Homework #5

Problem 1.

(1) We have an expected-polynomial time algorithm M for any language in BPP. Here goes an exchange protocol $\mathrm{IP}(\mathrm{P},\mathrm{V})$ as follow:

-1 For an input sequence x, V apply an algorithm M and sends a random bit b form $\{0,1\}$ to P.

-2 P sends the inverse bit $\bar{\mathrm{b}}$ back to V.

-3 V ignore the bit $\bar{\mathrm{b}}$, and apply algorithm to generate answer.

(2) For the protocol IP, it is a perfect zero-knowledge property because

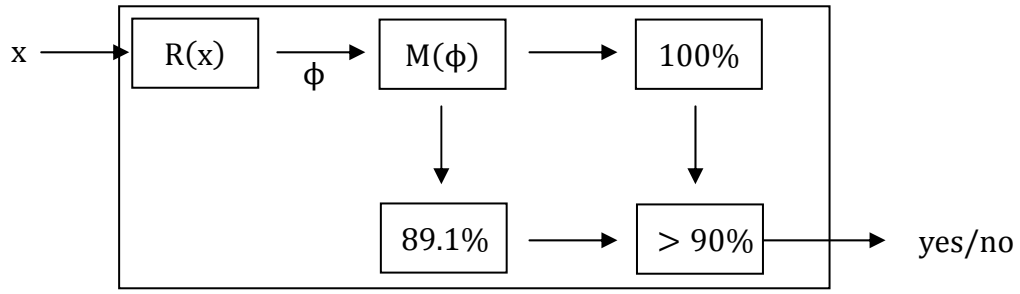
-1 M is in expected polynomial time.

-2 M on any x has the same probability distribution as the one that can be observed.

(3) Therefore, we claim that zero-knowledge proofs exist for every language in BPP.

Problem 2.

(1) Construct a Turning Machine T has the property follow:



We know the satisfied cluster at most achieve 89.1% of all clusters if the Boolean expression is unsatisfiable, so we set the threshold at 90%. Thus, we can decide any x is satisfiable or not in polynomial time. If there is more than 90% clusters is satisfy, than x is belong to SAT, otherwise, it isn't. Therefore, $\mathrm{SAT} \in \mathrm{P}$ under our assumption.