Does Competition Improve Information Quality: Evidence from the Security Analyst Market

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Abstract

This paper studies the effect of competition on the quality of information provided by experts. I estimate the incentives and the information structure of security analysts who compete to make earnings forecasts. Security analysts are rewarded for being more accurate than their peers, which creates competition. This reward for relative accuracy leads analysts to distort their forecasts to differentiate themselves, but also disciplines them to be less influenced by the prevailing incentive to report over-optimistic forecasts in the financial market. I structurally estimate a contest model with incomplete information that captures these two effects, disentangling the payoff for relative accuracy from the payoffs for optimism and absolute accuracy. Using the model, I conduct counterfactuals to evaluate policies that reduce the importance of relative accuracy in the current market, simulating their effect on the quality of information. I find that the disciplinary effect dominates: the reward for relative accuracy reduces individual and consensus forecast errors by 34.01% and 60.84% respectively, but at a cost of increasing individual and consensus forecast variances by 6.59% and 6.68% because of the distortionary effect. For each security, it is optimal to have moderate competition between the covering analysts, as competition generates more aggregate information but intensifies the distortionary effect.

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1 Introduction

Efficiency of many markets depends on information provided by experts. It is natural for consumers to compare experts and reward the most accurate ones, which generates competition. However, this focus on *relative accuracy* creates an incentive for distortion, leading experts to deviate from reporting their honest beliefs. In financial markets, such incentives are particularly prevalent. A security analyst that correctly predicts a stock boom (or bust), when no one else does, might rapidly gain better reputation, career and pay. This is exemplified by various "best analyst" contests organized financial news outlets such as the Institutional Investor Magazine (Bradshaw 2011, Groysberg et al. 2011, Craig 2011). However, such high reward for the most accurate may incentivize analysts to issue exaggerated forecasts to differentiate themselves — if they are wrong, the costs will be small, but if they are right, the rewards will be huge. As a result, the quality of information may be compromised.

Does this mean that rewarding relative accuracy impairs information quality in the financial market? Not necessarily. Evidence shows that security analysts may also face an incentive to optimistically bias their predictions (Hong and Kubik 2003, Hong and Kacperczyk 2010). They may predict a stock boom even when their beliefs are only mildly positive, and they would expect their rivals to do so as well. In this case, rewarding the more accurate may lead analysts to predict less optimistically to differentiate, which disciplines the optimism incentive and improves information quality.

This paper studies the effect of competition on information quality in the financial industry, where competition manifests through the reward for relative accuracy. I focus on security analysts and empirically estimate the importance of relative accuracy and other incentives, in particular optimism, in their payoffs.

Security analysts conduct research on security-issuing firms. They use industry expertise and connections to firms' management teams to obtain information. Then, they compete to make third-party forecasts on firms' earnings. They are the gateway between firms seeking capital and investors choosing investment opportunities. If they face incentives to distort their forecasts, investors may receive poor information, which can lead to bad investment decision. Therefore, it is crucial to ensure that analysts are incentivized to provide quality information.

Policy makers around the world have adopted many measures to regulate analysts' incentives. Policies in the United States (US) have focused on reducing the optimism incentive, the most influential of which is the Securities and Exchange Commission's (SEC) regulation on favorable research in 2002. It prohibits tying analysts' compensation with specific investment banking transactions and mandates disclosure of any conflict of interest related to investment banking. Meanwhile, in Europe, the Markets in Financial Instruments Directive II (MiFID II) announced in 2018 restricts paying for analyst research with commissions, which reduces the average number of analysts covering a security (Fang et al. 2020, Guo and Mota 2021). When analysts are rewarded for relative accuracy, these policies not only affect individual analysts' incentives, but also how they interact strategically with each other. Accounting for these strategic interactions is critical for the interpretation of these policies.

Rewarding relative accuracy could have two effects on information quality as discussed earlier: it may have a *distortionary effect* as analysts try to differentiate their forecasts, but it may also have a *disciplinary effect* on optimism. Meanwhile, this reward also results in strategic interaction between analysts, which implies that competition affects information quality. Therefore, the overall impact of the reward for relative accuracy is an empirical question: How important are relative accuracy and optimism in analysts' payoff? Which of the two effects dominates? From the perspective of investors and policy makers, what is the optimal level of competition for information quality?

To answer these questions, I specify and estimate a structural model that captures analysts' incentives and information structure. In the model, analysts compete in a contest to forecast the earnings of a security in a given year. First, analysts observe the number of analysts in the contest and receive private signals on the actual earnings of the security. Then, they choose forecasts simultaneously. After, the actual earnings are revealed and analysts receive payoffs based on all the forecasts and the actual earnings. Critically, my model allows analyst payoffs to depend on absolute accuracy, relative accuracy, and optimism in a flexible way.

Building on a long reduced-form literature, this paper is the first structural study on analysts' choice of forecasts. A major challenge to this literature is that incentives must be identified from observable proxies of analysts' payoffs, but these proxies are imperfect. For example, career outcomes are often used as proxies for analysts' long-term compensation. However, because there is limited data on compensation along the career path, it is hard to compare these long-term incentives from career outcomes with short-term incentives from bonuses and commission.[1] This paper takes a different approach and estimates analysts' incentives with revealed preference, agnostic about their sources, so it provides a more complete understanding of analyst incentives.

To estimate the model, I use indirect inference with detailed forecast data obtained from Institutional Brokers' Estimate System (IBES). IBES contains the history of analysts' earnings forecasts on publicly listed US companies as well as their corresponding actual earnings since 1982. The estimation proceeds as follows. For any given parameter value, I solve the model numerically using analysts' first order conditions and simulate equilibrium forecasts for every single contest in the data many times. Then, I construct a set of moments from the data and from the simulated forecasts. In order to capture the effect of competition on analysts' forecast strategy, I estimate the coefficients of a set of auxiliary regressions as moments, regressing functions of forecasts on observables such as the number of analysts and the actual earnings. Finally, I search over the parameter space to find values that minimize the difference between data moments and simulated moments.

This model is a common value model, similar to common value auctions, where rivals have

[1] Compensation data is highly confidential in this industry.

information relevant to a player's assessment of the outcome they are predicting. This creates a challenge for jointly identifying the payoff and the signal distribution (Athey and Haile 2007).[2] For example, it is difficult to identify whether a high earnings forecast is due to a high signal or a payoff that incentivizes exaggeration.

To tackle the challenge, I rely on additional data and structure of the security analyst market. First, being able to observe the actual earnings, that is, the outcome that analysts are predicting, is crucial to identification. This allows me to construct analysts' beliefs about the actual earnings as well as their rivals' forecasts given their signals, so that I can recover analysts trade-offs between choosing different forecast values. This is similar to observing ex post values in mineral rights auctions, which also helps identify the joint distribution of signals and valuations.

Second, I use variation in forecast strategy with the number of analysts to identify the payoff function, especially the reward for relative accuracy. For intuition, consider a winner-takes-all contest, where only the most accurate analyst gets a reward. In this contest, analysts will distort their forecasts to differentiate themselves. The more rivals they face, the harder it is to be the most accurate, so they will distort more. Hence, if analysts face a large reward for the most accurate, we, as econometricians, expect their forecast errors to increase with the number of rivals. This variation in forecast error with the number of analysts identifies the payoff function, and I capture this variation using auxiliary regressions in estimation. This identification strategy is similar to using variation in the number of bidders to test between common and private values in auctions.[3]

The estimates of the model show that analysts receive significant reward for being the most accurate. In addition, they are rewarded for optimism. Both of these rewards are strong relative to the payoff for absolute accuracy. I use the estimated model to conduct two counterfactuals to evaluate the effect of relative accuracy and competition on information quality. First of all, I simulate how information quality changes when analysts are less rewarded for relative accuracy, both in the current market and in a hypothetical scenario where the optimism incentive is removed. To measure information quality, I compute average forecast error and dispersion across individual forecasts, which reflect the accuracy and noise of information provided by any individual analyst. To evaluate the accuracy and noise of aggregate information, I also compute forecast error and variance of consensus forecast, defined as the mean of the individual forecasts in a security-year. Then, I simulate how these measures of information quality change with the number of analysts to find the optimal levels of competition in covering one security given different objectives of the policy maker.

I find that the *disciplinary effect* dominates in the current market. The existing optimism incentive results in high individual and consensus forecast errors, and the reward for relative accu-

[2] Payoff and forecasts in my model map into valuation and bids in auctions.

[3] In common value auctions, bidders shade their bids downward from their beliefs about the valuation to protect themselves from the winner's curse, which is not present in private value auctions. When bidders face more rivals, they will shade more as the winner's curse becomes more severe. This variation in shading with the number of bidders is used to identify between common and private value auctions.

racy reduces the individual forecast errors by 34.01% and the consensus forecast errors by 60.84%, resulting in more accurate information in the overall market. However, the distortionary effect is still present. Specifically, this improvement in information accuracy is at the cost of noisier information, as forecasts are 6.59% more dispersed and the consensus forecast variance increases by 6.68%. Also, this distortionary effect intensifies with competition, so the improvement in accuracy becomes smaller as more analysts cover a security.

Interestingly, in the hypothetical scenario without optimism, where only the distortionary effect is present, the reward for relative accuracy increases individual forecast errors by merely 0.07% and it also induces less noise than in the current market. In other words, the distortionary effect is amplified in the current market from its interaction with the optimism incentive. This is because the optimism incentive reduces the cost of distortion for all analysts, but more for analysts with optimistic signals than those with pessimistic signals before the actual earnings are revealed, so the noise in the market is magnified. This result implies that optimism regulations, such as SEC's regulation on favorable research in 2002, not only reduce forecasts' optimism bias, but also prevent analysts' from being involved in an "optimism contest" where they compete to stand out, thereby reducing forecast noise.[4]

The optimal level of competition depends on the policy maker's objective. If the goal is to improve the information quality of individual analysts, the least amount of competition with two analysts is optimal. Allowing two analysts to compete with each other is sufficient to discipline the optimism incentive and minimize the distortionary effect. However, if the goal is to improve the quality of aggregate information, which applies to the current market where analysts' forecasts are public information, I find that a medium level of competition with 10 analysts is optimal on average. The effect of competition on the consensus forecast quality is inverted-U-shaped. More analysts bring more aggregate information to the market, which improves information quality initially, but as the level of competition increases further, the distortionary effect intensifies and impairs information quality, canceling the positive effect of more aggregate information.

This challenges the common belief that more competition is always better for information quality. Policies and events that reduce the level of competition in this market do not necessarily impair information quality. A recent example is the Markets in Financial Instruments Directive (MiFID) II, which is found to reduce the number of analysts covering a security and improve information quality (Fang et al. 2020, Guo and Mota 2021). The inverted-U-shaped effect of competition offers a new mechanism that may explain this phenomenon.

Related literature This paper contributes to the literature on security analysts' incentives (see Bradshaw (2011) and Kothari et al. (2016) for surveys). Existing empirical studies have been

[4]In this "optimism contest", analysts are not competing to issue the most optimistic forecast per se. Instead, they are competing to be more accurate than their rivals, but in attempts to stand out, they have more tendency to distort in the optimistic direction due to its lower cost.

reduced-form: researchers are restricted to identify analysts' incentives from observable proxies of their payoffs, such career outcomes. However, these proxies are imperfect, as there is limited data on analysts' compensation, and some incentives such as reputation are also unobservable (e.g., Cowen et al. 2006, Ljungqvist et al. 2006). Therefore, analysts' overall incentives remain unknown. To resolve this issue of observability, this paper takes a revealed preference approach. I use a structural model to infer analysts' incentives from the distribution of their observed forecasts, agnostic about the sources of these incentive, so this offers a more complete understanding.

The model builds on the rich reduced-form studies, many of which point to relative accuracy and optimism as the main components of analysts' incentives. Analysts with higher relative accuracy and optimism are found to have better career outcomes (Hong et al. 2000, Hong and Kubik 2003, Clarke and Subramanian 2006, Cen et al. 2017). In addition, their compensation is influenced by their relative performance in "best-analyst" contests, as well as their contributions to investment banking and trading volume, which are also potentially consistent with rewarding relative accuracy and optimism (Groysberg et al. 2011, Maber et al. 2014).[5]

This paper also adds to the literature on the effect of competition on information provision (e.g., media competition in Gentzkow and Shapiro (2008) and Cagé (2020); credit rating agency competition in Becker and Milbourn (2011)). Several papers have studied this topic in the context of security analysts with mixed reduced-form results. For example, Hong and Kacperczyk (2010) and Merkley et al. (2017) use brokerage mergers and closures as exogenous shocks on competition and find that consensus forecast quality worsens with less competition, but Guo and Mota (2021) find the opposite using MiFID II as an exogenous shock on competition. While these studies propose their own mechanisms for these effects, my paper differs from and connects them by proposing a new mechanism, through which competition has an inverted-U-shaped effect on consensus forecast quality. Furthermore, I support this mechanism with empirical evidence at analyst-level, highlighting their strategic interaction.

Methodologically, the closest paper to mine in this literature is Camara (2015), which estimates a structural model of forecast timing where analysts choose when to release forecasts. She finds that competition increases forecast errors as analysts preempt rivals by releasing forecasts early. My paper differs from hers in two ways. First, I focus on the strategic choice of forecast values with longer forecast horizons (i.e., for annual earnings), departing from her focus of strategic forecast timing for quarterly earnings. To keep our models tractable, I assume simultaneity in timing while she assumes truth-telling in value. In reality, analysts issue forecasts asynchronously but are allowed to revise at any time before the forecasting period ends. Revision happens much more frequently with annual forecasts than with quarterly forecasts, so it is sensible to think that analysts compete more in values for annual forecasts but more in timing for quarterly forecasts. My modeling choice

[5] In their studies based on single-firm data, the effect of relative accuracy is small conditional on "best-analyst" contest outcomes, but the effect of theses contests are huge and significant and they often take forecast accuracy as one of the criteria (Bradshaw 2011).

is also consistent with theoretical papers in this literature (Ottaviani and Sørensen 2006, Clarke and Subramanian 2006, Banerjee 2020). Second, I explicitly model optimism, a big driver of forecast errors and an important target for regulations, while Camara (2015) abstracts away from it with normalization.

More broadly, this paper is related to the literature on incentive provision based on relative performance (e.g., Prendergast 1999, Nalebuff and Stiglitz 1983). The incentive to be the top performer is particularly pervasive in the financial industry, potentially leading to risk-taking behavior (Chevalier and Ellison 1997). This paper explicitly models this incentive for security analysts, generalizing the forecasting contest model proposed by Ottaviani and Sørensen (2005, 2006). Their papers establish the theoretical implications of a winner-takes-all contest model with infinite number of players (analytically) and finite number of players (numerically). They find that analysts put more weight on private information in their forecasts to outperform rivals. This paper extends their model to account for optimism, capturing the security analyst market more realistically. It also develops an efficient numerical procedure for solving this class of models with finite number of players that is feasible for estimation.

The estimation of common value models is an emerging area (e.g., Compiani et al. 2020, Ordin 2019). As mentioned earlier, a challenging identification problem for these models is to jointly identify the distribution of players' signals and payoffs. To resolve this challenge, I leverage additional structure and richness of data in the financial industry. A related paper is Bhattacharya et al. (2018) which studies mineral rights auctions. They resolve the identification challenge by explicitly linking auction design to post-auction economic activity, also providing more structure and data to the problem. Nevertheless, I depart from and contribute to this literature, which focuses almost entirely on auctions, by studying a novel contest model with common values.

