REINFORCEMENT LEARNING (RL) and Q-LEARNING

1. Introduction

RL is a method of learning that uses a reward function to guide the agent in the search for the best path to a goal. RL system has the follow components:

- s: **state** describes the observation of the environment.
- a: **action** describes how the agent interacts with the environment.
- $\pi(s)$: **policy** describes the rule that the agent decices to take an action a at the state s.
- r: **reward** describes the reward for the agent after performing an action.
- γ: **discount factor** describes how much the agent cares about future rewards.
- V: value function
- Q: (quality) action-value function

2. Agent's Learning Objectives:

- Each time step t, the agent chooses an action based on the current state, he will receive a reward r_t.
- Along the path, by following the policy π, the total reward R will be accumulated.

$$R_\pi = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \cdots \gamma^T r_{t+T}$$

  where γ is the discount factor, which emphasizes the recent rewards are more important than the future ones.
- The **ultimate goal** is to achieve the maximum acumulated reward R. However, since the environment is stochastic, the agent will not always get the same reward r_t for the same action a. Therefore, the accumulated reward R is expressed in term of expectation, which is called **Value**.

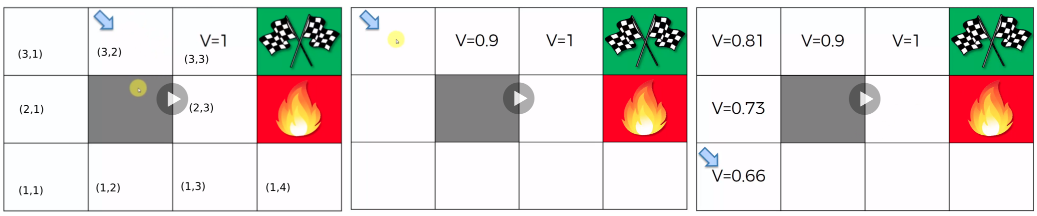
$$V_\pi(s) = E[r_t + r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \cdots \gamma^T r_{t+T} | \pi(s)]$$

3. How to choose the action?

- By the learning objective, we should of course choose the action that maximizes the value function. This is expressed by the Bellman equation:

$$V(s) = \max_a [r(s,a) + \gamma V(s')]$$

In the example below, the agent gets a reward $r = 1$ when reaching the goal, $r = -1$ when it gets into fire, and $r = 0$ otherwise. We use discount factor $\gamma = 0.9$.



- We start from near the goal $s = (3,3)$. The value of this state is: $V(s = (3,3)) = 1$, where reward $r = 1$ and stop (no further state).

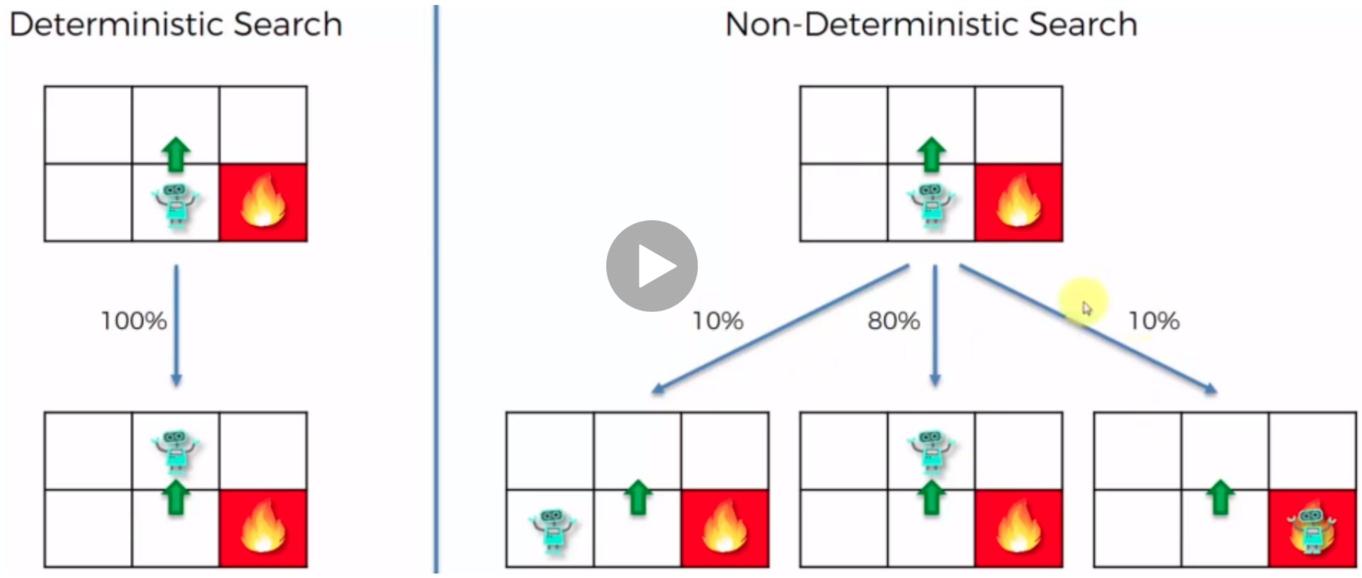
- Then, we move 1 step backward to the left $s = (3, 2)$. The value of this state is: $V(s = (3, 2)) = 0 + 0.9 * V(s = (3, 3)) = 0.9$.
- Then, we continue the steps to all other states.

This procedure is basically the "Dynamic Programing".

- We build the value map starting from the location nearing goal, and extend to all the other location in the map.
- From the Value map above, we can easily set the plan to move the agent.

