Firm Default Risks and Exchange Rate Puzzles *

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Abstract

Imperfect international risk sharing stems from imperfectness in the financial market. This paper introduces micro-founded financial frictions into a standard international real business cycle model to quantify their effects on risk-sharing with micro-level data. One empirical evidence of frictional consumption risk sharing is the Backus-Smith puzzle (Backus & Smith 1993) — real exchange rates is negatively correlated with cross-country consumption growth. In my model, demand effects induced by firm-level financial frictions are able to account for the puzzle. Moreover, I derive a novel empirical moment from the micro-foundation to quantify the transmission of financial frictions. I estimate the moment from bond-level data. Disciplined with my estimations and moments of macro variables, the model is consistent with international business cycle properties, and resolve the Backus-Smith puzzle.

Keywords: Exchange Rate Puzzles, Financial Frictions, International Risk Sharing.

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1 Introduction

Equilibrium exchange rates departures from their empirical patterns, begging the question of perfect consumption risk sharing (Obstfeld & Rogoff 2000). Perfect risk sharing predicts that consumption is relatively high where goods is relatively cheap. However, this correlation has an opposite sign in data. Higher consumption growth in home country than foreign country is accompanied with an appreciation in home currency. This is the well-known Backus-Smith puzzle documented in Backus & Smith (1993).

Imperfect risk sharing stems from imperfectness in the financial market. Literature finds that a bond economy with low trade elasticity (Corsetti et al. 2008) or ad-hoc financial frictions (Itskhoki & Mukhin 2021) has large demand effects to boosts the price of goods at good times. The negative correlation of exchange rate and relative consumption growth is consistent with that in data.

Object of this project is to quantify the transmission of financial frictions to imperfectness in risk sharing with micro-level data. In order to do so, I incorporate firm-level financial frictions into a standard international real business cycle (IRBC) model. My model generates similar demand effects as in literature. More importantly, I derive a novel moment from the micro-foundation to capture the transmission of financial frictions. I estimate it from bond-level data. Disciplined with the empirical estimations, my model is consistent with international business cycle properties, and is able to account for about 80% of the Backus-Smith correlation in data. The unexplained correlation can be attributed to other forces such as demand shocks.

I introduce the heterogeneous firms set-up from Gomes et al. (2016) into a two-country model with incomplete asset market (Backus et al. 1992). Each country is populated with a representative household and a continuum of heterogeneous firms. Firms issue bonds to finance investment. They can choose to default based on the realization of their idiosyncratic shocks. Firm-level financial frictions arise from default punishment to creditors and therefore reflect in bond prices. Households provide labor to local firms and consume a basket of home and foreign goods with home bias. The financial market is integrated. Households have access to both domestic and foreign corporate bonds to smooth consumption. There are two opposite forces to drive exchange rates dynamics. Direct effect of supply increase after a positive productivity shock lowers the home goods price and depreciates the home currency. On the other hand, the decrease in default risk boosts the demand for home bonds and appreciates the home currency. Whether the equilibrium exchange rates are consistent with the Backus-Smith puzzle depends on the relative strength of these two forces.

I use micro-level data to discipline the demand effects arising from financial frictions. A novel empirical moment is derived from the uncovered interest parity (UIP) condition containing default risks. Expected changes in the exchange rates equalize the discounted returns for all creditors on the same bond. Home creditors require a return of domestic risk-free rate, the compensation for the expected default loss, and the default risk premium measured by the domestic stochastic discount factors (SDFs). Similarly, foreign creditors require a return of foreign risk-free rate, the compensation for the expected default loss, the risk premium of exchange rate volatility, and the default risk premium measured by the foreign SDFs. Taking the difference, exchange rate changes are determined by three factors: the risk-free interest rate differentials, the relative default risk premium, and a residual of risk premium from exchange rate volatility. Demand effects of financial frictions is mapped to the relative default risk premium between home and foreign creditors.

I create a time-series measure of the relative default risk premium from bond-level data. In order to distinguish returns required by different investors, I collect data on pairs of bonds issued by the same U.S. firm and traded in two markets – dollar bonds in the US domestic market and euro bonds in the European market – as proxies for bonds held by domestic and foreign creditors. A time-series measure of relative risk premium between 2007 and 2019 is estimated from monthly cross-sectional regressions of bond spreads on currency denomination dummy. For each bond, their spreads are constructed as the difference between end-of-month secondary-market yield-to-maturity and risk-free rates of the same maturity in respective currencies. Corporate bond spreads are related to two factors: the expected default loss and default risk premium. To capture the expected default loss as much as possible, I include a bunch of bond characteristics and firm fixed effect in control variables. Firm fixed effect cancels out the common default risks from the same issuer. The estimated relative risk premium is negatively correlated with the relative output growth between the U.S. and G7 countries. Foreign investors' default risk premium is higher when home country is more productive, transmitting to a home currency appreciation in booms.

In the following quantitative exercise, I calibrate the model to quarterly data of the U.S. and G7 countries. Model parameters are disciplined to standard moments of macro aggregates and moments of firm behaviors, including average default rate, leverage ratio and the output-growth-correlation of relative risk premium from my empirical estimation. The model is able to reproduce standard domestic and international business cycle moments, and a Backus-Smith correlation similar to that in data.

I further investigate the role of financial frictions in the model. First, I eliminate default risks on bonds and the Backus-Smith puzzle re-emerges. Home currency depreciates in booms since supply effects of productivity shocks dominate. Second, to reproduce a Backus-Smith correlation close to data, the calibration target of the output-growth-correlation of relative risk premium is important. A more negative value of that moment indicates stronger effects of default risks, and generates a more negative Backus-Smith correlation that overshoots the data. Third, I confirm that the role of financial fictions to address the Backus-Smith puzzle is not confounded with that of trade elasticity (Corsetti et al. 2008). The business-cycle movements of exchange rates and trade balances are insensitive to variation of trade elasticity within a range suggested by literature.

Contributions of this project are three-folds. Theoretically, I build an international real business cycle (IRBC) model with heterogeneous firms and endogenous default risks. Empirically, I create a novel measure to capture the transmission of default risks to exchange rates. Quantitatively, firm-level financial frictions help to reproduce equilibrium exchange rates that are consistent with exchange rate puzzles.

Literature Review

The paper bridges two large literature. One is on equilibrium exchange rate dynamics and the other one is on macro effects of financial frictions. The puzzling co-movements of exchange rate with macro variables (Obstfeld & Rogoff 2000) and excess volatility of exchange rates (Backus & Smith 1993) are of central interests in international macroeconomics. In order to understand exchange rate puzzles, literature augments a standard two-country model with various features, such as time-varying risks premium, imperfect financial and goods market, deviations from full information rational expectations and bond convenience yield.

