A2Q1

1. $E(\tilde{\theta}_1) = E(2\overline{X}) = 2E(\overline{X}) = 2E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$

$= \frac{2}{n} E\left(\sum_{i=1}^{n} X_i\right) = \frac{2}{n}\sum_{i=1}^{n}(E(X_i)) = \frac{2}{n} n \left(\frac{0+\theta}{2}\right)$

$= 2\left(\frac{\theta}{2}\right) = \boxed{\theta}$

$\tilde{\theta}_1$ is an unbiased estimator of θ

2. $E(\tilde{\theta}_2) = E(\max(X_1, \cdots, X_n)) = E(X_i)$ where i is an integer $1 \le i \le n$

$= \frac{0+\theta}{2} = \theta/2 \neq \theta$

$\tilde{\theta}_2$ is a biased estimator of θ

3. $Var(\tilde{\theta}_1) = Var(2\overline{X})$

$= 2^2 Var(\overline{X})$

$= 4 Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$

$= \frac{4}{n^2} Var\left(\sum_{i=1}^{n} X_i\right)$

$= \frac{4}{n^2}\sum_{i=1}^{n}(Var(X_i))$ because X_i are independent

$$= \frac{4}{n^2} n\left(\frac{(\theta-0)^2}{12}\right)$$

$$= \frac{4}{n}\frac{\theta^2}{12} = \boxed{\frac{\theta^2}{3n}}$$

4. See simulation.R and simulation.txt

Comparison: The sample mean & sample variance of the 10000 samples of $\tilde{\theta}_1$ are closer to the theoretical results I've obtained for $E(\tilde{\theta}_1)$ and $Var(\tilde{\theta}_1)$

5. See simulation.R and simulation.txt

6. $\tilde{\theta}_1$ over $\tilde{\theta}_2$: The sample mean & sample variance of the 10000 samples of $\tilde{\theta}_1$ are closer to the theoretical results I've obtained for $E(\tilde{\theta}_1)$ and $Var(\tilde{\theta}_1)$. Also, $\tilde{\theta}_1$ is an unbiased estimator of θ whereas $\tilde{\theta}_2$ is biased.

$\tilde{\theta}_2$ over $\tilde{\theta}_1$: From simulation, $\widehat{MSE}(\tilde{\theta}_2) < \widehat{MSE}(\tilde{\theta}_1)$