Does Competition Improve Information Quality: Evidence From the Security Analyst Market

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Abstract

This paper studies the effect of competition on the quality of information provided by experts. I estimate the incentives and the information structure of security analysts who compete to make earnings forecasts. Security analysts are rewarded for being more accurate than their peers, which creates competition. This reward for relative accuracy leads analysts to distort their forecasts to differentiate themselves, but it also disciplines them to be less influenced by the prevailing optimism incentive. I structurally estimate a contest model with incomplete information that captures both effects, adapting the estimation of common value auctions to this setting. My model disentangles the payoff for relative accuracy from the payoffs for optimism and absolute accuracy. Using the model, I conduct counterfactuals to evaluate policies that reduce the importance of relative accuracy in analysts' payoff. I simulate the effect of these policies on the quality of information in terms of forecast error and variance across analysts. I find that the disciplinary effect of competition dominates in the current market, reducing forecast error by 33.37%, at a cost of a 3.74% increase in forecast variance. However, once the optimism incentive is removed, competition increases both forecast error and forecast variance.

1 Introduction

Efficiency of many markets depends on information provided by experts. It is natural for consumers to compare experts and reward the most accurate ones, which generates competition. However,

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this focus on *relative accuracy* creates an incentive for distortion, leading experts to deviate from reporting their honest beliefs of an outcome. For example, a financial analyst that correctly predicts a stock boom or bust when no one else does might gain reputation overnight, but such reward could also drive analysts to issue exaggerated forecasts to differentiate themselves. As a result, the quality of information may be compromised.

Does this mean that competition impairs information quality? Not necessarily. When experts face incentives for other systematic biases, competition can exacerbate the bias, but it can also discipline experts as they want to be relatively more accurate than their peers. For example, in the financial markets, many observers believe that analysts face an incentive to be optimistic, so analysts themselves also expect their rivals to predict more booms. In that case, they may increase the chance of being the most accurate by predicting even more booms, or perhaps better, by predicting *fewer* booms to differentiate. This latter behavior could improve information quality in a market with such systematic biases.

This paper studies the overall effect of competition on information quality in the financial industry, where competition manifests through the reward for relative accuracy. I focus on security analysts and empirically estimate the importance of relative accuracy and other incentives in their payoff.

Security analysts conduct research on firms. They use industry expertise and connections to firms' management teams to obtain information, and then compete to make third-party forecasts on firms' earnings. They are the gateway between firms seeking capital and investors choosing between investment opportunities. If security analysts face incentives to distort their forecasts, investors may receive poor information that can lead to bad investment decisions. Therefore, regulators around the world have adopted many measures to ensure that analysts face incentives to provide quality information, such as restricting sources of their compensation and mandating disclosure of conflict of interest.[1]

There is substantial evidence on the importance of relative accuracy in security analysts' payoff. Several papers find that analysts who are more accurate than their rivals face better career outcomes (Hong et al. (2000), Hong and Kubik (2003), Clarke and Subramanian (2006), Cen et al. (2017)). Anecdotally, it is also observed that analysts compete in various "best analyst" contests, often organized by financial news outlets, which rank analysts using forecast accuracy as one of the criteria. Winners of these contests often receive a boost in compensation (Bradshaw (2011), Groysberg et al. (2011)).

[1]For example, the Securities and Exchange Commission (SEC) prohibited tying analysts' compensation with specific investment banking transactions in its 2002 regulation in an attempt to curb optimism. The SEC also mandated disclosure of any conflict of interest related to investment banking. In terms of whether analysts' compensation could from commissions, regulators in the US and Europe have taken different stances in recent year. Section 28(e) of the US Securities Exchange Act of 1934 and its subsequent interpretive releases allow paying for analyst research with commissions in the US, whereas recently in Europe, the Markets in Financial Instruments Directive II (MiFiD II) announced in 2018 restricts it.

At the same time, analysts face incentives for a notable systematic bias: optimism. Security analysts are usually employed by brokerage houses, which generate revenue through trade execution and provision of investment banking services. Part of analysts' compensation comes from their contribution to these businesses. Optimistic forecasts could help their brokerage houses generate more trade and secure more underwriting deals, so analysts might have an incentive to be optimistic. Empirically, optimistic analysts are also more likely to receive better career outcomes (Hong and Kubik (2003), Hong and Kacperczyk (2010)).

Hence, competition could have two effects on information quality here: given that analysts are rewarded for relative accuracy, competition may have a *distortionary effect* as analysts try to differentiate their forecasts, but it may also have a *discplinary effect* on the optimism bias. The question is: which effect dominates? Does competition exacerbate or mitigate distortions? Moreover, from the perspective of investors, how would the quality of information change, in terms of forecast error, bias and variance, if the reward to top analysts were reduced?

To answer these questions, I specify and estimate a structural model that captures analysts' incentives and information structure. In the model, analysts compete in a contest to forecast the earnings of a security in a given year. First, they observe the number of analysts in the contest and receive private signals on the true earnings of the security (the truth). Then, they choose forecasts simultaneously. In the end, the truth is revealed and analysts receive payoffs based on all forecasts and the truth.

Critically, my model allows analysts' payoff to depend on relative accuracy and optimism. The payoff for relative accuracy is modelled in a flexible way, while optimism is modelled as a reward for higher forecasts, unaffected by competition.

In addition, I distinguish between relative accuracy and absolute accuracy by incorporating the latter in the payoff function as well. The payoff for absolute accuracy pins down analysts' forecasts in the absence of competition and moderates the distortionary effect of the payoff to relative accuracy as competition increases.

Building on a long reduced-form literature, this paper is the first structural study on analysts' choice of forecasts. A major challenge to this literature is that incentives must be identified from a specific payoff channel such as career outcome or compensation, but comprehensive data on these channels are limited.[2] This paper takes a different approach and estimates analysts' incentives with revealed preference, agnostic of the payoff channel. Therefore, it provides a more complete understanding of the incentives.

To estimate the model, I use indirect inference with detailed forecast data obtained from Institutional Brokers' Estimate System (IBES). IBES contains the history of analysts' earnings forecasts on publicly listed US companies as well as their corresponding true earnings since 1982. The estimation proceeds as follows. For any given parameter value, I solve the model numerically using

[2] In particular, compensation data is highly confidential in this industry.

analysts' first order conditions and simulate equilibrium forecasts for every single contest in the data many times. Then, I construct a set of moments from the data and from the simulated forecasts. In order to capture the effect of competition on analysts' forecast strategy, I estimate the coefficients of a set of auxiliary regressions as moments, regressing functions of forecasts on observables such as the number of analysts and the true earnings. Finally, I search over the parameter space to find values that minimize the difference between data moments and simulated moments.

This model shares many similarities with common value auctions, which also creates an identification challenge. In both models, rivals have information relevant to a player's assessment of her own expected payoff. As a result, it is impossible to jointly identify the distribution of signals and payoff based on forecast data with a fixed number of players (Athey and Haile (2007)).[3] Specifically in my model, when a fixed number of analysts make a distribution of forecast errors, it is impossible to distinguish whether they are rewarded only for absolute accuracy and the forecast errors reflect their signal precision, or whether they are rewarded for relative accuracy.

To identify the model, I rely on additional structures of the security analyst market, which are comparable to those used in the auction literature. First, being able to observe the true earnings, that is, the outcome that analysts are trying to predict, in the data is crucial to identification. This is similar to observing ex post values in mineral rights auctions, which helps identify the joint distribution of signals and valuations. In my model, observing the true earnings allows me to construct analysts' forecast errors. With those, for a given number of analysts, I can recover analysts' posterior distribution of receiving each rank of relative accuracy, which enters analysts' first order conditions.

Second, I use variation in forecast strategy with the number of analysts to identify the payoff, particularly the reward for relative accuracy. For intuition, consider a winner-takes-all contest. In this contest, analysts distort their forecast to differentiate themselves so as to increase the chance of being the most accurate. However, the more rivals they face, the harder it will be to be the most accurate, so analysts will distort more. Consequently, analysts' forecast errors will be larger with more analysts. So in estimation, I construct moments that reflect such variations to identify the payoff.

This identificaton strategy is similar to using variation in the number of bidders to test between private value and common value in auctions. In auctions, notice that the winner's curse is only present in common value auctions and that it makes bidders shade their bids (downward from expected valuations conditional on privates signals). The more rivals they face, the more severe the winner's curse is, and the more they shade. Therefore, this variation in shading with the number of bidders can be used similarly.

The estimates of the model show that analysts receive significant reward for being the most ac-

[3] Payoff and forecasts in my model map into valuation and bids in auctions. In auctions, signals are often normalized to be equal to private values, so the difficulty can also be thought of as distinguishing between private values and common values.

curate. In addition, they face an incentive for optimism, which manifests as a reward for optimistic forecast errors.

Using the estimated model, I conduct two counterfactuals to evaluate the effect of relative accuracy and competition on information quality. To quantify the *distortionary effect*, I first mute the *disciplinary effect* by removing the optimism incentive. Then, I simulate forecasts with and without the reward for relative accuracy and compute the changes in forecast error, bias, and variance. To quantify the *disciplinary effect*, I maintain the estimated optimism incentive and repeat the forecast simulation with and without the reward for relative accuracy. I perform both counterfactuals for every contest observed in the data to evaluate the overall effect on the market, as well as for abstract markets with different number of analysts, so as to assess the effect of competition.