The first branch of literature models time-varying risks premium in exchange rates arsing from recursive preferences and long-run risks (Bansal & Shaliastovich 2013, Colacito et al. 2018, Colacito & Croce 2011, 2013, Lustig & Verdelhan 2007), preferences with external habits (Verdelhan 2010), or rare disaster risks (Farhi & Gabaix 2016). A second branch of literature emphasizes imperfect financial or goods market. International capital flows alter the balance sheet or risk-bearing capacity of financial intermediaries who require excess returns for holding currency risks (Gabaix & Maggiori 2015, Itskhoki & Mukhin 2021, Mac Mullen & Woo 2023). Trade costs create wedges in the relative price of goods and drive exchange rate dynamics (Alessandria & Choi 2021, Obstfeld & Rogoff 2000). The third explanation casts doubts on the assumption of rational expectations on interest rate and inflation to determine exchange rates (Bacchetta & Van Wincoop 2006, Burnside et al. 2011, Gourinchas & Tornell 2004). A forth and recent literature conjectures bond convenience yield as an additional force in exchange rates determination (Bianchi et al. 2021, Jiang et al. 2021, Valchev 2020).

This paper is closely related to the literature on time-varying risk premium while stick to a standard preference with constant relative risk aversion (CRRA) preference, and first-order productivity shocks with normal distributions. The novelty is on introducing additional risks from firm-level financial frictions into exchange rate dynamics.

a vast literature studies macro effects of financial frictions (Bernanke et al. 1999, Carlstrom & Fuerst 1997, Cooley et al. 2004, Gertler & Karadi 2015, Gourio 2013, Kiyotaki & Moore 1997). I extend the firm-level financial frictions (Gomes et al. 2016) to an open economy set-up and study implications on exchange rates.

The paper is organized as follows. Section II lays out the model. Section III presents the empirical estimations. Section IV presents the quantitative results. Section V concludes.

2 Model

I introduce the heterogeneous firms set-up with endogenous default choices from (Gomes et al. 2016) into a standard international real business cycle (IRBC) model with incomplete asset market, home bias in consumption and productivity shocks. The baseline specification features no nominal rigidity as all prices and wages are flexibly adjusted. Compared to a standard two-country model, the difference is in that international risk sharing is conducted via default-able corporate bonds. Default punishment reflects in bond prices and transmits to exchange rate dynamics.

There are two countries – home (United States) and foreign (Europe, denoted with an asterisk). Each country has their own currency to quote local prices. The real exchange rate is expressed as the price of foreign currency in local currency; hence, an increase in $\mathcal{E}_t$ corresponds to a home currency depreciation. The following description focuses on the home country. The foreign country is symmetric and all foreign variables are denoted by an asterisk.

2.1 Firm

The structure of firms is the same as Gomes et al. (2016). Each country has a continuum of measure 1 of firms indexed by superscript j. They produce local final goods, accumulate capital, and issue 1-period real bonds to both foreign and home creditors. Firms are ex-ante identical and ex-post heterogeneous in the realization of idiosyncratic shocks.

Production. At time t, the aggregate productivity is A_t for home country and A_t^* for foreign country. Firm j employs capital k_t^j and labor l_t^j with wage w_t to produce final goods according to the following technology:

$$y_t^j = A_t k_t^{j,\alpha} n_t^{j,1-\alpha} \tag{1}$$

Firm's operating profit with optimal labor choice is given by:

$$R_t k_t^j = \max_{n_t^j} A_t (k_t^j)^\alpha (n_t^j)^{1-\alpha} - w_t n_t^j \tag{2}$$

Since the production function is constant return to scale, all firms choose identical capital-labor ratio k^j/n^j, therefore, R_t is identical across firms and is without a superscript j.

Idiosyncratic shock. It is assumed that firm-level profits are subject to idiosyncratic shock z_t^j at time t. That summarizes all firm-specific business risks. z_t^j is assumed to be i.i.d. across firm and time, and has a cumulative distribution $\Phi(z)$ over interval $[\underline{z}, \bar{z}]$. The realized operating profits are given by:

$$(R_t - z_t^j)k_t^j$$

Investment. In each period t, firm makes the choice of investment-to-capital ratio i_t^j. Capital accumulation with constant deprecation rate δ follows this law of motion:

$$k_{t+1}^j = (1 - \delta + i_t^j)k_t^j \equiv g(i_t^j)k_t^j \tag{3}$$

Financing. Firms finance themselves via issuing equity and 1-period real bonds. Let b_{t+1}^j denote the total unit of new bond issuance during period t. They are sold at common price to home and foreign creditors; hence the prices are expressed as q_t^j in home country, and $q_{H,t}^{j*} = q_{H,t}^j/\mathcal{E}_t$ in foreign country. Each unit of home (foreign) bond repays creditors 1 unit of home (foreign) goods in the next period if no default happens.

Dividends and Equity Value. The equity distribution to shareholders in terms of local consumption goods at the end of period t are equal to:

$$(1-\tau)(R_t - z_t^j) - (1-\tau)b_t^j - i_t^j k_t^j + \tau\delta k_t^j$$

where, τ is the effective capital tax rate. The first term captures the firm's operating profits after tax. The second term captures bond repayment. $(1-\tau)$ indicates the tax benefit of issuing bonds. The third term captures the investment decision. The last term captures the tax shields accrued through depreciation expenditures.

Default choice. Firms are owned by local shareholders who can choose to default based on the realization of idiosyncratic shocks. The present value of firm j to its shareholders is denoted by $J(.)$:

$$J(k_t^j, b_t^j, z_t^j, A_t, A_t^*) = \max[0, \underbrace{(1-\tau)(R_t - z_t^j)k_t^j}_{\text{operating profit after tax}} - \underbrace{(1-\tau)b_t^j}_{\text{debt repayment}} + \underbrace{V(k_t^j, A_t, A_t^*)}_{\text{continuation value}}] \tag{4}$$

Shareholders can simply walk away with zero value if choosing to default. If not default, shareholders can collect the operating profit, serve their debt, continue to operate with the continuation value $V(.)$.

$V(.)$ depends on the optimal choice of new debt issuance and investment:

$$\begin{aligned} V(k_t^j, A_t, A_t^*) = \max_{k_{t+1}^j, b_{t+1}^j} & \underbrace{q_t b_{t+1}^j}_{\text{new issuance}} - \underbrace{(i_t^j - \tau\delta)k_t^j}_{\text{investment}} \\ & + E_t M_{t,t+1} \int_{\underline{z}}^{\bar{z}} J(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j, A_{t+1}, A_{t+1}^*) d(\Phi(z_{t+1}^j)) \end{aligned} \tag{5}$$

where, expectation E_t are taken over the distribution of aggregate states (A_{t+1}, A_{t+1}^*) with home shareholders' stochastic discount factor (SDF) $M_{t,t+1}$ which is exogenous to firms and determined in the equilibrium.

