Parallel Algorithm Evaluation

Pangfeng Liu
National Taiwan University

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Introduction

- Algorithm describes the procedure that solves a problem.
- *Algorithmic thinking* is essential when one wants to use computers to solve problems, in the sense that computers can easily realize an algorithm with its powerful computation capability.
- This is not confined to problem-solving with computers, but we focus on algorithms for computers in this lecture.

Computers

- Some algorithms are particularly suitable for computers.
- Consider a Sudoku problem. You want to place 1 to 9 into a nine by nine matrix so that every row, every column, and the nine three by three sub-matrices all have numbers 1 to 9.
- We can use a *trial-and-error* algorithm to solve this problem. However, this is time-consuming and error-prone for the human to execute this algorithm.
- A computer programmer can easily convert the trial-and-error algorithm to a computer program that solves this problem in no time.

Definition

- An algorithm must describe the *detailed operations* one wants to perform.
- These operations must be *well-defined* within the underling model (more on this later).
- The algorithm must also describe the *temporal dependency* of these operations, i.e., loop, synchronization, etc.

Example

- How to pick the largest number from a set of numbers.
- Look through all the numbers one at a time.
- Compare a number with the current largest one you have seen.
- If the number is the larger, replace the current largest one with it.
- Finally, you have the largest number.

Example

- The operations, i.e., examine, compare, and replace, are well defined.
- The temporal sequences of these operations are also well defined – "look through all", "if something happens then does this".
- This is an algorithm indeed.

Discussion

- Give an example of algorithm.

Parallel Algorithms

- Now, we extend the concept to parallel computation.
- A *sequential* algorithm describes the procedure that solves a problem with a computer.
- A *parallel* algorithm describes the procedure that solves a problem with a parallel computer.
- In this course, we will focus on parallel algorithms.

Parallel Algorithms

- A sequential algorithm is relatively easy to describe.
  - The timing sequence is for one processor only.
- A parallel algorithm is *harder* to describe since we have to deal with multiple entities working concurrently.
  - The timing constraints are about multiple processors, hence much difficult to describe and analyze.
  - Different processors may need to access the same data and have race conditions.

Model

- Any algorithm has an underlining assumption on allowed operations. For example, a sorting algorithm may assume that it can compare two keys in a constant amount of time.
- We then follow these assumptions to estimate the *cost* of the algorithm we are considering.
- The purpose of the model is to estimate the cost accurately, so it must be realistic, which means it must resemble the real hardware to be meaningful.

Analysis

- The process of estimating the cost of an algorithm is *algorithm complexity analysis*.
- Note that we are not estimating the algorithm's running time since this is a moving target.
- Instead we measure *the number of times* certain operations (e.g., computation or communication), and use them as the estimate on the cost of the algorithm.
- We want to design algorithms will low costs.

Analysis

- A sequential algorithm is usually easy to analyze.
- A parallel algorithm is much more difficult to analyze since different models, i.e., the assumption on how the parallel computer can do, have different algorithmic characteristics.

Analysis

- A shared memory model algorithm for a multiprocessor can differ from a distributed memory algorithm for a multicomputer.
- Nevertheless, there are certain criteria that we may follow, mostly from what the actual CPU can do in a fixed amount of time.
- In this lecture, I will try my best to focus on the parallel computing issues instead of the computation models, i.e., I want to have generic analysis instead of model-specific analysis.

Discussion

- Give an example of algorithm analysis, like doing an barrier synchronization on a distributed memory parallel computer..

Speedup

- How to determine which parallel algorithm is good, and which is bad?
- We use *speedup* as a metric to evaluate parallel algorithms, which is the ratio between the best sequential time T_s and the parallel time T_p.

$$k = \frac{T_s}{T_p} \tag{1}$$

- Note that we need to use the *best* T_s for a meaningful comparison.

Speedup Improvement

- Remember that performance is paramount for parallel processing (remember the racecar?), so speedup is essential.
- There are two ways to improve speedup.
  - The *right* way is to reduce the parallel time.
  - The *wrong* way is to increase the sequential time. That is why we need to use the best sequential algorithm.

Lesson

When you hear people talking about speedup, always make sure that you know the definition of their "speedup".

Banana and Orange

- We should always calculate the speedup with *the same basis* (in my opinion).
- What do we get if we compare the sequential time of a sequential program running on a CPU with the parallel time of the same program running on five GPUs and get the speedup of 136?

Banana and Orange

- The comparison between one CPU ad five GPUs is not *quantitatively* meaningful because we cannot derive any *quantitative* conclusion on how well we are doing.
- We may have a terrible implementation and still get good speedup because the CPU is running slowly, or the GPUs are running fast like hell.
- You compare banana and orange.

