Homework #2: LQR

Deliverable: Report submitted to Gradescope by **Wednesday October 2nd, 23:59pm**. Your report should be a PDF containing the following:

Page 1: Solution to 1(a)

Page 2: Solution to 1(b)

Pages 3 onwards: PDF rendering of completed notebook provided in 2

1. Theoretical Exercises

(a) Consider the following optimal control problem, which considers a linear system with additive noise and quadratic cost:

$$\min_{x,u} \mathbb{E}[\sum_{t=0}^{T-1} x_t^\top Q x_t + u_t^\top R u_t] + \mathbb{E}[x_T^\top Q x_T]$$
$$\text{s.t. } x_{t+1} = Ax_t + Bu_t + w_t, \forall t = 0, 1, 2, ..., T-1$$

with w_t independent random vectors with $\mathbb{E}[w_t] = 0$, and $\mathbb{E}[w_t w_t^\top] = \Sigma_w$.

Find an LQR-like sequence of matrix updates that computes the optimal cost-to-go at all times and the optimal feedback controller at all times. Describe the expected cost incurred in excess of the expected cost in the case when there is no noise.

(b) Now consider a linear system with multiplicative noise and quadratic cost:

$$\min_{x,u} \mathbb{E}[x_T^\top Q x_T]$$
$$\text{s.t. } x_{t+1} = Ax_t + (B + W_t)u_t, \ \forall t = 0, 1, 2, ..., T-1$$

Here $Q \in R^{n_X \times n_X}, A \in R^{n_X \times n_X}, B \in R^{n_X \times n_U}$ are given and fixed. $W_t \in R^{n_X \times n_U}, t = 0, 1, ..., T-1$ are independent random matrices with $\mathbb{E}[W_t] = 0$. Higher-order expectations involving W_t will show up. These higher-order expectations are not assumed to be zero, and you should just keep these expectations around, i.e., no need to try to simplify these (and not possible anyway unless additional assumptions are made).

Find an LQR-like sequence of matrix updates that computes the optimal cost-to-go at all times and the optimal feedback controller at all times.

2. Programming Exercise

See `lqr.ipynb` and follow the instructions within which will walk you through each question. But first, set up your environment by following this link to install Anaconda and this link for MuJoCo installation –if you do not already have a key, please see Piazza to obtain one. After installation, navigate to your homework directory, create your environment via `conda env create -f environment.yml`, run `jupyter notebook` within the environment to start a server, and get started on the implementation.