Huahua's Tech Road
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花花酱 LeetCode 494. Target Sum

BY ZXI ON JANUARY 10, 2018

花花酱 LeetCode 494. Target Sum 上 - 刷题找工作 E…

题目大意：给你一串数字，你可以在每个数字前放置+或-，问有多少种方法可以使得表达式的值等于target。You are given a list of non-negative integers, a1, a2, ..., an, and a target, S. Now you have 2 symbols `+` and `-`. For each integer, you should choose one from `+` and `-` as its new symbol.

Find out how many ways to assign symbols to make sum of integers equal to target S.

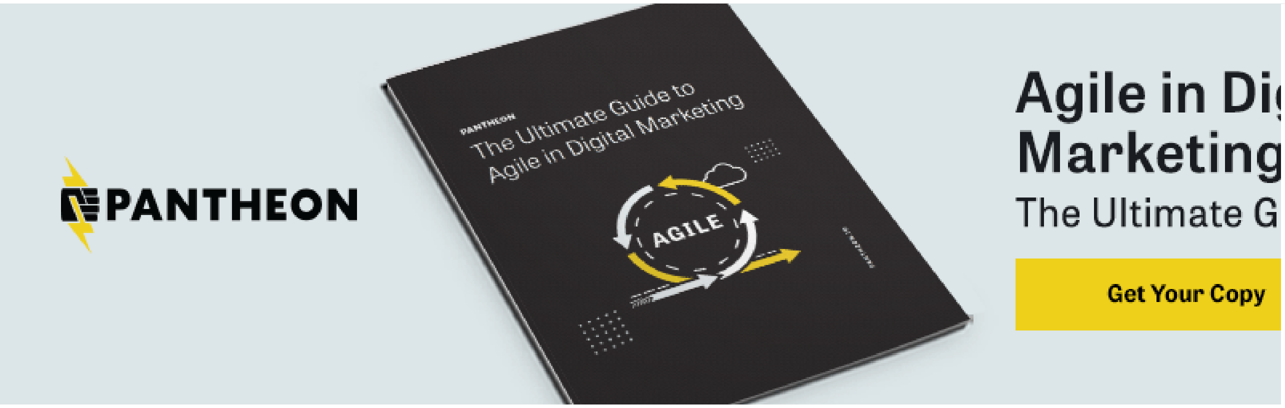
Example 1:

```
Input: nums is [1, 1, 1, 1, 1], S is 3.
Output: 5
Explanation:

-1+1+1+1+1 = 3
+1-1+1+1+1 = 3
+1+1-1+1+1 = 3
+1+1+1-1+1 = 3
+1+1+1+1-1 = 3
```

```

There are 5 ways to assign symbols to make the sum of nums be target 3.
```

Note:

1. The length of the given array is positive and will not exceed 20.
2. The sum of elements in the given array will not exceed 1000.
3. Your output answer is guaranteed to be fitted in a 32-bit integer.



Idea: DP

Why DP works?

Let's look at a simpler problem: Can the following equation can be true?

$\pm a_1 \pm a_2 \pm a_3 ... \pm a_n$ = target

$O(2^n)$ combinations, but
S = 2*sum(a) + 1 total possible sums, S <= 2000 + 1

We use V_i to denote the possible sums by using first i elements

$V_0 = \{0\}$
$V_i = \{V_{i-1} + a_i\} \cup \{V_{i-1} - a_i\}$
Check target in V_n

DP works because $|V_n| <= S << O(2^n)$

Time complexity is $Sum\{2*|V_i|\} <= n * S = O(n * S)$

input: [1,1,1,1,1], target = 3
2^5 = 32 combination, but total 6 distinct values, max is 11 (2*5 + 1)

i	a_i	V_i
0	-	{0}
1	1	{-1, 1}
2	1	{-2, 0, 2}
3	1	{-3, -1, 1, 3}
4	1	{-4, -2, 0, 2, 4}
5	1	{-5, -3, -1, 1, 3, 5}

