84. Largest Rectangle In Histogram (/problems/largest-rectangle-in-histogram/)

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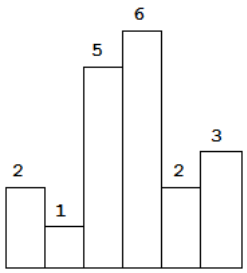
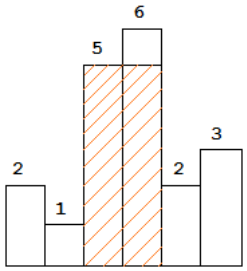
Given *n* non-negative integers representing the histogram's bar height where the width of each bar is 1, find the area of largest rectangle in the histogram.



Above is a histogram where width of each bar is 1, given height = `[2,1,5,6,2,3]`.



The largest rectangle is shown in the shaded area, which has area = `10` unit.

For example,
Given heights = `[2,1,5,6,2,3]`,
return `10`.

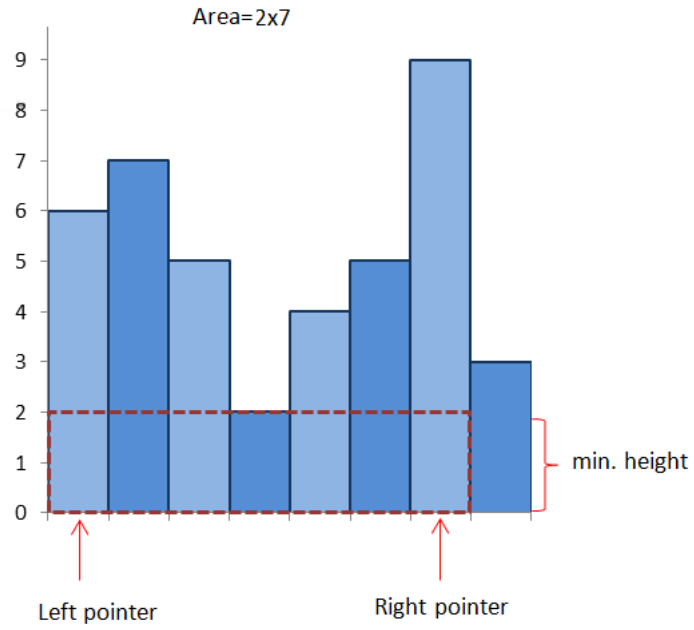
Summary

We need to find the rectangle of largest area that can be formed by using the given bars of histogram.

Solution

Approach #1 Brute Force [Time Limit Exceeded]

Firstly, we need to take into account the fact that the height of the rectangle formed between any two bars will always be limited by the height of the shortest bar lying between them which can be understood by looking at the figure below:



Thus, we can simply start off by considering every possible pair of bars and finding the area of the rectangle formed between them using the height of the shortest bar lying between them as the height and the spacing between them as the width of the rectangle. We can thus, find the required rectangle with the maximum area.

Java

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```
public class Solution {
    public int largestRectangleArea(int[] heights) {
        int maxarea = 0;
        for (int i = 0; i < heights.length; i++) {
            for (int j = i; j < heights.length; j++) {
                int minheight = Integer.MAX_VALUE;
                for (int k = i; k <= j; k++)
                    minheight = Math.min(minheight, heights[k]);
                maxarea = Math.max(maxarea, minheight * (j - i + 1));
            }
        }
        return maxarea;
    }
}
```

Complexity Analysis

- Time complexity : $O(n^3)$. We have to find the minimum height bar $\left(O(n)\right)$ lying between every pair $\left(O(n^2)\right)$.
- Space complexity : $O(1)$. Constant space is used.

Approach #2 Better Brute Force[Time Limit Exceeded]

Algorithm

We can do one slight modification in the previous approach to optimize it to some extent. Instead of taking every possible pair and then finding the bar of minimum height lying between them everytime, we can find the bar of minimum height for current pair by using the minimum height bar of the previous pair.

In mathematical terms, $minheight = \min(minheight, heights(j))$, where $heights(j)$ refers to the height of the jth bar.

Java

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```java
public class Solution {
    public int largestRectangleArea(int[] heights) {
        int maxarea = 0;
        for (int i = 0; i < heights.length; i++) {
            int minheight = Integer.MAX_VALUE;
            for (int j = i; j < heights.length; j++) {
                minheight = Math.min(minheight, heights[j]);
                maxarea = Math.max(maxarea, minheight * (j - i + 1));
            }
        }
        return maxarea;
    }
}
```


Complexity Analysis

- Time complexity : $O(n^2)$. Every possible pair is considered
- Space complexity : $O(1)$. No extra space is used.