The remainder of the paper proceeds as follows. Section 2 introduces the background of security analysts. Section 3 outlines the data and how some key variables are constructed. Some reduced-form evidence is provided in Section 4 to guide the setup of the model. Section 5 formally presents the model and characterizes the equilibrium, followed by Section 6, detailing the estimation procedure and the results. Section 7 performs the counterfactual analyses to and Section 8 concludes.

2 Industry Background: Security Analysts

Security analysts are typically employed by brokerage houses, commonly referred to as the sell-side. They are experts in the securities they cover and are connected to the securities' management teams, which allows them to obtain information. Using the information, they publish research reports on these securities, where they make forecasts on the earnings and stock prices, and recommend whether to buy, sell or hold. Besides these reports, analysts also communicate with investors via other means such as organizing visits to the security-issuing firms. The information they provide is crucial to both institutional and retail investors.

In this paper, I focus on earnings forecasts as one of the most common types of information provided by security analysts. Compared to stock prices, which could be drastically affected by analysts' recommendations, a firm's earnings are determined by its own profitability and less endogenous to analysts' actions. Moreover, the actual earnings are announced after analysts make their forecasts, so we can evaluate the forecast quality ex post.

In the rest of this section, I will introduce the institutional details of security analysts' payoffs and competition, as well as existing policies on their incentives.

2.1 Payoffs

Compensation is the main source of payoff for security analysts. In general, analysts are not compensated directly for their research.[6] Instead, their compensation is composed of fixed wages, trading commissions and bonuses, which depend heavily on the revenue of their employing brokerage houses. Brokerage houses generate revenue by providing investment banking services to firms and executing trades for investors, and they compensate analysts for their ability to indirectly increase these revenues (Cowen et al. 2006).

One way analysts can increase the revenues is through marketing — analysts can provide investors with information that increase their brokerage house's trading volume, or promote securities linked to its investment banking businesses. In return, they may be rewarded with higher trading commissions and bonuses. Besides, analysts also play a signaling role for their brokerages house. To investors, having reputable analysts signals a brokerage house's expertise in identifying good investment opportunities. Then to firms, this signals the brokerage house's ability to sell, potentially leading to more investment-banking businesses.

Most of the existing studies on analysts' incentives use their career outcomes, i.e., job transfers across brokerage houses, as a proxy for compensation, because detailed compensation data is hard to obtain. This proxy is reasonable because there is a well-defined prestige hierarchy in the sell-side. High-prestige brokerage houses, which are major investment banks such as JPMorgan and Merill Lynch, hire more analysts, take on larger deals, generate more revenue, and therefore, offer much higher compensation for analysts than smaller brokerage houses. Job transfers up the prestige hierarchy imply higher future payoffs (Hong and Kubik 2003).[7]

[6] Recently, some security analysts also provide fee-based research and subscription-based research services. The former is compensated by the subject firms hoping to disclose information more efficiently to investors. The latter is compensated by report readers on a subscription or pay-per-view basis. However, those accounts for a very small fraction of all forecasts.

[7] Groysberg et al. (2011) and Maber et al. (2014) use proprietary data from two brokerage houses and find analysts at the lower-prestige house are paid about $250k (30%) less.

However, this proxy is also incomplete. It doesn't account for analysts' immediate commissions and bonuses, which are affected by the information they provide and sometimes make up over half of analysts' compensation (Groysberg et al. 2011). Meanwhile, analysts may also be incentivized by intangible payoffs such as reputation concerns. These complex and unobservable payoffs motivate my revealed preference approach to study analyst' incentives.

2.2 Competition and Reward for Relative Accuracy

Security analysts compete with other analysts covering the same security. They are usually from different brokerage houses, as each brokerage house assigns only one analyst to cover one security. This competition is exemplified by various "best analyst" contests, the most influential of which being the All-America Research Team contest held by the Institutional Investors Magazine. In this contest, security analysts are ranked based on democratic votes by buy-side analysts and portfolio managers, and earning forecast accuracy enters as one of the performance criteria (Bradshaw 2011). Similar contests are also organized by Thomson-Reuters and the Wall Street Journal to rank analysts based on their earnings forecast and stock-picking performance. Winners of these "best-analyst" contests make headlines at media outlets and receive disproportionally large compensation and better job offers (Groysberg et al. 2011, Craig 2011). Consistent with the literature, these anecdotes also show that analysts are rewarded for relative accuracy.

2.3 Policies

In the past 20 years, there were two major policies on security analysts' incentives. The first is US regulations on optimism incentives after the dotcom bubble. Because analysts can increase their brokerage houses' revenues through marketing, they tend to provide over-optimistic forecasts in order to earn more commissions and bonuses. To untie analysts' compensation from brokerage houses' investment banking business, the Global Analyst Research Settlement (the Settlement) in 2003 mandated that ten of the country's largest brokerage houses block their research department from their investment banking department and pay \$850 millions of penalties in total.[8] At the same time, the SEC also put forward regulations to prohibit tying analysts' compensation to specific investment banking transactions and require disclosure if it was linked to investment banking revenues.

The second is MiFID II implemented in the European Union (EU) in 2018. MiFID II builds on MiFID, which is a broad financial industry reform announced by the EU in 2007, aiming to promote fairness, transparency, efficiency and integration in the financial markets. One of the most controversial aspects of MiFID II is that it requires sell-side research to be priced separately from

[8]The Settlement revealed that these brokerage houses could be using compensation to induce over-optimistic research from their analysts. The brokerage houses and analysts in question neither accepted nor denied the charges but agreed to pay the penalties.

the execution of financial instruments, which is found to reduce the number of security analysts covering European firms. Many observers expressed concerns that this reduced level of competition would impair security analysts' information quality (Preece 2019, Hyatt 2022), but recent empirical evidence finds analysts' forecast accuracy improved post-MiFID II on average (Fang et al. 2020, Guo and Mota 2021).

This paper offers new interpretations of these policies focusing on their impact on analysts' strategic forecasting behavior, potentially providing a new mechanism to explain the change in information quality with reduced analyst coverage post-MiFID II.

3 Data and Summary Statistics

I obtain data on analysts' forecasts and employment history from Institutional Brokers' Estimate System(IBES). IBES's Detail Earnings Estimate History contains analysts' forecasts of US companies since 1982. I focus on annual forecasts of Earnings Per Share (EPS) of publicly listed US companies as they are published the most frequently. For each forecast, IBES contains both the forecast value and the actual value so I can measure its accuracy; it also contains identifiers for securities, analysts, and brokerage houses, so I can trace out the forecast and employment history of each analyst. Finally, analysts can issue and revise their forecasts at any time before the earnings are announced. The IBES data also contains when forecasts are issued and earnings are announced.

I match securities in IBES to Center for Research in Security Prices (CRSP) database to obtain daily stock prices, stock returns, number of outstanding shares, and whether it is included in S&P500. In addition, I match them to Compustat to obtain financial information on the issuing companies such as book value and operating income.

My sample covers forecasts announced between January 1st, 1984 and December 31st, 2016 for securities that are present in all three datasets. When an analyst issues multiple forecasts over a year, I keep the most recent forecast. Moreover, I restrict the sample to the security-years that are covered by at least two analysts so that relative accuracy is well-defined. Finally, I focus on securities that are present in the data for more than 20 years so that I have enough observations to account for security-level heterogeneity. In total, there are 209,320 forecasts, 697 securities, 12,267 analysts, and 837 brokerage houses in my sample.

Brokerage House and Analyst Characteristics Table 1 presents the summary statistics for brokerage houses, showing the prestige hierarchy mentioned in Section 2. An average brokerage house employs 15 analysts and covers 77 securities, but the median brokerage house employs only 7 analysts and covers 27 securities, implying that there is significant skewness in brokerage house sizes. This sknewness is driven by the cross-sectional heterogeneity across brokerage houses, rather than the times series fluctuation of the entire market, as the average number of analysts employed by a brokerage house remains stable overtime (Figure A1).

Table 1: Characteristics of Brokerage Houses

Statistic	Mean	Pctl(25)	Median	Pctl(75)
Number of Securities Covered per Year	77	9	27	84
Number of Industries Covered per Year	5	3	5	8
Number of Analysts per Year	15	2	7	17
Number of Analysts per Industry-year	3	1	2	4
Number of Analysts per Security-year	1	1	1	1

Table 2: Characteristics of Analysts

Statistic	Mean	Pctl(25)	Median	Pctl(75)
Number of Securities Covered per Year	7	3	6	10
Number of Securities Covered	12	4	8	17
Number of Industries Covered per Year	2	1	1	2
Number of Industries Covered	2	1	2	3
Number of Brokerage Houses	2	1	1	2

Table 2 present the summary statistics for analysts. There is a lot of persistence in the securities analysts cover: an average analyst covers 7 securities from 2 industries in a year, and 12 securities from 2 industries throughout her career. This is consistent with the anecdote that analysts specialize and their security coverage decisions are driven more by their expertise and connections, rather than temporary information shocks. Meanwhile, an average analyst works at 2 brokerage houses and experiences one job transfer throughout her career, which implies that analysts' labor market is reasonably dynamic. However, this dynamism is skewed, as more than half of the analysts stay at the same brokerage house. Combined with the prestige hierarchy, this shows that job transfers are a reasonable but imperfect proxy for analysts' compensation.

Competition The competition I focus on is at the security-level. I consider each security as a separate market, where analysts compete to make forecasts every year. Analysts' rivals are from other brokerage houses rather than their own because each brokerage house typically hires only one analyst to cover a security in any given year (Table 1). Moreover, the competition is intense, as about 54% of security-years are covered by more than 10 analysts (Table 3).

Standardizing Analysts' Forecast Problems The actual earnings evolve in very different patterns across securities: some may be more volatile, so analysts may receive noisier information; some others may be growing, so analysts tend to issue forecasts that are higher than the previous year. This creates challenge for comparing analysts' forecasts across securities. To resolve this, I

Table 3: Most Analysts Face Significant Competition When Making Forecasts on Securities

Number of Analysts	Number of Securities-years	Percent
2-10	6,590	45%
11-20	4,363	30%
21-30	2,325	16%
31-40	932	6%
40+	318	2%
Total	14,528	100%

Notes: Table presents the distribution of the level of competition across security years. The first column are bins for the number of analysts. The second and the third columns are the number and the percentage of security-years that fall into each bin.

standardize analysts' forecast problems across securities and over time by assuming that analysts know the Markov process that governs the actual earnings time series, so they only need to make forecasts on the time series shocks based on their private signals of them. I formalize this procedure in Appendix B. After this standardization, analysts across all security-years have the same common prior of the actual earnings, which is distributed i.i.d. standard normal, and they forecast this random variable based on their private signals.

4 Reduced-form Evidence

Now I document two data patterns that guide the model setup and the parametric specifications for estimation. First, I present evidence on the importance of relative accuracy in analysts' incentives. Second, I distinguish between two different payoff structures for relative accuracy. One is a reward for having the lowest forecast errors (or "reward for the top"), and the other is a punishment for having the highest forecast errors (or "punishment for the bottom"). I show that analysts' observed behavior is more consistent with a reward for the top rather than a punishment for the bottom.

My analysis relies on the following two observations. Suppose analysts covering the same security-year face a common prior for the actual earnings, and then receive private signals before making forecasts simultaneously. Upon receiving the private signals, analysts update their posterior beliefs of the earnings, which fall between the common prior and their private signals.

Observation 1. If analysts are rewarded for being ranked in the top or punished for being ranked in the bottom, their forecast errors will be higher when they face more rivals.

Observation 2. If they are rewarded for the top, they will distort their forecast towards their private signals. If they are punished for the bottom, they will distort their forecast towards the common prior. The distortion intensifies with more rivals.

Ottaviani and Sørensen (2005, 2006) show these observations in a simplified setting analytically with the number of players going to infinity, and in numerical examples with finite numbers of players. Later in Section 5, I extend their model to have a more general payoff function and develop an efficient new algorithm to compute its equilibrium numerically, which allows me to verify these observations. In this section, I take them as given to motivate the reduced-form analysis.

The intuition is as follows: suppose an analyst believes based on the common prior and her private signal that the earnings of a security-year will be \$1. This \$1 is the analyst's posterior, which is between the common prior and her private signal. If the analyst reports her posterior belief honestly, she will make a forecast of \$1. However, if the analyst is rewarded for the top, she will deviate from \$1 towards her private signal in order to differentiate herself from her rivals. This way, if her forecast turns out to be correct, she will be more likely enjoy the reward for the top all by herself. Conversely, if the analyst is punished for being ranked in the bottom, she will deviate from \$1 towards the common prior in order to herd herself with her rivals, so that she will be less likely to be singled out and punished as the worst. The more rivals she faces, the more she will need to deviate from \$1 in either direction in order to differentiate or herd herself (Observation 2). And more deviation from her belief leads to higher forecast error (Observation 1).

4.1 Importance of Relative Accuracy

Motivated by Observation 1, I study the effect of competition on analysts' forecast error to see whether they are rewarded for relative accuracy. For analyst i covering security j in year t, I consider the following panel data regression,

$$|X_{ijt} - Z_{jt}| = \alpha_1 \log(N_{jt}) + Controls_{ijt} + e_{1ijt}, \tag{1}$$

where X_{ijt} is analyst i's earnings forecast, Z_{jt} is the actual earnings, or the actual earnings, and $|X_{ijt} - Z_{jt}|$ is the forecast error. The variable N_{jt} is the number of analysts covering security j in year t and e_{1ijt} is an error term with clustering at the security-year level. α_1 is the coefficient of interest. If analysts are rewarded for relative accuracy, Observation 1 will imply α_1 to be positive.

A potential threat to identification is that the number of analysts covering a security may be endogenous due to unobserved heterogeneity, which may affect analysts' entry into covering a security. For example, more analysts may enter to cover a security for which it is easier to obtain information, and better information may lead to smaller forecast errors. I adopt various measures to deal with this endogeneity.

First, I include various fixed effects to control for heterogeneity that may affect both competition and analysts' forecasts. For example, one might be concerned that years and securities with more uncertainty would be covered by more analysts; or that the share of bad analysts would be higher when there are more analysts covering a security. The fixed effects address these concerns: year fixed effects account for overall uncertainty in the financial market over time; security fixed effects ensure that the regression evaluates the effect of competition on the same security; security-analyst pair fixed effects further ensure that the evaluated effect is on the same analyst covering the same security.

Second, I also control for time-varying security and analyst characteristics, as well as forecast timing to complement the fixed effects. The security characteristics include firm size, profitability, return volatility, average monthly return and an indicator for whether the security is included in the S&P500 index. The analyst characteristics include the analyst's experience with the security, with the industry and with this profession in general, as well as her prestige, which is an indicator for whether she is hired by a high-prestige brokerage house. As the indicator for prestige is included, I also control for the percentage of analysts covering the security that are high-prestige. To make sure that the results are not driven by the timing of forecast and earnings report, I also include the order of the forecast and earnings report delay.

Finally, I instrument the number of analysts and the percentage of high-prestige analysts with their one-year-lagged counterparts. The identifying assumption is that conditional on the controls, the lagged variables on the level of competition is uncorrelated with shocks that affect forecast errors this year. Table A2 presents the first-stage results.

The results are presented in the first block of Table 4. The ordinary-least-squares (OLS) estimates are in Columns (1)-(3) and the two-stage-least-squares (2SLS) estimates are in Columns (4)-(6). All columns include year fixed effects. Columns (1) and (4) include security fixed effects and the rest of the columns include security-analyst pair fixed effects. I find that across all specifications, forecast errors significantly increase with the number of analysts covering the security. The coefficient ranges from 0.033 to 0.096. To put this range in perspective, it means that for the same analyst-security, when the number of analysts increases from 6 (the first quartile) to 12 (median), forecasts on average are further from the actual earnings by 10.8% to 31.5%.