The equity value $J(.)$ is bounded below by zero implying limited liability. That is, shareholders can default on their debts whenever their idiosyncratic profit shock z_t^j are above the default cut-off $\hat{z}_t^j \leq \bar{z}$. The cut-off $\hat{z}_t^j$ is expressed as follows:

$$(1-\tau)(R_t - \hat{z}_t^j)k_t^j - (1-\tau)b_t^j + V(k_t^j, A_t, A_t^*) = 0 \tag{6}$$

Restructure and Credit Value. I assume that there is no segmentation and no partial default in the financial market. In default, shareholders walk away while creditors take over and restructure the firm. Based on the number of their shares, creditors equally divide after-tax operating profits and revenues from selling off the equity portion to new owners after restructuring. Restructuring entails the loss of ξk_t^j in units of local consumption goods, where $\xi \in [0,1]$. To wrap up, the valuation of corporate debt holdings at the end of period

t is the expected value of bond repayments and default collections. It is:

$$b^j_{t+1} q^j_{H,t} = E_t M_{t,t+1} \{ \underbrace{\Phi(\hat{z}^j_{t+1}) b^j_{t+1}}_{\text{not default}} + \underbrace{\int_{\hat{z}^j_{t+1}}^{\bar{z}} [(1-\tau)](R_{t+1} - z^j_{t+1}) k^j_{t+1} + V(k^j_{t+1}, A_{t+1}, A^*_{t+1}) - \xi k^j_{t+1}] d\Phi(z_{t+1}) \}}_{\text{default, value of restructure}} \tag{7}$$

to home creditors in units of home consumption goods ; and

$$b^j_{t+1} \frac{q^j_{H,t}}{\mathcal{E}_t} = E_t M^*_{t,t+1} \frac{1}{\mathcal{E}_{t+1}} \{ \Phi(\hat{z}^j_{t+1}) b^j_{t+1} + \int_{\hat{z}^j_{t+1}}^{\bar{z}} [(1-\tau)](R_{t+1} - z^j_{t+1}) k^j_{t+1} + V(k^j_{t+1}, A_{t+1}, A^*_{t+1}) - \xi k^j_{t+1}] d\Phi(z_{t+1}) \} \tag{8}$$

to foreign creditors in units of foreign consumption goods. $\mathcal{E}_{t+1}$ is the future real exchange rate. $M^*_{t,t+1}$ is the foreign SDF determined in equilibrium.

The right-hand-side of both equations can be divided into two parts. The first-term reflects full debt repayment if no default happens; while the integral expresses returns in default net of restructure costs.

Foreign creditor's valuation are different from home creditor's valuation in two ways. First, their expectations are taken over foreign SDF $M^*_{t,t+1}$ instead of home SDF $M_{t,t+1}$. Second, firm payments with and without default are in units of home goods; hence, they are converted by the exchange rate $\mathcal{E}$ for foreign creditor's consumption.

2.2 Households

Households have constant-relative-risk-aversion (CRRA) preferences over consumption and leisure. Domestic and foreign households' utility at time 0 are given by:

$$E_0[\sum_{t=0}^{\infty} \beta^t u(C_t, N_t)] \tag{9}$$

$$E_0[\sum_{t=0}^{\infty} \beta^t u(C^*_t, N^*_t)]$$

Goods consumption C_t and labor N_t are aggregated with Greenwood–Hercowitz–Huffman (GHH) preference:

$$u(C_t, N_t) = \frac{(C_t - \varphi N_t^{1+\frac{1}{f}})^{1-\gamma}}{1-\gamma} \tag{10}$$

$$u(C_t^*, N_t^*) = \frac{(C_t^* - \varphi N_t^{*1+\frac{1}{f}})^{1-\gamma}}{1-\gamma}$$

where, γ is the relative risk aversion coefficient, f is the Frisch elasticity of labor supply and φ is a scale of labor disutility.

Consumption basket C_t bundles over home and foreign final goods $C_{H,t}$ and $C_{F,t}$ with home bias $\lambda > 0.5$:

$$C_t = C_{H,t}^{\lambda} C_{F,t}^{1-\lambda} \tag{11}$$

$$C_t^* = C_{H,t}^{*1-\lambda} C_{F,t}^{*\lambda}$$

Households' flow budget constraint is given by:

$$C_t + q_{H,t}B_{H,t+1} + q_{F,t}B_{F,t+1} = R_{H,t}B_{H,t} + R_{F,t}B_{F,t} + W_tN_t + \Pi_t + T_t \tag{12}$$

where,

$$P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$$

P_t is the price of local consumption basket normalized to be 1, $P_{H,t}$ is the price of local goods, and $P_{F,t}$ is the price of foreign goods. W_tN_t, Π_t and T_t are labor income, dividends and capital tax rebate respectively. $R_{H,t}$ and $R_{F,t}$ are returns of home and foreign corporate bonds in unit of home goods. They are included in the right-hand-side of Equation (7) and (8). I rewrite them here:

$$R_{H,t} \equiv \Phi(z_{H,t}^*) + \int_{z_{H,t}^*}^{\bar{z}} [(1-\tau)](R_t - z_t)k_t/b_t + V(k_t, A_t)/b_t - \xi k_t/b_t] d\Phi(z_t)$$

$$R_{F,t} \equiv \mathcal{E}_t\{\Phi(z_{F,t}^*) + \int_{z_{F,t}^*}^{\bar{z}} [(1-\tau)](R_t^* - z_t)k_t^*/b_t^* + V(k_t^*, A_t^*)/b_t^* - \xi k_t^*/b_t^*] d\Phi(z_t)\}$$

Households' optimal choice of labor, consumption, home and foreign bond holdings are characterized by their first-order conditions:

$$W_t = -\frac{\partial u(C_t, N_t)/\partial N_t}{\partial u(C_t, N_t)/\partial C_t} \tag{13}$$

$$\frac{P_{F,t}}{P_{H,t}} = \frac{\partial u(C_t, N_t)/\partial C_{F,t}}{\partial u(C_t, N_t)/\partial C_{H,t}} \tag{14}$$

$$q_{H,t} = E_t[\frac{\partial u(C_{t+1}, N_{t+1})/\partial C_{t+1}}{\partial u(C_t, N_t)/\partial C_t} R_{H,t+1}] \tag{15}$$

$$q_{F,t} = E_t[\frac{\partial u(C_{t+1}, N_{t+1})/\partial C_{t+1}}{\partial u(C_t, N_t)/\partial C_t} R_{F,t+1}] \tag{16}$$

2.3 Productivity Process

Aggregate productivity A_t, A_t^* follows an AR(1) process as:

$$\begin{aligned} log(A_t) &= \rho_a log(A_{t-1}) + \epsilon_{a,t} \\ log(A_t^*) &= \rho_a log(A_{t-1}^*) + \epsilon_{a,t}^* \end{aligned} \tag{17}$$

where, ρ_a is the persistence of productivity process, $\epsilon_{a,t}$ and $\epsilon_{a,t}^*$ are exogenous productivity shocks following jointly log-normal distribution:

$$[\epsilon_{a,t}, \epsilon_{a,t}^*] \sim iidN(0, \Sigma)$$

where

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho_{sr}\sigma^2 \\ \rho_{sr}\sigma^2 & \sigma^2 \end{bmatrix}$$

σ is the common volatility of productivity shocks and ρ_{sr} is cross-country correlation of shocks.