Approach #3 (Divide and Conquer Approach) [Time Limit Exceeded]

Algorithm

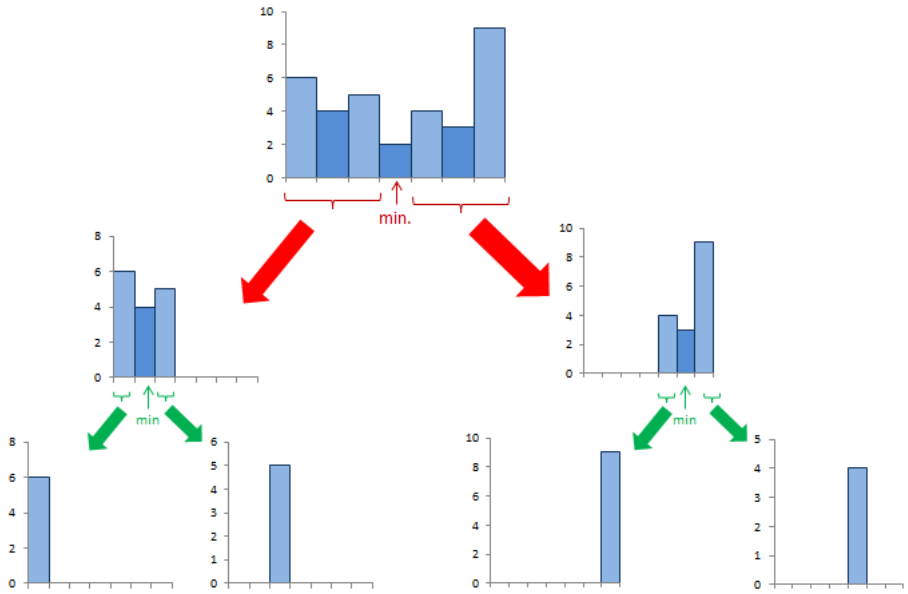
This approach relies on the observation that the rectangle with maximum area will be the maximum of:

1. The widest possible rectangle with height equal to the height of the shortest bar.
2. The largest rectangle confined to the left of the shortest bar(subproblem).
3. The largest rectangle confined to the right of the shortest bar(subproblem).

Let's take an example:

```
[6, 4, 5, 2, 4, 3, 9]
```

Here, the shortest bar is of height 2. The area of the widest rectangle using this bar as height is 2x8=16. Now, we need to look for cases 2 and 3 mentioned above. Thus, we repeat the same process to the left and right of 2. In the left of 2, 4 is the minimum, forming an area of rectangle 4x3=12. Further, rectangles of area 6x1=6 and 5x1=5 exist in its left and right respectively. Similarly we find an area of 3x3=9, 4x1=4 and 9x1=9 to the left of 2. Thus, we get 16 as the correct maximum area. See the figure below for further clarification:



Divide and Conquer

Java Copy

```java
public class Solution {
    public int calculateArea(int[] heights, int start, int end) {
        if (start > end)
            return 0;
        int minindex = start;
        for (int i = start; i <= end; i++)
            if (heights[minindex] > heights[i])
                minindex = i;
        return Math.max(heights[minindex] * (end - start + 1), Math.max(calculateArea(heights, start, minindex - 1), calculateArea(heights, minindex + 1, end)));
    }
    public int largestRectangleArea(int[] heights) {
        return calculateArea(heights, 0, heights.length - 1);
    }
}
```


Complexity Analysis

- Time complexity :

  Average Case:$O\big(n \log(n)\big)$.

  Worst Case: $O(n^2)$. If the numbers in the array are sorted, we don't gain the advantage of divide and conquer.

