Homework #3

Problem 1.

(1) The logarithmic-space reduction can be generate as follow

-1 For each node in the graph, duplicate it without the edges.

-2 Draw the new node with color 2.

Because all of new nodes didn't contain any edge, is clearly that we can solve the new graph if only if we solve the original graph. We claim this reduction is in log-space because we only have to know the iteration times that can encoded by log n bits. Thus, we find a logarithmic-space reduction from 3-COLORING to MAJORITY-3-3COLORING.

Problem 2.

(1) By Fermat's `little' Theorem, we have

$$a\Phi(p) = \Phi(p) \tag{2.1}$$

Where, a = 2 in our case, we reduce the problem to

$$3p - 1 > \phi(3p) \tag{2.2}$$

(2) We know that p is an odd prime the phi function can represent in

$$\phi(3p) = \begin{cases} 2p * \left(1 - \frac{1}{p}\right) = 3p * \left(1 - \frac{1}{3}\right) * \left(1 - \frac{1}{p}\right), \text{when } p > 3 \\ 6 = 9 * \left(1 - \frac{1}{3}\right), \text{when } p = 3 \end{cases} \tag{2.3}$$

(3) Combine inequality (2.2) and equation (2.3), we get the inequality below

$$3p - 1 - \phi(3p) = \begin{cases} 3p - 1 - (2p - 2) > 0, \text{when } p > 3 \\ 8 - 6 > 0, \text{when } p = 3 \end{cases} \tag{2.4}$$

Now we satisfy the inequality.