494. Target Sum

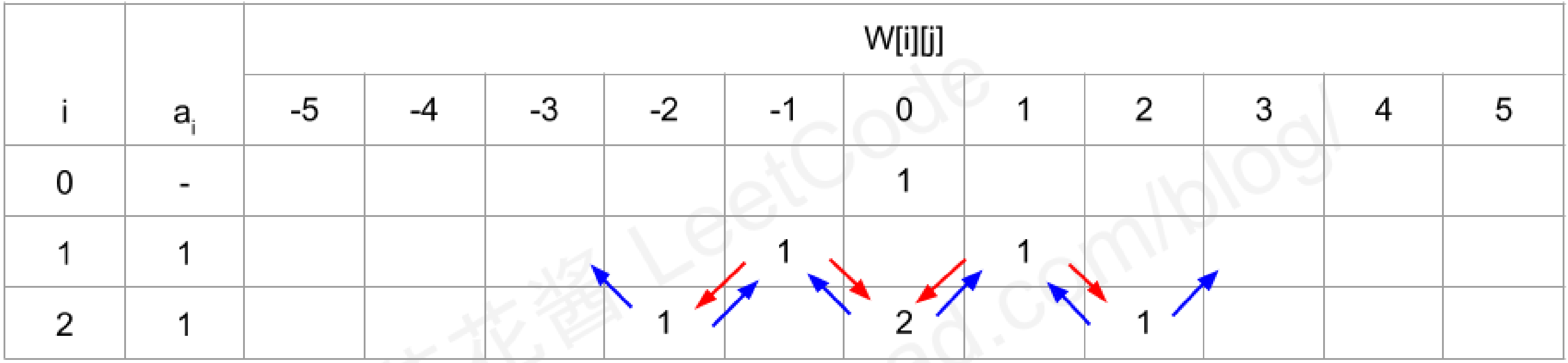
Solution 1:
Brute force: DFS
Time complexity $O(2^n)$, n = 20, AC 585 ms
Space complexity: O(n)

```
Solution 2:
DP

ways[i][j] # of ways to sum up to j using nums[0~i]
ways[i][j] =   ways[i - 1][j - nums[i]]
             + ways[i - 1][j + nums[i]]

Init: ways[-1][0] = 1, one way to sum up 0, do nothing
Ans:  ways[n-1][S]

Time complexity O(n * sum), 16 ms
Space complexity: O(n * sum) -> O(sum)
```

input: [1,1,1,1,1], target = 3
sum = 5, range = -5 ~ 5

		W[i][j]										
i	a_i	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	-						1					
1	1					1		1				
2	1				1		2		1			
3	1			1		3		3		1		
4	1		1		4		6		4		1	
5	1	1		5		10		10		5		1

Sum of (W[5][j]) is 32 = 2^5



i	a_i	W[i][j]										
		-5	-4	-3	-2	-1	0	1	2	3	4	5
0	-						1					
1	1					1		1				
2	1				1		2		1			

Transition 1: Push

```
Scan j for W[i - 1]

W[i][j - num_i] += W[i - 1][j]
W[i][j + num_i] += W[i - 1][j]
```

Transition 2: Pull

```
Scan j for W[i]

W[i][j] =    W[i - 1][j - num_i]
           + W[i - 1][j + num_i]
```

Optimization: Subset sum

Let **P** denotes a set of nums have a + sign in front of it

Let **N** denotes a set of nums have a - sign in front of it

$P \cup N = \{a_1, a_2, \ldots, a_n\} \mid P \cap N = \emptyset$

sum(**P**) - sum(**N**) = target

sum(**P**) ~~- sum(**N**)~~ + sum(**P**) + ~~sum(**N**)~~ = target + sum(**P**) + sum(**N**)

2*sum(**P**) = target + sum(a)