The key findings are threefold. First, the *disciplinary effect* dominates in the current market. The reward for relative accuracy reduces forecast error by 33.37% overall. However, it is at the cost of noisier information to investors as forecast variance increases by 3.74%. Second, without the optimism incentive, the reward for relative accuracy results in a small increase in forecast error and forecast variance, reflecting a mild *distortionary effect*. Lastly, the effect of competition is non-linear. Overall, allowing analysts to compete for relative accuracy improves information quality by disciplining optimism, however, the improvement is the greatest with two analysts. When more analysts enter to cover a security, the *distortionary effect* becomes more pronounced, impairing the information quality.

Related literature This paper contributes to understanding security analysts' incentives in making forecasts. Prior empirical studies have been reduced-form. One limitation is that researchers are restricted to identify analysts' incentives from one single channel, such as career or compensation, and some channels such as reputation may be unobservable, so it is unclear what the overall incentives are. Instead, this paper takes a revealed preference approach. My structural model studies analysts' incentives agnostic about the channels, hence offering a more complete understanding. The model builds on the rich reduced-form studies, many of which point to relative accuracy and optimism as main components of the incentive. Analysts with higher relative accuracy and optimism are found to have better career outcomes (Hong et al. (2000), Hong and Kubik (2003), Cen et al. (2017)). In addition, their compensation is influenced by their performance in "best-analyst" contests and internal votings, which is consistent with the emphasis on relative performance in this paper. The compensation also heavily depends on their investment banking and trading volume contributions, which supports the optimism incentive (Bradshaw (2011), Groysberg et al. (2011), Maber et al. (2014)).

This paper also adds to the literature on the effect of competition on information provision (e.g., media competition in Gentzkow and Shapiro (2008) and Cagé (2020); credit rating agency competition in Becker and Milbourn (2011)). Several papers have studied this topic in the context of security analysts. The closest paper to mine is Camara (2015), which estimates a structural model of forecast timing. In her model, analysts choose when to release forecasts and are truth-telling at all times. She finds that for quarterly earnings, competition increases forecast errors as analysts preempt rivals by releasing forecasts early.

My paper differs from hers in two ways. First, I focus on annual earnings forecasts and assume that their values are chosen strategically but at the same time. In reality, analysts issue forecasts asynchronously but are allowed to revise at any time before the forecasting period ends. Revision happens much more frequently with annual forecasts than with quarterly forecasts. So it is sensible to think that analysts are competiting in values for annual forecasts but in timing for quarterly forecasts. My modeling choice is also consistent with theoretical papers in this literature (Ottaviani and Sørensen (2006), Clarke and Subramanian (2006), Banerjee (2020)). Second, I model optimism explicitly, while Camara (2015) removes its effect with normalization. Optimism is a big source of forecast error and a seminal paper on this topic is Hong et al (2010), which find that optimism bias decreases with competition. My paper connects the seemingly contradictory results in these two papers by showing that competition decreases forecast error if optimism is present, but increases forecast error otherwise.

More broadly, this paper is related to the literature on incentive provision based on relative performance, either in terms of career benefits or compensation (e.g., Prendergast (1999) and Nalebuff and Stiglitz (1983)). The incentive to be the top performer is particularly pervasive in the financial industry, potentially leading to risk-taking behavior (Chevalier and Ellison (1997)). This paper explicitly models this incentive for security analysts, generalizing the forecasting contest model proposed by Ottaviani and Sørensen (2005, 2006). Their papers establish the theoretical implications of a winner-takes-all contest model with infinite number of players (analytically) and finite number of players (numerically). They find that analysts put more weight on private information in their forecasts to outperform rivals. This paper extends their model to acount for optimism, so that it fits the security analyst market more realistically. It also develops an efficient numerical procedure for solving this class of models with finite number of players that is feasible for estimation.

The estimation of common value models is an emerging area (Compiani et al. (2020), Ordin (2019)). As mentioned earlier, a challenging identification problem for these models is to jointly identify the distribution of players' signals and payoffs. To resolve this challenge, I leverage additional structure and richness of data in the financial industry. A related paper is Bhattacharya et al. (2018), which studies mineral rights auctions. We both solve the equilibrium and use indirect inference to estimate the model. Their paper resolves the identification challenge by explicitly linking auction design to post-auction economic activity, which provides more structure and data to the problem as well.

The remainder of the paper proceeds as follows. Section 2 introduces the background of security analysts. Section 3 outlines the data and how some key variables are constructed. Some reduced-form evidence is provided in Section 4 to guide the setup of the model. Section 5 formally presents the model and characterizes the equilibrium, followed by Section 6, detailing the estimation procedure and the results. Section 7 performs the counterfactual analyses to and Section 8 concludes.

2 Industry Background: Security Analysts

In this section, I lay out the institutional details of the security analyst market. Security analysts publish reserch reports on securities where they make earnings forecasts and stock recommendations. They also communicate with investors through other means such as organizing company visits and giving presentations on the prospects of the industries they cover. I focus on earnings forecasts as one of the most common types of information provided by analysts. Compared to the stock prices that could be affected by analsyts' recommendations, a firm's earnings are mostly determined by its own profitability, hence exogenous to analysts' forecasts, so they better serve the purpose of this paper.

In the rest of this section, I first describe security analysts' payoff channels, detailing the determinants of their compensation and career prospects. Second, I introduce how analysts compete and the importance of relative accuracy in this industry. These two asepects motivate the focus of this paper and the set-up of the model. Lastly, I present the regulatory background on analysts' incentives.

2.1 Analysts' Payoff Channels

Security analysts are typically employed by brokerage houses, referred to as the *sell-side*. Their main payoff channels consist of current compensation paid by their employers and career prospect in the financial industry, also interpretable as future compensation. The determinants of these payoff channels are key considerations when modelling analysts' incentives.

To start with, I describe how analysts are compensated by their employing brokerage houses. Brokerage houses generate revenue through provision of investment banking services and trade execution. In general, analysts are not paid directly for their research.[4] Instead, their compensation is composed of a fixed wage, as well as bonuses and trading commissions that depend heavily on their employers' revenue (Cowen et al. (2006)).

For brokerage houses, the information that analysts provide is valuable, but analysts are worth beyond their information. In a way, security analysts can be viewed as a marketing device for brokerage houses. 'Good' analysts signal the buy-side that their brokerage houses have the expertise

[4] Recently, some security analysts also provide fee-based research and subscription-based research services. The former is compensated by the subject firms hoping to disclose information more efficiently to investors. The latter is compensated by report readers on a subscription or pay-per-view basis. However, those accounts for a very small fraction of all forecasts (Cowen et al. (2006)).

to identify quality investment opportunities. Subsequently, with the buy-side reputation, they convince firms to direct sell-side business to these brokerage houses.

As a result of this "marketing" role, analysts are compensated for their ability to indirectly boost revenue, which is influenced by their forecast accuracy and optimism. Brokerage houses may offer lucrative compensation to the most accurate analysts because they value analysts' buy-side reputation. But they may also reward analysts who help attract immediate investment banking businesses or increase trading volume, leadings to a reward for optimism for analysts.[5]

The career prospects of security analysts is reflected in the possibility of moving to a better job in the financial industry, often at a more pretigous brokerage house.[6] There is a well-defined prestige hierarchy in the sell-side (Hong and Kubik (2003)). High-prestige brokerage houses, sometimes referred to as "bulge brackets", are major investment banks such as JPMorgan and Merill Lynch, as opposed to low-prestige brokerage houses which are smaller and more specialized. High-prestige brokerage houses hire more analysts, take on larger deals, generate more revenue and hence offer much higher compensation for analysts.[7] Therefore, a job transfer up the prestige hierarchy is considered a career advancement in the sell-side and serves as a strong incentive for analysts.

Besides these tangible channels, analysts can be incentivized by intangible payoffs such as reputatioin concerns. However, the effect of these channels are difficult to observe directly, which also motivates the revealed preference approach I take in this paper to study analysts' incentives.

2.2 Analyst Competition

Security analysts compete with analysts in different brokerage houses covering the same security as brokerages houses usually hire only one analyst to cover one security. The competition could be explicit, as exemplified by the presence of various "best analyst" contests and their huge impact on analysts' payoffs. The most influential contest is the All-America Research Team held by the Institutional Investors Magazine, which dates back to as early as 1972 (*All-America* hereafter). Security analysts are ranked based on democratic votes by buy-side analysts and portfolio managers and earnings forecast accuracy is one of the traits valued by the voters (Bradshaw (2011)). Top analysts make headlines at media outlets, receive disproportionally large compensation package and better career offers. Besides, Thomson-Reuters and the Wall Street Journal also organize contests that rank analysts based on their earnings forecast accuracy and stock-picking performance, whose outcome have also been shown to significantly influence analysts' compensation (Groysberg et al. (2011), Craig (2011)).

[5] A brokerage house could promise favorable research from its in-house analysts to companies seeking investment banking services, while providing bonuses to the analysts involved in the deal.

[6] In recent years, buy-side options such as private equity investment firms and hedge funds are becoming increasingly attractive as well and they look for similar qualities in security analysts as prestigous sell-side firms (Cen et al. (2017)).

[7] Groysberg et al. (2011) and Maber et al. (2014) use proprietary data from two brokerage houses at different prestige levels and find analysts at the lower-prestige house are paid about \$250,000 (30%) less.

