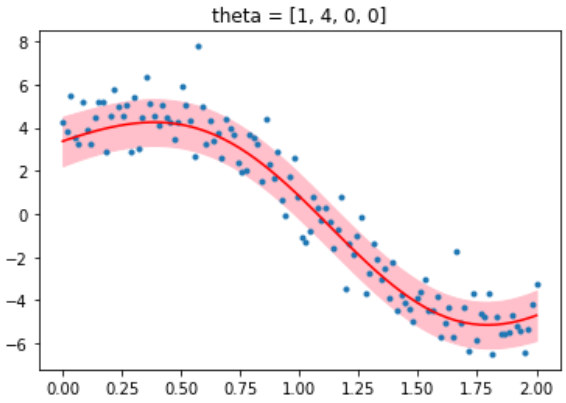
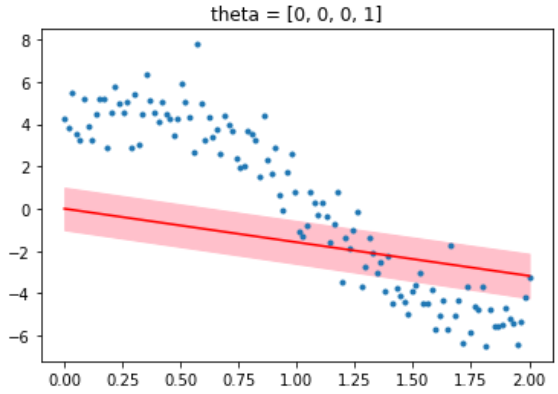
Machine Learning (Homework 3)

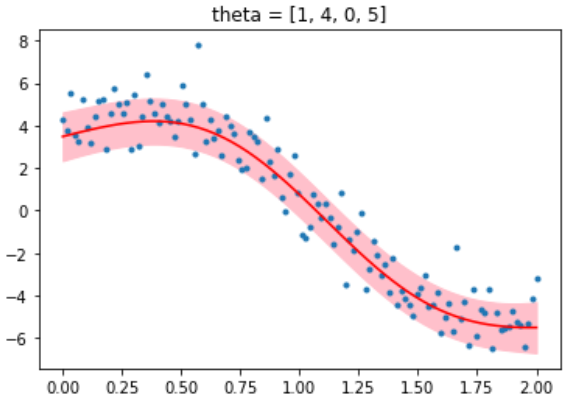
Due date : 1/4

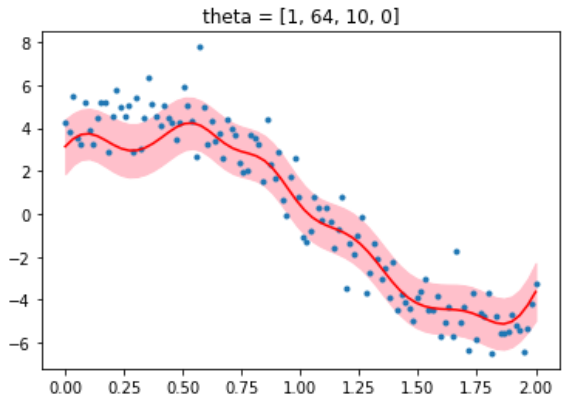
1 Gaussian Process (60%)

