Deep Q-Learning (DQN)

1. Motivation

- In Q-Learning, we discretize the state spaces, and use a table to record and update the Q-function.
- However, most of interesting problems are too large to learn all action values in all states separately. Instead, we can learn a value function $Q_{\theta_t}(a|s)$ parameterized with θ, to predict the value of (discrete) action a at the state s.

2. Deep Q-Learning

Deep Q-learning uses a Deep Neural Network to approximate Q-function.

- For an $n-$dimentional state space $s \in R^n$ and an action space containing m discrete actions, $Q_{\theta_t}(s) : R^n \rightarrow R^m$.
- So, the value of action a_i is $q(a_i) = Q_{\theta_t}(s)[i]$, where i is the row index $i = 1 : m$. For convenience, we use the notation $Q_{\theta_t}(a|s)$

Similar to the original Q-Learning, DQN updates the parameters after taking action a_t in the state s_t, and observing the immediate reward r_t and resulting state s_{t+1}. Concretely:

\theta_{t+1} = \theta_t + \alpha \frac{1}{2}\nabla *{\theta}(Q*{target} - Q_{\theta}(a_t|s_t))^2

,

\text{where:} \hspace{1cm} Q_{target}=r_t + \gamma \max_a Q_{\theta}(a|s_{t+1})

3. What makes DQN work?

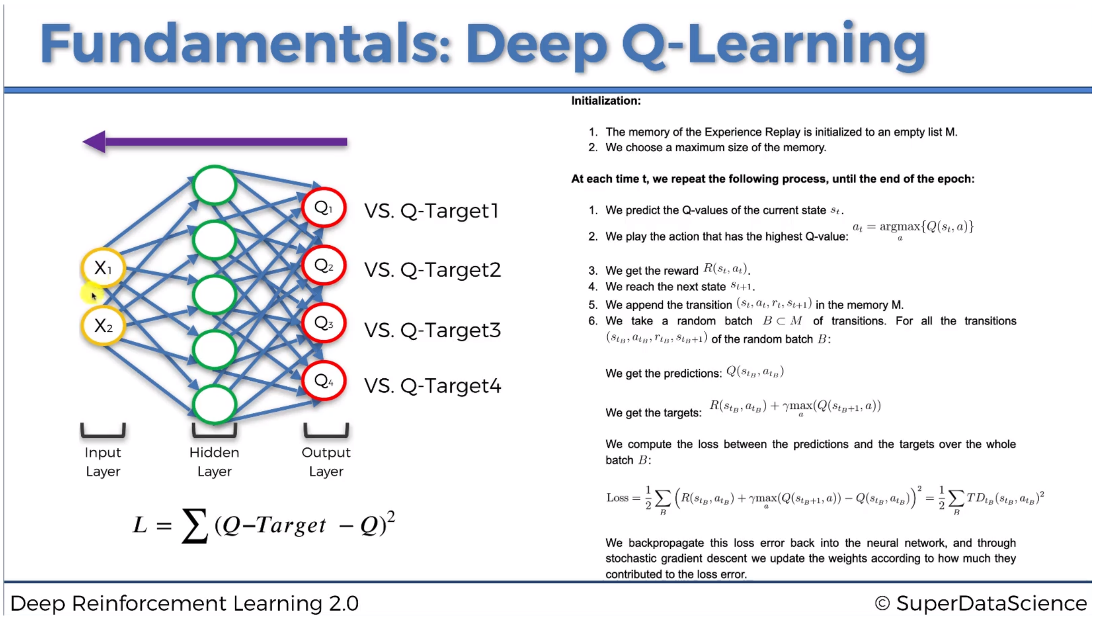
Two important ingredients of DQN algorithm as proposed by Mnih et. al. (2015) are the use of:

- **Target Network**, with parameters $\bar{\theta}$, is the same as the online network except that its parameters are copied every τ steps from the online network, so that $\bar{\theta} \leftarrow \theta$ if t, and keep fixed on all other steps. The target used by DQN is then:

  $$Q_{target} = r_t + \gamma \max_a Q_{\bar{\theta}}(a|s_{t+1})$$

  In other words, we **freeze the target network** for τ steps.
- **Experience Replay**: observed transitions (s_t, a_t, r_t, s_{t+1}) are stored for some time and **sampled uniformly** from this memory bank to update the network. This is because, DNN needs to be trained with mini-batch and SDG-like optimizer. If we use only a single sample, e.g the most current one, the network will be easily overfitted, and it cann't generalizes to all the states it saw in the past.

4. DQN Pseudo Code



5. Double DQN:

- The idea of Double Q-Learning is to reduce the overestimation by decomposing the $\max_a$ operation in the target into: **action selection** and **action evaluation**.
- Although not fully decoupled, the target network in the DQN architecture provides a natural candidate for the second value function, without having to introduce additional networks.
- Double DQN's update is the same as for DQN, but replace the target

$$Q_{target} = r_t + \gamma Q_{\bar{\theta}}(a_{opt}|s_{t+1}), \qquad \text{where} \qquad a_{opt} = \arg\max_a Q_\theta(a|s_{t+1})$$

- Here, the **optimal action** is **selected** by the main network $Q_\theta(a|s_{t+1})$, and the **action value** is **evaluated** by the target-network $Q_{\bar{\theta}}(a_{opt}|s_{t+1})$. This leaves the DQN intach, and only modify the way to compute target.

Reference:

1. Mnih, Volodymyr, et al. "Human-level control through deep reinforcement learning." nature 518.7540 (2015): 529-533.
2. Van Hasselt, Hado, Arthur Guez, and David Silver. "Deep reinforcement learning with double q-learning." Proceedings of the AAAI conference on artificial intelligence. Vol. 30. No. 1. 2016.