2.3 Regulation on Analysts' Incentives

Most of the regulation on security analysts have focused on addressing analysts' optimism bias resulting from the link between investment banking services, trading commissisions, and analysts' compensation. In the United States, a series of legal and regulatory measures were taken after the dotcom bubble, the most prominent of which being the Global Analyst Research Settlement (the Settlement) in 2003. This settlement mandated that ten of the country's largest brokerage houses block their research department from investment banking department and pay $850 millions of penalties in total.[8] At the same time, the SEC also put forward regulations to restrict favorable research. It prohibited tying analysts' compensation to specific investment banking transactions and required disclosure if it was linked to investment banking revenues.

However, the effectiveness of these policies is still under debate. While the investment banking business could benefit from optimistic forecasts thereby giving analysts an optimism incentive, it also relies heavily on analysts' reputation with the investors. If the reputation is damaged by optimism, it could ultimately spoil the investment banking business (Cowen et al. (2006)). So it remains an empirical question whether these policies have successfully reduced the optimism incentive for security analysts.

3 Data and Summary Statistics

I obtain data on analysts' forecasts and employment history from Institutional Brokers' Estimate System(IBES). IBES's Detail Earnings Estimate History contains analysts' forecasts of US companies since 1982. I focus on annual forecasts of Earnings Per Share (EPS) of publicly listed US companies because they are published the most frequently. For each forecast, IBES contains both the forecast value and the realized earnings so I can measure its accuracy. It also contains identifying information of the security, the forecasting analyst, and the employing brokerage house. Therefore, I can trace out the employment history of each analyst using IBES's unique identifiers of analysts and brokerage houses. Finally, it contains when forecasts and earnings are announced so I can account for the possible influence of timing.

The data on US companies comes from IBES, Center for Research in Security Prices (CRSP) database, and Compustat database. IBES contains an industry identifier, known as Thomson Reuters Business Classifications (TRBC). It groups companies that offer products and services into similar end markets globally into 10 industries (Economic Sectors).

I match securities in IBES to CRSP to obtain daily stock prices, stock returns, number of outstanding shares, and whether it is included in S&P500. In addition, I match them to Compustat to obtain financial information on the issuing companies such as book value and operating income.

[8] The Settlement revealed that these brokerage houses could be using compensation to induce over-optimistic research from their analysts. The brokerage houses and analysts in question neither accepted nor denied the charges but they agreed to pay the penalties.

My sample covers forecasts announced between January 1st, 1984 and December 31st, 2016 for securities that are present in all three datasets. When an analyst issues multiple forecasts over a year, I keep the most recent forecast. Moreover, I restrict the sample to the security-years that are covered by at least two analysts so that relative accuracy is well-defined. Finally, I focus on securities that are present in the data for more than 20 years so that I have enough observations to account for security-level heterogeneity.

Table 1: Characteristics of Brokerage Houses

Statistic	Mean	Pctl(25)	Median	Pctl(75)
Number of Securities Covered per Year	77.282	9	27	84
Number of Industries Covered per Year	5.482	3	5	8
Number of Analysts per Year	15.069	2	7	17
Number of Analysts per Industry-year	3.299	1.333	2.222	4.000
Number of Analysts per Security	1.062	1.000	1.014	1.083

Table 2: Characteristics of Analysts

	Mean	Pctl(25)	Median	Pctl(75)
Number of Securities Covered per Year	6.995	3	6	10
Total Number of Securities Covered	12.015	4	8	17
Number of Industries Covered per Year	1.688	1	1	2
Total Number of Industries Covered	2.233	1	2	3
Total Number of Brokerage Houses Worked at	1.797	1	1	2

Table 1 and 2 present the summary statistics of brokerage houses and analysts respectively.[9] The data confirms the anecdotal belief that brokerage houses and analysts specialize in industries. An average brokerage house covers 5 out of 10 industries in a year, with 3 analysts in each industry (Table 1). An average analyst covers 2 industries and 7 securities in a year (Table 2).

In total, there are 209,320 forecasts, 697 securities, 12,267 analysts, and 837 brokerage houses in the sample.

The prestige hierarchy mentioned in Section 2 is reflected in the significant variation in brokerage house size. An average brokerage house employs 15 analysts and covers 77 securities, but the median brokerage house employs only 7 analysts and covers 27 securities (Table 1). The variation is driven by the presence of many large brokerage houses, rather than the expansion and shrinkage of the entire industry over time. The average number of analysts remains between 9 and 13 from 1984 to 2016 (Table A1).

[9]The characteristics for each brokerage house and analyst are computed without the sample restrictions and then

Table 3: Most Analysts Face Significant Competition When Making Forecasts on Securities

Number of Analysts	Number of Securities-years (%)
1-10	6,590 (45.361%)
11-20	4,363 (30.032%)
21-30	2,325 (16.004%)
31-40	932 (6.415%)
40+	318 (2.189%)
Total	14,528

I consider each security as a separate market where analysts compete to make forecasts. Two characteristics of the competition are worth noting. First, analysts' rivals are from other brokerage houses rather than their own. From each brokerage house, only one analyst makes forecast on a given security-year (Table 1). Moreover, the competition is intense, as more than 54% of security-years are covered by more than 10 analysts (Table 3).

Prestige Measure Following Hong and Kubik (2003), I measure an analyst's prestige by the size of its employer.[10] For each brokerage house in each year, I compute the number of analysts it hires as a proxy for size. Then, I classify a brokerage house as high-prestige if it has more analysts than 90% of the brokerage houses in that year and low-prestige otherwise. This measure of prestige has variation over time for the same brokerage house. The prestige of an analyst is defined to be the same as that of the employing brokerage house for a given year. Appendix B illustrates how the measure is computed with a hypothetical example. This procedure results in, on average, 24 high-prestige brokerage houses per year and 48.7% analysts with high-prestige per market-year.

Normalizing the Forecast Problem I normalize analysts' forecast problems across securities and over time by assuming that analysts know the true time series process and only need to make a forecast on the (normalized) shock. Then, for each security-year, analysts have the same common prior of the true earnings that is standard normal.

In practice, I first run a unit root test on the EPS of each security (MZ test with GLS-detrended data). Then, a unit root process is fitted if it is not rejected and an AR(1) with time trend is fitted otherwise. Finally, I subtract the fitted values from the truths and the forecasts and divide them by the estimates of the standard deviations.

summarized within the sample.

[10] What I refer to as "prestige" is called "status" in Hong and Kubik (2003).

4 Reduced-form Evidence

In this section, I document two data patterns that guide the setup of the model and parametric specifications in the estimation. First, I present evidence on the importance of relative accuracy in analysts' incentives in general. Second, I show that analysts' observed behavior is more consistent with a reward for being ranked at the top rather than a punishment for being ranked at the bottom.

My analysis relies on the following two observations.

Observation 1. If analysts are rewarded for being ranked in the top or punished for being ranked in the bottom, their forecast errors will be higher when they face more rivals.

Observation 2. If analysts are rewarded for being ranked in the top, they will distort their forecast towards their signals. Conversely, if analysts are punished for being ranked in the bottom, they will distort their forecast towards their common prior. The distortion intensifies with more rivals.

The intuition is as follows: suppose all analysts covering a security have a common prior for the earnings of the security and also receive private signals. An analyst believes based on her private signal that the earnings of the covered security will be \$1. If she is rewarded for being ranked in the top, she will deviate from making a forecast of \$1 because she will find it likely that her rivals will make similar forecasts. In order to differentiate herself and outperform her rivals, she will deviate towards her private signal.

Conversely, if she is punished for being ranked in the bottom, she will deviate from \$1 as well because she will find it likely that her private signal is particularly bad and singles her out as one of the worst analysts. In order to hide in the herd and avoid underperforming against her rivals, she will deviate towards the common prior.

The more rivals she faces, the more she will need to deviate from \$1 in order to differentiate or to hide herself. And more deviation from her belief leads to higher forecast error.

Ottaviani and Sørensen (2006) show these observations analytically in the simplified theoretical setting of a winner-takes-all game with an infinite number of players. In Ottaviani and Sørensen (2005), they also present a few numerically-simulated equilibria with finite numbers of players as examples, which are also consistent with these observations. In this paper, I develop a robust and efficient new algorithm to compute equilibrium numerically for any finite number of players with more general payoff functions. The algorithm allows me to analyze richer comparative statics and estimate the model.

4.1 Importance of Relative Accuracy

Motivated by Observation 1, I investigate the effect of competition on analysts' forecast error to see whether they are rewarded for relative accuracy. For analyst i covering security j in year t, I consider the following panel data model,

$$|X_{ijt} - Z_{jt}| = \alpha_1 \log(N_{jt}) + Controls_{ijt} \tag{1}$$

where X_{ijt} is analyst i's earnings forecast, Z_{jt} is the actual earnings, or the truth, and $|X_{ijt} - Z_{jt}|$ is the forecast error. N_{jt} is the number of analysts covering security j in year t and α_1 is the coefficient of interest. Following Obesrvation 1, if analysts are rewarded for relative accuracy, α_1 will be expected to be positive.

A potential threat to identification is that the number of analysts covering a security could be endogenous due to entry decisions. For example, more analysts may enter to cover a security if it is easier to obtain information on, while better information also leads to smaller forecast error. I adopt various measures to deal with the endogeneity issue.