The magnitude of the coefficient remains comparable when the pair fixed effects, time-varying characteristics, forecast timing controls, and lag instruments are added to the regression. This implies that the impact of cross-analyst and time-varying heterogeneity within security is limited. The coefficients on the characteristics and timing controls are detailed in Table A1. Most notably, forecast errors are smaller in security-years that "perform well" and possibly more predictable, with high market capitalization, profit and returns, and lower volatility. Analysts in high-prestige brokerage houses tend to have lower forecast errors, but the difference is small and weak in significance. Finally, earlier forecasts have larger forecast errors, which is consistent with Camara (2015), but controlling for it does not undermine the effect of competition.

In summary, analysts' forecast errors increase with the level of competition they face. It is consistent with the strategic behavior when they are rewarded for relative accuracy.

Table 4: Effect of Competition on Analysts' Forecasts

	OLS			2SLS		
	(1)	(2)	(3)	(4)	(5)	(6)
	Dependent Variable: Forecast Error					
$\log(N_{jt})$	0.041	0.033	0.093	0.047	0.061	0.096
	(0.006)	(0.007)	(0.008)	(0.011)	(0.023)	(0.037)
	Dependent Variable: Forecast					
$\log(N_{jt}) \times Z_{jt}$	0.029	0.026	0.025	0.029	0.025	0.024
	(0.007)	(0.007)	(0.007)	(0.008)	(0.007)	(0.007)
Z_{jt}	0.794	0.804	0.810	0.793	0.806	0.812
	(0.020)	(0.018)	(0.018)	(0.022)	(0.020)	(0.020)
$\log(N_{jt})$	0.030	0.007	0.006	0.031	0.020	0.003
	(0.007)	(0.008)	(0.009)	(0.012)	(0.024)	(0.041)
Year FE	Y	Y	Y	Y	Y	Y
Security FE	Y			Y		
Security-analyst FE		Y	Y		Y	Y
Characteristics			Y			Y
Timing Controls			Y			Y
Observations	209,320	209,320	209,320	209,320	209,320	209,320

Notes: This table presents the reduced-form results on the effect of competition on analysts' forecasts. The first block uses individual analysts' forecast errors as the dependent variable. The second block uses individual analysts' forecasts as the dependent variable. Columns (1)-(3) present the OLS regressions. Columns (4)-(6) present the 2SLS regressions using lag number of analysts covering the security $\log(N_{jt-1})$ as instrument. All columns include year fixed effects. Columns (1) and (4) include security fixed effects and the rest of the columns include security-analyst pair fixed effects. Column (3) and (6) also include time-varying security characteristics (firm size, profitability, return volatility, average monthly return, an indicator for being included in the S&P500 index, and the share of covering analysts that are high-prestige), analyst characteristics (experience with the security, with the industry and with this profession in general, prestige) and forecast timing controls (order of the forecast and earnings report delay). All standard errors are heteroscedasticity-consistent and clustered at the security-year level.

4.2 Reward for the Top or Punishment for the Bottom

Now I present evidence to distinguish between reward for the top and punishment for the bottom. Inspired by Observation 2, I study the correlation between analysts' forecasts and their private signals. Observation 2 implies that as analysts become more inclined to differentiate themselves, their forecasts move towards their signals. If they are rewarded for the top, they will distort their forecasts more towards their private signals as they face more rivals, so the forecasts will be more correlated with the signals; conversely, if analysts are punished for being in the bottom, their forecasts will be less correlated with the signals as they face more rivals.

Ideally, if analysts' private signals were observable, I could regress forecasts directly on signals and examine how the coefficient on signals varies with the number of analysts. However, private signals are unobservable, so instead, I regress forecasts on actual earnings, which are correlated with private signals, using the following regression,

$$X_{ijt} = \lambda_1 \log(N_{jt}) + \lambda_2 Z_{jt} + \lambda_3 \log(N_{jt}) \times Z_{jt} + Controls_{ijt} + e_{2ijt}. \tag{2}$$

A similar endogeneity issue as in Section 4.1 could arise with the number of analysts in this regression, so I adopt the same controls and instrumental variables.

The results are presented in the second block of Table 4. I find that the coefficient on the interaction between the actual earnings and the number of analysts is positive and significant. This means that when analysts face more rivals, their forecasts are more correlated with the actual earnings, and consequently their private signals, so they are more likely to be rewarded for the top. The magnitude of the coefficient is also robust across specifications, again implying the limited impact of heterogeneity within security. To put the coefficients in perspective, in Column (6) where all control variables and the lag instrument are included, an increase in the number of analysts from 6 to 12 increases the coefficient on the actual earnings from 0.855 to 0.872. This implies that a unit increase in analysts' signals translates to 0.017 or 2% increase in forecast.

To sum up, the reduced-form evidence shows that analysts' payoffs depend on relative accuracy, predominantly in the form of a reward for the top, and analysts respond strategically to these payoffs. Building on this, I set up a formal model.

5 Model

I consider each security-year as a forecasting contest. For ease of exposition, the security subscript j and time subscript t are omitted in this section. A contest has N analysts, $i = 1, ..., N$. Analysts do not know the actual earnings $Z \in \mathcal{Z} \subset \mathbb{R}$ when they enter the contest, but they share a common prior distribution of Z. At the beginning of a contest, analysts observe the number of analysts in the contest N. Then, they draw private signals of the earnings, denoted by $S_i \in \mathcal{S} \subset \mathbb{R}$ from a continuous distribution, parameterized with τ. After that, they simultaneously make forecasts $X_i \in \mathcal{Z}$. Let $S = (S_1, ..., S_N)$, $X = (X_1, ..., X_N)$, $S_{-i} = S \backslash S_i$, and $X_{-i} = X \backslash X_i$. Finally, Z is realized and analysts' payoffs are determined by a function $u : \mathcal{Z}^{N+1} \to \mathbb{R}$, defined as the following,

$$u(X_i, X_{-i}, Z) = \underbrace{\sum_{k=1}^{N} \gamma_k(N) \mathbf{1}(\#\{j \ s.t. |X_j - Z| \leq |X_i - Z|\} = k)}_{\text{Relative Accuracy}} + \underbrace{\gamma_c (X_i - Z)^2}_{\text{Absolute Accuracy}} + \underbrace{\gamma_o X_i}_{\text{Optimism}} .$$

This payoff function has three components: relative accuracy, absolute accuracy, and optimism. Analyst i's forecast error is defined as absolute difference between i's forecast and the actual earnings $|X_i - Z|$. If an analyst has k-th smallest forecast error, I say that this analyst has the k-th rank. Then, the top (bottom) of a contest discussed in Section 4 can be formally defined as being ranked higher (lower) than more than half of the analysts, i.e., $k < \frac{N}{2}$ ($k > \frac{N}{2}$).

Then, the payoff for relative accuracy is expressed with a series of weakly decreasing reward for having the k-th rank $\gamma_1(\cdot) \geq \gamma_2(\cdot) \geq \cdots \geq \gamma_N(\cdot)$, which are allowed to depend on N. For example, $\gamma_1(\cdot)$ is the reward for being the closest to the actual earnings or being ranked top 1 in the contest. Without loss of generality, I set $\gamma_N(\cdot) = 0$.

This payoff for relative accuracy is flexible enough to encompass a number of common contests. Here are a few examples:

- <u>winner-takes-all</u> $\gamma_1(N) = C$, $\gamma_k(N) = 0$ for $k = 2, ..., N$;
- <u>reward for top 10% analysts</u> $\gamma_k(N) = \begin{cases} C \text{ for } k = 1, ...\lfloor 0.1N \rfloor, \\ 0 \text{ for } k = \lfloor 0.1N \rfloor + 1, ..., N; \end{cases}$
- <u>loser-loses-all</u> $\gamma_k(N) = C$ for $k = 1, ...N - 1$, $\gamma_N(N) = 0$,

where C is a constant and positive real number.

The payoff for absolute accuracy is represented by a cost of squared forecast error, γ_c, which I normalize to -1. By normalizing, I am imposing the assumption that analysts are always punished for forecast errors, which is empirically supported. If they are not, we will expect them to issue extremely high (low) forecasts when there is the slightest incentive for optimism (pessimism). Such patterns are not observed in the data. The absolute accuracy component sets an *honest* payoff. If it is the only component in the payoff function, analysts will maximize payoffs by choosing the posteriors $\mathbb{E}(Z|S_i)$ as their forecasts, which I call the *honest* forecasts.

Finally, I model the payoff for optimism as a linear function of forecast with coefficient γ_o. If $\gamma_o > 0$ and there is no payoff for relative accuracy, analysts will forecast higher than $\mathbb{E}(Z|S_i)$.

5.1 Equilibrium Existence

Analyst i's strategy is denoted by $\beta_i : \mathcal{S} \to \mathcal{Z}$. Given any set of rivals' strategies, β_{-i} , analyst i chooses forecast X_i to maximize the expected payoff conditional on receiving signal S_i defined as follows

$$U(X_i, S_i; \beta_{-i}(\cdot)) \equiv \int_Z \int_{S_{-i}} u_i(X_i, \beta_{-i}(S_{-i}), Z) h(S_{-i}, Z|S_i) dS_{-i} dZ, \tag{3}$$

where $h(S_{-i}, Z|S_i)$ represents the joint distribution of rivals' signals and the actual earnings conditional on analyst i's own signal.

Equilibrium β characterizes a pure-strategy Bayesian Nash equilibrium if and only if for all i and $S_i \in \mathcal{S}$,

$$\beta^*(S_i) = \arg\max_{X_i \in \mathcal{Z}} U(X_i, S_i; \beta^*_{-i}(\cdot)).$$

To guarantee the existence of an equilibrium, I assume that analysts' private signals are continuously distributed and correlated only through their correlation to the actual earnings. Formally,

Assumption 1 (Conditional Independence)**.** $S_i \perp\!\!\!\perp S_j|Z$ for all $j \neq i$.

Then, I can express the conditional density of the actual earnings given i's private signal as $f(Z|S_i)$ and the conditional density of rival j's signal given the actual earnings as $g(S_j|Z)$. Equation (3) can be rewritten as

$$U(X_i, S_i; \beta_{-i}(\cdot)) \equiv \int_Z \int_{S_{-i}} u_i(X_i, \beta_{-i}(S_{-i}), Z) g(S_{-i}|Z) f(Z|S_i) dS_{-i} dZ. \tag{4}$$

Assumption 1 is sufficient for the existence of an equilibrium when forecast space $\mathcal{Z}$ is finite, formalized in the following proposition.

Proposition 1. If Assumption 1 holds, analysts' private signals S_i are continuously distributed, and $\mathcal{Z}$ is finite, there exists a pure strategy equilibrium.

Proof. See Appendix C.1. ■

This proof can be extended to allow analysts to receive a common signal and an equilibrium exists as long as analysts' private signals are independent conditional on both the actual earnings and the common signal, so the conditional independence assumption is not driving the main results of this paper (proof in Appendix C.2).[9] Nonetheless, I maintain this assumption as it highlights the key strategic interactions.

[9] Intuitively, the common signal updates analysts' common prior distribution of the actual earnings. Once it received, analysts will be playing the same contest with private signals, only with a new common prior, so all strategic interactions will remain the same.

While the equilibrium existence is established for finite $\mathcal{Z}$, I will focus on a continuous state space in the rest of the paper, where the equilibrium is more tractable and faster to compute numerically, enabling estimation. Now I will characterize the equilibrium using first order conditions and introduce the numerical procedure for equilibrium computation.

5.2 Equilibrium Characterization

The relative accuracy component of the objective function can be simplified as follows. First, consider analyst i and an arbitrary rival j. I say that j is better than i when j's forecast is more accurate than i's forecast, that is, $|X_j - Z| < |X_i - Z|$. This is equivalent to

$$\begin{aligned} X_i < & X_j < 2Z - X_i, \text{ if } X_i \geq Z \\ X_i > & X_j > 2Z - X_i, \text{ if } X_i > Z. \end{aligned}$$

I assume that analysts' forecast strategies are symmetric and strictly increasing in their private signals. This assumption allows us to infer analysts' private signals from their forecasts and guarantees the identification of the model (discussed further in Section 6.1).[10] Leveraging this, I write the probability of j being better than i as

$$P(j \text{ is better than } i|X_i, Z, S_i) = \left| \int_{\beta^{-1}(X_i)}^{\beta^{-1}(2Z-X_i)} g(S_j|Z)dS_j \right| \equiv 1 - p(X_i, Z),$$

where $p(X_i, Z)$ is the probability of i being better than one arbitrary rival.

Then, the probability of i being ranked the k-th is equal to the probability of i being better than $N-k$ rivals and worse than $k-1$ rivals. The expected value of sending forecast X_i after observing signal S_i can be rewritten as

$$\begin{aligned} U(X_i, S_i; \beta_{-i}(\cdot)) = \int_{-\infty}^{\infty} \Big[\sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} p(X_i, Z)^{N-k} (1 - p(X_i, Z))^{k-1} + \\ - (X_i - Z)^2 + \gamma_o X_i \Big] f(Z|S_i)dZ. \end{aligned}$$

The first order condition describing the symmetric and increasing equilibrium is given by

$$\begin{aligned} & \underbrace{\sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} \int_{-\infty}^{\infty} \Xi(X_i, Z, N, k) f(Z|S_i)dZ}_{\text{Relative Accuracy}} \\ & \underbrace{-2[X_i - \mathbb{E}(Z|S_i)]}_{\text{Absolute Accuracy}} + \underbrace{\gamma_o}_{\text{Optimism}} = 0, \end{aligned} \tag{5}$$

[10] While this is a strong assumption, it is verified numerically and also mirrors similar assumptions in the common value auction literature to guarantee identification (Athey and Haile 2007).

to be solved for a strictly increasing $\beta^*(\cdot)$, where $X_i = \beta^*(S_i)$ and

$$\begin{aligned}\Xi(X_i, Z, N, k) = & \big[\mathbf{1}(X_i \le Z) - \mathbf{1}(X_i > Z)\big] \\ & \big[\frac{g(\beta^{*-1}(X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(X_i))} + \frac{g(\beta^{*-1}(2Z - X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(2Z - X_i))}\big] \\ & \big[\mathbf{1}(N > k)(N-k)p(X_i, Z)^{N-k-1}(1 - p(X_i, Z))^{k-1} - \\ & \mathbf{1}(k > 1)p(X_i, Z)^{N-k}(k-1)(1 - p(X_i, Z))^{k-2}\big] \end{aligned} \tag{6}$$

The first order condition has the same three components as the payoff function. Consider the last two components. The payoff for absolute accuracy creates an incentive for analysts to forecast close to their posteriors $\mathbb{E}(Z|S_i)$. Meanwhile, if analysts are rewarded for optimism, deviations above the posterior will bring higher expected payoff until the marginal loss from accuracy (relative or absolute) is no longer justified by the marginal gain in optimism γ_o.

The relative accuracy component is the sum of the expected marginal reward for each rank. For a given rank k in N analysts, its marginal reward is its reward $\gamma_k(N)$ multiplied by the marginal probability of receiving this rank $\binom{N-1}{N-k} \int_{-\infty}^{\infty} \Xi(X_i, Z, N, k) f(Z|S_i) dZ$. Analysts choose forecasts to increase the probability of receiving better rewarded ranks.

