Spring 2018
Peking University
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Computer Programming in Financial Engineering
Midterm Project

Assigned on 2pm, May 6th. Due by 10pm, May 6th. The project should be completed independently. You may only discuss the project with the instructor and the teaching assistant. Do not discuss with your classmates. Your responsibilities are described in `http://dean.pku.edu.cn`. Your solutions consist of both your answers, written or submitted as a Word or pdf file, and computer codes. You must submit both.

1. Cox, Ingersoll and Ross (CIR) model specifies the instantaneous interest rate follows

$$dr_t = \gamma(\bar{r} - r_t)dt + \sqrt{\alpha r_t}dX_t \tag{1}$$

Equation (1) is known as the risk-natural (physical) dynamics for the instantaneous interest rates.

The risk-neutral dynamics for the instantaneous interest rates follows

$$dr_t = \gamma^*(\bar{r^*} - r_t)dt + \sqrt{\alpha r_t}dX_t \tag{2}$$

Let $Z(r_t, t, T)$ denote the time-t price for a zero coupon bond (ZCB) that matures at time T with the current instantaneous interest rate r_t. The ZCB has terminal payoff

$$Z(r_T, T, T) = 1$$

The solution to the fundamental bond pricing equation yields

$$Z(r, t; T) = e^{A(\tau)-B(\tau)r_t}$$

$$B(\tau) = \frac{2(e^{\psi\tau} - 1)}{(\gamma^* + \psi)(e^{\psi\tau} - 1) + 2\psi}$$

$$A(\tau) = 2\frac{\gamma^* \bar{r^*}}{\alpha} \log\left(\frac{2\psi(e^{(\psi+\gamma^*)(\tau/2)})}{(\gamma^*+\psi)(e^{\psi\tau}-1)+2\psi}\right)$$

$$\psi = \sqrt{(\gamma^*)^2 + 2\alpha}$$

where $\tau = T - t$ is the time to maturity. The excel spreadsheet USTreasurySpotRate contains monthly data on continuously compounded annualized interest rates from 1961 to 2014.

(a) [15 points] From the data on continuously compounded interest rates, calculate the corresponding ZCB bond prices (face values are 100 units of the currency) (You don't need to report all prices). Report means and standard deviations of bond prices with different maturities.

(b) [15 points] Estimate the CIR model for a panel of interest rates using nonlinear least squares. The data is from USTreasurySpotRates.xls. The goal is to minimize the sum of squared pricing errors, that is,

$$\min_{\gamma^*, \bar{r^*}, \alpha} J(\gamma^*, \bar{r^*}, \alpha) \equiv \sum_{t=1}^{T} \sum_{i=1}^{d} (P_{i,t}^{data} - P_{i,t}^{CIR})^2$$

where d is the number of bonds in the dataset and d equals to 12. (Note: we are minimizing the sum of squared pricing errors instead of interest rates errors.) Also, treat the 3-month interest rate as the instantaneous interest rate when using the solution of the CIR model.

(c) [10 points] If the current instantaneous interest rate is 0.02, plot yield curves of the CIR model in the same figure for $\alpha = 1.2373$ and α estimated from question (b). You could treat $\gamma^* = 1.24$, $\bar{r}^* = 0.04$.

(d) [10 points] Simulate Equation (2) by the Euler method. Set the parameters as $\alpha = 1.2373$, $\gamma^* = 1.24$, $\bar{r}^* = 0.04$. For the square root diffusion process, r_t doesn't drop below zero in continuous time. However, in discretized simulation, it may drop below 0. So replace the diffusion term by $\sqrt{\alpha \max(r_t, 0)}$

if r_t drops below 0. Simulate the process with a time step of 252 periods per unit interval. Set the random number seed to be 123. Also, simulate the process from r_t=0.02 at $t = 0$ till $t = 1$ for 10,000 times and report the sample mean and variance for the simulated distribution at time 1.

(e) [15 points] Rolling Window Estimation: A fund manager takes the course and wants to use the CIR model to build a quantitative investment strategy. To this end, she proposes the following rolling estimation procedure. Instead of estimating the parameters of equation (2) using all historical interest rates data, estimate the the CIR model using the previous 24 months bond yields data (not including the current month) for each month. Again, minimize the sum of squared pricing errors. Each month after estimation, use the estimated CIR model to produce the relative pricing errors for the cross-section of bonds. (Relative pricing errors = (Actual Prices - Theoretical Prices) /Theoretical Prices) All ZCBs have face values of 100. The idea is to dynamically update the CIR model parameters to find potential arbitrage opportunities. After the rolling estimation, report your relative pricing results for 10-year bonds from January 2007 to January 2008. For estimation in (e) and (f), also treat the 3-month interest rate as the instantaneous interest rate.

(f) [15 points] A dynamic arbitrage strategy is to sell the bond with largest relative overpricing (when actual price is larger than theoretically predicted) and buy the bond with the largest underpricing (when actual price is smaller than theoretically predicted) each month. Again, all ZCBs have face values of 100. The concept of relative pricing is defined in (e). The parameters of equation (2) are updated each month by methods in question (e). The strategy is to select Δ_t to make the portfolio value $\Pi(r_t, t)$ insensitive to changes in the interest rate.

$$\Pi(r_t, t) = -P_{overpricing} + \Delta_t \times P_{underpricing}$$

where $P_{overpricing}$ and $P_{underpricing}$ are prices of the most overpriced and underpriced bonds. Note in the above portfolio, only 1 unit of the most overpriced bond is sold. The hedge ratio Δ_t is chosen according to the CIR model. Start this strategy from June 1963 till the end of 2014. Report Δ_t and the maturity of the most overpriced bond from June 1990 to December 1990.

Bonus part: Note in the strategy described above $\Pi(r_t, t)$, only 1 unit of the most overpriced bond is sold. Suppose investors have wealth W_t and are only buying and selling bonds with mispricing as described, the budget constraint is then $\Pi(r_t, t) * U_t = W_t$ where U_t is units of the most overpriced bond sold. Implement the strategy subject to the budget constraint starting from 100 dollar at June 1963 till the end of 2014. What is the historical return of this strategy? Report the mean and standard deviation of the return.

(g) [20 points] Pricing of callable bonds. Consider a callable bond that matures in 5 years. This callable bond is a standard fixed coupon bonds, but ones in which the issuer retains the option to buy back the bond for its par value (face value) at specific time before maturity. The annual coupon rate of the bond is 5% and coupons are paid out semi-annually. The callablle bond has face value of 100. The current instantaneous interest rate is 0.04. The issuer has the option to call the bond at the end of 3 years immediately after the coupon is paid out. Use Monte-Carlo simulation methods (Euler method) and CIR model to price this callable bond. Keep in mind that the callable bond can be viewed as a portfolio of the non-callable bond minus the value of the option to call the bond. The issuer will call the bond only if the value of the bond is higher than the face value. The non-callable part can be directly priced according to the CIR model. The parameters of equation (2) is taken as $\alpha = 1.2373$, $\gamma^* = 1.24$, $\bar{r}^* = 0.04$.