- Space complexity : $O(n)$. Recursion with worst case depth n.

Approach #4 (Better Divide and Conquer) [Accepted]

Algorithm

You can observe that in the Divide and Conquer Approach, we gain the advantage, since the large problem is divided into substantially smaller subproblems. But, we won't gain much advantage with that approach if the array happens to be sorted in either ascending or descending order, since every time we need to find the minimum number in a large subarray $\big(O(n)\big)$. Thus, the overall complexity becomes $O(n^2)$ in the worst case. We can reduce the time complexity by using a Segment Tree to find the minimum every time which can be done in $O\big(\log(n)\big)$ time.

For implementation, click here (https://discuss.leetcode.com/topic/45822/segment-tree-solution-just-another-idea-o-n-logn-solution).

Complexity Analysis

- Time complexity : $O\big(n \log(n)\big)$. Segment tree takes $log(n)$ n times.
- Space complexity : $O(n)$. Space required for Segment Tree.

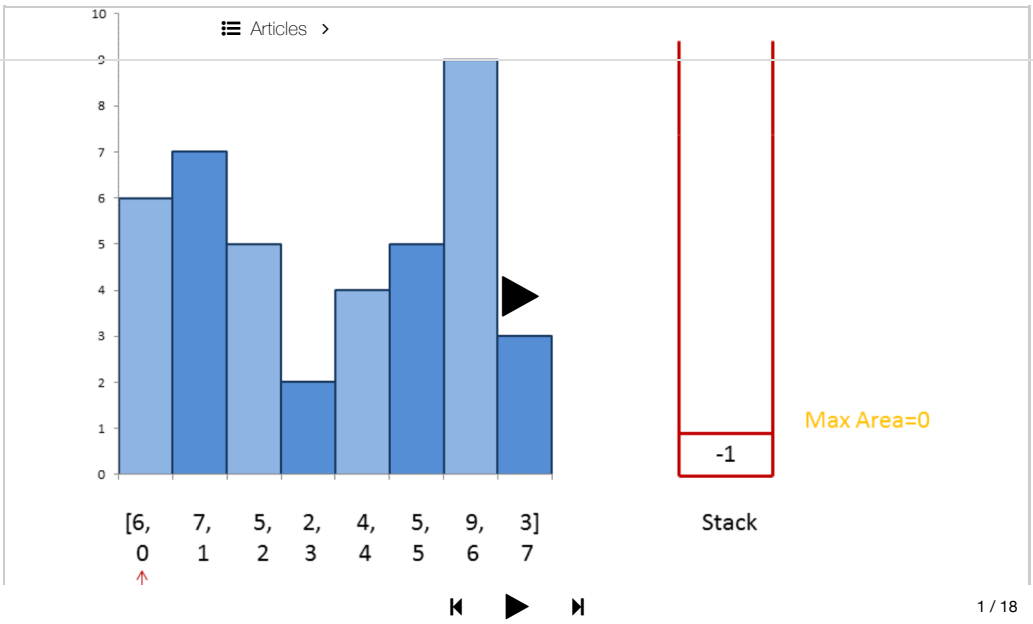
Approach #5 (Using Stack) [Accepted]

Algorithm

In this approach, we maintain a stack. Initially, we push a -1 onto the stack to mark the end. We start with the leftmost bar and keep pushing the current bar's index onto the stack until we get two successive numbers in descending order, i.e. until we get $a[i]$. Now, we start popping the numbers from the stack until we hit a number $stack[j]$ on the stack such that $a\big[stack[j]\big] \leq a[i]$. Every time we pop, we find out the area of rectangle formed using the current element as the height of the rectangle and the difference between the the current element's index pointed to in the original array and the element $stack[top-1]-1$ as the width i.e. if we pop an element $stack[top]$ and i is the current index to which we are pointing in the original array, the current area of the rectangle will be considered as $(i - stack[top-1] - 1) \times a\big[stack[top]\big]$.

Further, if we reach the end of the array, we pop all the elements of the stack and at every pop, this time we use the following equation to find the area: $(stack[top] - stack[top-1]) \times a\big[stack[top]\big]$, where $stack[top]$ refers to the element just popped. Thus, we can get the area of the of the largest rectangle by comparing the new area found everytime.