$\Xi(X_i, Z, N, k)$ denotes the marginal probability of being better than a given set of $N-k$ rivals and worse than the rest. The first bracket of equation (6) shows that this can be further split into two terms, for when the actual earnings is higher than the forecast ($X_i \le Z$), and when it is lower ($X_i > Z$). These two terms reflect the trade-off analysts face when they want to be better than the rivals. For example, a higher forecast improves the chance of being better than the rivals in realizations where the actual earnings is higher than the forecast, but worsens the chance otherwise. The term $\big[\frac{g(\beta^{*-1}(X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(X_i))} + \frac{g(\beta^{*-1}(2Z-X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(2Z-X_i))}\big]$ represents the marginal gain or loss in the chance of being better than one rival conditional on the actual earnings.

Observation 2 introduces the intuition that when analysts are rewarded for the top, they will forecast close to their private signals, away from the common prior. Proposition 2 proves this formally in a winner-takes-all game with two players. I show that even without the payoff to optimism, reporting the posterior is not an equilibrium. Moreover, analysts have an incentive to deviate towards their signals.

Proposition 2. Suppose the actual earnings Z and the signals S are normally distributed. In a winner-takes-all game with two players, when analysts reports their posterior means as forecasts, there exists a threshold $\bar{S} > 0$ such that for all $|S_i| < \bar{S}$, an analyst's utility increases from a deviation in the direction of the signal.

Proof. See Appendix C.3. ∎

5.3 Numerical Procedure for Equilibrium Computation

In this subsection, I introduce the numerical procedure used to solve the symmetric equilibrium. An analytical solution is difficult to obtain for this game. The equilibrium is characterized by the first order conditions in equation (5) for every signal in $\mathcal{S}$, which form a system of differential equations involving the inverse function and the derivative of the equilibrium strategy. With a change of variable, this system can be rewritten to be free of the inverse function of the equilibrium strategy (see Appendix D.1 for details). Even so, this system does not have a well-established solution to my knowledge.

Therefore, I numerically approximate the equilibrium strategy by projecting it onto a family of pre-specificed functions, such as polynomials. The projection is parameterized with θ. First, I solve the derivatives of the pre-specified function analytically. Then, I substitute the equilibrium strategy and its derivatives in the first order conditions with those implied by the function family. After that, I find the θ^* that minimizes a residual function, which is defined as the sum of squared first order condition at a number of representative signals in $\mathcal{S}$ (also called "collocation points" in Judd (2020)). Finally, I verify whether the function with θ^* is a good approximation of the equilibrium strategy by checking if the first order conditions are indeed close to zero.

This procedure, namely the projection method, is commonly used in the macroeconomics literature to solve continuous-time life-cycle consumption models and growth models (Judd 2020). Conceptually, it is similar to setting players' strategy to be symmetric when analytically solving a Cournot or Bertrand game.

The projection method has three benefits compared to other commonly used equilibrium computation algorithms. First, it allows researchers to focus on equilibria with characteristics of interest, such as symmetry and monotonicity. Such restrictions are difficult to impose with, for example, an iterated best response algorithm. Second, this procedure can be computed quickly thanks to its parsimonious specification. Last but not least, the method can be extended to be fully non-parametric. The functional form can be chosen to flexible enough to describe any arbitrary strategy in the strategy space.

I choose monotonic polynomials as the function family and experiment with up to the 7th order polynomial to simulate equilibria under the parametric assumptions of the game, which I will introduce in Section 6 (details of the experimentation are provided in Appendix D.2). In practice, for any given set of parameters, this algorithm always converges to the same symmetric equilibrium.

6 Identification, Estimation and Results

In this section, I discuss identification and estimation of the model. The parameters of interest are the signal distribution and the payoff function. First, I provide a discussion of the identification of the parameters. I argue that for many contests of interest, the signal distribution and the payoff function are non-parametrically identified, so the results are not driven by parametric assumptions. Then, I introduce how I estimate the model with indirect inference, detailing the parametric specifications imposed to simplify computation and the choice of moments. Finally, I present the results.

6.1 Identification

To identify the model, I first show that the signal distribution is non-parametrically identified. Then, given the identified signal distribution, I use the first order conditions implied by the Bayesian Nash Equilibrium (equation (5)) to identify the payoff function.

6.1.1 Signal Distribution

The signal is only of information value in this model, just as in auction models. That is, any strictly monotonic transformation of the signal does not change the information it contains (Athey and Haile 2007). For ease of exposition, I adopt the strictly increasing equilibrium forecast strategy under the true payoff $\beta^*(S_i; N)$ as the transformation in my identification argument. For a given number of analysts N, the set of signals can then be transformed to

$$\tilde{\mathcal{S}} \equiv \{\tilde{S} : \tilde{S} = \beta^*(S; N), S \in \mathcal{S}\}. \tag{7}$$

In other words, under the true payoff function, there is a bijection between the set of observed forecasts and the set of signals for a given N.

This transformation allows me to back out two conditional distributions relevant for the equilibrium characterization of this model: 1) the density of the actual earnings conditional on the signal, $f(Z|S_i)$; 2) the density of the signal conditional on the actual earnings, $g(S_i|Z)$. For a given N, the density of the earnings conditional on the signal $f(Z|S_i)$ is equivalent to the density of the earnings conditional on the equilibrium forecast and the number of analysts $\tilde{f}(Z|\beta^*(S_i; N), N)$. Because Z, $\beta^*(S_i; N)$, and N are observable, the conditional density $\tilde{f}(Z|\beta^*(S_i; N), N)$ is identified. Similarly, the density of signal conditioning on the earnings $g(S_i|Z)$ is equivalent to the density of the equilibrium forecast conditioning on the earnings and the number of analysts $\tilde{g}(\beta^*(S_i; N)|Z, N)$, and thus is identified as well. Therefore, the signal distribution is non-parametrically identified.

6.1.2 Payoff Function

Given the signal distribution, I use variation in equilibrium forecast strategy with the number of analysts to identify the payoffs for optimism and relative accuracy. The intuition follows from Observation 1 and 2. Consider a winner-takes-all contest, analysts' forecast errors will be higher when they face more rivals; and this effect becomes more pronounced the more rewarded the top analyst is. Therefore, the change in forecast error with the number of analysts identifies the reward for the most accurate analyst in the payoff function.

More formally, I establish a sufficient condition for the non-parametric identification of the payoff function using the first order conditions. Let γ denote the vector of γ_o and $\gamma_k(N)$ for all k and N. Following the transformation in equation (7), $f(Z|S_i)$ can be rewritten using the equilibrium forecast $X = \beta^*(S_i; N)$ as $\tilde{f}(Z|X, N)$. As X has a continuous state space, for all N and integer $L_N > 0$, there exist L_N intervals in the support of X denoted as $\{(\underline{x}_{lN}, \overline{x}_{lN}) : l = 1, 2, ..., L_N$ and $\underline{x}_{lN}, \overline{x}_{lN} \in \mathcal{Z}\}$. Integrating equation (5) over any interval $(\underline{x}_{lN}, \overline{x}_{lN})$ gives

$$\begin{aligned} &\gamma_o(\overline{x}_{lN} - \underline{x}_{lN}) + \sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} \int_{\underline{x}_{lN}}^{\overline{x}_{lN}} \int_{-\infty}^{\infty} \Xi(X, Z, N, k) \tilde{f}(Z|X, N) dZ dX \\ =& 2\int_{\underline{x}_{lN}}^{\overline{x}_{lN}} [X - \mathbb{E}(Z|X, N)] dX, \end{aligned} \tag{8}$$

which is linear in γ. Then, equations (8) for any set of intervals $\{(\underline{x}_{lN}, \overline{x}_{lN}) : l = 1, 2, ..., L_N, N \in \mathcal{N},$ and $\underline{x}_{lN}, \overline{x}_{lN} \in \mathcal{Z}\}$ form a linear moment restriction of γ, $A\gamma = b$, where A denotes the Jacobian matrix. I impose the following assumption on A as a sufficient condition for the global identification of γ.

Assumption 2. There exists a set of intervals $\{(\underline{x}_{lN}, \overline{x}_{lN}) : l = 1, 2, ..., L_N, N \in \mathcal{N},$ and $\underline{x}_{lN}, \overline{x}_{lN} \in \mathcal{Z}\}$ such that the corresponding Jacobian matrix A has full rank.

Under this assumption, the rank-order condition for identification is satisfied, so γ is identified. In addition, as the moment restriction is linear in γ, the identification is global.

Intuitively, this assumption requires that there is sufficient variation in first order conditions across different numbers of analysts and signals. As a more concrete example, I derive a more primitive sufficient condition for a special case where the reward for any rank of relative accuracy is homogeneous across different N's in Appendix C.4. I show that only mild conditions on the marginal probabilities of receiving some relative accuracy ranks are needed to guarantee identification in this case.

Moreover, Assumption 2 is potentially testable as all components of A can be computed from the data. In equation (8), $\Xi(X, Z, N, k)$ is the marginal probability of receiving relative accuracy rank k with forecast X, conditioning on the actual earnings being Z; $\tilde{f}(Z|X, N)$ and $\mathbb{E}(Z|X, N)$ are the density and the expectation of the actual earnings, conditioning on equilibrium forecast X and the number of analysts N. Therefore, while it is out of scope for this paper to establish a general primitive condition that guarantees Assumption 2, one can test this assumption in the data for future applications.

6.2 Estimation Method

The parameters to be estimated are γ in the payoff function and τ in the signal distribution. I estimate the model with indirect inference in four steps:

1. I run the auxiliary regressions in the data and use the regression coefficients as data moments (denoted $\hat{m}$).

2. Starting from some parameter values $\{\tilde{\gamma}, \tilde{\tau}\}$, I use the numerical procedure described in Section 5.3 to compute the equilibrium strategies for games in the data, which are characterized by the number of players N. An important assumption here is that if the model has multiple equilibria, all observations in the given data set are from the same equilibrium; otherwise, the structural estimates are not identified.[11] In practice, the numerical procedure for equilibrium computation always gives the same symmetric monotone equilibrium.

3. I simulate signals 100 times and use the equilibrium strategy to find the corresponding forecasts. For each simulation, I run the auxiliary regressions in the simulated data and use the average of the regression coefficients across simulations as simulated moments (denoted $m(\tilde{\gamma}, \tilde{\tau})$).

4. I match the simulated moments to the data moments to update $\{\tilde{\gamma}, \tilde{\tau}\}$. For some weighting matrix matrix W, I repeat step 2 and 3 until the weighted distance between the simulated moments and the data moments $(\hat{m} - m(\gamma, \tau))' W (\hat{m} - m(\gamma, \tau))$ is minimized.

The identification strategy relies on the variation in the number of analysts to identify the payoff function. However, the number of analysts could be endogenous due to unobserved heterogeneity in payoff functions and signal distributions, so I normalize the data and estimate a homogeneous "average contest" in the sample. I focus on removing the heterogeneity at the security-year level and assume analysts to be homogeneous otherwise, as the impact of cross-analyst heterogeneity within a security is limited (Table 4). First, I regress $(X_{ijt}, Z_{jt}, N_{jt})$ on security fixed effects and year fixed effects and take the residuals as the corresponding homogenized variables, assuming that the fixed effects sum to 0. This removes the heterogeneity at the security and the year level. Then, similar to in Table 4, I instrument the number of analysts with the lag number of analysts when running the auxiliary regressions in the data (see Table A2 for first stage results). This ensures that the data moments capture the causal and strategic effect of the number of analysts. Further details on the homogenization are presented in Appendix D.4. For simplicity and with a slight abuse of notation, I use $(X_{ijt}, Z_{jt}, N_{jt})$ to denote the corresponding homogenized variables in the rest of the paper.

Now I present the parametric specifications for estimation and discuss my choice of auxiliary regressions and moments in detail.

[11] See Athey and Haile (2007) for a similar assumption in the discussion of first-price auctions.

6.2.1 Parametric Specifications

Signal Distribution I assume that the signal is normally distributed around the actual earnings as the following,

$$S_i = Z + \epsilon_i, \quad \epsilon_i \sim N(0, 1/\tau)$$

where τ denotes the precision of the signal. Then, the posterior distribution of the actual earnings conditional on observing signal S_i is given by,

$$Z|S_i \sim N(\frac{\tau S_i}{1+\tau}, \frac{1}{1+\tau}).$$

Note that the posterior mean is the *honest* forecast, whose correlation with the signal and the actual earnings is given by $\frac{\tau}{1+\tau}$, which is increasing in τ. This means that the honest forecast puts more weight on the signal if the signal is more precise.

Payoff Function Reduced-form evidence suggests that analysts are rewarded for being in the top, so I estimate a winner-takes-all specification of relative accuracy as the baseline. For robustness, I also estimate a specification where I relax the winner-takes-all assumption and allow the convexity of the reward for each rank to be flexible (Appendix E).

Equilibrium Computation I experimented with polynomials up to the 7th order and found that a simple linear function, $X_i = \theta_1 + \theta_2 S_i$, is a great approximation of the equilibrium strategy under normality (Appendix D.2). This is also consistent with the analytical equilibrium in some special cases, such as the *honest* case, and a winner-takes-all game with infinite number of players (Ottaviani and Sørensen 2006). Hence, I adopt the linear approximation.

To reduce the computation burden, I use a cubic spline to interpolate the equilibrium strategies for different numbers of analysts. I compute the equilibrium strategies with the projection method for representative games, where the number of analysts are at the 5th percentiles $\{0\%, 5\%, 10\%, ..., 100\%\}$, and interpolated for the rest (Appendix D.3).

6.2.2 Choice of Moments

Following the identification argument, I use the variation in forecast strategy with the level of competition to identify the structural parameters. Specifically, I estimate the following auxiliary regressions:

$$\mathbf{1}(X_{ijt} > Z_{jt}) = \tilde{\eta}_0 + \tilde{\eta}_1 \log(N_{jt}) + \varepsilon_{1ijt} \tag{9}$$

$$|X_{ijt} - Z_{jt}| = \tilde{\alpha}_0 + \tilde{\alpha}_1 \log(N_{jt}) + \varepsilon_{2ijt} \tag{10}$$

$$X_{ijt} = \tilde{\lambda}_0 + \tilde{\lambda}_1 \log(N_{jt}) + \tilde{\lambda}_2 Z_{jt} + \tilde{\lambda}_3 \log(N_{jt}) \times Z_{jt} + \varepsilon_{3ijt} \tag{11}$$

These regressions are similar to the ones used for reduced-form evidence in Section 4, but with additional emphasis on the effect of the optimism incentive. $\mathbf{1}(X_{ijt} > Z_{jt})$ in equation (9) is an indicator which is equal to 1 when $X_{ijt} > Z_{jt}$, capturing whether analysts are more likely to make optimistic "mistakes" with more competition. Equation (10) matches equation (1), showing how analysts' forecast errors changes with competition. In addition to estimating this regression in the full sample, I also estimate it conditional on whether $X_{ijt} > Z_t$ to reflect how optimistic and pessimistic forecast errors respond to competition. Finally, equation (11) matches equation (2), showing how the correlation between the forecast and the actual earnings changes with competition.

I use the coefficients $\tilde{\eta}, \tilde{\alpha}, \tilde{\lambda}$, as well as the forecast residual variance from regression equation (11) to construct moments in the data and in the simulated data. The weighting matrix W is constructed as a block-diagonal matrix. Each block that corresponds to the coefficients of an auxiliary regression is the sample variance-covariance matrix heteroskedastic-consistent standard errors clustered at the security-year level; and the forecast residual variance is weighed equally as the average of the rest of the moments.

