Homework #2

Problem 1.

(1) Denote n is the length of input x in L, n' is the length of input x' in L'.

We decide a Deterministic Turing Machine M:

-1 If x didn't belong in x' at the beginning, return False.

-2 Cut the string at length of x, if the rest of bits have 1, return False.

-3 If the length of rest of 0's not equal to 100 time $|x|$, return False.

-4 Return True.

(2) Now we proof M is run in $O(2^n)$.

The relation of size between L' and L can be showed as the equation

$$|x'| = n' = n^{100} + n \tag{1.1}$$

By equation (2.1), we get

$$n \approx (n')^{1/100} \tag{1.2}$$

We measure the computation time in M, we get

$$\text{Time} = O\left(2^{n^{10}}\right) + O(n^{100}) \tag{1.3}$$

Substitute n' into n in (2.3), we get

$$O\left(2^{(n')^{1/10}}\right) + O(n') \tag{1.4}$$

Formula (2.4) bounded by $O(2^n)$. Thus, we show that $L' \in \text{TIME}(2^n)$.

Problem 2.

(1) By definition, the configuration of NTM M is $C(M, x)$, and a DTM A, outputs a set $S(x)$ such that $C(M, x) \subseteq S(x)$. The truth tells us that $C(M, x)$ occupy $O(n^c)$ space, where c is a finite constant.

We now use DTM M'

-1 Use depth-first-search parse all possible computing path, if there is a `Yes', than return True.

-2 If no path is `Yes', than return False.

to simulated NTM M. Because the size of $C(M, x)$ is polynomial, the DTM M' decide L within a polynomial time. Thus, $L(M) = L(M') \in P$.