First, I include analyst-security pair fixed effects and year fixed effects for all specifications. The analyst-security fixed effect ensures that the model evaluates the effect of competition on the same analyst covering the same security. The year fixed effect accounts for overall uncertainty in the financial market over time. For example, one might be concerned that more bad analysts will be covering a security as the number of analysts increases, or that years that are harder to predict are covered by more analysts. The fixed effects address these concerns.

Second, I control for many time-varying security and analyst characteristics that could be correlated with analysts' coverage decision and forecast behavior. The security characteristics include firm size, profitability, return volatility, average monthly return and an indicator for whether the security is included in the S&P500 index. The analyst characteristics include the analyst's experience with the security, with the industry and in general, as well as her prestige, which is an indicator for whether she is hired by a high-prestige brokerage house. Whenever the indicator for prestige is included, I also control for the percentage of analysts covering the security that are high-prestige. To make sure that the results are not driven by the timing of forecast and earnings report, I also run a specification with the order of the forecast and earnings report delay.

Finally, I instrument the number of analysts and the percentage of high-prestige analysts with their one-year-lagged counterparts. The identifying assumption is that conditional on the controls, the lagged variables on the level of competition is uncorrelated with shocks that affect forecast errors this year.

Table 4 presents the results. The OLS estimates are in Columns (1)-(4) and the 2SLS estimates are in Columns (5)-(8). I find that across all specifications, forecast errors significantly increase with the number of analysts covering the security. The coefficient ranges from 0.033 to 0.096. To put this range in perspective, it means that for the same analyst-security, when the number of analysts increases from 12 (the first quartile) to 21 (the second quartile), forecasts are 0.018 to 0.054 (9.869% to 28.710%) further from the truth on average.

In addition, when more controls are included in the regression and when the lag instrument is

Table 4: Effect of Competition on Forecast Error

	Dependent variable: Forecast Error							
	OLS				2SLS			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log(N_{jt})$	0.033***	0.039***	0.037***	0.093***	0.061***	0.063***	0.046***	0.096***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.008)	(0.009)	(0.010)	(0.010)
log(Market Cap)		−0.034***	−0.034***	−0.027***		−0.039***	−0.037***	−0.029***
		(0.004)	(0.004)	(0.004)		(0.005)	(0.005)	(0.005)
log(Book Value)		0.009*	0.008*	0.006		0.007	0.007	0.006
		(0.005)	(0.005)	(0.005)		(0.005)	(0.005)	(0.005)
Profitability		−0.001***	−0.001***	−0.001***		−0.001***	−0.001***	−0.001***
		(0.0002)	(0.0002)	(0.0002)		(0.0002)	(0.0002)	(0.0002)
Volatility		55.118***	54.963***	54.308***		54.623***	54.583***	54.056***
		(3.380)	(3.374)	(3.326)		(3.372)	(3.355)	(3.309)
Average Monthly Return		−0.335***	−0.332***	−0.363***		−0.285***	−0.290***	−0.334***
		(0.050)	(0.050)	(0.049)		(0.053)	(0.053)	(0.052)
SP500		0.036***	0.035***	0.032***		0.034***	0.035***	0.032***
		(0.007)	(0.007)	(0.007)		(0.007)	(0.007)	(0.007)
Experience (Security)			0.018***	0.012***			0.018***	0.012***
			(0.002)	(0.002)			(0.002)	(0.002)
Experience (Industry)			−0.015**	−0.016**			−0.016**	−0.016**
			(0.007)	(0.006)			(0.007)	(0.006)
Experience (Overall)			0.014**	0.013**			0.014**	0.013**
			(0.006)	(0.006)			(0.006)	(0.006)
High Prestige			−0.005	−0.006			−0.010**	−0.010**
			(0.004)	(0.004)			(0.004)	(0.004)
log(% High Prestige Rivals)			0.002	0.001			0.023***	0.020***
			(0.002)	(0.002)			(0.006)	(0.006)
Forecast Order				−0.007***				−0.007***
				(0.0002)				(0.0002)
EPS Report Delay				0.002***				0.002***
				(0.0002)				(0.0002)
Observations	209,320	209,320	209,320	209,320	209,320	209,320	209,320	209,320

Analyst-security and year fixed effects are included. The standard errors are clustered at the security-year level.
*p<0.1; **p<0.05; ***p<0.01.

adopted, the coefficient on the number of analysts increases. It implies that most of the endogeneity would bias the coefficient downwards. In other words, analysts might have initiated coverage on securities where they expected to make small forecast errors.

The effect of the control variables on forecast errors are consistent across specifications. Analysts make smaller forecast errors for securities that have higher market capitalization, higher profit, higher returns and lower volatility. Given market capitalization, a firm's book value has a positive but sometimes insignificant effect on forecast error. These findings suggest that forecast errors are generally smaller with security-years that are "well-performing" and possibly more predictable. Perhaps more surprisingly, forecast errors are bigger with securities that are included in the S&P500 index. Additional robustness analysis show that both optimistic and pessimistic forecast errors are higher with S&P500 securities but optimistic forecast errors rise more (not presented). One possible explanation is that the trend towards index funds in recent years has led investors to pay more attention to securities comprising the index. But as the funds are passively managed, analysts' absolute accuracy is of less importance. As a result, only the incentive for distortion is heightened.

In terms of analyst characteristics, forecast errors increase with security experience and overall experience, but decrease with industry experience. High-prestige analysts make smaller errors but a high percentage of high-prestige analysts increases forecast errors.

Timing affects analysts' forecasts but accounting for it does not undermine the effect of interest. I find that forecast errors are bigger in security-years where there is a longer delay in earnings report after the fiscal year end. As discussed in Camara (2015), this could be due to less commitment to transparent financial reporting in these security-years. In addition, later forecasts have smaller forecast errors. The coefficient on the number of analysts remains positive with the inclusion of forecast order in the regression. [11]

In summary, analysts' forecast errors increase with the level of competition they face. It is consistent with the strategic behavior when they are rewarded for relative accuracy, either in terms of a reward for the top or a punishment for the bottom. Now I will discuss the important difference between the two.

[11] If anything, the coefficient on the number of analysts becomes bigger with the inclusion of forecast order. It suggests that when there are many analysts, some of them might gather information from earlier forecast to be accurate. In other words, if the analysts were to make forecasts at the same time, they would distort more. Camara (2015) interpret the positive coefficient on the number of analysts and the negative coefficient on forecast order as evidence for preemption. That is, analysts might issue forecasts earlier when they face more competition, resulting in worse information at the time of forecast and higher forecast errors. However, Camara (2015) focuses on forecasts on quarterly earnings forecasts, where timeliness is more important and revisions are less common compared to annual earnings forecasts that I am focusing on. With annual earnings forecasts, if the higher forecast error with competition does result from preemption, we would expect forecasts to be less correlated with the truth when there are more analysts because analysts would be issuing forecast when there is not enough information. However, it is not the case, as I will show in the next section.

4.2 Reward for the Top or Punishment for the Bottom

I study the relationship between forecast and truth to distinguish reward for the top and punishment for the bottom.

Ideally, if analysts' private signals are observable, we will want to study forecast strategies, that is, the relationship between analysts' forecast and private signals. Observation 2 implies that if analysts are rewarded for being in the top, an increase in the number of rivals will lead analysts' to put more weight on their private signals as they issue forecasts. So when forecasts are regressed on private signals, I would expect the slope to be closer to one as the number of analysts increases. Conversely, if analysts are punished for being in the bottom, a increase in the number of analysts will drive the slope closer to 0. Hence, to further distinguish between these two payoff structures, I can examine how this correlation varies with the number of analysts using the following regression,

$$X_{ijt} = \lambda_1 \log(N_{jt}) + \lambda_2 S_{ijt} + \lambda_3 \log(N_{jt}) \times S_{ijt} + Controls_{ijt} \tag{2}$$

where S_{ijt} stands the signal received by analyst i on security j in year t.

However, private signals are unobservable. Therefore, I assume them to be mean-preserving-spread of the truth and replace S_{ijt} with the truth Z_{jt}

$$X_{ijt} = \lambda_1 \log(N_{jt}) + \lambda_2 Z_{jt} + \lambda_3 \log(N_{jt}) \times Z_{jt} + Controls_{ijt}. \tag{3}$$

If analysts are rewarded for being in the top (punished for being in the bottom), we expect λ_3 to be positive. A similar endogeneity issue could arise with the number of analysts in this regression, so I adpot the same controls and instrumental variables.

The results are presented in Table 5. I find that the coefficient on the interaction between the truth and the number of analysts is positive and significant, which implies that analysts are rewarded for being ranked in the top. The magnitude of the coefficieint is robust across specifications [12]. When all control variables and the lag instrument are considered in Column (8), an increase in the number of analysts from 12 to 21 raise the correlation between forecasts and truths from 0.872 to 0.885.

To sum up, the reduced-form evidence shows that analysts' payoffs depend on relative accuracy, predominently in the form of a reward for the top, and analysts respond strategically to these payoffs. Building on these observations, I now formally introduce the model.

[12] The coefficients on the controls are difficult to interpret here because their correlation with the forecast could be from their effect on analysts' strategy, their signals, or the truth. For example, analysts' forecasts increase with the market capitalization of a security. It could be because analysts have more optimism incentive securities with large market capitalization. Or, it could be because security-years with large market capitalizaton also have high earnings per share. Additional analysis shows that analysts' optimism bias actually decrease with market capitalization, suggesting that it is unlikely for analysts to face more optimism incentive for securities with large market captalization.