6.3 Results

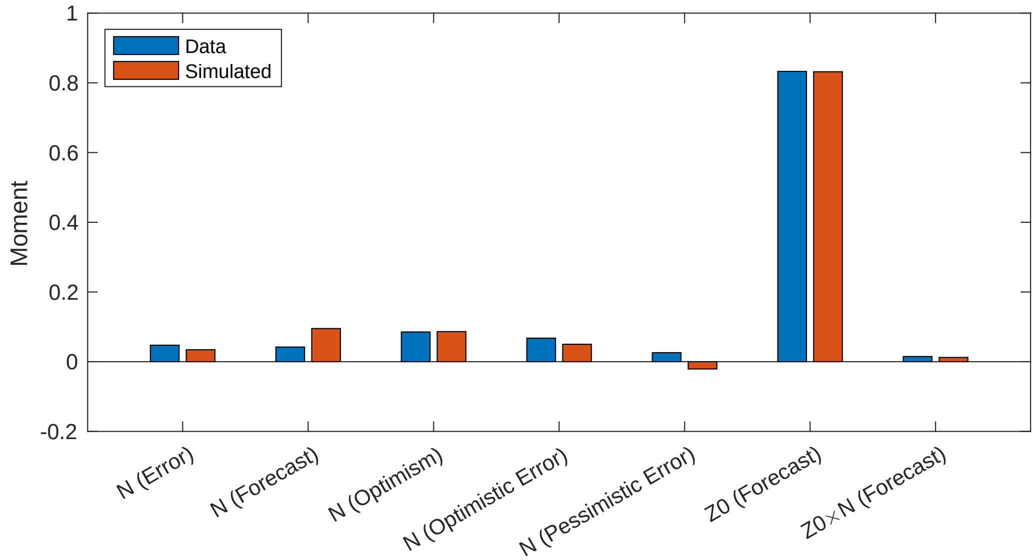
Table 5: Estimates of Analysts' Payoff

	Estimate	Standard Error
The Most Accurate	11.719	(3.092)
Optimism	0.955	(0.101)
Signal Precision	5.297	(0.169)

Notes: Table presents estimates of analysts' payoff function.

The estimation results are presented in Table 5. I find large and significant rewards for being the most accurate analyst. This reward compensates the loss in absolute accuracy of a forecast that is up to 3.423 standard errors away from the actual earnings. In addition, there is significant incentive for optimism. The optimism coefficient implies that in the absence of the relative accuracy reward, reporting an optimistic forecast up to 0.955 standard error higher than the actual earnings will yield higher payoff than reporting the actual earnings as the forecast. When I relax the winner-

Figure 1: Moment Fit



takes-all assumption, the estimated reward is still very concentrated on the most accurate, which is consistent with the baseline model and implies that the result is not driven by this assumption (Appendix E).

Figure 1 presents the fit of the coefficients on the state variables in the auxiliary regressions. The model fits the data reasonably well. The structural estimates captures the two reduced-form observations that analysts' forecast errors increase and their forecasts become more correlated with the actual earnings when they face more rivals. Also, consistent with the data, the simulated probability of reporting an optimistic forecast increase with competition. The only difference in sign between the data moments and the simulated moments is how pessimistic forecast errors change with competition. Nevertheless, this moment is close to zero both in the data and the simulation.

7 Counterfactuals

Now I use the estimated model to perform two analyses. First, I study the effect of the reward for relative accuracy. Specifically, I consider how information quality changes when analysts are less rewarded for relative accuracy. In Section 7.1, I focus on a hypothetical scenario where analysts are no longer rewarded for optimism. This illustrates the mechanism of the *distortionary* effect with the *disciplinary* effect muted. In Section 7.2, I add back the optimism incentive to show how the two effects interact and how the reward for relative accuracy affects the current market overall. A comparison of these two scenarios also illustrate how optimism regulations affect analysts' strategic interactions, providing a new interpretation to existing regulations. Second, in Section 7.3, I investigate the optimal level of competition in covering one security by exploring how measures of information quality changes with the number of analysts. Many policies may affect the level of competition in the security analyst market, such as MiFID II and merger approvals for investment banks. I show that the effect of these policies on information quality depends on the current level of competition.

To quantify information quality under any payoff and market structure, I simulate analysts' forecasts as follows: 1) For each contest, I draw a random number from the standard normal distribution as the actual earnings. 2) For each analyst in the contest, I draw a random noise and add it to the actual earnings to generate a signal. 3) I solve the equilibrium strategy for the contest to simulate forecasts from the signals.

Using the simulated forecasts, I compute the following measures of information quality. For information accuracy and noise from any individual analyst, I consider average forecast error and forecast dispersion across *individual* forecasts in a security-year. For the accuracy and noise of aggregate information, I consider forecast error and forecast variance of *consensus* forecast X^*, which is defined as the mean of the individual forecasts in a security-year.[12]

I repeat this procedure for every contest in the data 20 times and average across them to measure information quality in the overall market. Then, to measure information quality under different levels of competition, I follow the same procedure and simulate 1 million contests for any given number of analysts.

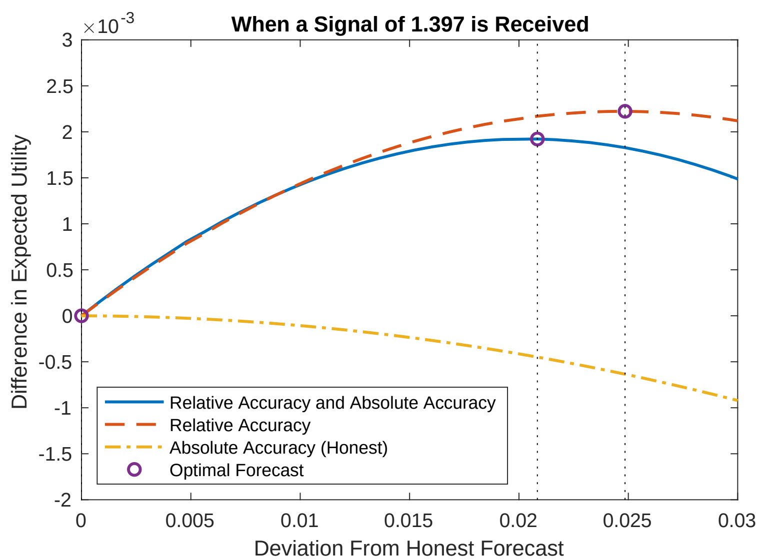
It is worth noting that forecast strategies optimizing quality of *individual* forecasts do not necessarily optimize quality of the *consensus*, because each analyst is limited by their private information. For example, suppose the goal is to minimize forecast errors. The ex ante forecast error of analyst i's individual forecast is $\mathbb{E}(|X_i - Z|\ |S_i)$, which is conditional on her private signal S_i and optimized at the honest forecast $X_i = E(Z|S_i) = \frac{\tau}{1+\tau}S_i$. In contrast, the ex ante forecast error of the consensus forecast is $\mathbb{E}(|X^* - Z|\ |S)$, which is optimized at $X^* = E(Z|S) = \frac{1}{N}\sum_{i=1}^{N}\frac{N\tau}{1+N\tau}S_i \neq \frac{1}{N}\sum_{i=1}^{N}E(Z|S_i)$. To achieve the optimal consensus forecast jointly, each analyst needs to forecast $\frac{N\tau}{1+N\tau}S_i$, putting more weight on their private signals than the optimal individual forecast $\frac{\tau}{1+\tau}S_i$. Therefore, if investors care only about the consensus quality, they will prefer analysts to incorporate more private information in their forecasts even if it is noisy, as the noises of different analysts "cancel out" in the consensus. This example demonstrates the trade-off between optimizing individual forecast quality and consensus forecast quality, which I will take into account to interpret the counterfactual results.

[12]The consensus can also be defined as the median of the individual forecasts, which yield similar counterfactual results.

7.1 Without Optimism

Now I present the impact of competition without the optimism incentive to separate out the distortionary effect. To provide intuition, I start with an example that illustrates analysts' equilibrium forecast strategies under different incentive structures: Figure 2 shows the gains and losses in expected utility when an analyst deviates from the honest forecast under different payoff functions. Here, the analyst competes with 9 other analysts and receives a signal of 1.397, which implies an honest forecast of 1.175. This signal is at the 90th percentile of the signal distribution, so I call it an optimistic signal.

Figure 2: Analyst's Problem with 10 Analysts:
Difference in Expected Utility When Deviating from Honest Forecast



This figure shows that analysts deviate in the direction of their signals under the estimated reward for relative accuracy. When analysts are rewarded only for relative accuracy (red dashed line), the analysts make forecasts that are more optimistic than the honest forecast upon receiving an optimistic signal. And when the reward for absolute accuracy is added (blue solid line), the difference between the equilibrium and the honest forecast remains positive but smaller. Though not illustrated, analysts will behave similarly when pessimistic signals are received, making forecasts more pessimistic than honest forecasts.

Table 6 presents how the reward for relative accuracy affects information quality without optimism by showing the differences in information measures between contests when analysts are rewarded for relative accuracy versus when they are not. Overall, the reward for relative accuracy increases individuals' average forecast error by a small amount of 0.07% and increases their forecast dispersion by 3.16%. Meanwhile, this reward decreases consensus forecast error by 6.10% and

Table 6: How Does Reward for Relative Accuracy Affect Information Quality (Without Optimism)

	Individual		Consensus	
N	Forecast Error	Forecast Dispersion	Forecast Error	Forecast Variance
2	0.001 (0.00%)	0.190 (0.14%)	-0.124 (-0.05%)	1.097 (0.14%)
5	0.219 (0.07%)	4.288 (3.21%)	-5.715 (-3.15%)	23.583 (3.21%)
10	0.259 (0.08%)	4.735 (3.55%)	-8.347 (-5.32%)	25.535 (3.55%)
20	0.219 (0.07%)	4.283 (3.20%)	-8.884 (-6.24%)	22.876 (3.20%)
40	0.134 (0.04%)	3.387 (2.53%)	-7.690 (-5.71%)	18.008 (2.53%)
60	0.093 (0.03%)	2.784 (2.08%)	-6.530 (-4.94%)	14.788 (2.08%)
Overall	0.217 (0.07%)	4.219 (3.16%)	-8.726 (-6.10%)	22.565 (3.16%)

Notes: Table shows the difference in information quality between two payoff structures: when analysts are rewarded for relative and absolute accuracy versus when they are rewarded for only absolute accuracy. The differences in levels are in the unit of 10^{-3} and the percentage differences are in parentheses.
For a given contest, the forecast error of individual forecasts is computed as $\sum_i |X_i - Z|/N$; the forecast variance of individual forecasts is computed as $\sum_i (X_i - \bar{X})^2/(N-1)$; the forecast error of the consensus is computed as $|X^* - Z|$. The mean of these measures across contests in the simulated sample are reported in this table. The variance of the consensus forecast is computed as the variance of X^* across contests in the simulated sample.

increases consensus forecast variance by 3.16%. Note that the effect is opposite on individual and consensus forecast accuracy, highlighting the trade-off between these two quality dimensions mentioned above. Rewarding the most accuracy induces analysts to differentiate themselves and put more weight on their private signals, resulting in a forecast strategy that is closer to the one that optimizes consensus forecast error. However, this improvement is at the cost of making individual forecasts more dispersed and consensus forecast more noisy.

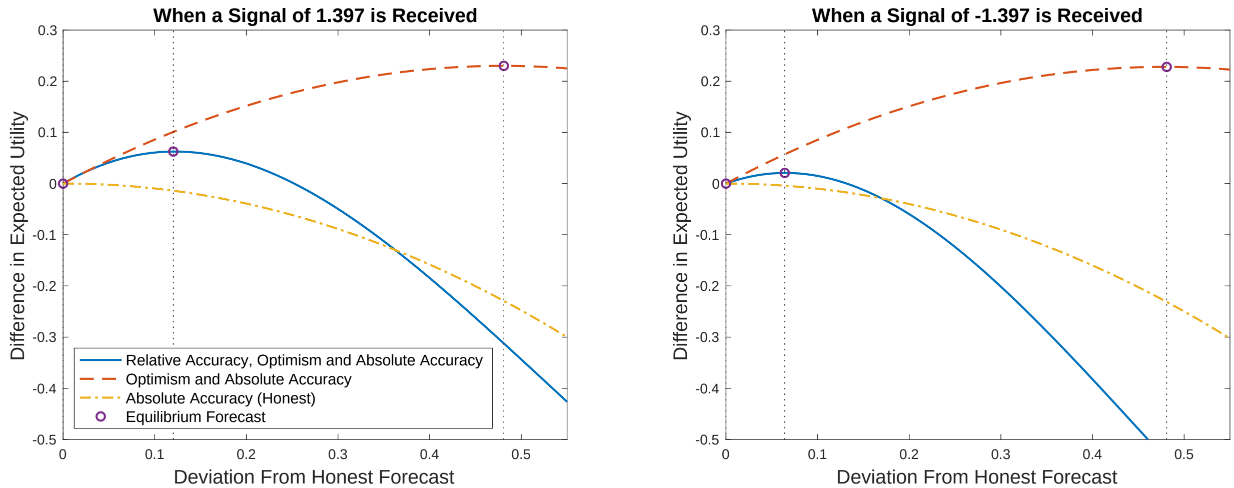
Comparing across different numbers of analysts, I find that the effect of the reward for relative accuracy is non-linear in the level of competition: its magnitude first increases then decreases with the number of analysts. This is because when the number of analysts is small, more analysts means that it is more difficult to be the most accurate, so analysts will need to distort more to out-perform their rivals. But as the number of analysts grows further, the chance of being the most accurate becomes so low that the expected gains in relative accuracy is outweighed by the expected loss from absolute accuracy. As a result, the effect of the reward for relative accuracy becomes smaller.

7.2 With Optimism

Now I add the optimism reward back to analysts' payoff, so both the distortionary and the disciplinary effect are present. This case reflects the effect of the reward for relative accuracy in the current market. If analysts' forecast errors decreases with the addition of relative accuracy to the payoff, I will say that the disciplinary effect dominates and vice versa.

Figure 3 again illustrates the gains and losses in expected utility when an analyst competes

Figure 3: Analyst's Problem with 10 Analysts:
Difference in Expected Utility When Deviating from Honest Forecast



with 9 other analysts and deviates from the honest forecast, now with the optimism incentive. Here, I present the difference in expected utility from deviation upon receiving an optimistic signal of 1.397 (the 90th percentile) and in addition, a pessimistic signal of -1.397 (the 10th percentile), which correspond to honest forecasts of 1.175 and -1.175.

With both signals, the analyst benefit from a deviation towards a higher forecast once the optimism incentive enters the payoff function (red dashed lines). The reward for relative accuracy disciplines the amount of optimism in the forecast (blue solid lines). In contrast to the case without optimism (where the analyst always deviates in the direction of their signal), here the analyst will benefit from an optimistic deviation even when a pessimistic signal is received. Nevertheless, the deviation with an optimistic signal is larger than that with a pessimistic signal, because the optimism reward reduces the cost of making an optimistic distortion. In other words, those who receive optimistic signals will exaggerate their signals more than those who receive pessimistic signals, because they are less worried about being punished for optimistic "mistakes".

Table 7 presents how the reward for relative accuracy affect information quality with optimism by showing the differences in information measures when analysts are rewarded for relative accuracy versus when they are not. Overall, I find that in the reward for relative accuracy substantially decreases individuals' average forecast error by 34.01% and the consensus's forecast error by 60.84%, which implies that the disciplinary effect dominates.