2.4 Equilibrium

Definition 2.1. Let $s = \{A, A^*, K, K^*, b, b^*\}$ denotes the aggregate state, where aggregate productivity $\{A, A^*\}$ are exogenous processes, and aggregate capital and bond supply

$\{K, K^*, b, b^*\}$ is an endogenous state variable. A recursive competitive equilibrium is defined as a set of home firms' policy functions $\{n^j(s), i^j(s), \omega^j(s)\}$, foreign firms' policy functions $\{n^{j*}(s), i^{j*}(s), \omega^{j*}(s)\}$, home households' policy functions $\{N(s), C_H(s), C_F(s), B_H(s), B_F(s)\}$, foreign households' policy functions $\{N^*(s), C_H^*(s), C_F^*(s), B_H^*(s), B_F^*(s)\}$, goods prices $\{w(s), s(s)\}$, and bond prices $\{q_H(s), q_H^*(s), q_F(s), q_F^*(s)\}$ such that:

1. $\{n^j(s), i^j(s), \omega^j(s)\}$ satisfy home firms' optimal labor choice, borrowing and investment (2), (30),(31); analogusly, $\{n^{j*}(s), i^{j*}(s), \omega^{j*}(s)\}$ satisfy the same optimal labor, borrowing and investment decision for foreign firms;

2. $\{N(s), C_H(s), C_F(s), B_H(s), B_F(s)\}$ satisfy household's optimal conditions (13), (14), (15), (16), and balance of payment condition (22);

3. Wage $\{w(s)\}$ clears labor market (20). Goods prices $\{P_H(s), P_F(s)\}$ clear local and foreign goods market (18), (19).

4. Endogenous investment choice is consistent with aggregate capital accumulation in equilibrium: $k(i(s_{-1})) = K$.

5. Endogenous bond demand is consistent with aggregate bond supply in equilibrium: $B_H(s_{-1}) + B_H^*(s_{-1}) = b$ and $B_F(s_{-1}) + B_F^*(s_{-1}) = b^*$.

Market clearing conditions are specified as follows. Home goods production satisfies domestic, foreign consumption demand, plus domestic investment demand:

$$Y_t = C_{H,t} + C_{H,t}^* + i_t k_t \tag{18}$$

where, aggregate output is firm's output net of default costs:

$$Y_t = y_t - [1 - \Phi(z_{H,t}^*)]\xi k_t$$

the subscript j are ignored as firms' problem is linearly homogeneous in capital under constant returns to scale.

Similarly, foreign goods market clears as:

$$Y_t^* = C_{F,t} + C_{F,t}^* + i_t^* k_t^* \tag{19}$$

$$Y_t^* = y_t^* - [1 - \Phi(z_{F,t}^*)]\xi k_t^*$$

Labor market clears so that labor demand equals local labor supply:

$$n_t = N_t, n_t^* = N_t^* \tag{20}$$

Corporate bond supply equals to demand from both home and foreign creditors:

$$\begin{aligned} b_{t+1} &\equiv \int b_{t+1}^j dj = B_{H,t+1} + B_{H,t+1}^* \\ b_{t+1}^* &\equiv \int b_{t+1}^{j*} dj* = B_{F,t+1} + B_{F,t+1}^* \end{aligned} \tag{21}$$

Balance of payment condition requires that net imports equal net capital inflows:

$$q_{F,t}B_{F,t+1} + q_{H,t}(B_{H,t+1} - b_{t+1}) - R_{F,t}B_{F,t} - R_{H,t}(B_{H,t} - b_t) = NX_t = P_{H,t}C_{H,t}^* - P_{F,t}C_{F,t} \tag{22}$$

2.5 Characterize Exchange Rates

The equilibrium path of exchange rate is derived from combining Equation (7) and (8), the home and foreign creditors' pricing to home bonds:

$$E_t\{[M_{t,t+1} - M_{t,t+1}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}]R_{H,t+1}\} = 0 \tag{23}$$

I rewrite it in log terms as:

$$E_t[exp(m_{t,t+1} + r_{H,t+1}) - exp(m_{t,t+1}^* + r_{H,t+1} - \Delta e_{t+1})] = 0$$

and make second-order Taylor expansion around the steady state:

$$
\begin{aligned}
&E_t\{1 + m_{t,t+1} + r_{H,t+1} + \frac{1}{2}(m_{t,t+1} + r_{H,t+1})^2 \\
&- [1 + m^*_{t,t+1} + r_{H,t+1} - \Delta e_{t+1} + \frac{1}{2}(m^*_{t,t+1} + r_{H,t+1} - \Delta e_{t+1})^2)]\} = 0
\end{aligned}
$$

The exchange rate path is therefore determined as follows: [1]

$$
E_t[\Delta e_{t+1}] = \underbrace{E_t[m^*_{t,t+1} - m_{t,t+1}]}_{\text{risk-free interest rate differentials}} - \underbrace{E_t[m^*_{t,t+1}\Delta e_{t+1}]}_{\text{exchange rate risk premium}} + \underbrace{E_t[-m_{t,t+1}r_{H,t+1} + m^*_{t,t+1}r_{H,t+1}]}_{\text{relative default risk premium}} + resid \tag{24}
$$

Exchange rate changes depends on the risk-free interest rate differentials, the risk premium from exchange rate volatility and the relative default risk premium (RRP) between home and foreign creditors. The risk-free interest rate differentials capture the direct effect of supply increase after a positive productivity shock that lowers the home goods price and depreciates the home currency. The relative default risk premium captures the demand effect of decrease in default risk to appreciates the home currency. The equilibrium exchange rates are consistent with the Backus-Smith puzzle if demand effects are large enough. The relative default risk premium (RRP) is the empirical moment to quantify such effects from data.

3 An Empirical Measure of Relative Bond Spreads

In this section, I create a time-series measure of the relative default risk premium (RRP) from secondary-market corporate bond prices to quantify the degree of financial frictions. Home country is taken as the U.S. and foreign country as the remaining G7 countries. Home currency is dollar and foreign currency is euro.

[1] *resid* is the additional second-order terms:

$$
resid = E_t[-r_{H,t+1}\Delta e_{t+1} + \frac{1}{2}(\Delta e_{t+1})^2 + + \frac{1}{2}(m^*_{t+1})^2 - \frac{1}{2}(m_{t+1})^2]
$$

3.1 Data

In order to distinguish returns required by different investors, I collect data on pairs of bonds issued by the same U.S. firm and traded in two markets – dollar bonds in the US domestic market and euro bonds in the European market – as proxies for bonds held by domestic and foreign creditors. Data of dollar bonds are extracted from WRDS and that of euro bonds are extracted from Bloomberg. Bonds with the same parent entity are linked by firm ticker across these two datasets. The sample period spans from 2007 January to 2019 December at a monthly frequency.

