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STAT 305

Assignment 1

Question 4

4.1 Operative group: $n = 27$

sample mean $\bar{y} = 93.5$

sample sd $s = 4.2$

$$\bar{Y} \sim N(\mu, \sigma^2/n)$$

$$\frac{\bar{Y} - \mu}{\sqrt{\sigma^2/n}} \sim Z \quad \text{where } Z \sim N(0,1)$$

$$Pr\left(-z_{0.975} < \frac{\bar{y} - \mu}{\sqrt{s^2/n}} < z_{0.975}\right) = 5\%$$

$$Pr\left(-z_{0.975}\sqrt{s^2/n} < \bar{y} - \mu < z_{0.975}\sqrt{s^2/n}\right) = 5\%$$

$$Pr\left(z_{0.975}\sqrt{s^2/n} > \mu - \bar{y} > -z_{0.975}\sqrt{s^2/n}\right) = 5\%$$

$$Pr\left(z_{0.975}\sqrt{s^2/n} + \bar{y} > \mu > -z_{0.975}\sqrt{s^2/n} + \bar{y}\right) = 5\%$$

95% confidence interval:

$$\vec{y} \pm z_{0.975}\sqrt{s^2/n} = 93.5 \pm 1.96\sqrt{4.2^2/27} \approx \boxed{[91.9,\ 95.1]}$$

4.2 I will write a denote non-op & b to denote op

Show: If we assume $\mu_a \overset{①}{=} \mu_b$ and $\overset{②}{\mu_a \in I_a}$ and $\overset{③}{\mu_b \in I_b}$, then $I_a \cap I_b \neq \emptyset$ ④

Prove by contradiction:

Given ①②③

Assume ④ doesn't hold, i.e. $I_a \cap I_b = \emptyset$

But from ①②③, we can conclude that $I_a \cap I_b$ must have at least one element in common (i.e. $\mu_a = \mu_b$), which contradicts our assumption that $I_a \cap I_b = \emptyset$

Therefore, given ①②③, ④ must hold

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① and ② and ③ → ④

⇒ not ④ → (not ①) or (not ②) or (not ③)

⇒ If $I_a \cap I_b = \emptyset$, then $\mu_a \neq \mu_b$ or $\mu_a \notin I_a$ or $\mu_b \notin I_b$

But we are given (b) $\mu_a = \mu_b$

⇒ If $\underbrace{I_a \cap I_b = \emptyset}_{M}$, then $\underbrace{\mu_a \notin I_a \text{ or } \mu_b \notin I_b}_{N}$

$Pr(N) \geq Pr(M)$

$$P(R) = P(I_a \cap I_b = \emptyset) \leq P(\mu_a \notin I_a \cup \mu_b \notin I_b)$$

$$= P(\mu_a \notin I_a) + P(\mu_b \notin I_b) - \underbrace{P(\mu_a \notin I_a \cap \mu_b \notin I_b)}_{= P(\mu_a \notin I_a) P(\mu_b \notin I_b) \text{ because of independence}}$$

$= 0.05 + 0.05 - 0.05^2 = 0.1 - 0.025 = 0.0975$

$P(R) \leq 0.0975$ upper bound for $P(R)$ is $\boxed{0.0975}$

4.3

If independence condition is removed,
we can no longer conclude

$$P(\mu_a \notin I_a \cap \mu_b \notin I_b) = P(\mu_a \notin I_a) P(\mu_b \notin I_b)$$

However, a less precise upper bound can still be obtained

$$P(R) \leq P(\mu_a \notin I_a) + P(\mu_b \notin I_b) - P(\mu_a \notin I_a \cap \mu_b \notin I_b)$$

$$\leq P(\mu_a \notin I_a) + P(\mu_b \notin I_b)$$

$$= 0.5 + 0.5 = \boxed{0.1}$$