1. Please implement the Gaussian process based on an exponential-quadratic kernel function given by







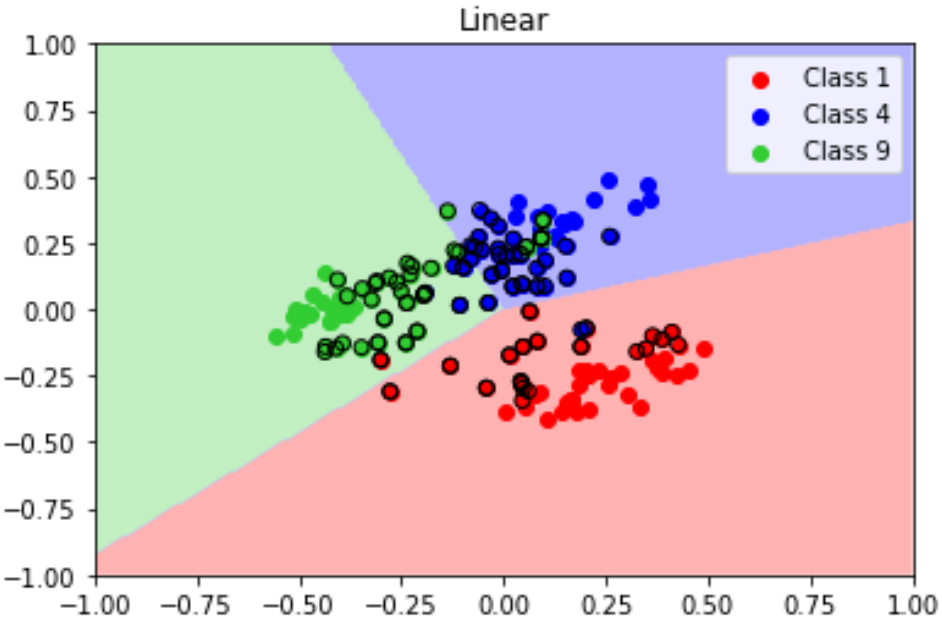


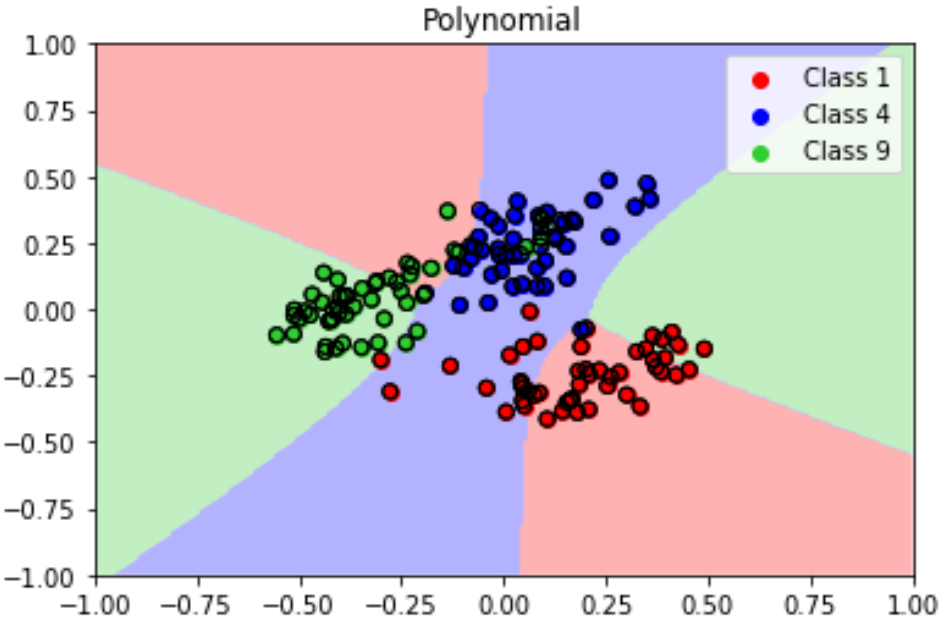
Theta[1,4,0,0]	Theta[0,0,0,1]	Theta[1,4,0,5]	Theta[1,64,10,0]
【RMSE】 train：1.085908 test：1.016760	【RMSE】 train：3.375501 test：3.849775	【RMSE】 train：1.076836 test：1.057627	【RMSE】 train：1.060665 test：1.243346

發現隨著 theta 值的調整，可以用來檢視自己是在哪一個 kernel 出錯，例如：當 theta 為 [0,0,0,1] 時，可以從數學式上看出來，只使用到 linear kernel，就可以藉此檢視 linear kernel 的正確性，可以看到 Gaussian kernel 的預測能力很強，有使用到 Gaussian kernel 的部分就可以大部分概括 sample。且 train 的 loss 通常都比 test 的 loss 低，合情合理。

2 Support Vector Machine (40%)

1. Use the dataset to build a SVM with linear kernel to do multi-class classification. Then plot the corresponding decision boundary and support vector.
2. Repeat (1) with polynomial kernel (degree = 2).
3. Please discuss the difference between (1), (2).