I restrict the sample to investment-grade bonds issued by non-financial, non-government firms between 2004m9 and 2016m12 to have enough observations at the beginning of 2007. I follow the standard selection process to include bonds that are (1) senior unsecured; (2) with fixed coupon payment; (3) with issuance amount ≥ 1 million USD; (4) with maturity at issuance between 1 to 30 years. I remove outliers by keeping spreads with risk-free rates between 5 and 3500 basis points and dropping illiquid bonds whose remaining maturity is less than 1 year. Finally, I have 59,044 end-of-month observations from 991 bonds issued by 53 firms.

I can observe end-of-month secondary-market bond yields, and various bond characteristics – maturity, amount, ratings, and issuance firm. Table B1 summarizes the sample statistics. Four-fifths of sampled bonds are denominated in dollars. Dollar bonds are of shorter maturity, higher coupon rates, and higher yield-to-maturity at issuance. A concern may rise on market liquidity differentials if the dollar and euro bond markets are segmented. I can observe bid-ask spreads for most of the sampled bonds. On average, dollar bonds are slightly more liquid than euro bonds.

3.2 Construct the Relative Default Risk Premium

I match each bond with risk-free rates of similar remaining maturity to construct bond spreads. Dollar risk-free rates are taken from the Treasury rates and euro risk-free rates are taken from the ECB yield curve. A time-series measure of the relative default risk premium

(RRP) is estimated from the following regression for each month t:

$$y_{itk} = \alpha_t + \phi_t USD_k + \delta_t Amount_{itk} + \gamma_t Maturity_{itk} + \xi_t Rate_{itk} + FE_{it} + \epsilon_{itk} \tag{25}$$

all standard errors are robust.

y_{itk} is the spreads of bond k issued by firm i at the end of month t. USD_k is a dummy equal to 1 if bond k is denominated in the dollar; equal to 0 if it is denominated in the euros. In order to control as much bond characteristics as possible, I include the firm fixed effect FE_{it}, issuance amount $Amount$, remaining maturity $Maturity$ and bond ratings at issuance $Rate$ on the right-hand side.

$\{\hat{\phi}_t\}$ is the time-series measure of RRP in interest. The reason is as follows. Corporate bond spreads are related to two factors: the expected default loss and default risk premium. To capture the expected default loss as much as possible, I include a bunch of bond characteristics and firm fixed effect in control variables. Firm fixed effect cancels out the common default risks from the same issuer.

Table 1 reports its business cycle correlations with the relative output growth between the US and the G7 countries. The whole sample period is separated according to the 2008 global financial crisis (GFC). As predicted, the negative correlations imply that foreign investors' default risk premium is higher when home country is more productive, transmitting to a home currency appreciation force at good times.

Table 1: Correlation of RRP and Relative Output Growth

		All 2007Q1-2017Q4	GFC 2008Q1-2009Q4	Post-GFC 2010Q1-2017Q4
US firms	$\rho(RRP_t, \Delta y_t - \Delta y_t^*)$	-0.13	-0.11	-0.10

Notes: Quarterly RRP is the series of $\{\hat{\phi}_t\}$ estimated from regression(25) for the US and EU firm sample, respectively. They are averaged to the quarterly frequency. Δy is quarterly real GDP growth rate of the U.S. collected from FRED, Δy^* is the growth rate of Real GDP growth for the G7 countries collected from OECD dataset.

4 Quantitative Results

To quantitatively evaluate the role of financial frictions in international risk sharing, I calibrate the model to standard international business-cycle moments, and moments of firm behaviors including default rate, leverage ratio and the RRP-output-growth correlation from the previous estimation.

4.1 Calibration

The model is calibrated to quarterly data for the U.S. and the rest of world (G7 countries excluding the U.S.) between 1975Q1-2017Q4. Data for growth rate of real GDP, consumption and investment are taken from OECD dataset. Leverage ratio is constructed by Gomes et al. (2016), defined as the ratio of credit market instruments to real assets plus cash and cash equivalent holdings for the US non-financial business sector. The U.S. real narrow effective exchange rate are from BIS dataset. I average the monthly estimation of the relative default risk premium to a quarterly series.

There are two sets of parameters to be chosen in the model. The first set is exogenously assigned (Table 2 Panel A). Time discount β is set to match a 4% annual interest rate. Capital share α, capital depreciation rate δ, relative risk aversion γ and Frisch labor-consumption elasticity f all correspond to fairly common values in the literature.

The second set of parameters in Panel B is calibrated to match data moments. Consumption home bias λ are set to match the US imports-to-GDP ratio. Following Gomes et al. (2016), default restructure costs ξ is calibrated to match Moody's average default rate. Capital tax rate τ is calibrated to match the average leverage ratio.

The distribution of idiosyncratic shock to firm profits governs default risks. Its probability density function (PDF) is approximated as a general quadratic function:

$$\phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2$$

The zero mean assumption implies $\eta_2 = 0$. The upper and lower bound of distribution are set as $\bar{z} = -\underline{z} = 1$, therefore $2\eta_1 + \frac{2}{3}\eta_3 = 1$. There is one free parameter η_1 to be calibrated.

η_1 matters for the volatility in default probabilities and is therefore closely related with the default risk premium. I calibrate it to the correlation between the estimated RRP and the relative output growth for the whole sample period (-0.13 in Table 1). Parameters for productivity process are set to match the empirical persistence, volatility and cross-country correlation of real GDP growth.

Table 2: Calibration

Parameters		Values	Target
Panel A: Exogenous assigned			
β	Time discount factor	0.99	4% annual interest rate
α	Capital income share	0.3	
γ	Relative risk aversion	5	
f	Frisch elasticity	1.5	
δ	Capital depreciation rate	0.06	
Panel B: Moment matching			
λ	Consumption home bias	0.8	The US imports-GDP ratio
τ	Capital tax rate	0.28	Average default probability
ξ	Default restructure cost	0.29	Average leverage ratio
η_1	Profit shock distribution	0.22	Correlation of RBP and output growth
ρ_a	Productivity persistence	0.93	Output persistence $\rho(\Delta y)$
σ	Productivity shock volatility	0.005	Output volatility $\sigma(\Delta y)$
ρ_{sr}	Productivity shock correlation	0.85	Output cross-correlation $\rho(\Delta y, \Delta y^*)$

4.2 Results

Table 3 reports the quantitative results. The model is featured with firm-level financial frictions. Panel A compares firm-level moments from model simulations with data. Leverage ratio is defined as the bond-to-capital ratio b/k in model [2] The model is well calibrated to the average default rate, leverage ratio and output correlation of RRP. Benchmark model delivers a less volatile leverage ratio and more volatile RRP but of similar persistence.

Panel B exhibits moments of real exchange rate properties and trade balances. The benchmark model is able to reproduce counter-cyclical exchange rates that is consistent with the Backus-Smith condition ("Benchmark" rows) and the large empirical volatility.

