1. Any CDF $F(x)$ should satisfy

① $\lim_{x \to \infty} F(x) = 1$ ② $F(x)$ is right continuous

③ $\lim_{x \to -\infty} F(x) = 0$ ④ $F(x)$ is non-decreasing

Clearly ③ and ④ are satisfied.

Now consider ①:

$$\lim_{x \to \infty} F_X(x) = 1$$

$$\lim_{x \to \infty} \left[1 - \left(\frac{2}{x}\right)^{\theta}\right] = 1$$

$$\lim_{x \to \infty} \left(\frac{2}{x}\right)^{\theta} = 0$$

$$\Rightarrow \boxed{\theta > 0}$$

Clearly ② is also satisfied when $\theta > 0$

2. $f_X(x) = \frac{d}{dx} F_X(x) = \frac{d}{dx}\left(1-\left(\frac{2}{x}\right)^{\theta}\right) 1[x \geq 2]$

$$= \begin{cases} \frac{d}{dx}\left(1-\left(\frac{2}{x}\right)^{\theta}\right) & \text{for } x \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{\theta 2^{\theta}}{x^{\theta+1}} & \text{for } x \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

3. Since $X_1, \cdots, X_n$ are independent

$$f_{X_1,\cdots,X_n}(x_1,\cdots,x_n) = \prod_{i=1}^{n}\left[f_{X_i}(x_i)\right] = \prod_{i=1}^{n}\left(\frac{\theta 2^\theta}{x^{\theta+1}}\right)$$

$$= \begin{cases} \dfrac{(\theta 2^\theta)^n}{\prod_{i=1}^{n}(x_i^{\theta+1})} & \text{if } x_k \geq 2 \quad \forall\ 1 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$$

Now consider the case $x_k \geq 2 \ \forall \ 1 \leq k \leq n$

$$\frac{d}{d\theta} f_{X_1,\cdots,X_n}(x_1,\cdots,x_n)$$

$$= \frac{d}{d\theta}\left[\frac{(\theta 2^\theta)^n}{\prod_{i=1}^{n}(x_i^{\theta+1})}\right] = \frac{d}{d\theta}\left[\theta^n \left(\frac{2^n}{\prod_{i=1}^{n} x_i}\right)^\theta \frac{1}{\prod_{i=1}^{n} x_i}\right]$$

$$= \frac{1}{\prod_{i=1}^{n} x_i}\left[n\theta^{n-1}\left(\frac{2^n}{\prod_{i=1}^{n} x_i}\right)^\theta + \theta^n \cdot \left(\frac{2^n}{\prod_{i=1}^{n} x_i}\right)^\theta \cdot \ln\left(\frac{2^n}{\prod_{i=1}^{n} x_i}\right)\right]$$

$$= 0$$

$$\Rightarrow \quad n + \theta \ln\left(\frac{2^n}{\prod_{i=1}^{n} x_i}\right) = 0$$

$$\Rightarrow \quad n + \theta\left(\ln(2^n) - \ln\left(\prod_{i=1}^{n} x_i\right)\right) = 0$$

$$\Rightarrow \quad n + \theta \left[n \ln 2 - \sum_{i=1}^{n} \ln x_i \right] = 0$$

$$\Rightarrow \quad L(\theta) = \boxed{\frac{n}{\sum_{i=1}^{n} [\ln(x_i)] - n \ln 2}}$$