However, the distortionary effect is still present, as evident in the higher noise measures. In fact, the distortionary effect generates more noise with the optimism incentive than without, as the increase in individual forecast dispersion and consensus forecast variance is higher with optimism (6.59% and 6.68%) than without optimism (3.16% and 3.16%). This is precisely because the

Table 7: How Does Reward for Relative Accuracy Affect Information Quality (With Optimism)

	Individual		**Consensus**	
N	Forecast Error	Forecast Dispersion	Forecast Error	Forecast Variance
2	-0.203 (-38.84%)	0.000 (0.15%)	-0.249 (-50.54%)	0.001 (0.15%)
5	-0.201 (-38.44%)	0.005 (3.61%)	-0.299 (-62.28%)	0.027 (3.61%)
10	-0.194 (-37.24%)	0.007 (4.90%)	-0.313 (-65.46%)	0.035 (4.90%)
20	-0.178 (-34.18%)	0.009 (6.58%)	-0.294 (-61.50%)	0.047 (6.58%)
40	-0.147 (-28.21%)	0.011 (8.50%)	-0.233 (-48.90%)	0.060 (8.50%)
60	-0.123 (-23.65%)	0.012 (9.07%)	-0.188 (-39.47%)	0.064 (9.07%)
Overall	-0.178 (-34.01%)	0.009 (6.59%)	-0.291 (-60.84%)	0.048 (6.68%)

Notes: Table shows the difference in information quality between two payoff structures: when analysts are rewarded for relative accuracy, absolute accuracy and optimism versus when they are rewarded for absolute accuracy and optimism. The differences in levels are in the unit of 1 and the percentage differences are in parentheses.

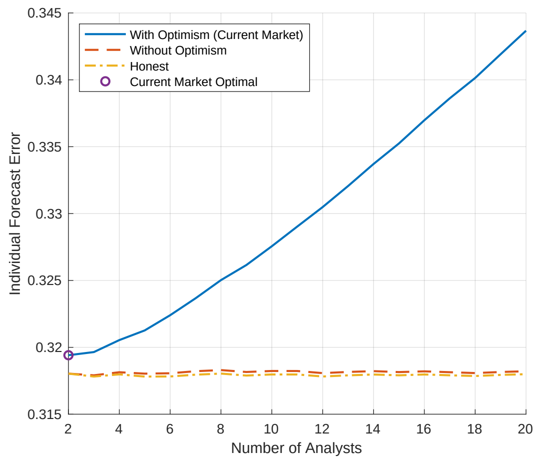
For a given contest, the forecast error of individual forecasts is computed as $\sum_i |X_i - Z|/N$; the forecast variance of individual forecasts is computed as $\sum_i (X_i - \bar{X})^2/(N-1)$; the forecast error of the consensus is computed as $|X^* - Z|$. The mean of these measures across contests in the simulated sample are reported in this table. The variance of the consensus forecast is computed as the variance of X^* across contests in the simulated sample.

optimism reward leads to asymmetric cost in making an optimistic versus a pessimistic distortion (Figure 3). This observation implies a new mechanism for the existing optimism regulations to take effect: targeting individual analysts' optimism incentive not only improves analysts' forecast accuracy by reducing the optimism bias, but it also reduces noise, because optimistic signals are less over-exaggerated.

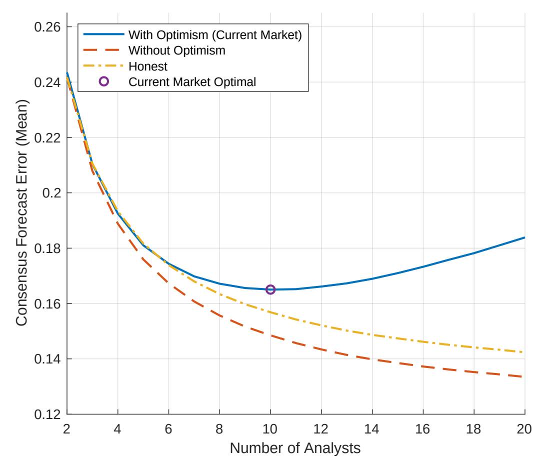
Comparing across different numbers of analysts, the biggest improvement in individual forecast quality is generated with 2 analysts. As analysts face more rivals, the distortionary effect becomes more pronounced, which increases individual forecast error and dispersion. In contrast, the improvement in consensus forecast quality is not monotone in the level of competition. It first increases as analysts put more weight on their private signals, but as the level of competition increases further, the distortionary effect becomes more pronounced so that analysts, especially those who receive optimistic signals, over-weigh their private signals in their forecasts. As a result, the improvement in consensus forecast error becomes smaller. Meanwhile, the effect on consensus forecast variance is always greater with more analysts.[13]

Figure 4: The Effect of Competition

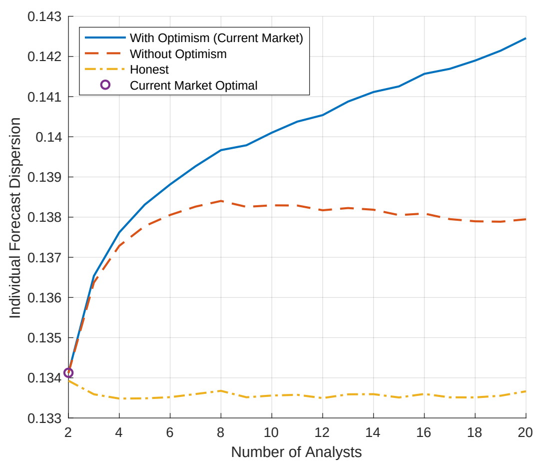
(a) Individual: Forecast Error



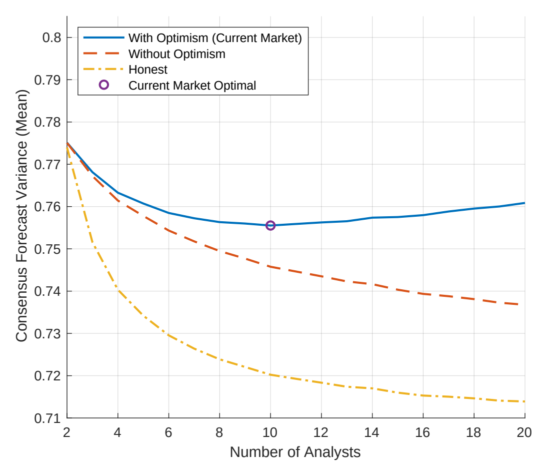
(b) Consensus: Forecast Error



(c) Individual: Forecast Dispersion



(d) Consensus: Forecast Variance



7.3 Optimal Level of Competition

Figure 4 presents how measures of information quality changes with the level of competition under different incentive structures. The optimal level of competition in the current market depends on which measures the investors or policy makers are optimizing over.

Figure 4a and 4b show how the accuracy measures, individual average forecast error and consensus forecast error, change with the level of competition. Under the honest payoff (yellow dash-dot line), the individual forecast error does not change with competition. Meanwhile, the consensus forecast error decreases with competition because there is more aggregate information.

When analysts are in addition rewarded for relative accuracy (red dashed line), the individual forecast error is very similar to that of the honest payoff. The consensus forecast error also decreases with competition in this case but is generally lower than that of the honest payoff. As discussed at the beginning of this section, this is again because analysts adopt a forecast strategy that puts more weight on their private signals and improves consensus forecast error when rewarded for relative accuracy.

When analysts are further rewarded for optimism as well (blue solid line), the individual forecast error increases with competition because the distortionary effect intensifies. The consensus forecast error first decreases with competition as with the other two payoff structures because there is more aggregate information, but once the number of analysts reaches 10, it starts to increase with competition because the distortionary effect offsets the benefit of more aggregate information.

Therefore, in the current market, if the goal is to optimize individual forecast accuracy, the optimal level of competition is 2 analysts. However, if the goal is to optimize consensus forecast accuracy, the optimal level is 10 analysts. Any combinations of the two goals will result in an optimal level that lies between 2 and 10 analysts per security.

Figure 4c and 4d show how the noise measures, individual forecast dispersion and consensus forecast variance, change with the level of competition. Overall, they exhibit very similar patterns as individual forecast error and consensus forecast error respectively. The most notable difference is that the noise measures without optimism (red dashed line) is much higher under the honest payoff (yellow dash-dot line), whereas the accuracy measures are relatively similar between the two cases, again highlighting that the reward for relative accuracy increases the level of noise in the market. In the current market, the optimal level of competition for individual forecast dispersion is 2 analysts, and the optimal level of competition for consensus forecast variance is 10 analysts. Any combinations of the two will result in an optimal level between 2 and 10 analysts. Interestingly, the optimal level of competition for accuracy and noise coincides with each other.

These results challenges two common beliefs about information quality provided by security

[13] Similar to the case without optimism, when the number of analysts approaches infinity, the chance of being the most accurate becomes very small, so any effect of the reward for relative accuracy eventually declines. However, in the case with optimism, this doesn't happen until the level of competition is really high and such high level of competition doesn't happen in my sample.

analysts. First, analysts forecast dispersion is often used as a measure of uncertainty in the market. However, my results suggest that forecast dispersion can also be driven by analysts' strategic incentives and the level of competition should be taken into consideration when using forecast dispersion as an uncertainty measure.

Second, it is commonly believed that more competition is always better for information quality thanks to more aggregate information. However, my results show that when analysts are rewarded for relative accuracy, this is no longer the case. While a small amount of competition disciplines the optimism incentive, too much competition leads to information that is less accurate and more noisy. The quality of the consensus is inverted-U-shaped in the level of competition, optimized with moderate level of competition.

This implies that policies that increase coverage of less popular securities and decrease coverage of popular ones may improve information quality in the overall market. This adds a new explanation to why information quality improved after MiFID II, complementing existing studies. In particular, MiFID II reduces analyst coverage and improve information quality for securities with large market capitalization, which are also on average covered by more analysts (Guo and Mota 2021). This is consistent with my result that reducing analyst coverage for popular securities improves information quality.

8 Conclusion

In this paper, I estimate the incentives of security analysts to study how competition affects the quality of information in the financial market.

Analysts are rewarded for relative accuracy, specifically, for issuing forecasts that are closer to the actual earnings than their rivals. This reward generates competition and leads to two opposing effects. On the one hand, this incentivizes analysts to differentiate their forecasts from their rivals', so they exaggerate their private signals in their forecasts, which may increase forecast errors and variances. I call this the *distortionary effect*. On the other hand, when analysts are also rewarded for optimism, the relative accuracy reward disciplines them to forecast less optimistically, which may decrease forecast errors. I call this the *disciplinary effect*.

I find that the *disciplinary effect* dominates in the current market. The estimated analyst payoff implies a huge reward for being the most accurate in covering one security, as well as a significant reward for optimism. Counterfactual experiments further show that this reward for the most accurate contributes to a 34.01% and 60.84% decrease in analysts' individual and consensus forecast errors. Meanwhile, the *distortionary effect* is still present, so this improvement is at a cost of a 6.59% increase in individual forecast dispersion and a 6.68% increase in consensus forecast variance.

When the number of analysts covering a security increases, the *distortionary effect* becomes more pronounced and the positive effects of the reward for relative accuracy weakens. The optimal level of competition depends on the policy maker's objective and the availability of analysts' forecasts to the investors. More analysts bring more information to the market but also intensifies the *distortionary effect*. In the US market where analysts' forecasts are public information, the policy makers may care more about aggregate information. In this case, the optimal level of competition is medium, about 10 analysts per security, which balances the positive impact of more information and the negative impact of the distortionary effect.

This paper provides two new perspectives on regulating analysts' incentives in the financial industry. First, I provide a new mechanism for existing optimism regulations to take effect —they not only reduce each analyst's independent incentive to issue optimistic forecast, but also prevents them from involving in an "optimism contest", reducing forecast noise. Second, policies that reduce the level of competition between security analysts, i.e., analyst coverage, do not necessarily impair information quality. The marginal effect of competition on information quality depends on the current level of competition. This is consistent with the recent empirical evidence on MiFID II and guides evaluation of future policies.

A limitation of this paper is that it takes analysts' incentives as given and focuses on their behavior in this partial equilibrium. Modeling the general equilibrium will require a much more complex model involving security-issuing firms, investors, brokerage houses, and analysts. Some advances have been made in the theory literature in this direction, investigating the optimal reward mechanism (e.g., Inderst and Ottaviani 2012, Deb et al. 2018). I consider my paper as a step towards joining the theory and empirical literature on this topic to offer policy insights that best suit the current market.

More broadly, experts in many markets face incentive to outperform each other, as well as to bias their information in a systematic direction. For example, media outlets may benefit both from providing more accurate news reports and telling stories that accommodate their readers' political preference. When approached by a home seller, real estate brokers may want to make not only an accurate and achievable evaluation of houses to home sellers but also a high evaluation that will be attractive to the seller. The notion that competition has both a distortionary and a disciplinary effect is widely applicable and more analyses need to be done to understand the nuanced effect of competition in these markets.

Methodologically, I contribute to the identification and estimation of common value models. I structurally estimate a contest model where analysts with asymmetric information simultaneously make forecasts on a firm's earnings. Because analysts' payoff depends on relative accuracy, this model shares a defining characteristic of common value auctions: rivals' signals are relevant to players' assessment of the outcome. An important challenge for this class of models is to identify between signals and payoffs. To tackle the challenge, I use the variation in the number of analysts in a contest and the resulting variation in forecast strategy to identify payoff. Also, being able to observe firms' actual earnings in this market helps with identification as it allows me to compute the trade-offs with different forecast strategies. This identification method is similar to using variation in the number of bidders and ex post valuation to identify common value auctions. With this paper, I extend the current knowledge about common value auctions to a broader class of common value models, an exciting area for future research.

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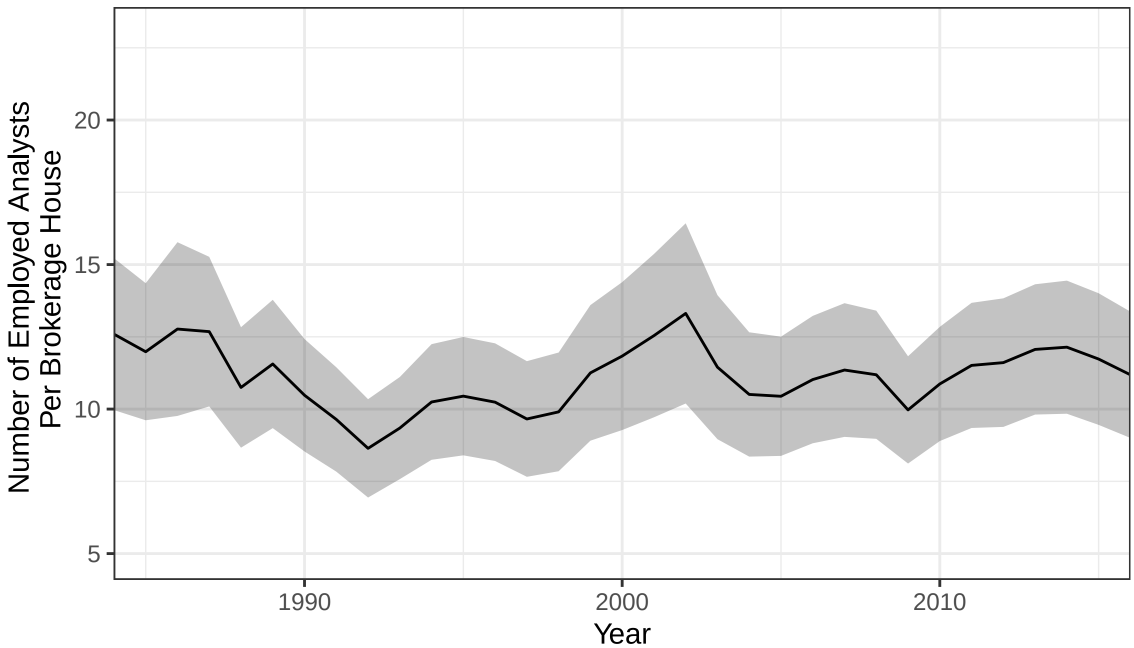
A Supplementary Tables and Figures

Figure A1: Average Number of Employed Analysts Per Brokerage House Overtime



Notes: Figure plots the average number of employed analysts per brokerage house overtime. The solid line represents the mean and the gray ribbon represents the 95% confidence interval.