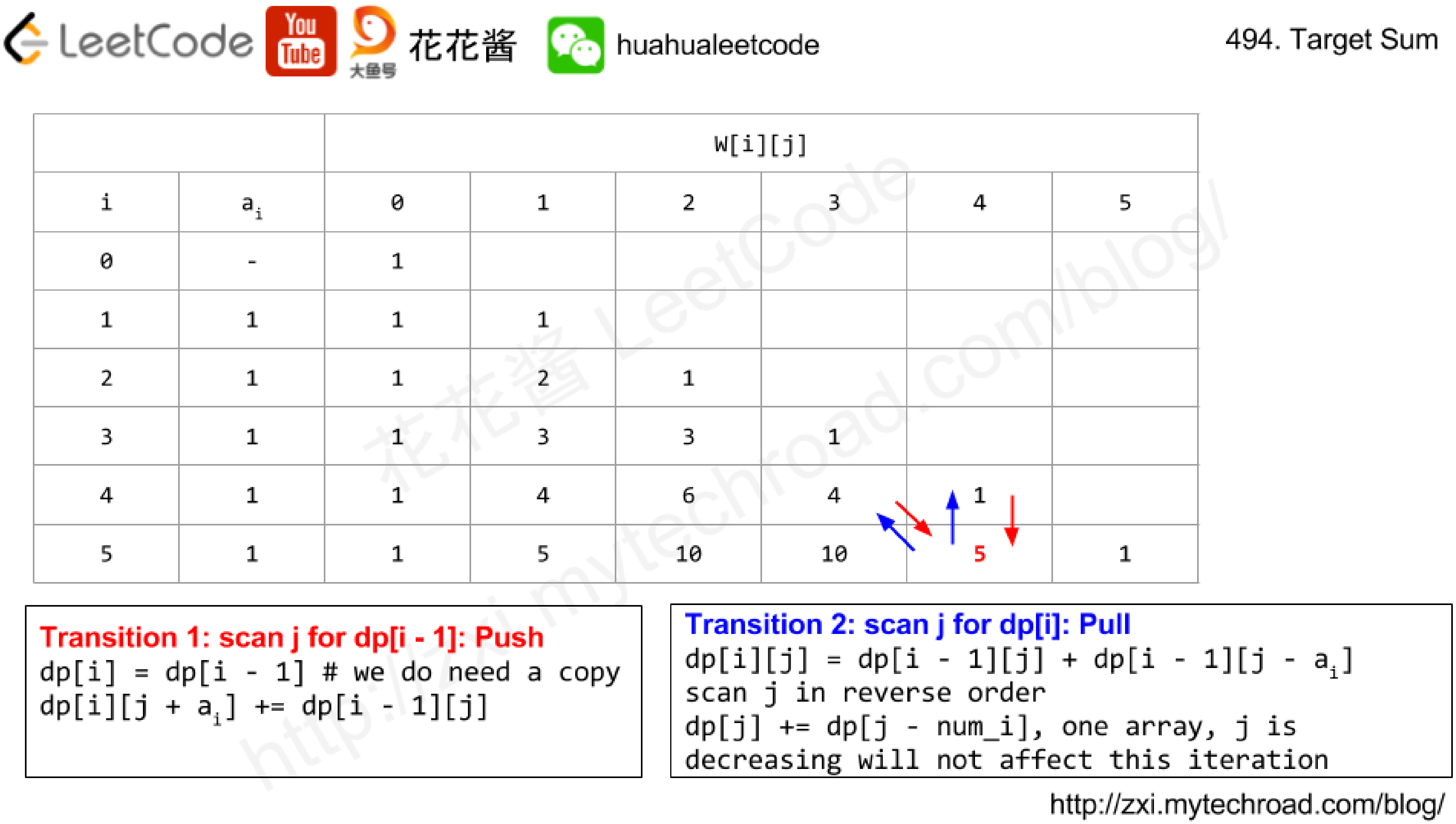
sum(**P**) = (target + sum(a)) / 2 <- 0-1 Knapsack problem