Table 5: How Much Do Private Signals Weigh in Forecasts

	Dependent variable: Forecast							
	OLS				2SLS			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log(N_{jt})$	0.007	−0.017***	−0.016***	0.006	0.020**	−0.010	−0.017	0.003
	(0.005)	(0.006)	(0.006)	(0.006)	(0.009)	(0.011)	(0.011)	(0.012)
Z_{jt}	0.804***	0.807***	0.807***	0.810***	0.806***	0.809***	0.809***	0.812***
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
$\log(N_{jt}) \times Z_{jt}$	0.026***	0.025***	0.025***	0.025***	0.025***	0.025***	0.025***	0.024***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
log(Market Cap)		0.032***	0.032***	0.036***		0.030***	0.031***	0.036***
		(0.005)	(0.005)	(0.005)		(0.005)	(0.005)	(0.005)
log(Book Value)		−0.008	−0.008	−0.009*		−0.008	−0.008	−0.009*
		(0.005)	(0.005)	(0.005)		(0.005)	(0.005)	(0.005)
Profitability		−0.001*	−0.001*	−0.0005*		−0.001*	−0.001*	−0.001*
		(0.0003)	(0.0003)	(0.0003)		(0.0003)	(0.0003)	(0.0003)
Volatility		20.636***	20.627***	19.843***		20.513***	20.528***	19.783***
		(3.889)	(3.888)	(3.873)		(3.884)	(3.880)	(3.865)
Average Monthly Return		−0.714***	−0.716***	−0.737***		−0.701***	−0.703***	−0.730***
		(0.055)	(0.055)	(0.055)		(0.058)	(0.058)	(0.057)
SP500		−0.001	−0.001	−0.003		−0.001	−0.0005	−0.003
		(0.007)	(0.007)	(0.007)		(0.007)	(0.007)	(0.007)
Experience (Security)			0.004	0.001			0.004	0.001
			(0.002)	(0.002)			(0.002)	(0.002)
Experience (Industry)			−0.006	−0.006			−0.006	−0.006
			(0.007)	(0.007)			(0.007)	(0.007)
Experience (Overall)			0.011	0.010			0.011	0.011
			(0.007)	(0.007)			(0.007)	(0.007)
High Prestige			−0.008*	−0.008*			−0.011**	−0.011**
			(0.004)	(0.004)			(0.005)	(0.005)
log(% High Prestige Rivals)			−0.004**	−0.005**			0.009	0.008
			(0.002)	(0.002)			(0.006)	(0.006)
Forecast Order				−0.003***				−0.003***
				(0.0002)				(0.0002)
EPS Report Delay				0.002***				0.002***
				(0.0002)				(0.0002)
Observations	209,320	209,320	209,320	209,320	209,320	209,320	209,320	209,320

Analyst-security and year fixed effects are included. The standard errors are clustered at the security-year level.
*p<0.1; **p<0.05; ***p<0.01.

5 Model

I define each security-year as a forecasting contest. For ease of exposition, I omit the subscript j for security and t for time in this section. A contest has N analysts, $i = 1, ..., N$. Analysts do not know the EPS, or the truth $Z \in \mathcal{Z} \subset \mathbb{R}$ when they enter the contest. First, they observe the number of analysts in the contest N and their own private signals of the truth, denoted by $S_i \in \mathcal{S} \subset \mathbb{R}$. Then, they simultaneously make forecasts $X_i \in \mathcal{Z}$. Let $S = (S_1, ..., S_N)$, $X = (X_1, ..., X_N)$, $S_{-i} = S \backslash S_i$, and $X_{-i} = X \backslash X_i$. Finally, Z is realized and analysts' payoffs are determined by a function $u : \mathcal{Z}^{N+1} \to \mathbb{R}$, defined as the following,

$$u(X_i, X_{-i}, Z) = \underbrace{\sum_{k=1}^{N} \gamma_k(N) 1(\#\{j \ s.t. |X_j - Z| \leq |X_i - Z|\} = k)}_{\text{Relative Accuracy}} + \underbrace{\gamma_c (X_i - Z)^2}_{\text{Absolute Accuracy}} + \underbrace{\gamma_o X_i}_{\text{Optimism}} .$$

This payoff function has three components: relative accuracy, absolute accuracy, and optimism. I define analyst i's forecast error as the absolute difference between i's forecast and the truth $|X_i - Z|$. If an analyst has k-th smallest forecast error, I define this analyst as having the k-th rank. If $k < N/2$, this rank is described as *in the top*; if $k > N/2 + 1$, it is described as *in the bottom*. Then, the payoff for relative accuracy is expressed with a series of weakly decreasing reward for having the k-th rank $\gamma_1(\cdot) \geq \gamma_2(\cdot) \geq \cdots \geq \gamma_N(\cdot)$, which are also allowed to depend on N. For example, $\gamma_1(\cdot)$ is the reward for being closest to the truth in the contest, or ranked as top 1. Assume without loss of generality that $\gamma_N(\cdot) = 0$.

The payoff for relative accuracy is flexible enough to encompass a number of common contests. Here are a few examples. Let C denote a constant and positive number.

- Winner-takes-all: $\gamma_1(N) = C, \quad \gamma_k(N) = 0 \quad \text{for} \quad k = 2, ..., N$
- Reward for top 10% players: $\gamma_k(N) = \begin{cases} C & \text{for} \quad k = 1, ... \lfloor 0.1N \rfloor \\ 0 & \text{for} \quad k = \lfloor 0.1N \rfloor + 1, ..., N \end{cases}$
- Loser-loses-all: $\gamma_k(N) = C \quad \forall k = 1, ...N - 1, \quad \gamma_N(N) = 0,$

Payoff for absolute accuracy is represented by a cost of squared forecast error, γ_c, which I normalize to -1. By normalizing, I am imposing the assumption that analysts are always punished for high forecast errors. This is a natural assumption that is also supported by empirical evidence. If analysts are not punished for forecast errors, we will expect them to issue extremely high (low) forecasts when there is the slightest incentive for optimism (pessimism). Such patterns are not observed in the data.

Given the normalization, the absolute accuracy component sets an *honest* payoff. If it is the only component in the payoff function, analysts will find it optimal to choose the posterior $\mathbb{E}(Z|S_i)$ as their forecast, which I call the *honest* forecast.

Finally, I model the payoff for optimism as a linear function of forecast with coefficient γ_o. If $\gamma_o > 0$ and all else equal, analysts will find it optimal to report higher than $\mathbb{E}(Z|S_i)$.

Now I specify the signal distribution and its relationship with the distribution of the truth.

Assumption 1. (Conditional Independence) $S_i \perp\!\!\!\perp S_j|Z$ for $i \neq j$.

The private signals are assumed to be independent of each other conditional on the truth as in Assumption 1. I express the conditional density of the truth given i's private signal as $f(Z|S_i)$ and the conditional density of rival j's signal given the earnings as $g(S_j|Z)$.

Analyst i's strategy is denoted by $\beta_i : \mathcal{S} \to \mathcal{Z}$. Given any set of rivals' strategies, β_{-i} , analyst i chooses forecast X_i to maximize the expected payoff conditional on receiving signal S_i defined as follows

$$U(X_i, S_i; \beta_{-i}(\cdot)) \equiv \int_Z \int_{S_{-i}} u_i(X_i, \beta_{-i}(S_{-i}), Z) g(S_{-i}|Z) f(Z|S_i) dS_{-i} dZ.$$

Equilibrium β characterizes a pure-strategy Bayesian Nash equilibrium if and only if for all i and $S_i \in \mathcal{S}$,

$$\beta^*(S_i) = \arg\max_{X_i \in \mathcal{Z}} U(X_i, S_i; \beta^*_{-i}(\cdot)).$$

I prove the existence of an equilibrium when the signal space $\mathcal{S}$ and the truth space $\mathcal{Z}$ are finite, making use of Assumption 1 (details in Appendix C). Equilibrium with continuous state space is difficult to characterize analytically, but it is easier and faster to compute numerically with parametric assumptions, which enables estimation. Therefore, I restrict to settings with a continuous state space in the rest of this paper. I assume that the fundamentals of the model are such that there exists an equilibrium where all analysts play a symmetric and strictly increasing forecast strategy $\beta^*(S_i)$.

Now I characterize the equilibrium using first order conditions and introduce the numerical procedure for computation.