Table A1: Effect of Competition on Analysts' Forecasts

	Forecast Error		*Forecast*	
	(OLS)	(2SLS)	(OLS)	(2SLS)
$\log(N_{jt})$	0.093	0.096	0.006	0.003
	(0.008)	(0.037)	(0.009)	(0.041)
Z_{jt}			0.810	0.812
			(0.018)	(0.020)
$\log(N_{jt}) \times Z_{jt}$			0.025	0.024
			(0.007)	(0.007)
log(Market Cap)	−0.027	−0.029	0.036	0.036
	(0.007)	(0.009)	(0.008)	(0.011)
log(Book Value)	0.006	0.006	−0.009	−0.009
	(0.007)	(0.008)	(0.007)	(0.007)
Profitablity	−0.001	−0.001	−0.0005	−0.001
	(0.0005)	(0.0005)	(0.0003)	(0.0003)
Volatility	−0.363	−0.334	−0.737	−0.730
	(0.080)	(0.092)	(0.087)	(0.104)
Average Monthly Return	0.032	0.032	−0.003	−0.003
	(0.010)	(0.010)	(0.011)	(0.011)
SP500	0.012	0.012	0.001	0.001
	(0.002)	(0.002)	(0.002)	(0.002)
Experience (Security)	−0.016	−0.016	−0.006	−0.006
	(0.005)	(0.005)	(0.005)	(0.005)
Experience (Industry)	0.013	0.013	0.010	0.011
	(0.005)	(0.005)	(0.005)	(0.005)
Experience (Overall)	−0.006	−0.010	−0.008	−0.011
	(0.003)	(0.008)	(0.003)	(0.009)
High Prestige	0.001	0.020	−0.005	0.008
	(0.002)	(0.032)	(0.002)	(0.034)
log(% High Prestige Analysts)	−0.007	−0.007	−0.003	−0.003
	(0.0002)	(0.0003)	(0.0002)	(0.0003)
Forecast Order	54.308	54.056	19.843	19.783
	(5.497)	(5.534)	(6.283)	(6.320)
EPS Report Delay	0.002	0.002	0.002	0.002
	(0.001)	(0.001)	(0.001)	(0.001)
Observations	209,320	209,320	209,320	209,320

Notes: This table presents the reduced-form results on the effect of competition on analysts' forecasts, with the coefficients on control variables. Column (1)-(2) uses individual analysts' forecast errors as the dependent variable. Column (3)-(4) uses individual analysts' forecasts as the dependent variable. Columns (1) and (3) present the OLS regressions. Columns (2) and (4) present the 2SLS regressions using lag number of analysts covering the security $\log(N_{jt-1})$ as instrument. All columns include year fixed effects and security-analyst pair fixed effects. All standard errors are heteroscedasticity-consistent and clustered at the security-year level.

Table A2: First-stage Results

	$\log(N_{jt})$			
$\log(N_{j-1t})$	0.681	0.492	0.416	0.731
	(0.001)	(0.002)	(0.002)	(0.001)
Year FE	Y	Y	Y	
Security FE	Y			
Security-analyst FE		Y	Y	
Homogenized				Y
Observations	209,320	209,320	209,320	208,587

Notes: Table presents the first stage results when the log number of analysts is instrumented with its lagged value. Column (1)-(3) shows the results when we include controls and fixed effects corresponding to columns (4)-(6) in Table 6. Column (4) shows the result for the homogenized variables.

B Details on Standardizing Analysts' Forecast Problems

For a given security, denote the Markov process of its actual earnings as $Z_t = F(Z_{t-1}) + \xi_t$, where $\xi_t \sim N(0, \sigma^2)$. Assume analysts who cover a security know the Markov process of the actual earnings its $F(\cdot)$. Then every period, the game is equivalent to forecasting of ξ_t/σ. We can normalize forecast X_{it} and actual earnings Z_t using the estimated time series $\hat{F}(\cdot)$ as the following,

$$\begin{aligned}\tilde{Z}_t =& (Z_t - \hat{F}(Z_{t-1}))/\hat{\sigma} \\ \tilde{X}_{it} =& (X_{it} - \hat{F}(Z_{t-1}))/\hat{\sigma}.\end{aligned}$$

In practice, I first run a unit root test on the actual earnings of each security (MZ test with GLS-detrended data). Then, if a unit root process is not rejected, a unit root process will be fitted for $\hat{F}(\cdot)$. Otherwise, an AR(1) with time trend will be fitted for $\hat{F}(\cdot)$. Finally, for each security-year, I standardize the actual earnings and forecasts by subtracting the corresponding fitted value of the actual earnings from them, and divide them by the estimated standard deviation of the time series shock. I rename the standardized forecasts and actual earnings to X_{it} and Z_t.

C Proofs

C.1 Proof of Proposition 1

I follow Proposition 3 of Ottaviani and Sørensen (2005) and incorporate absolute accuracy and optimism in the payoff function.

Proof. Consider an arbitrary N. I apply results from Milgrom and Weber (1985) (henceforth MW) and show that conditions (i)-(vi) of their Theorem 4 are satisfied. $\mathcal{Z}$ in my model corresponds to both the metric space of the environmental variable T_0 and the action space A_i in MW. Assumption 1 ensures the players' types are independent conditional on the environmental variable, so (i) is satisfied. Analysts' signals are continuously distributed, so (ii) is satisfied. An analyst's payoff function depends only on the actual earnings Z and the vector of forecasts X so (iii) is satisfied. Finally, because $\mathcal{Z}$ is finite, analysts' payoffs are equicontinuous by Proposition 1(a) of MW and the metric space of the environmental variable is also finite, i.e., (iv), (v), and (vi) are satisfied. By Theorem 4 of MW, there exists an equilibrium in pure strategies. ■

C.2 Extention of Proposition 1 to Allow Common Signals

Now suppose analysts signals are correlated through a common signal $C \in \mathcal{C} \subset \mathbb{R}$ where $\mathcal{C}$ is finite. Specifically, let $S_i = (C, \tilde{S}_i)$ where the common signal C is observed by all analysts in the contest and $\tilde{S}_i$ is analyst i's private signal. In this case, Assumption 1 is no longer satisfied. To guarantee that an equilibrium exists, it is sufficient to assume that analysts' private signals are uncorrelated conditional on the actual earnings and the common signal:

Assumption 3. $\tilde{S}_i \perp\!\!\!\perp \tilde{S}_j | C, Z$ for all $j \neq i$.

Corollary 1. If Assumption 3 holds, analysts' private signals $\tilde{S}_i$ are continuously distributed, and $\mathcal{C}$ and $\mathcal{Z}$ are finite, there exists a pure strategy equilibrium.

Proof. Now (C, Z) corresponds to the environmental variable T_0 in MW and $\mathcal{C} \times \mathcal{Z}$ corresponds to its metric space. The rest of the argument follows the proof for Proposition 1. ■

Intuitively, given Assumption 3, the model with a common signal is equivalent to a two-stage game. In the first stage, analysts receive a common signal and update their common prior for the actual earnings. In the second stage, analysts receive private signals and choose forecasts. The second stage is the same as the model described in 5. The common signal only alters analysts common prior for the actual earnings.

C.3 Proof of Proposition 2

Assume without loss of generality that $Z \sim N(0,1)$, $S_i|Z \sim N(Z, 1/\tau)$, and that analyst i has received a positive signal $S_i > 0$. In a winner-takes-all game, the reward for issuing a forecast with the lowest forecast error is a positive constant, so the first order condition at the posterior simplifies to,

$$\int_{\frac{\tau}{1+\tau}S_i}^{\infty} \Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)dZ - \int_{-\infty}^{\frac{\tau}{1+\tau}S_i} \Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)dZ = 0$$

where $\Xi(\frac{\tau}{1+\tau}S_i|Z) = \frac{1+\tau}{\tau}[g(S_i|Z) + g(\frac{1+\tau}{\tau}(2Z - \frac{\tau}{1+\tau}S_i)|Z)]$.

$\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)$ can be simplified using normality

$$
\begin{aligned}
\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i) =& \frac{1+\tau}{\tau}[g(S_i|Z) + g(\frac{1+\tau}{\tau}(2Z - \frac{\tau}{1+\tau}S_i)|Z)]f(Z|S_i) \\
=& \frac{1+\tau}{\tau}\sqrt{\tau(1+\tau)}[\phi(\frac{Z-S_i}{\frac{1}{\sqrt{\tau}}}) + \phi(\frac{\frac{2+\tau}{\tau}Z - S_i}{\frac{1}{\sqrt{\tau}}})]\phi(\frac{Z - \frac{\tau}{1+\tau}S_i}{\frac{1}{\sqrt{1+\tau}}}) \\
=& \frac{(1+\tau)^{\frac{3}{2}}}{2\tau^{\frac{1}{2}}\pi}\Big[\exp\big(-\frac{1}{2}[\tau(Z-S_i)^2 + (1+\tau)(Z - \frac{\tau}{1+\tau}S_i)^2]\big) + \\
& \exp\big(-\frac{1}{2}[\tau(\frac{2+\tau}{\tau}Z - S_i)^2 + (1+\tau)(Z - \frac{\tau}{1+\tau}S_i)^2]\big)\Big]
\end{aligned}
$$

Let $t = Z - \frac{\tau}{1+\tau}S_i$.

$$
\begin{aligned}
\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i) =& \frac{(1+\tau)^{\frac{3}{2}}}{2\tau^{\frac{1}{2}}\pi}\Big[\exp\big(-\frac{1}{2}[\tau(t - \frac{1}{1+\tau}S_i)^2 + (1+\tau)t^2]\big) + \\
& \exp\big(-\frac{1}{2}[\tau(\frac{2+\tau}{\tau}t + \frac{1}{1+\tau}S_i)^2 + (1+\tau)t^2]\big)\Big]
\end{aligned}
$$

Completing the squares within the exponential function, we can obtain

$$
\begin{aligned}
\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i) = \frac{(1+\tau)^{\frac{3}{2}}}{\sqrt{2\pi\tau}}\Big[&\frac{1}{\sqrt{1+2\tau}}\exp\big(-\frac{1}{2}\frac{\tau}{(1+\tau)(1+2\tau)}S_i^2\big)\cdot\sqrt{1+2\tau}\,\phi(\frac{t - \frac{\tau}{(1+\tau)(1+2\tau)}S_i}{\frac{1}{\sqrt{1+2\tau}}}) + \\
&\frac{\sqrt{\tau}}{\sqrt{4+5\tau+2\tau^2}}\exp\big(-\frac{1}{2}\frac{\tau^2}{(1+\tau)(4+5\tau+2\tau^2)}S_i^2\big)\cdot\frac{\sqrt{4+5\tau+2\tau^2}}{\sqrt{\tau}}\phi(\frac{t + \frac{(2+\tau)\tau}{(1+\tau)(4+5\tau+2\tau^2)}S_i}{\sqrt{\frac{\tau}{4+5\tau+2\tau^2}}})\Big]
\end{aligned}
$$

Then, the first order condition can be simplified to

$$
\begin{aligned}
&\frac{(1+\tau)^{\frac{3}{2}}}{\sqrt{2\pi\tau}}\Big\{\frac{1}{\sqrt{1+2\tau}}\exp\big(-\frac{1}{2}\frac{\tau}{(1+\tau)(1+2\tau)}S_i^2\big)\cdot(2\Phi(\frac{\tau}{(1+\tau)\sqrt{1+2\tau}}S_i) - 1) - \\
&\frac{\sqrt{\tau}}{\sqrt{4+5\tau+2\tau^2}}\exp\big(-\frac{1}{2}\frac{\tau^2}{(1+\tau)(4+5\tau+2\tau^2)}S_i^2\big)\cdot(2\Phi(\frac{(2+\tau)\sqrt{\tau}}{(1+\tau)\sqrt{4+5\tau+2\tau^2}}S_i) - 1)\Big\}.
\end{aligned}
$$

Note that the first order condition is equal to 0 when $S_i = 0$ and increasing in S_i. Therefore, there exists $\bar{s}$ such that for $0 < S_i < \bar{s}$, the first order condition is positive and a deviation towards the signal, or exaggeration, increases expected utility for two players.

C.4 Sufficient Condition for Identification

The section provides a sufficient condition for identification when the reward for any rank of relative accuracy is homogeneous across different N's. Specifically, consider $\mathcal{N} = \{2, 3, ..., N_{max}\}$, where

$N_{max} \geq 4$. I impose the following constraints on the payoff parameters,

$$\gamma_k(N) = \begin{cases} \gamma_1 & \textit{if } k = 1, \\ \gamma_k & \textit{if } 2 \leq k \leq N-2 \textit{ and } N \geq 3, \\ 0 & \textit{if } k = N-1 \textit{ and } N \geq 3, \end{cases}$$

where $\gamma_k \in \mathbb{R}$ for all k. Integrate equation (5) over $\mathbb{R}$ and denote the coefficient on $\gamma_k(N)$ with $a_{N,k}$, that is,

$$a_{N,k} = \binom{N-1}{N-k} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Xi(X, Z, N, k) \tilde{f}(Z|X, N) dZdX.$$

Proposition 3. The payoff function is globally identified if $a_{2,1} \neq a_{3,1}$ and $a_{N,N-2} \neq 0$ for all $N \geq 4$.

Proof. Integrating equation 5 over X, I can rewrite the first order conditions as

$$A\gamma = 2[\mathbb{E}(X|N) - \mathbb{E}(Z|N)],$$

where

$$A = \begin{pmatrix} 1 & a_{2,1} & 0 & \dots & 0 \\ 1 & a_{3,1} & 0 & \dots & 0 \\ 1 & a_{4,1} & a_{4,2} & \dots & 0 \\ & & & \dots & \\ 1 & a_{N_{max}-1,1} & a_{N_{max}-1,2} & \dots & 0 \\ 1 & a_{N_{max},1} & a_{N_{max},2} & \dots & a_{N_{max},N_{max}-2} \end{pmatrix}, \gamma = \begin{pmatrix} \gamma_o \\ \gamma_1 \\ \gamma_2 \\ \dots \\ \gamma_{N_{max}-2}. \end{pmatrix}$$

To show that the payoff function is globally identified, it suffices to show that there is a unique solution to γ, that is, A is invertible. Note that A is a square matrix and if $a_{2,1} = 0$, it would be an upper triangular matrix. Therefore, we can compute its determinant easily as

$$\det(A) = (a_{3,1} - a_{2,1}) \prod_{N=4}^{N_{max}} a_{N,N-2}.$$

Because $a_{2,1} \neq a_{3,1}$ and $a_{N,N-2} \neq 0$ for all $N \geq 4$, this is not equal to zero, so there is a unique solution to γ and the payoff function is globally identified. ■

The sufficient condition for identification in this proposition means that the marginal probability of being the most accurate is different between contests with two analysts and three analysts, and that the marginal probability of receiving rank $N-2$ is nonzero for $N \geq 4$.