Simpler questions: using the given nums, can be sum up to target?
We use V_i to denote the possible sums by using any subset of the first i elements
$V_0 = \{0\}$
$V_i = \{V_{i-1}\} \cup \{V_{i-1} + a_i\}$ # V_i contains V_{i-1} , do we need a copy?
Ans: Check target in V_n
dp[i][j] := whether we can use the first i elements to sum up to j / j in V_i

Init: dp[0][0] = True

```
Push: scan j for dp[i - 1]
for i in 1..n:
  for j in 0..S:
   if dp[i - 1][j]:
     dp[i][j + $a_i$] = True
```

```
Pull: scan j for dp[i]
for i in 1..n:
  for j in 0..S:
    dp[i][j] = dp[i-1][j] or dp[i-1][j - $a_i$]
```



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494. Target Sum

		W[i][j]					
i	a_i	0	1	2	3	4	5
0	-	1					
1	1	1	1				
2	1	1	2	1			
3	1	1	3	3	1		
4	1	1	4	6	4	1	
5	1	1	5	10	10	5	1

Transition 1: scan j for dp[i - 1]: Push

dp[i] = dp[i - 1] # we do need a copy
dp[i][j + a_i] += dp[i - 1][j]

Transition 2: scan j for dp[i]: Pull

dp[i][j] = dp[i - 1][j] + dp[i - 1][j - a_i]
scan j in reverse order
dp[j] += dp[j - num_i], one array, j is
decreasing will not affect this iteration

http://zxi.mytechroad.com/blog/

Solution 1: DP

Time complexity: O(n*sum)

Space complexity: O(n*sum)

C++

```cpp
// Author: Huahua
// Running time: 16 ms
class Solution {
public:
    int findTargetSumWays(vector<int>& nums, int S) {
      const int n = nums.size();
      const int sum = std::accumulate(nums.begin(), nums.end(), 0);
      if (sum < S) return 0;
      const int offset = sum;
      // ways[i][j] means total ways to sum up to (j - offset) using nums[0] ~ nums[i - 1].
      vector<vector<int>> ways(n + 1, vector<int>(sum + offset + 1, 0));
      ways[0][offset] = 1;
      for (int i = 0; i < n; ++i) {
        for (int j = nums[i]; j < 2 * sum + 1 - nums[i]; ++j)
          if (ways[i][j]) {
            ways[i + 1][j + nums[i]] += ways[i][j];
            ways[i + 1][j - nums[i]] += ways[i][j];
          }
      }

      return ways.back()[S + offset];
    }
};
```

C++ SC O(n)

```cpp
// Author: Huahua
// Running time: 12 ms
class Solution {
public:
    int findTargetSumWays(vector<int>& nums, int S) {
      const int n = nums.size();
      const int sum = std::accumulate(nums.begin(), nums.end(), 0);
      if (sum < std::abs(S)) return 0;
      const int kOffset = sum;
      const int kMaxN = sum * 2 + 1;
      vector<int> ways(kMaxN, 0);
      ways[kOffset] = 1;
      for (int num : nums) {
        vector<int> tmp(kMaxN, 0);
        for (int i = num; i < kMaxN - num; ++i)
          if (ways[i]) {
            tmp[i + num] += ways[i];
            tmp[i - num] += ways[i];
          }
        std::swap(ways, tmp);
      }
      return ways[S + kOffset];
    }
};
```

Java

```
// Author: Huahua
// Running time: 28 ms
class Solution {
  public int findTargetSumWays(int[] nums, int S) {
    int sum = 0;
    for (final int num : nums)
      sum += num;
    if (sum < S) return 0;
    final int kOffset = sum;
    final int kMaxN = sum * 2 + 1;
    int[] ways = new int[kMaxN];
    ways[kOffset] = 1;
    for (final int num : nums) {
      int[] tmp = new int[kMaxN];
      for (int i = num; i < kMaxN - num; ++i) {
        tmp[i + num] += ways[i];
        tmp[i - num] += ways[i];
      }
      ways = tmp;
    }
    return ways[S + kOffset];
  }
}
```

