_V^*\Psi_\gamma](Q^2,\boldsymbol{r}_\perp,z)N_\Omega(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp,z,x) \tag{7}$$

Then

$$\mathcal{A} = i\int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{2\pi} e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}F(\boldsymbol{b}_\perp,x) \tag{8}$$

Then

$$|\langle\mathcal{A}\rangle_\Omega|^2 = \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{2\pi} e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\langle F(\boldsymbol{b}_\perp,x)\rangle_\Omega \int \frac{\mathrm{d}^2\boldsymbol{b}'_\perp}{2\pi} e^{i\boldsymbol{b}'_\perp\cdot\boldsymbol{\Delta}_\perp}\langle F^*(\boldsymbol{b}'_\perp,x)\rangle_\Omega \tag{9}$$

The integral over $\boldsymbol{\Delta}_\perp$ is trivial as it only appears on the phase giving a $(2\pi)^2\delta^{(2)}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$, we then integrate over $\boldsymbol{b}'_\perp$

$$\begin{aligned}
\int \mathrm{d}^2\boldsymbol{\Delta}_\perp|\langle\mathcal{A}\rangle_\Omega|^2 &= \int \mathrm{d}^2\boldsymbol{\Delta}_\perp \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{2\pi} e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\langle F(\boldsymbol{b}_\perp,x)\rangle_\Omega \int \frac{\mathrm{d}^2\boldsymbol{b}'_\perp}{2\pi} e^{i\boldsymbol{b}'_\perp\cdot\boldsymbol{\Delta}_\perp}\langle F^*(\boldsymbol{b}'_\perp,x)\rangle_\Omega & (10)\\
&= \int \mathrm{d}^2\boldsymbol{b}_\perp\langle F(\boldsymbol{b}_\perp,x)\rangle_\Omega\langle F^*(\boldsymbol{b}_\perp,x)\rangle_\Omega & \\
&= \int \mathrm{d}^2\boldsymbol{b}_\perp|\langle F(\boldsymbol{b}_\perp,x)\rangle_\Omega|^2 & (11)
\end{aligned}$$

Finally, we have

$$\sigma^{\gamma^*+A\to V+A} = \frac{1}{16\pi^2}\int \mathrm{d}^2\boldsymbol{b}_\perp|\langle F(\boldsymbol{b}_\perp,x)\rangle_\Omega|^2 \tag{12}$$

[1] Off-forward phase has been absorbed into the dipole coordinates

where

$$F(\boldsymbol{b}_\perp, x) = \int \mathrm{d}^2\boldsymbol{r}_\perp \mathrm{d}z [\Psi_V^* \Psi_\gamma](Q^2, \boldsymbol{r}_\perp, z) N_\Omega(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, z, x) \tag{13}$$

Event-by-event $F(\boldsymbol{b}_\perp, x)$ might depend on the angle of $\boldsymbol{b}_\perp$ but after average it only depends on $|\boldsymbol{b}_\perp|$, so for a given x, $\langle F(|\boldsymbol{b}_\perp|, x)\rangle_\Omega$ can be stored in a one-dimensional table. $\langle F(|\boldsymbol{b}_\perp|, x)\rangle_\Omega$ should be a smooth function of $|\boldsymbol{b}_\perp|$, it can be understood as an x-dependent "effective" thickness function [2].

$$\sigma^{\gamma^*+A\to V+A} = \frac{1}{8\pi} \int |\boldsymbol{b}_\perp| \mathrm{d}|\boldsymbol{b}_\perp| |\langle F(|\boldsymbol{b}_\perp|, x)\rangle_\Omega|^2 \tag{14}$$

Then the total cross-section is computed from a one-dimensional integral over $|\boldsymbol{b}_\perp|$.

The incoherent cross-section can be computed very easily as well, this time we need

$$\sigma^{\gamma^*+A\to V+A^*} = \frac{1}{16\pi^2} \int \mathrm{d}^2\boldsymbol{b}_\perp \left[\langle |F(\boldsymbol{b}_\perp, x)|^2\rangle_\Omega - |\langle F(\boldsymbol{b}_\perp, x)\rangle_\Omega|^2\right] \tag{15}$$

Once again note $\langle |F(\boldsymbol{b}_\perp, x)|^2\rangle_\Omega$ can only depend on $|\boldsymbol{b}_\perp|$ so the integral is again 1D.

$$\sigma^{\gamma^*+A\to V+A^*} = \frac{1}{8\pi} \int |\boldsymbol{b}_\perp| \mathrm{d}|\boldsymbol{b}_\perp| \left[\langle |F(|\boldsymbol{b}_\perp|, x)|^2\rangle_\Omega - |\langle F(|\boldsymbol{b}_\perp|, x)\rangle_\Omega|^2\right] \tag{16}$$

[2] In the dilute limit in which $\langle N_\Omega(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, z, x)\rangle_\Omega = \langle N_\Omega(\boldsymbol{r}_\perp, x)\rangle_\Omega T(\boldsymbol{b}_\perp)$, then $\langle F(\boldsymbol{b}_\perp, x)\rangle_\Omega \propto T(\boldsymbol{b}_\perp)$