D Computation Details

D.1 Removal of Inverse Function in the First Order Condition

We can write the first order condition to be free of inverses with a change of variable, similar to the transformation of equation (5.4) to equation (5.5) in Ottaviani and Sørensen (2005). Let $Y = \beta^{*-1}(2Z - \beta^*(S_i))$. Then

$$\begin{aligned} Z &= \frac{\beta^*(Y) + \beta^*(S_i)}{2} \\ dZ &= \frac{1}{2}\beta^{*'}(Y)dY. \end{aligned}$$

Also, note that since β^* is the equilibrium strategy, $X_i = \beta^*(S_i)$. Then we can write

$$\begin{aligned} p(X_i, Z) =& \tilde{p}(S_i, Y) \\ =& 1 - |\int_{S_i}^{Y} g\left(S_j | \frac{\beta^*(Y) + \beta^*(S_i)}{2}\right) dS_j| \\ \Xi(X_i, Z, N, k) =& \tilde{\Xi}(S_i, Y, N, k) \\ =& [1 - 2 \cdot \mathbf{1}(S_i > Y)][\frac{g\left(S_i | \frac{\beta^*(Y)+\beta^*(S_i)}{2}\right)}{\beta^{*'}(S_i)} + \frac{g\left(Y | \frac{\beta^*(Y)+\beta^*(S_i)}{2}\right)}{\beta^{*'}(Y)}] \\ & \big[\mathbf{1}(N > k)(N-k)\tilde{p}(S_i, Y)^{N-k-1}(1 - \tilde{p}(S_i, Y))^{k-1} - \\ & \mathbf{1}(k > 1)\tilde{p}(S_i, Y)^{N-k}(k-1)(1 - \tilde{p}(S_i, Y))^{k-2}\big]. \end{aligned}$$

The first order condition can be written as

$$\begin{aligned} & \sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} \int_{-\infty}^{\infty} \tilde{\Xi}(S_i, Y, N, k) f(\frac{\beta^*(Y) + \beta^*(S_i)}{2} | S_i) \frac{1}{2} \beta^{*'}(Y) dY \\ & - 2[\beta^*(S_i) - \mathbb{E}(Z|S_i)] + \gamma_o = 0, \end{aligned} \tag{12}$$

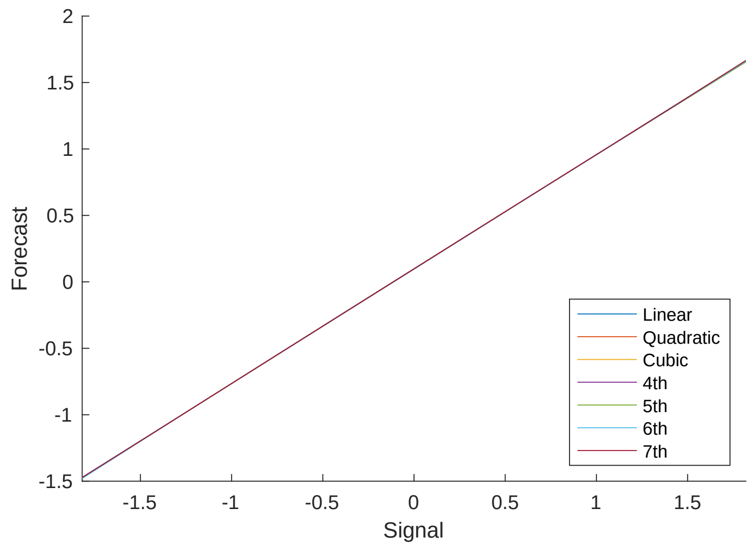
which is free of the inverse function of the equilibrium strategy.

D.2 Projection Method

In this section, I provide the details of the equilibrium computation and the experimentation with different polynomial orders as the pre-specified function family in the projection method. I show that a linear strategy is a great approximation to the equilibrium strategy.

I choose the representative signals (collocation points) to be 20 finite and evenly distributed quantiles between 0 and 1. Then, under the parametric assumption that $S_i = Z + \epsilon_i$ where

Figure D2: Equilibrium Strategy Approximated With Polynomials up to the 7th Order



$Z \sim N(0,1)$ and $\epsilon_i \sim N(0, 1/\tau)$, these 20 collocation points are

$$\left\{ \sqrt{1+\frac{1}{\tau}}\Phi^{-1}(\frac{1}{21}), \sqrt{1+\frac{1}{\tau}}\Phi^{-1}(\frac{2}{21}), \cdots, \sqrt{1+\frac{1}{\tau}}\Phi^{-1}(\frac{20}{21}) \right\}.$$

The equilibria are computed following the the algorithm described in Section 5.3. I experiment with polynomials up to the 7th order as the pre-specified function at various model parameters and compare the performance of these different polynomial orders using the residual function, i.e., the sum of squared first order conditions.

Table D3: Equilibrium Approximation Performance of Polynomials up to the 7th Order

Polynomial Order	Residual Function	Computation Time (s)
1	2.21e-02	1.27
2	4.07e-04	1.36
3	2.65e-04	2.35
4	2.68e-03	4.92
5	9.97e-10	8.64
6	6.04e-03	11.61
7	8.31e-04	19.65

Figure D2 plots the approximated equilibria with 10 players using polynomials up to the 7th order at the estimated parameters in Table 5. Table D3 presents their corresponding residual functions and computation time with Matlab on a macOS machine with 16GB RAM and Apple M1. Higher polynomial orders generally achieve smaller residual functions, but the estimated polynomials are very close to the linear function and cost much more computation time. This is observed consistently at different model parameters. Therefore, I use a linear function to approximate the equilibrium in both estimation and counterfactual analysis.

D.3 Interpolation of Equilibrium Strategy

I interpolate the equilibrium forecast strategy to alleviate the computation burden. For a given set of parameter values, I first use the projection method to compute the equilibria at all 5th percentiles of the number of analysts N. For a given number of analysts N, denote the parameters in the equilibrium strategy by θ_N^*. Then, for each element in θ_N^*, I use a cubic spline to interpolate across N to find the equilibrium strategy with any other number of analysts.

D.4 Data Normalization

I assume that the unobserved heterogeneity in analysts' forecasts X_{ijt}, the actual earnings Z_{jt}, and the number of analysts N_{jt} only lies in the security-year level. Specifically,

$$\begin{aligned} X_{ijt} =& X_{ijt}^h + \delta_j^X + \delta_t^X \\ N_{jt} =& N_{jt}^h + \delta_j^N + \delta_t^N \\ Z_{jt} =& Z_{jt}^h + \delta_j^Z + \delta_t^Z, \end{aligned}$$

where δ_j's and δ_t's are unobserved heterogeneity at the security-level and year-level respectively. The h superscript denotes the corresponding homogenized variables, which I compute by regressing X_{ijt}, N_{jt} and Z_{jt} on security fixed effects and year fixed effects and taking the residuals.

As analysts demonstrate persistence in the securities they cover, I model the homogenized number of analysts with a Markov process such that

$$N_{jt}^h = \rho_N(N_{jt-1}^h, \delta_{jt}^N),$$

where δ_{jt}^N is the shock on the number of analysts covering security j in year t. The homogenized forecast X_{ijt}^h is a function of the analyst's signal as implied by the model. In addition, I allow it be subject to time-varying unobserved heterogeneity δ_{jt}^X such that

$$X_{ijt}^h = \rho_X(S_{ijt}, \delta_{jt}^X).$$

If δ_{jt}^N and δ_{jt}^X are correlated, the homogenized number of analysts will be endogenous in the auxiliary regressions in equation (9) to (11). Therefore, I adopt the lag number of analysts N^h_{jt-1} as the instrument for N^h_{jt}, with the assumption that the lag number of analysts is uncorrelated with the shocks this period δ^N_{jt} and δ^X_{jt}.

After the normalization, I keep the security-years covered by more than two normalized analysts, which accounts for 99.7% of the sample, so that the game remains well-defined. For simplicity and with a slight abuse of notation, I drop the h superscript and use $(X_{ijt}, Z_{jt}, N_{jt})$ to denote the normalized variables from Section 6.2.1 onward in the paper.

Table A2 presents the first-stage results for the lag instrument. Column (4) shows the result for the normalized variable and Column (1)-(3) shows the results when we include controls and fixed effects corresponding to columns (4)-(6) in Table 6. Across all specifications, the coefficients on the log lag number of analysts are statistically and economically significant, which confirms the persistence in the level of competition and the strength of the instrument.

E Alternative Specification: Flexible Reward Convexity

In this section, I present and estimate an alternative specification where I allow flexible reward convexity in relative accuracy. Specifically,

$$\gamma_k(N) \equiv v(k, N) = C_1(1 - \frac{k-1}{N-1})^{\frac{1}{C_2}},$$

where $C_1 \geq 0$ and $C_2 > 0$. Note that $\gamma_1(N) = C_1$, so C_1 is the payoff for the most accurate. The reward for other k's are scaled with $(1 - \frac{k-1}{N-1})^{\frac{1}{C_2}}$, which is decreasing in k.

Proposition 4. Consider any positive integers k and l such that $k < l$. (a) If $C_2 \in (0, 1)$, $\gamma_k(N) - \gamma_{k+1}(N) > \gamma_l(N) - \gamma_{l+1}(N)$; if $C_2 \in (1, \infty)$, $\gamma_k(N) - \gamma_{k+1}(N) < \gamma_l(N) - \gamma_{l+1}(N)$; if $C_2 = 1$, $\gamma_k(N) - \gamma_{k+1}(N) = \gamma_l(N) - \gamma_{l+1}(N)$. (b) $\gamma_k(N) - \gamma_{k+1}(N) - (\gamma_l(N) - \gamma_{l+1}(N))$ is decreasing in C_2.

Proof. (a) $v(k, N)$ is continuous and differentiable with respect to k on $\mathbb{R}$, so by mean value theorem, there exist $k^* \in (k, k+1)$ and $l^* \in (l, l+1)$ such that

$$v(k, N) - v(k+1, N) = -v'(k^*, N),$$
$$v(l, N) - v(l+1, N) = -v'(l^*, N).$$

Because k and l are integers and $k < l$, $k^* < k+1 \leq l < l^*$.

Differentiating $v(k, N)$ twice with respect to k, we have

$$\frac{\partial^2 v(k, N)}{\partial k^2} = \frac{C_1}{C_2(N-1)^2}(\frac{1}{C_2} - 1)(1 - \frac{k-1}{N-1})^{\frac{1}{C_2}-2}.$$

Note that when $C_2 \in (0,1)$, $\frac{\partial^2 v(k,N)}{\partial k^2} > 0$, so v' is increasing. As $l^* > k^*$,

$$\begin{aligned}
& v'(l^*,N) > v'(k^*,N) \\
\Leftrightarrow & -v'(k^*,N) > -v'(l^*,N) \\
\Leftrightarrow & v(k,N) - v(k+1,N) > v(l,N) - v(l+1,N) \\
\Leftrightarrow & \gamma_k(N) - \gamma_{k+1}(N) > \gamma_l(N) - \gamma_{l+1}(N).
\end{aligned}$$

Similarly, when $C_2 \in (1,\infty)$, $\frac{\partial^2 v(k,N)}{\partial k^2} < 0$, so v' is decreasing and $\gamma_k(N) - \gamma_{k+1}(N) < \gamma_l(N) - \gamma_{l+1}(N)$; when $C_2 = 1$, $\frac{\partial^2 v(k,N)}{\partial k^2} = 0$, so v' is constant and $\gamma_k(N) - \gamma_{k+1}(N) = \gamma_l(N) - \gamma_{l+1}(N)$.

(b) Differentiating $v(k,N) - v(k+1,N) - (v(l,N) - v(l+1,N))$ with respect to C_2,

$$\begin{aligned}
& \frac{\partial v(k,N) - v(k+1,N) - (v(l,N) - v(l+1,N))}{\partial C_2} \\
= & \log C_2 \cdot \frac{C_1}{C_2^2} \left((1 - \frac{k-1}{N-1})^{\frac{1}{C_2}} - (1 - \frac{k}{N-1})^{\frac{1}{C_2}} - (1 - \frac{l-1}{N-1})^{\frac{1}{C_2}} + (1 - \frac{l}{N-1})^{\frac{1}{C_2}} \right) \\
= & \log C_2 \cdot \frac{1}{C_2^2} \left(v(k,N) - v(k+1,N) - v(l,N) + v(l+1,N) \right),
\end{aligned}$$

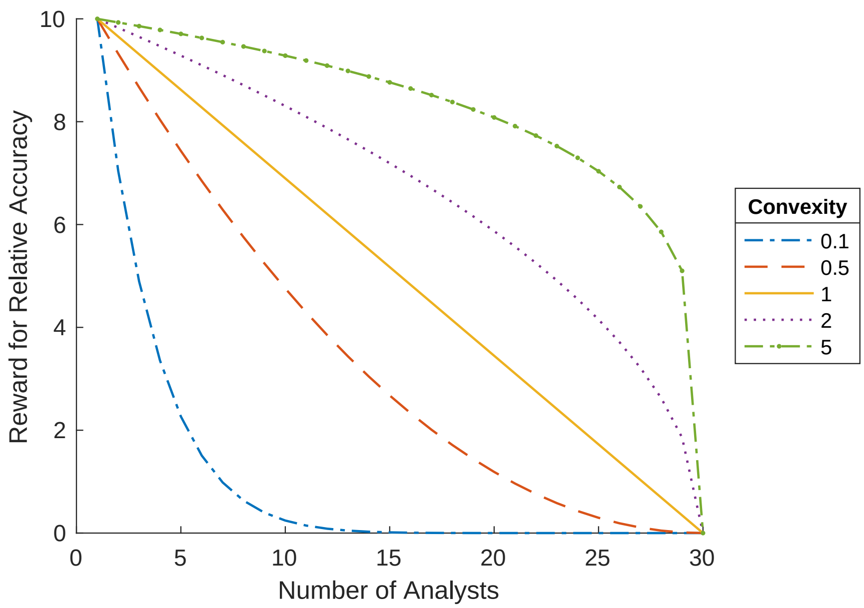
which I denote as Δ. When $C_2 \in (0,1)$, $v(k,N) - v(k+1,N) - v(l,N) + v(l+1,N) > 0$, as shown in (a), and $\log C_2 < 0$, so $\Delta < 0$. Similarly, when $C_2 \in (1,\infty)$, $v(k,N) - v(k+1,N) - v(l,N) + v(l+1,N) < 0$, as shown in (a), and $\log C_2 > 0$, so $\Delta < 0$. When $C_2 = 1$, $\Delta = 0$. Hence, $\Delta \leq 0$ for all $C_2 > 0$, which implies that $\gamma_k(N) - \gamma_{k+1}(N) - (\gamma_l(N) - \gamma_{l+1}(N))$ is decreasing in C_2.

■

Proposition 4 shows that C_2 determines the convexity of the payoff function, so I call C_2 the convexity parameter. With the payoffs fixed for the most and the least accurate, the smaller the C_2, the more convex is the relative accuracy payoff. To illustrate, Figure E3 plots the relative accuracy payoffs at different values of C_2 when there are 30 players. As $C_2 \to 0$, this payoff approaches the winner-takes-all specification. As $C_2 \to \infty$, this approaches the loser-loses-all specification.

Table E4 presents the payoff estimates with this payoff specification. I find that the convexity parameter is close to 0, suggesting that analysts are indeed playing a winner takes all game. The other parameters are similar to the results of the main specification in Table 5.

Figure E3: Relative Accuracy Payoffs with Different Convexity Parameters When $N = 30$



Notes: Figure plots the relative accuracy payoffs with different convexity parameters when there are 30 analysts in the market. The reward for the most accurate is set at 10.

Table E4: Estimates of Analysts' Payoff

	Estimate	Standard Error
The Most Accurate	10.823	(3.999)
Optimism	1.134	(0.217)
Signal Precision	5.178	(0.164)
Convexity	0.020	(0.005)

Notes: Table presents estimates of analysts' payoff function, allowing for a non-negative parameter to capture convexity. When the Convexity parameter is small, the analysts' payoff is close to a winner-takes-all game. The larger the parameter is, the closer the payoff is to a loser-loses-all game. When Convexity is exactly 1, the payoff is linear in analysts' forecast error rank k as defined in Section 5.