5.1 Equilibrium Characterization

The relative accuracy component of the objective function can be simplified as follows. The probability of being ranked the k-th is equal to the probability of $k-1$ rivals making more accurate forecasts (being better) and $N-k$ making less accurate ones (being worse). Consider analyst i and j. If $X_i < Z$, then j is better than i if $X_i < X_j < 2Z - X_i$. If $X_i > Z$, then j is better than i if

$2Z - X_i < X_j < X_i$. The probability j being better than i can be written as

$$P(j \text{ is better than } i | X_i, Z, S_i) = \left| \int_{\beta^{-1}(X_i)}^{\beta^{-1}(2Z-X_i)} g(S_j|Z) dS_j \right| \equiv 1 - p(X_i, Z)$$

With symmetric analysts, the expected value of sending forecast X_i after observing signal S_i can be rewritten using these probabilities,

$$\begin{aligned} U(X_i, S_i) = & \int_{-\infty}^{\infty} \Big[\sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} p(X_i, Z)^{N-k} (1 - p(X_i, Z))^{k-1} + \\ & - (X_i - Z)^2 + \gamma_o X_i \Big] f(Z|S_i) dS. \end{aligned}$$

Proposition 1. (First Order Condition with Symmetric Analysts) The first order condition describing the symmetric and increasing equilibrium is given by

$$\begin{aligned} & \underbrace{\sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} \int_{-\infty}^{\infty} \Xi(X_i, Z, N, k) f(Z|S_i) dZ}_{\text{Relative Accuracy}} \\ & \underbrace{-2[X_i - \mathbb{E}(Z|S_i)]}_{\text{Absolute Accuracy}} + \underbrace{\gamma_o}_{\text{Optimism}} = 0, \end{aligned} \tag{4}$$

to be solved for a strictly increasing $\beta^*(\cdot)$, where $X_i = \beta^*(S_i)$ and

$$\begin{aligned} \Xi(X_i, Z, N, k) = & \big[1 - 2 \cdot \mathbf{1}(X_i > Z)\big] \Big[\frac{g(\beta^{*-1}(X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(X_i))} + \frac{g(\beta^{*-1}(2Z - X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(2Z - X_i))} \Big] \\ & \big[\mathbf{1}(N > k)(N - k) p(X_i, Z)^{N-k-1} (1 - p(X_i, Z))^{k-1} - \\ & \mathbf{1}(k > 1) p(X_i, Z)^{N-k} (k-1)(1 - p(X_i, Z))^{k-2}\big] \end{aligned}$$

The first order condition has the same three components as the payoff function. Consider the last two components. The payoff for absolute accuracy creates an incentive for analysts to bring forecasts closer to their posteriors $\mathbb{E}(Z|S_i)$. If analysts are rewarded for optimism, deviations above the posterior will bring higher expected payoff until the marginal loss from accuracy is no longer justified by the marginal gain in optimism γ_o.

The relative accuracy component is the sum of the marginal changes in expected reward from each rank. Analysts choose forecasts to increase the probability of receiving the ranks that are better rewarded. For each rank, the marginal change in the probability of receiving it is further split into two terms, representing the cases where the truth is higher than the forecast and where it is lower.

These two terms reflect the trade-off analysts face when they want to be better than the rivals.

Consider a higher forecast. This improves the chance of being better than the rivals in realizations where the truth is higher than the forecast and reduces it otherwise. The term $\left[\frac{g(\beta^{*-1}(X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(X_i))} + \frac{g(\beta^{*-1}(2Z-X_i)|Z)}{\beta^{*\prime}(\beta^{*-1}(2Z-X_i))}\right]$ represents the marginal gain or loss in the chance of being better than one rival conditional on the truth.

If analysts are incentivized to be better than the rivals, they will move their forecasts towards their signals as long as their signals are not too extreme.[13] This move generates more gain than loss in the chance of being better than the rivals, because analysts expect their rivals to receive information correlated with truth thereby issuing similar forecasts. Lemma 1 illustrates this intuition more technically in a winner-takes-all game with two players. I show that even without the payoff to optimism, reporting the posterior is not an equilibrium and analysts have an incentive to deviate towards their signals.[14]

Lemma 1. Suppose the truth Z and the signals S are normally distributed. In a winner-takes-all game with two players, when analysts reports their posterior means as forecasts, there exists a threshold $\bar{S} > 0$ such that for all $|S_i| < \bar{S}$, an analyst's utility increases from a deviation in the direction of the signal.

Proof. See Appendix D. ∎

5.2 Numerical Procedure for Equilibrium Computation

In this subsection, I introduce the numerical procedure that I use to solve the symmetric equilibrium. An analytical solution is difficult to obtain for this game. The equilibrium is characterized by the first order conditions in equation (4) for every signal in $\mathcal{S}$, which form a system of differential equations involving the inverse function and the derivative of the equilibrium strategy. With a change of variable, this system can be rewritten to be free of the inverse function of the equilibrium strategy (see Appendix E for details). Even so, to my knowledge, this system does not have a well-established solution.

Therefore, I numerically approximate the equilibrium strategy by projecting it onto a family of pre-specificed functions, such as polynomials. The projection is parameterized with θ. First, I solve the derivatives of the pre-specified function analytically. Then, I substitute the equilibrium strategy and its derivatives in the first order conditions with those implied by the function family. After that, I find the θ^* that minimizes a residual function, which is defined as the sum of squared first order condition at a number of representative signals (also called "collocation points") in $\mathcal{S}$.

[13]When the signal is in the tails of the distribution, the marginal gain in the probability of being better than a rival may not justify the deviation because the truth the signal implies is unlikely.

[14]Note that the payoff structure may not always encourage analysts to be better than the rivals. For example, in a loser-loses-all game with more than two players, a higher chance of being better than a rival implies a higher probability of being ranked the first, but also a lower chance of receiving other ranks with the same reward. In fact, in this case, the expected payoff decreases with the probability of being better than the rivals, so analysts will deviate away from the signal.

Finally, I verify whether the function with θ^* is a good approximation of the equilibrium strategy by checking if the first order conditions are indeed close to zero.

This procedure, namely the projection method, is commonly used in the macroeconomics literature to solve continuous-time life-cycle consumption models and growth models (Judd (1998)). Conceptually, it is similar to setting players' strategy to be symmetric when analytically solving a Cournot or Bertrand game.

The projection method has three benefits compared to other commonly used equilibrium computation algorithms. First, it allows researchers to focus on equilibria with characteristics of interest, such as symmetry and monotonicity. Such restrictions are difficult to impose with, for example, an iterated best response algorithm. Second, this procedure can be computed quickly thanks to its parsimonious specification. Last but not least, the method can be extended to be fully non-parametric. The functional form can be chosen to be so flexible that it can describe any arbitrary strategy in the strategy space.

In practice, I choose monotonic polynomials as the function family and experiment with up to the 7th order polynomial to simulate equilibria under the parametric assumptions of the game, which I will introduce in Section 6 (details of the experimentation are provided in Appendix F).

6 Identification, Estimation and Results

In this section, I discuss identification and estimation of the model in Section 5. The parameters of interest are the signal distribution and the γ's in the payoff function. First, I provide a discussion of the identification of the parameters. I argue that for many contests of interest, the signal distribution and the payoff function are non-parametrically identified. Then, I introduce how I estimate the model with indirect inference, detailing the parametric specifications imposed to ease computation and the choice of moments. Finally, I present the results.

6.1 Identification

To identify the model, I first show that the signal distribution can be normalized without loss of generality and that the normalized distribution is non-parametrically identified. Then, given the identified signal distribution, I use the first order conditions implied by the Bayesian Nash Equilibrium (equation (4)) to identify the payoff function. The non-parametric identification implies that the results are not driven by parametric assumptions.

6.1.1 Signal Distribution

The signal distribution is only of information value in this model, similar to that in auction models. In other words, any strictly monotonic transformation of the signal does not change the information it contains (Athey and Haile (2007)). Therefore, we can normalize the set of signals without loss of generality using a strictly monotonic transformation. For any given number of analysts N, one such transformation is the strictly increasing equilibrium forecast strategy under the true payoff function $\beta^*(S_i; N)$, so the set of signals can be normalized as

$$\tilde{\mathcal{S}} \equiv \{\tilde{S} : \tilde{S} = \beta^*(S; N), S \in \mathcal{S}\}. \tag{5}$$

Then, the set of observed forecasts is equivalent to the set of signals for a given N and the true payoff function.

As implied by the equilibrium characterization, the signal distribution has only two properties of economic interest in this model: 1) the density of the truth conditional on the signal, $f(Z|S_i)$; 2) the density of the signal conditional on the truth, $g(S_i|Z)$.

With the normalization, for a fixed number of analysts N, the density of the truth conditional on the signal $f(Z|S_i)$ is equivalent to the density of the truth conditional on the equilibrium forecast and the number of analysts $\tilde{f}(Z|\beta^*(S_i; N), N)$. Because Z, $\beta^*(S_i; N)$, and N are observable, the conditional density $\tilde{f}(Z|\beta^*(S_i; N), N)$ is identified. Similarly, the density of signal conditioning on the truth $g(S_i|Z)$ is equivalent to the density of the equilibrium forecast conditioning on the truth and the number of analysts $\tilde{g}(\beta^*(S_i; N)|Z, N)$, and thus is identified as well from the same observables.

To clarify, this identification result *does not* imply that analysts' private signals are equal to their forecasts, because the equivalence between the set of forecasts and signals must be established with a *fixed* monotonic transformation (in this case, the *fixed* equilibrium strategy under the true payoff function). For example, suppose an analyst is observed to make a forecast X^* in a contest with 5 analysts after receiving signal $S \in \mathcal{S}$. Then, the corresponding normalized signal is $\tilde{S} \equiv \beta^*(S; 5) = X^* \in \tilde{\mathcal{S}}$. Denoting the equilibrium strategy on the normalized signal set by $\tilde{\beta}^* : \tilde{\mathcal{S}} \to \mathcal{Z}$, we have $\tilde{\beta}^*(\tilde{S}) = \tilde{S}$. Now, if the payoff function is changed to have higher optimism incentive and the new equilibrium strategy becomes $\tilde{\beta}' : \tilde{\mathcal{S}} \to \mathcal{Z}$, then $\tilde{\beta}'(\tilde{S}) > \tilde{S}$. There is no longer an equality between the forecast and the signal.