The following example will clarify the process further: `[6, 7, 5, 2, 4, 5, 9, 3]`



Java Copy

```java
public class Solution {
    public int largestRectangleArea(int[] heights) {
        Stack < Integer > stack = new Stack < > ();
        stack.push(-1);
        int maxarea = 0;
        for (int i = 0; i < heights.length; ++i) {
            while (stack.peek() != -1 && heights[stack.peek()] >= heights[i])
                maxarea = Math.max(maxarea, heights[stack.pop()] * (i - stack.peek() - 1));
            stack.push(i);
        }
        while (stack.peek() != -1)
            maxarea = Math.max(maxarea, heights[stack.pop()] * (heights.length - stack.peek() -1));
        return maxarea;
    }
}
```

Complexity Analysis

- Time complexity : $O(n)$. n numbers are pushed and popped.
- Space complexity : $O(n)$. Stack is used.

Analysis written by: @vinod23 (https://leetcode.com/vinod23)

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Signed in as **tan7**.

Post a Reply

go2ready commented last month

I guess there is a typo in the description of the algorithm. The second equation used to find area when for loop ends seems differ from the Java implementation down below. In the implementation, the formula used is: (len(a) - stack[top–1] - 1) * a[stack[top]]. Can anyone else confirm this for me? Whether it is a typo? Or I have misunderstood sth.

Andrewli commented last month

actually, I think in divide and conqure, you can use two pointers technique in every level. From the middle, one pointer moves backward, the other forward. Initial considered height is heights[middle], every time we find the left and right boundaries the current height, get the max rectangle for this height, then update the height

li.xiong.391 commented 3 months ago

If you append 0 to the end of heights, you can omit the last while loop that checks .peek() != -1.

ulyx commented 4 months ago

the last loop can be removed, and modify the range of the first loop can do the same thing:

```
public int largestRectangleArea(int[] heights) {
Stack < Integer > stack = new Stack < > ();
stack.push(-1);
int maxarea = 0;
for (int i = 0; i <= heights.length; ++i) {
while (stack.peek() != -1 && (i == heights.length || heights[stack.peek()] >= heights[i]))
maxarea = Math.max(maxarea, heights[stack.pop()] * (i - stack.peek() - 1));
stack.push(i);
}
return maxarea;
}
```

n_leetcode commented 4 months ago

Has anyone managed to pass this problem using an O(n lgn) java solution? Not sure if thats possible.

lincolnhuj commented 4 months ago

@vinod23 (https://discuss.leetcode.com/uid/1362)
In 15th slides in the example of the stack solution, should the red expression be 3>2 rather than 7>3?

vinod23 commented 8 months ago

@andreyst (https://discuss.leetcode.com/uid/86166) THanks for your suggestion. I have removed the extra variable.

A

andreyst commented 8 months ago

First of all - great article!

I think Java algo towards the end could be simplied a little bit to make it even more straightforward:

```
public class Solution {
    public int largestRectangleArea(int[] heights) {
        Stack<Integer> stack = new Stack<> ();
        stack.push(-1);
        int maxarea = 0;
        for (int i = 0; i < heights.length; ++i) {
            while (stack.peek() != -1 && heights[stack.peek()] >= heights[i])
                maxarea = Math.max(maxarea, heights[stack.pop()] * (i - stack.peek() - 1));
            stack.push(i);
        }
-        int lastIndex = stack.peek();
        while (stack.peek() != -1)
-            maxarea = Math.max(maxarea, heights[stack.pop()] * (lastIndex - stack.peek()));
+            maxarea = Math.max(maxarea, heights[stack.pop()] * (heights.length - stack.peek
() - 1));


        return maxarea;
    }
}
```

Because at the end of first cycle last element (lastIndex) will always be equal to heights.length - 1, since it is pushed onto the stack in the last line of the first loop which is unconditionally executed in the last iteration. So last term in the calculation could be presented in almost the same way as the one in the body loop, plus no need for additional var.

R

rucTeam commented 9 months ago

The last step of animation is wrong.

View original thread (https://discuss.leetcode.com/topic/71069) Load more comments...