In order to investigate the role of firm-level financial frictions to exchange rate puzzles, I re-calibrate an alternative model specification without financial frictions where firms have

[2] Appendix A rewrites the firm's problem in terms of leverage ratio.

to repay all their debt at any time ("No default" rows). Exchange rate is pro-cyclical in the no-default specification since it reflects the interest rate differentials, plus risk premium of exchange rate volatility which is of small magnitude. Backus-Smith puzzle reemerges, similar to the case of a standard international real business cycle model with incomplete market. Moreover, the volatility of exchange rates shrinks to about one-tenth of that in data and one-fifth of that in benchmark model due to the lack of default risk premium.

The domestic and international business cycle statistics are reported in Panel C. Both benchmark and no-default specifications deliver the empirical volatility and correlations of macro aggregates within and across countries.

Table 3: Aggregate Moments

Panel A: Firm-level moments						
	Default rate	Leverage ratio		Relative default risk premium		
	$E[1-\Phi(z^*)]^\dagger$	$E[\omega]^\dagger$	$\sigma(\omega)_{\times 100}$	$\sigma(.)_{\times 100}$	ACF(1)	$\rho(., \Delta y - \Delta y^*)^\dagger$
Data	0.0026	0.42	1.70	0.42	0.68	-0.13
Benchmark	0.0026	0.42	0.32	0.06	0.71	-0.21
Panel B: Real exchange rate properties and trade balance				Backus-Smith correlation		
	ACF(1)(e)	$\sigma(\Delta e)/\sigma(\Delta y)$	$\rho(\Delta e, \Delta y)$	$\rho(\Delta \frac{nx}{y}, \Delta y)$	$\rho(\Delta e, \Delta c - \Delta c^*)$	$\rho(e, c - c^*)$
Data	0.95	4.07	-0.14	-0.06	-0.26	-0.12
Benchmark	0.71	2.13	-0.43	-0.25	-0.20	-0.11
No-default	0.71	0.42	0.12	-0.27	0.45	0.74
Panel C: International business cycle moments						
	$\sigma(\Delta y)^\dagger_{\times 100}$	ACF(1)$(\Delta y)^\dagger$	$\sigma(\Delta c)/\sigma(\Delta y)$	$\sigma(\Delta i)/\sigma(\Delta y)$		
Data	0.80	0.74	0.79	2.60		
Benchmark	0.85	0.71	0.85	1.77		
No default	0.85	0.71	0.86	1.61		
	$\rho(\Delta c, \Delta y)$	$\rho(\Delta i, \Delta y)$	$\rho(\Delta c, \Delta i)$	$\rho(\Delta y, \Delta y^*)$	$\rho(\Delta i, \Delta i^*)$	$\rho(\Delta c, \Delta c^*)$
Data	0.63	0.79	0.61	0.84	0.84	0.82
Benchmark	0.99	0.99	0.99	0.85	0.74	0.83
No default	0.99	0.99	0.99	0.85	0.75	0.83

Notes: Data moments: the US v.s. G7 countries, 1975Q1-2017Q4, at quarterly frequency. Model moments: simulations of order 1 deviation from steady states, HP filer=1600. **Benchmark** refers to the benchmark model with firm-level financial frictions. **No default** refers to the alternative model specification without firm-level financial frictions. *Relative default risk premium* in Panel A corresponds to the empirical estimation in (25), using the first month estimation of each quarter. †: targeted moments in calibration.

4.3 Mechanism

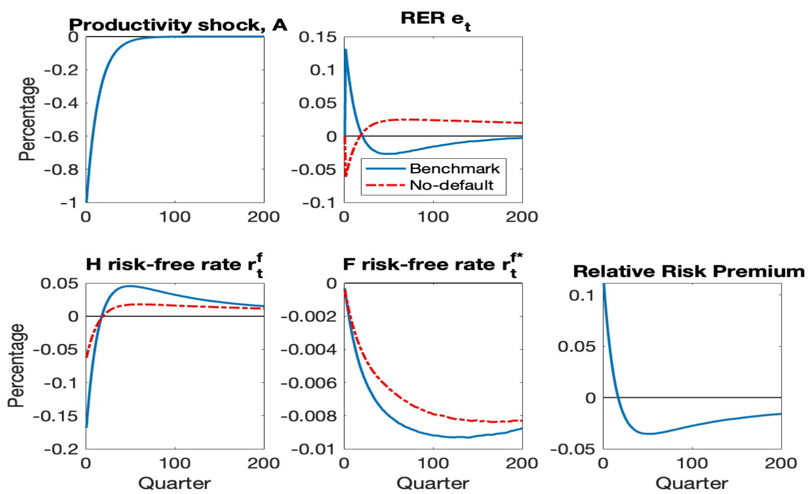
This section explores the mechanism of firm-level financial frictions to address Backus-Smith puzzle. Figure 1 and 2 plots impulse responses to a negative home productivity shock from the benchmark and no-default specification together to highlight the role of financial frictions.

Without firm default risks, real exchange rate (RER) experiences an expected appreciation (RER decreases) which is a reflection of increase in real interest rate differentials ($r^f - r^{f*}$) and exchange rate risk premium. This is the direct effects of productivity shock to decrease home goods supply and boost its price.

Once firm-level financial frictions are brought into play, the demand effects arise through the default risk premium. An increase in default risks dampens the demand for home bond and depreciates the home currency. Capital outflows ($NX/Y \uparrow$) to the more productive country and boosts foreign investment and consumption ($C^* \uparrow$, $I^* \uparrow$). The benchmark model is consistent with the Backus-Smith puzzle.

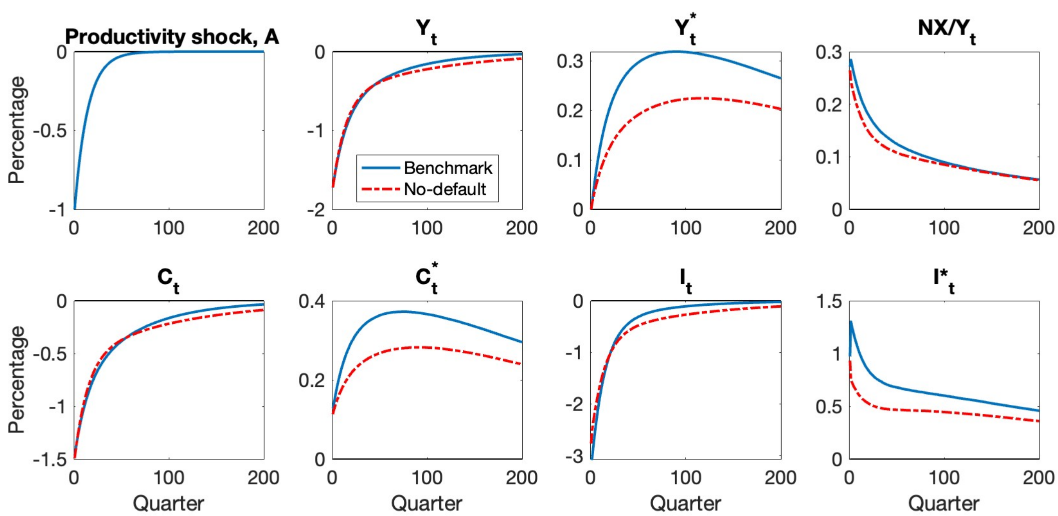
Table 4 presents the importance of choosing the output-RRP-correlation as a calibration target. Re-calibrated a more negative correlation with $\eta_1 = 0.99$ generates a more negative Backus-Smith correlation that departures from the empirical moment. That is because η_1 governs the volatility of default probability. A larger changes in default risks leads to more volatile leverage ratio and exchange rate. The full set of moments is in Appendix Table C1. This result confirms that the output-RRP-correlation is an key moment to capture the degree of financial friction transmission to exchange rate dynamics.