To conclude, the normalized signal distribution is non-parametrically identified.

6.1.2 Payoff Function

Given the signal distribution, the payoffs for optimism and relative accuracy can be non-parametrically identified through variation in equilibrium forecast strategy with the number of analysts, under reasonable assumptions. For intuition, first consider a winner-takes-all game. Consistent with Observation 1, when analysts face more rivals, it will be more difficult for them to be the most accurate, so they will distort their forecasts more to differentiate, leading to higher forecast errors. In addition, this effect becomes more pronounced when the top 1 analyst is more heavily rewarded. Therefore, the change in forecast error with the number of analysts could be used to identify the reward for the most accurate analyst.

More formally, I show non-parametric identification using the first order conditions. Following the normalization in equation (5), I rewrite $f(Z|S_i)$ using the equilibirum forecast $X = \beta^*(S_i; N)$ as $\tilde{f}(Z|X, N)$. Then, integrating over X, equation (4) can be rewritten as

$$\sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Xi(X, N, k|Z) \tilde{f}(Z|X, N) dZ dX \\ - 2[\mathbb{E}(X|N) - \mathbb{E}(Z|N)] + \gamma_o = 0. \tag{6}$$

For any N in the data, equation (6) is a linear equation in γ whose coefficients can be computed from the data. $\mathbb{E}(X|N)$ and $\mathbb{E}(Z|N)$ are the expectations of the forecast and the truth. $\tilde{f}(Z|X, N)$ is the density of the truth conditioning on equilibrium forecast X and the number of analysts N. Finally, $\Xi(X, N, k|Z)$ is the marginal probability of receiving rank k with forecast X conditioning on the truth being Z. Therefore, equation (6) is well-defined as long as contests with N analysts are observed repeatedly

Given this observation, the first order conditions of all contests with N in the support $\mathcal{N}$ yield a linear system with $|\mathcal{N}|$ such equations, where $|\cdot|$ denotes of number of elements in the set. The rank of the linear system is denoted by $m \leq |\mathcal{N}|$, so it identifies at most m parameters. For example, if the payoff for relative accuracy is allowed to vary flexibly with the number of players, it will not be identified, as it requires $\sum_{N \in \mathcal{N}} N + 1 > m$ parameters. Therefore, I make the following assumption to ensure that there is enough variation in the number of competitors for the parameters in the payoff function.

Assumption 2. The parameters in the payoff function γ are subject to at least $\sum_{N \in \mathcal{N}} N + 1 - m$ linear equality constraints.

This assumption implies that after simplifications, the payoff function can be characterized by at most m parameters. It restricts the attention to contests that present some consistency in the payoff function as the number of analysts changes (see examples in Section 5, which all satisfy this assumption). Under this assumption, the payoff function is identified from the rank-m linear system.

In the identification arguments of both the signal distribution and the payoff function, being able to observe the truth Z is crucial. On the one hand, it enables the identification of the conditional densities $f(Z|S_i)$ and $g(S_i|Z)$, which quantify the amount of noise in analysts' signals, similar to observing ex post values in mineral rights auctions. On the other hand, it helps with recovering first order condition in the identification of the payoff. Forecast contests differ from mineral rights auctions in that observations of forecasts and truths are both needed to recover analysts' probabilities of receiving each rank.[15]

[15] In mineral rights auctions, observations of bids are sufficient for identification of the winning probability.

In summary, the model is non-parametrically identified and the identification benefits from additional structures of the security analyst market, including the rich variation in the number of analysts across different securities over time; as well as the additional data on the truth.

6.2 Estimation Method

I estimate the model with indirect inference, in four steps: 1) I compute data moments using auxiliary regressions. The moments are chosen to reflect the variation in forecast strategy with the number of analysts, following the identification argument. 2) Starting from some parameter values $\{\tilde{\gamma}, \tilde{\tau}\}$, I use the numerical procedure described in Section 5.2 to compute the equilibrium strategies for games in the data, which are characterized by the number of players N.[16] An important assumption here is that if the model has multiple equilibria, all observations in the given data set are from the same equilibrium; otherwise, the structural estimates are not identified.[17] 3) I simulate signals 100 times and use the equilibrium strategy to find the corresponding forecasts and simulated moments. 4) I match the average of the simulated moments to the data moments. I update $\{\tilde{\gamma}, \tilde{\tau}\}$ and repeat step two and three until the distance between the simulated moments and the data moments is minimized.

To address the potential heterogeneity in forecast contests, I normalize them in two steps and estimate a homogeneous contest (Compiani et al. (2020)). First, as detailed in Section 3, I assume that analysts know the true time series process of the EPS and only make forecasts on the shock each period, distributed standard normal. This allows me to homogenize the distribution of the truth, removing the heterogeneity from volatility and time trend of security earnings. Second, I use linear panel models with fixed effects and instruments as auxiliary models to construct data moments. This is equivalent to normalizing the variables of interest to a baseline analyst-security-year before running indirect inference estimation. This accounts for the heterogeneity in the signal distribution and the payoff function across securities and analysts over time.

Now I provide the parametric specifications used in estimation and discuss my choice of moments in detail.

6.2.1 Parametric Specifications

Signal Distribution To ease computation, I assume that the signal is normally distributed around the truth as the following,

$$S_i = Z + \epsilon_i, \text{ where } \epsilon_i \sim N(0, 1/\tau).$$

[16] To reduce the computation burden, I compute the equilibrium strategy for representative games and interpolated for the rest (detail in Appendix G).

[17] See Athey and Haile (2007) for a similar assumption in the discussion of first-price auctions. In practice, the numerical procedure for equilibrium computation gives the same symmetric monotone equilibrium irrespective of initial values.

where τ denotes the precision of the signal and is the parameter to be estimated.

Then, the posterior distribution of the truth conditional on observing signal S_i is given by,

$$Z|S_i \sim N(\frac{\tau S_i}{1+\tau}, \frac{1}{1+\tau}).$$

Note that the posterior mean is the *honest* forecast. Its correlation with the signal, and therefore the truth, is given by $\frac{\tau}{1+\tau}$, which is increasing in τ. This means that the honest forecast puts more weight on the signal if the signal is more precise.

Payoff Function Reduced-form evidence suggests that analysts are rewarded for being in the top, so I estimate a winner-takes-all specification of relative accuracy as the baseline.

Equilibrium Computation I experimented with polynomials up to the 7th order and found that a simple linear function, i.e., $X_i = \theta_1 + \theta_2 S_i$, is a great approximation to the equilibrium strategy under normality (details in Appendix F). Therefore, I use a linear strategy to compute equilibrium in the rest of the paper. This is also consistent with the analytical equilibrium strategy in some special cases. For example, the honest forecast is a linear function of the signal and a winner-takes-all game with infinite number of players also has an equilibrium in linear strategy (Ottaviani and Sørensen (2006)).

6.2.2 Choice of Moments

Consistent with the identification argument in Section 6.1, I use the variation in forecast strategy with the level of competition to identify the structural parameters. Specifically, I estimate the following auxiliary regressions,

$$X_{ijt} = \tilde{\lambda}_0 + \tilde{\lambda}_1 \log(N_{jt}) + \tilde{\lambda}_2 Z_{jt} + \tilde{\lambda}_3 \log(N_{jt}) \times Z_{jt} + Controls_{ijt}$$

$$\left.\begin{array}{r} X_{ijt} - Z_{jt} \\ 1(X_{ijt} > Z_{jt}) \\ \text{If } X_{ijt} > Z_{jt}, \quad |X_{ijt} - Z_{jt}| \\ \text{If } X_{ijt} \leq Z_{jt}, \quad |X_{ijt} - Z_{jt}| \end{array}\right\} = \tilde{\alpha}_0 + \tilde{\alpha}_1 \log(N_{jt}) + Controls_{ijt},$$

where $X_{ijt} - Z_{jt}$ denotes optimism bias and $1(X_{ijt} > Z_{jt})$ is an indicator function which is 1 $X_{ijt} > Z_{jt}$. Here I also separately regress the forecast error on the number of analysts for when $X_{ijt} > Z_{jt}$ and otherwise to capture the changes in opstimic forecast errors and pessimistic forecast errors with competition. The coefficients on the functions of the state variables Z_{jt} and N_{jt}, $\tilde{\lambda}$ and $\tilde{\alpha}$, are used as moments.

I adopt analyst-security fixed effects, year fixed effects and the lag instrument as in Section 4 to normalize the forecast contests across securities and analysts over time and also to address the potential endogeneity concerns with the number of analysts, similar to those discussed in the reduced-form evidence. These controls and instruments ensure that coefficients in the auxiliary regressions reflect analysts' behavior in homogeneous contests with exogenous entry, so that they match the model in Section 5.[18]

I use the variance-covariance matrix of the auxiliary regression as the weighting matrix. The standard errors are clustered at the security-year level.

6.3 Results

The estimation results are presented in Table 6. I find large and significant rewards for being the most accurate analyst. This reward is equivalent to a 2.614 standard error deviation from the truth in the honest scenario. In addition, there is significant incentive for optimism as well. Without the reward for relative accuracy, an optimistic forecast up to 0.816 standard error higher than the truth will yield higher payoff than the truth.