C++ / V2

```
// Author: Huahua
// Running time: 17 ms
class Solution {
```

```
public:
  int findTargetSumWays(vector<int>& nums, int S) {
   const int n = nums.size();
   const int sum = std::accumulate(nums.begin(), nums.end(), 0);
   if (sum < std::abs(S)) return 0;
   const int kOffset = sum;
   const int kMaxN = sum * 2 + 1;
   vector<int> ways(kMaxN, 0);
   ways[kOffset] = 1;
   for (int num : nums) {
    vector<int> tmp(kMaxN, 0);
    for (int i = 0; i < kMaxN; ++i) {
     if (i + num < kMaxN) tmp[i] += ways[i + num];
     if (i - num >= 0)    tmp[i] += ways[i - num];
    }
    std::swap(ways, tmp);
   }
   return ways[S + kOffset];
  }
};
```

Solution 2: DFS

Time complexity: O(2^n)

Space complexity: O(n)

C++

```
// Author: Huahua
// Running time: 422 ms
class Solution {
public:
 int findTargetSumWays(vector<int>& nums, int S) {
  const int sum = std::accumulate(nums.begin(), nums.end(), 0);
  if (sum < std::abs(S)) return 0;
  int ans = 0;
  dfs(nums, 0, S, ans);
  return ans;
 }
private:
  void dfs(const vector<int>& nums, int d, int S, int& ans) {
    if (d == nums.size()) {
      if (S == 0) ++ans;
      return;
    }
    dfs(nums, d + 1, S - nums[d], ans);
    dfs(nums, d + 1, S + nums[d], ans);
  }
};
```

Java

```
// Author: Huahua
// Running time: 615 ms
class Solution {
  private int ans;
  public int findTargetSumWays(int[] nums, int S) {
    int sum = 0;
    for (final int num : nums)
      sum += num;
    if (sum < Math.abs(S)) return 0;
    ans = 0;
    dfs(nums, 0, S);
    return ans;
  }

  private void dfs(int[] nums, int d, int S) {
    if (d == nums.length) {
      if (S == 0) ++ans;
      return;
    }
    dfs(nums, d + 1, S - nums[d]);
    dfs(nums, d + 1, S + nums[d]);
  }
}
```

Solution 3: Subset sum

Time complexity: O(n*sum)

Space complexity: O(sum)

C++ w/ copy

```
// Author: Huahua
// Running time: 7 ms
class Solution {
public:
    int findTargetSumWays(vector<int>& nums, int S) {
      S = std::abs(S);
      const int sum = std::accumulate(nums.begin(), nums.end(), 0);
      if (sum < S || (S + sum) % 2 != 0) return 0;
      const int target = (S + sum) / 2;
      vector<int> dp(target + 1, 0);
      dp[0] = 1;
      for (int num : nums) {
        vector<int> tmp(dp);
        for (int j = 0; j <= target - num; ++j)
          tmp[j + num] += dp[j];
        std::swap(dp, tmp);
      }

      return dp[target];
    }
};
```

C++ w/o copy

```
// Author: Huahua
// Running time: 6 ms
class Solution {
public:
    int findTargetSumWays(vector<int>& nums, int S) {
      S = std::abs(S);
      const int n = nums.size();
      const int sum = std::accumulate(nums.begin(), nums.end(), 0);
      if (sum < S || (S + sum) % 2 != 0) return 0;
      const int target = (S + sum) / 2;
      vector<int> dp(target + 1, 0);
      dp[0] = 1;
      for (int num : nums)
        for (int j = target; j >= num; --j)
          dp[j] += dp[j - num];
      return dp[target];
    }
};
```

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combination DFS sum target

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