Figure 1: Impulse Responses to a Home Shock: Exchange Rates



Notes: Impulse responses of real exchange rate e, home and foreign risk-free rate r^f and r^{f*}, the relative default risk premium $= -E[mr^b] + E[m^*r^b]$ to a 1 percentage negative shock to home productivity shock A. Blue solid lines indicate responses from the benchmark model *with* firm default risks. Red dashed lines indicate responses from the no-default model *without* firm default risks.

Figure 2: Impulse Responses to a Negative Shock: Macro Aggregates



Notes: Impulse responses of home and foreign output Y and Y^*, consumption C and C^*, investment I and I^* and net-export-to-output ratio NX/Y to a 1 percentage negative shock to home productivity shock A. Blue solid lines indicate responses from the benchmark model *with* firm default risks. Red dashed lines indicate responses from the no-default model *without* firm default risks.

Table 4: Sensitivity Analysis of RRP-output Correlation as Calibration Target

	$\rho(RRP, \Delta y - \Delta y^*)$	$\frac{\sigma(\Delta e)}{\sigma(\Delta y)}$	$\rho(\Delta e, \Delta y)$	$\rho(\Delta \frac{nx}{y}, \Delta y)$	$\rho(\Delta e, \Delta c - \Delta c^*)$	$\rho(e, c - c^*)$
Data	-0.13	4.07	-0.14	-0.06	-0.26	-0.12
Model						
$\eta_1 = 0.22$ (Benchmark)	-0.21	2.13	-0.43	-0.25	-0.20	-0.11
$\eta_1 = 0.99$	-0.48	6.53	-0.47	-0.31	-0.63	-0.16

4.4 The Role of Trade elasticity

A low trade elasticity in Corsetti et al. (2008) generates similar wealth effects to address the Backus-Smith puzzle. In this section, I compare the role of financial fictions and trade elasticity in my model.

The consumption bundle of home and foreign final goods is revised to a general CES aggregator as follows:

$$C_t = [\lambda^{1-\eta} C_{H,t}^{\eta} + (1-\lambda)^{1-\eta} C_{F,t}^{\eta}]^{1/\eta}$$

$$C_t^* = [(1-\lambda)^{1-\eta} C_{H,t}^{*\eta} + \lambda^{1-\eta} C_{F,t}^{*\eta}]^{1/\eta}$$

The price elasticity of tradable goods echos the elasticity of substitution between home and foreign goods which is $\phi = \frac{1}{1-\eta}$. Home and foreign price levels and real exchange rates are as follows:

$$P_t = [\lambda P_{H,t}^{\frac{\eta}{\eta-1}} + (1-\lambda) P_{F,t}^{\frac{\eta}{\eta-1}}]^{\frac{\eta-1}{\eta}}$$

$$P_t^* = [\lambda P_{F,t}^{\frac{\eta}{\eta-1}} + (1-\lambda) P_{H,t}^{\frac{\eta}{\eta-1}}]^{\frac{\eta-1}{\eta}}$$

$$\mathcal{E}_t = \frac{P_t^*}{P_t}$$

Table 5 reports the real exchange rate properties and trade balances when trade elasticity varies from 1, the value in the benchmark model, to 5, close to the upper bound suggested by the literature *(cite)*. The volatility of exchange rate relative to output increases in trade elasticity as the future consumption growth become more volatile. It leads to a more negative Backus-Smith correlation. Meanwhile, investment becomes more volatile with a higher trade elasticity. The full set of moments is in Appendix Table C1. In summary, the role of financial frictions is not confounded with that of trade elasticity in addressing the

Backus-Smith puzzle.

Table 5: The Role of Price Elasticity of Tradables

		Value of Trade elasticity ϕ				
	Data	1	2	3	4	5
$\sigma(\Delta e)/\sigma(\Delta y)$	4.07	2.13	4.42	4.68	3.48	5.01
$\rho(\Delta e, \Delta y)$	-0.14	-0.43	-0.47	-0.48	-0.47	-0.49
$\rho(\Delta \frac{nx}{y}, \Delta y)$	-0.06	-0.25	-0.19	-0.13	-0.12	-0.06
$\rho(\Delta e, \Delta c - \Delta c^*)$	-0.26	-0.20	-0.86	-0.84	-0.80	-0.80

Notes: Data moments: the US v.s. G7 countries, 1975Q1-2017Q4, at quarterly frequency. Model moments: simulations of order 1 deviation from steady states, HP filer=1600.

5 Conclusion

In this paper, I incorporate an incomplete-market IRBC model with micro-founded financial frictions in endogenous default choices. Financial frictions have large demand effects to address the Backus-Smith puzzle. I derive a novel data moment from the micro-foundation to quantify the degree of transmission of financial frictions to exchange rates, the relative default risk premium.

I empirically estimate the relative default risk premium from bond-level data and use its cyclicality to discipline the model. With the existence of financial frictions, the model is able to quantitatively reproduce counter-cyclical exchange rates consistent with the Backus-Smith puzzle. I also find that the role of trade elasticity is diminished as firm-level financial frictions play a major role on exchange rates.

Uncovering sources of frictions in international risk sharing has significant policy implications. Default risks other than segmented international financial market should make policymakers more cautious in using foreign exchange interventions to improve risk-sharing across boarders.

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A Equations and Proof

A.1 Rewrite firm's problem in leverage

I define firm's leverage ratio as $\omega_t^j \equiv b_t^j / k_t^j$. It is the single endogenous state variable in firm's problem. The value function, default cut-off, bond valuation and firm first-order conditions are rewritten in terms of leverage ratio as follows.