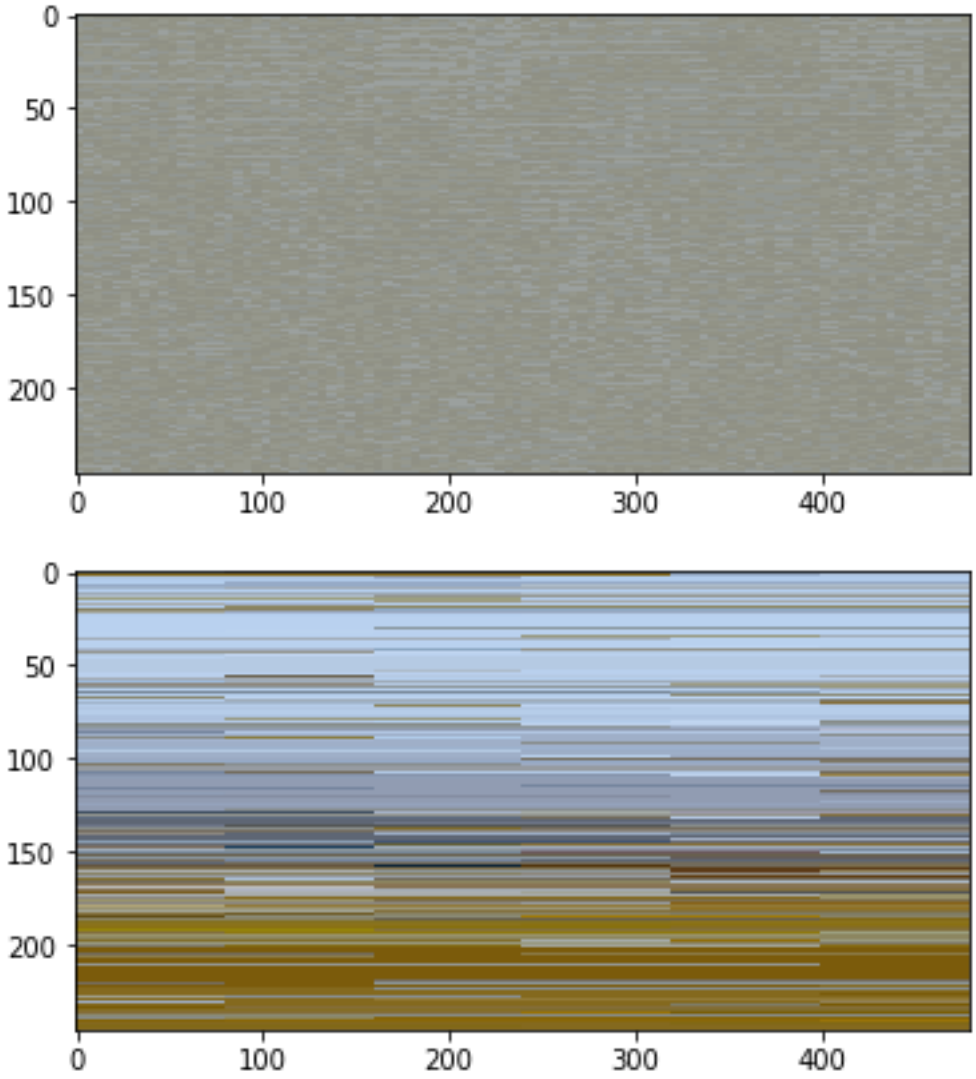
圖片裡面有加上黑框的點為 support vector，根據 predict 結果有些分類錯誤的情況，在上圖中也有觀察到這樣的錯誤。比較兩者，在做 linear svm 的時候，雖然有高比例的點為 support vector，但是到 polynomial svm 的時候，每個點都是 support vector(中間還一度誤以為自己寫錯了)。第二個不同點是，由於 kernel 計算的時候一個是有經過 phi process，一個是直接計算，因此出來的圖形會有線性跟非線性區塊的差異。從兩圖來看，感覺針對這份資料使用 linear svm 會是比較好的選擇。

3 Gaussian Mixture Model (30%)

1. Please build a *K*-means model by minimizing

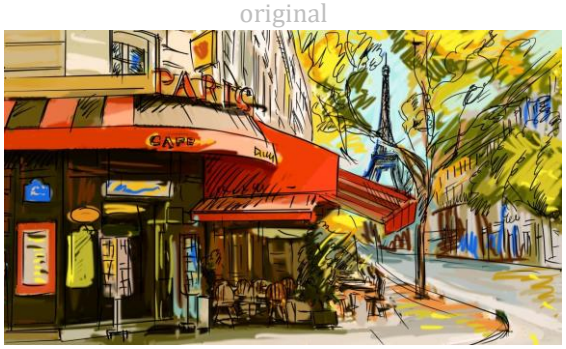
$$J = \sum_{n=1}^{N}\sum_{k=1}^{K}\gamma_{nk}||x_n - \mu_k||_2$$

and show the table of estimated $\{\mu_k\}_{k=1}^{K}$.



因為 Reshape 的部分失誤，因此出現的圖片似乎不太正確

2. Use $\{\mu_k\}_{k=1}^{K}$ calculated by the *K*-means model as means, and calculate the corresponding variances σ_k^2 and mixing coefficient π_k for the initialization of GMM $p(x) = \sum_{k=1}^{K}\pi_k\mathcal{N}(x|\mu_k, \sigma_k^2)$.
   Optimize the model by maximizing the log likelihood function $\log p(x|\pi,\mu,\sigma^2)$ over all training pixels through EM algorithm. Plot the log likelihood curve of GMM. (Please terminate EM algorithm when the iteration arrives 100)
3. Repeat step (1) and (2) for $K = 2, 3, 5$, and 20 respectively. Please show the resulting images in your report. Below are some examples.

K=3	*K*=5
	
K=7	original
	

– The input image is with licence free for personal and commercial use. Image from: https://www.pexels.com/photo/white-and-blue-house-under-cumulus-nimbus-clouds-906755/