4. What if the environment is uncertain ?

In the example above, the action is 100% certain, e.g `turn-left` command will definitely move the agent to the left. However, in practice, there are always uncertainties, e.g `turn-left` command will move the agent to the left in 80% cases but $0% cases it can move to the right or forward.



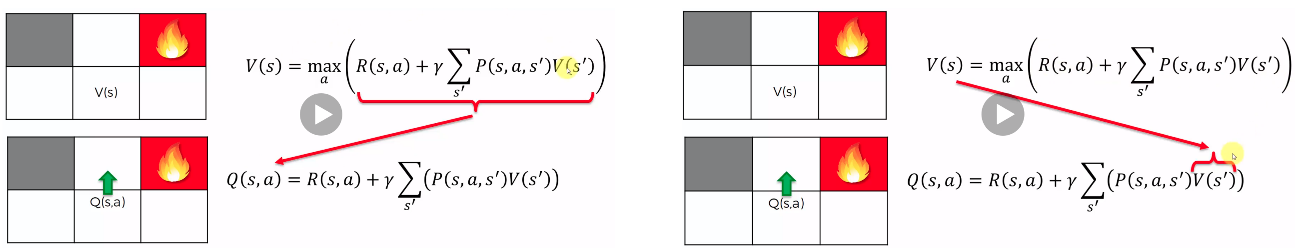
To make the problem easier, we requires the process to be an `Markov Process`, that is:

> In Markow process, the future state only depends on the current state, not the sequence in the past.

And the BellMan is rewritten in the stochastic form:

$$V(s) = \max_a [r(s, a) + \gamma \sum_{s'} P(s'|s, a) V(s')]$$

5. Q-Learning Intuition



- From the Bellman equation, let we define the quality of an action (left-figure) as:

$$Q(s,a) = r(s,a) + \gamma \sum_{s'} P(s'|s,a)V(s') \Rightarrow V(s) = \max_a[Q(s,a)]$$

> The value of the state $V(s)$ is the maximum quality of an action $Q(s,a)$.

- Then substitue $V(s') = \max_a[Q(s',a)]$ (right-figure), we write the Q-function independent of $V(s)$:

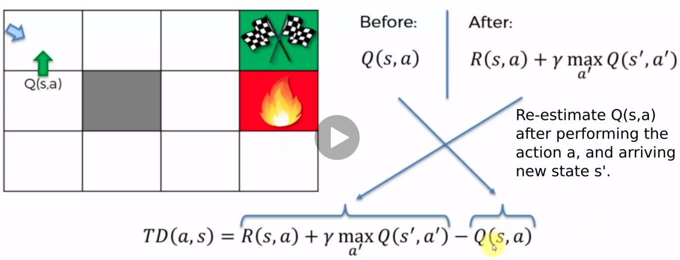
$$Q(s,a) = r(s,a) + \gamma \sum_{s'} P(s'|s,a) \max_a[Q(s',a)]$$

- For short notation, we can ignore the "Stochastic Process" and simplify the Q-function as follows.

$$Q(s,a) = r(s,a) + \gamma \max_a[Q(s',a)]$$

6. Temporal Difference (TD)

- Theoretically, if $Q(s,a)$ is known ahead, then every thing is very easy, we just choose the action that maximizes the quality.
- However, $Q(s,a)$ is just an estimation, which hopefuylly is getting closer and closer to the true value along the learning path. Therefore, after taking an action, there is always a mismatched from the previous and the re-estimated quality. This ammount of mismatch is called **temporal difference (TD)**.

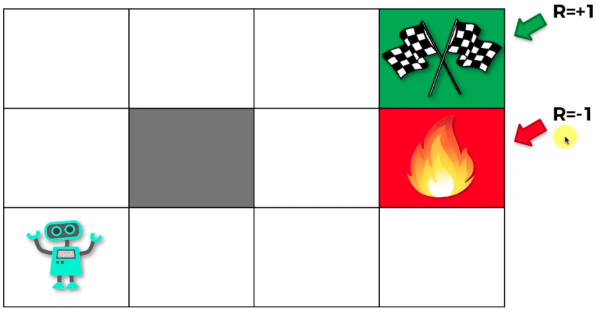


- And we update the Q-function as follows:

$$Q_t(s,a) = Q_{t-1}(s,a) + \alpha TD_t(s,a)$$

Note that, when $\alpha = 1$, this reduces to the ideal Bellman equation, i.e. we discard the whole experience learned in the past, and update the Q-function with the latest quality. However, this is not a good idea, because the new estimated value is not neccesarily the best value representing the state. For example, it may be a rare event due to the random process.

7. Q-Learning Algorithm



Initialization (First iteration):

For all couples of states s and actions a, the Q-values are initialized to 0.

Next iterations:

At each iteration t ≥ 1, we repeat the following steps:

1. We select a random state s_t from the possible states.
2. From that state, we play a random action a_t.
3. We reach the next state s_{t+1} and we get the reward $R(s_t, a_t)$.
4. We compute the Temporal Difference $TD_t(s_t, a_t)$:

   $$TD_t(s_t, a_t) = R(s_t, a_t) + \gamma \max_a(Q(s_{t+1}, a)) - Q(s_t, a_t)$$

5. We update the Q-value by applying the Bellman equation:

   $$Q_t(s_t, a_t) = Q_{t-1}(s_t, a_t) + \alpha TD_t(s_t, a_t)$$

- The algorithm is implemented at q_learning.py.
- An example of cartpole is shown in cartpole.py.
- An example of mountain car is shown in mountain_car.py.