Value function:

$$\begin{aligned} v_t = \frac{V(k_t^j)}{k_t^j} &= \max_{\omega_{t+1}^j, i_t^j} \omega_{t+1}^j g(i_t^j) q_t - (i_t^j - \tau\delta) - \frac{\lambda_k}{2}(g(i_t^j) - 1)^2 \\ &+ g(i_t^j) E_t M_{t,t+1} \int_{\underline{z}}^{z_{t+1}^{j*}} \{(1-\tau)(R_{t+1} - z_{t+1}^j) - [(1-\tau)c + 1]\omega_{t+1}^j + v_{t+1}\} d\Phi(z_{t+1}) \end{aligned} \tag{26}$$

Default cut-off:

$$z_{t+1}^{j*} = R_{t+1} - (c + \frac{1}{1-\tau})\omega_{t+1}^j + \frac{1}{1-\tau} v_{t+1} \tag{27}$$

Bond prices:

$$\begin{aligned} \omega_{t+1}^j q(\omega_{t+1}^j) = E_t M_{t,t+1} \{ &\Phi(z^*(\omega_{t+1}^j))(c+1)\omega_{t+1}^j \\ &+ \int_{z^*(\omega_{t+1}^j)}^{\bar{z}} [(1-\tau)(R_{t+1} - z_{t+1}^j) + v_{t+1} - \xi] d\Phi(z_{t+1}) \} \end{aligned} \tag{28}$$

$$\begin{aligned} \omega_{t+1}^j \frac{q(\omega_{t+1}^j)}{\mathcal{E}_t} = E_t M_{t,t+1}^* \{ &\frac{1}{\mathcal{E}_{t+1}} \Phi(z^*(\omega_{t+1}^j))(c+1)\omega_{t+1}^j \\ &+ \int_{z^*(\omega_{t+1}^j, \gamma_t^j)}^{\bar{z}} [(1-\tau)(R_{t+1} - z_{t+1}^j) + v_{t+1} - \xi] d\Phi(z_{t+1}) \} \end{aligned} \tag{29}$$

Firm's first-order-conditions are derived as follows (ignore superscript j as firms are ex-ante identical).

Optimal investment $\partial v_t / \partial i_t$:

$$\begin{aligned} &1 - \omega_{t+1} q_t - \lambda_k (g(i_t) - 1) \\ &= E_t M_{t,t+1} \int_{\underline{z}}^{z_{t+1}^*} \{(1-\tau)(R_{t+1} - z_{t+1}) - [(1-\tau)c + 1]\omega_{t+1} + v_{t+1}\} d\Phi(z_{t+1}) \end{aligned} \tag{30}$$

Optimal leverage $\partial v_t / \partial \omega_{t+1}$:

$$q_t + \omega_{t+1} \frac{\partial q_t}{\partial \omega_{t+1}} = -(1-\tau) E_t [M_{t,t+1} \Phi(z^*_{t+1}) \frac{\partial z^*_{t+1}}{\partial w_{t+1}}] \tag{31}$$

B Data

Table B1: Summary Statistics of the US Bond Sample

	USD Bonds	EUR Bonds
N	887	104
Amount (MM USD)	990,960.36	899.02
Maturity (year)	9.55	11.56
Coupon rate (%)	3.66	2.05
Yield-to-maturity (%)	3.39	1.89
Bid-ask spreads (%)	0.38	0.54

Note: *Amount* is the average amount at issuance, with unit of million U.S. dollars. *Bid-ask spreads* is the average trade-weighted spreads between ask and bid prices. *Yield-to-maturity* is the middle yield-to-maturity at bond issuance.

C Quantitative Results

Table C1: Aggregate Moments

Panel A: Firm-level moments						
	Default rate	Leverage ratio		Relative default risk premium		
	$E[1-\Phi(z^*)]^\dagger$	$E[\omega]^\dagger$	$\sigma(\omega)_{\times 100}$	$\sigma(.)_{\times 100}$	ACF(1)	$\rho(., \Delta y - \Delta y^*)^\dagger$
Data	0.0026	0.42	1.70	0.42	0.68	-0.13
Benchmark ($\phi = 1, \eta_1 = 0.22$)	0.0026	0.42	0.32	0.06	0.71	-0.21
Different η_1						
$\eta_1 = 0.99$	0.0026	0.42	1.56	0.66	0.71	-0.48
Different elasticity ϕ						
$\phi = 2$	0.0026	0.42	0.44	0.07	0.71	-0.21
$\phi = 3$	0.0026	0.42	0.44	0.08	0.71	-0.23
$\phi = 4$	0.0026	0.42	0.33	0.06	0.71	-0.23
$\phi = 5$	0.0026	0.42	0.44	0.08	0.71	-0.22
Panel B: Real exchange rate properties and trade balance					Backus-Smith correlation	
	ACF(1)(e)	$\sigma(\Delta e)/\sigma(\Delta y)$	$\rho(\Delta e, \Delta y)$	$\rho(\Delta \frac{nx}{y}, \Delta y)$	$\rho(\Delta e, \Delta c - \Delta c^*)$	$\rho(e, c - c^*)$
Data	0.95	4.07	-0.14	-0.06	-0.26	-0.12
Benchmark	0.71	2.13	-0.43	-0.25	-0.20	-0.11
Different η_1						
$\eta_1 = 0.99$	0.71	6.53	-0.47	-0.31	-0.63	-0.17
Different elasticity ϕ						
$\phi = 2$	0.71	4.42	-0.47	-0.18	-0.86	-0.18
$\phi = 3$	0.71	4.68	-0.47	-0.13	-0.84	-0.20
$\phi = 4$	0.71	3.48	-0.47	-0.13	-0.80	-0.18
$\phi = 5$	0.71	5.01	-0.48	-0.06	-0.80	-0.22
Panel C: International business cycle moments						
	$\sigma(\Delta y)^\dagger_{\times 100}$	ACF(1)(Δy)†	$\sigma(\Delta c)/\sigma(\Delta y)$	$\sigma(\Delta i)/\sigma(\Delta y)$		
Data	0.80	0.74	0.79	2.60		
Benchmark	0.85	0.71	0.85	1.77		
Different η_1						
$\eta_1 = 0.99$	0.87	0.71	0.85	2.67		
Different elasticity ϕ						
$\phi = 2$	0.84	0.70	0.70	2.79		
$\phi = 3$	0.84	0.70	0.71	2.84		
$\phi = 4$	0.84	0.70	0.72	2.82		
$\phi = 5$	0.84	0.70	0.73	3.10		
	$\rho(\Delta c, \Delta y)$	$\rho(\Delta i, \Delta y)$	$\rho(\Delta c, \Delta i)$	$\rho(\Delta y, \Delta y^*)$	$\rho(\Delta i, \Delta i^*)$	$\rho(\Delta c, \Delta c^*)$
Data	0.63	0.79	0.61	0.84	0.84	0.82
Benchmark	0.99	0.99	0.99	0.85	0.74	0.83
Different η_1						
$\eta_1 = 0.99$	0.99	0.90	0.92	0.85	0.46	0.79
Different elasticity ϕ						
$\phi = 2$	0.99	0.96	0.96	0.85	0.71	0.69
$\phi = 3$	0.99	0.93	0.93	0.85	0.58	0.69
$\phi = 4$	0.99	0.93	0.93	0.85	0.58	0.70
$\phi = 5$	0.99	0.81	0.82	0.85	0.26	0.69