Relative Speedup

- A speedup comparison is quantitatively meaningful if we can relate the speedup with the extra amount of resources we use in the parallel computation.
- Another speedup metric is to compare the parallel time of using k processors with the *parallel time* of using a single processor. This ratio is *relative speedup*.
- As the definition points out, a relative speedup is how well we parallelize a computation.

Discussion

- Describe the concept of speedup by examples.

Overheads

- Note that the execution time of a sequential algorithm may be shorter than a parallel program using a single processor.
- There are inherent overheads in parallel program execution, even if you use only one processor.

Overheads

- A parallel system may need to start up.
- The parallel program may use a parallel library, which may incur extra overhead.
- There may be synchronization overhead, even if only one processor is involved.
- To keep the following theoretical discussion (on efficiency and work) simple, we will assume that these two are the same, but keep in mind that there are always overheads in parallel programming.

Parallelism

- The speedup k alone is not sufficient to evaluate the algorithm, since we do not know how many *processors* are used.
- Let p be the number of processors used in the parallel algorithm.
- The speedup k is between 0 and p.

$$0 < k \leq p \tag{2}$$

- If the speedup is close to p then we have a *linear speedup*.

Theoretical Bounds

We can argue mathematically that speedup k is between 1 and p.

$$1 \leq k \leq p \tag{3}$$

Proof

- We can use one processor to simulate the sequential algorithm, hence $1 \leq k$.
  - Recall that we assume that we do not have overheads due to parallelization.

Proof

- We can simulate one step of the parallel algorithm with p steps on one processor.
  - If that takes less than k steps, we have an algorithm that runs faster than our optimal sequential algorithm, which is a contradiction.
  - Note that we need to assume that the sequential algorithm is optimal.

In Practice

- In rare occasion we may have $T_s < T_p$. This anomaly is usually the result of a minimal workload and a tremendous amount of overheads in parallel execution.
- Recall that we have all the overheads due to parallelization. We do not have any speedup if parallelization benefits are not enough to compensate for the overheads.
- Some problems are better left alone (e.g., inherently sequential problems).

In Practice

- Also in rare occasion we may even have $k > p$, which is *super-linear speedup*.
- This is usually due to the size of the working set of the program.

Caching Effects

- If a problem has a huge working set, then it is impossible to fit it into the cache of a single computer. As a result, frequent cache misses degrades performance significantly.
- If we divide the problem into many small sub-problems and run them in parallel, then it is likely that each working set of these small sub-problems will fit into the cache of a processor, hence having amazing speedup.

Lesson

Always question theory in the context of the *real world*.

Discussion

- Describe the relation of speedup and the number of processors.

Efficiency

Another metric to evaluate parallel algorithms is *efficiency*, which is defined as the speedup divided by the number of processors.

$$e = \frac{k}{p} = \frac{T_s}{pT_p} \tag{4}$$

Efficiency

- It is easy to see that efficiency e is between 0 and 100%, if the speed up is between 0 and p, where p is the number of processors.
- If we fully parallelize a computation, the efficiency is 100%.
- One can think of the efficiency as "how well we parallelize the computation per processor?" – that is, it is the CP value, or performance/cost ratio.

Discussion

- Describe the relation of efficiency and speedup.

Work

Another metric for parallel algorithm evaluation is *work*, which is the product of the number of processors and the parallel execution time.

$$w_p = pT_p \tag{5}$$

Energy

- The concept of *work* is like person-month; you used this many processors for this period.
- It is like that the energy is the product of power and time; you used this much power for a period of time, then you use this much energy.

Extra Work

- The work done by a sequential algorithm is $w_s = T_s$, and the work done by a parallel algorithm is $w_p = pT_p$.
- We assume that all the work done by the sequential algorithm is *necessary*; that means all the work done by the sequential algorithm is *essential*.
- Now, if the work done by the parallel algorithm is larger than the work done by the sequential algorithm, then the difference is *non-essential* in solving this problem.
- This is what we called *overheads*.

Extra Work

- If we parallel a computation well, the two works done by the sequential algorithm and the parallel algorithm should be similar, which implies three things.
  - The Efficiency e should be close to 1.
  - The speedup k should be close to p.
  - w_s should be close to w_p.

$$w_p = pT_p = p\frac{T_s}{k} = \frac{w_s}{e} \tag{6}$$

Discussion

Compute the speedup, efficiency, work for the following, and describe the circumstance where each of these actions is suitable.

- Using one processor and the time is 40 minutes.
- Using two processors and the time is 24 minutes.
- Using four processors and the time is 16 minutes.

Amdahl's Law

- The speedup of a program using multiple processors in parallel computing is limited by the time needed for the sequential fraction of the program.

Amdahl's law

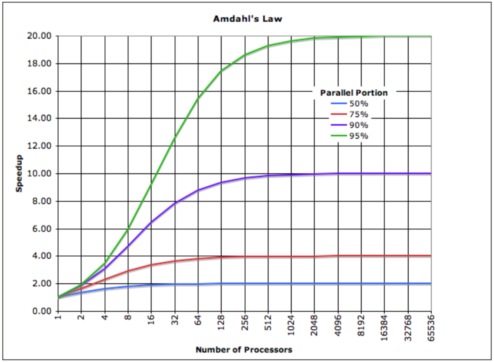
- Let the portion of inherent sequential in a computation be x, the part that can be parallelized will be $1 - x$.
- Assume that the sequential time is 1, then the parallel time will be at least $x + \frac{1-x}{p}$ while using p processors.

$$k = \frac{1}{x + \frac{1-x}{p}} \leq \frac{1}{x} \tag{7}$$

Amdahl's Law

- Amdahl's law says that if you have 20% of you code is inherently sequential, the speedup could not be more than 5.
- That hurts!

Amdahl's Law



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[1] http://upload.wikimedia.org/wikipedia/commons/6/6b/AmdahlsLaw.png

Implications

- Every program has an *inherently* sequential part, which limits the speedup.
- If the inherent part is large, the program is inherently sequential. We cannot parallelize these computations efficiently.
- The lesson here is recognizing that these cases have only limited parallelism.

Discussion

- Describe Amdahl's Law.

Two Ways

- Remember there are two ways to improve speedup. We either increase the sequential time, or we decrease the parallel time.
- In many cases, we can increase the sequential time by increasing the *problem size*. If the impact of the increased problem size on sequential time is larger than on the parallel time, then we have a *better* speedup.

Gustafson's Law

- Computations involving *arbitrarily* large data sets can be efficiently parallelized.
- This is true only when the "inherently sequential" part is a constant, i.e., it is not a function of the problem size.

Gustafson's Law

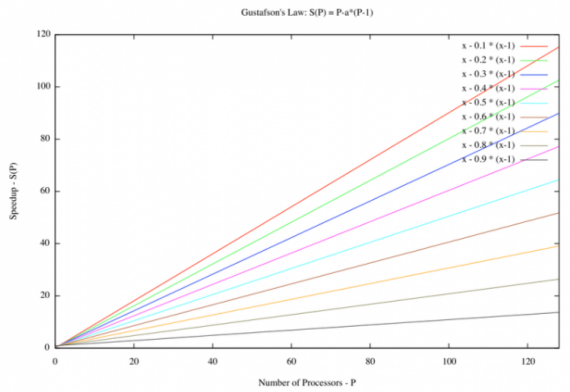
- Let the portion of inherent sequential in a computation be x, the part that can be parallelized will be $1 - x$.
- Instead of a fixed problem size, we increase the number of processors to p. Then after x of inherent sequential work, p processors does $p(1 - x)$ amount of work.

Gustafson's Law

- The sequential time is now at least $x + p(1-x)$, since the amount of work done by p processors is $p(1-x)$, and the sequential program only has one processor to do it.
- The parallel time is $x + (1-x) = 1$ with p processors.
- We can have speedup close to p for any x by increasing p.

$$k = p(1-x) = p - px \tag{8}$$

Gustafson's Law



[2]

[2] http://slashdot.org/topic/wp-content/uploads/2012/08/Screen-Shot-2012-08-29-at-12.19.53-PM-618x425.png

Notes

- We assume that the x portion of the sequential part is still sufficient even if we increase the work p times.
- In many cases, this is true if x only involves setting up the systems, and we can read data in parallel.
- If we cannot read data in parallel, then x is a function of the problem size, which invalidates the analysis.

Notes

- Despite the possible complications, this observation is generally useful because x is usually a small constant if only system setup is involved. In addition, most computation has complexity like $O(n^2)$ so the work of reading the data, even if done sequentially, is not comparable to the actual computation.

Notes

- If we want to have a clear picture about speedup, we would also like to know the problem size when the speedup is measured.
- If we have a *good* speedup even if the problem size is small, then we have a good parallel implementation. Otherwise, it could be a fabricated phenomenon with Gustafson's Law's use (or abuse).
- When you examine a speedup report, always ask for the baseline for comparison and the problem size.

Discussion

- Describe Gustafson's Law.