Table 6: Estimation Results

	The Most Accurate	Optimism	Signal Precision
Estimates	6.833***	0.816***	7.056***
Standard Error	(2.312)	(0.142)	(0.232)

7 Counterfactuals

When security analysts are rewarded based on relative accuracy, competition could have two opposing effects on information quality. On the one hand, it could have a *distortionary* effect as analysts differentiates their forecasts. On the other hand, it could have a *disciplinary* effect on analysts' optimistic behavior. The purpose of this section is see which of the two effects dominates in reality and how the effect of competition change under different incentive schemes.

To do that, I consider how information quality changes when the reward for relative accuracy is reduced in two cases: without and with the optimism incentive. The case without optimism

[18]This is equivalent to the following three-step approach. In the first step, I homogenize the the dependent, independent, and instrumental variables by regressing them on the controls and keeping the residuals as the normalized variables. The normalized variables are then free of the influence of the controls. In the second step, I regress the residuals on the instrumental variables and keep the predicted value. After the first two steps, the variables can be considered as generated by homogeneous game with exogenous entry. In the third step, I run a simple OLS analysis using the predicted-normalized variables without any controls. I choose an average analyst-year as the baseline by assuming that all analyst-security fixed effects sum to 0 and I choose the most recent year in the sample, 2016, as the baseline year.

separates out the *distoritionary* effect, whereas the case with optimism pins down the *disciplinary* effect.

To evaluate the impact on the overall market, I simulate the counterfactuals for every single contest in the sample. For each analyst in a security-year , I draw a random noise and add it to the ex-post-observed truth to generate a signal. Then, I compute the equilibrium strategy at the given number of analysts and simulate the forecast. To measure information quality, I compute forecast error, forecast bias, and forecast variance for each contest and average across all contests to integrate out the joint distribution of the number of players and the truth.

In addition, I investigate the optimal level of competition by comparing information quality under different numbers of analysts. For any given number of analysts, I simulate $10,000$ abstract contests with randomly drawn truths and compute the measures of information quality following the same procedure.

I find that the disciplinary effect of competition dominates. The reward for relative accuracy reduces forecast error by 33.37%, which is generated by the optimism incentive. However, this improvement in forecast accuracy is at the cost of more noise in the information, as forecast variance increases by 3.74%. Competition improves information quality overall, but small number of competitors generates bigger improvement. In contrast, without optimism, the reward for relative accuracy increases forecast error and variance albeit by a small amount, and competition impairs information quality.

7.1 Without Optimism

Most of the regulations on security analysts have been targeting the optimism incentive because it is considered as the most prominent source of bias, but very little is known about analysts' other incentives. How will they interact with competition if the optimism incentive is removed? This section answers this question.

To start with, I illustrate the trade-offs an analyst faces when choosing a forecast in Figure 1. Here, the analyst competes with 9 other analysts and receives a signal of 1.369, which implies an honest forecast of 1.199. The signal is at the 90th percentile of the signal distribution, so I call it an optimistic signal. This figure shows the gains and losses in expected utility when the analysts deviates from the honest forecast under different payoff functions.

Analysts benefit from a deviation in the direction of the signal under the estimated reward for relative accuracy. When the reward for relative accuracy is only element in the payoff (red dashed line), the analyst's optimal forecast will be higher than the honest forecast. When the reward for absolute accuracy is added (blue solid line), the optimal forecast will be slightly lower but still higher than the honest forecast. This is because an optimistic signal not only implies that the truth will be high, but also that the rivals will likely issue high forecasts. The latter creates additional incentive to issue high forecasts when analayts are rewarded for outperforming their rivals.

Figure 1: Analyst's Problem: Difference in Expected Utility When Deviating from Honest Forecast with 10 Analysts

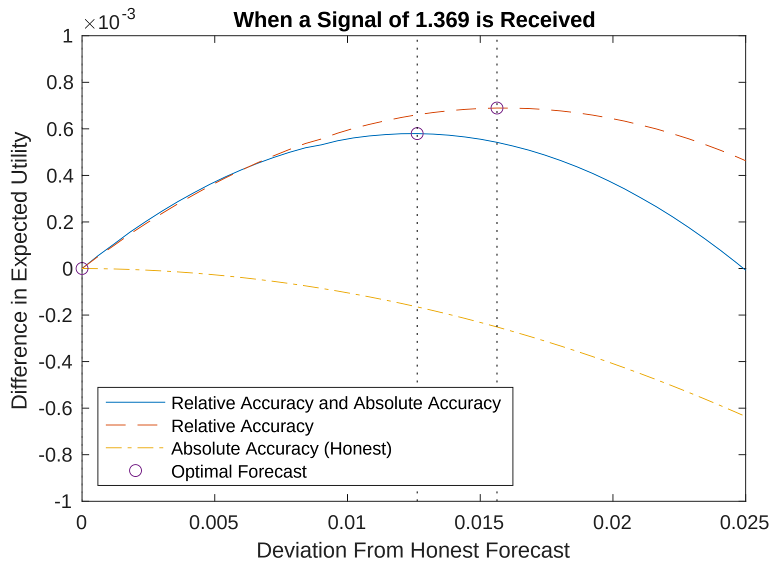


Table 7 presents the effect on information quality when the reward for relative accuracy is added to the honest payoff with only the absolute accuracy component. The reward for relative accuracy increases forecast error by a small amount of 0.08% and increases forecast variance by 1.64% in the sample. Comparing the results with different numbers of analysts, we find that the increase in forecast error and variance is bigger when there are more analysts in the market. These results show that without the optimism incentive, competition has a distortionary effect on information quality.

7.2 With Optimism

When analysts face the optimism incentive, both the distortionary and the disciplinary effect of competition are present. If analysts' forecast errors decreases with the addition of relative accuracy to the payoff, I will consider that the disciplinary effect dominates and vice versa.

An analyst's trade-offs in choosing a forecast is shown in 2. The analyst again competes with 9 other analysts. This time, I illustrate the difference in expected utility from deviation upon receiving an optimistic signal of 1.369 (the 90th percentile) and a pessimistic signal of -1.369 (the 10th percentile), which correspond to honest forecasts at 1.199 and -1.199.

With both signals, the analyst benefit from a deviation towards a higher forecast once the

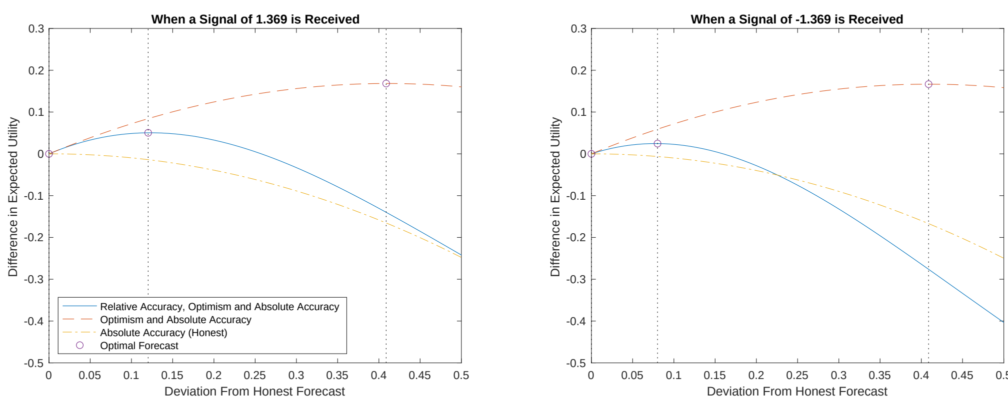
Table 7: The Effect on Information Quality of Payoff for Relative Accuracy Without Optimism (e-3)

N	Forecast Error	Forecast Variance
2	0.025 (0.01%)	0.581 (0.56%)
10	0.069 (0.02%)	1.700 (1.57%)
20	0.147 (0.05%)	2.170 (2.01%)
Sample	0.221 (0.08%)	1.760 (1.64%)

% change in parentheses.

*For a given contest, the **Forecast Error** is computed as* $\sum_i |X_i - Z|/N$*. The **Forecast Variance** is computed as* $\sum_i (X_i - \bar{X})^2/(N-1)$

Figure 2: Analyst's Problem: Difference in Expected Utility When Deviating from Honest Forecast with 10 Analysts



optimism incentive enters the payoff function (red dashed lines). The reward for relative accuracy disciplines the amount of optimism in the forecast (blue solid lines). In contrast to the case without optimism where the analyst deviate in the direction of their signal, here the analyst will still benefit from an optimistic deviation even if a pessimistic signal is received due to the large optimism incentive. Nevertheless, the deviation with an optimistic signal is larger than the deviation with a pessimistic signal.

Table 8: The Effect on Information Quality of Payoff for Relative Accuracy With Optimism

N	Forecast Error	Bias	Forecast Variance
2	-0.170 (-37.77%)	-0.378 (-92.67%)	0.001 (0.71%)
10	-0.153 (-34.03%)	-0.287 (-70.20%)	0.004 (3.54%)
20	-0.141 (-31.34%)	-0.247 (-60.59%)	0.005 (4.78%)
Sample	-0.150 (-33.37%)	-0.281 (-69.05%)	0.004 (3.74%)

% change in parentheses.
For a given contest, ***Forecast Error*** *is computed as* $\sum_i |X_i - Z|/N$. ***Bias*** *is computed as* $\sum_i (X_i - Z)/N$. ***Forecast Variance*** *is computed as* $\sum_i (X_i - \bar{X})^2/(N-1)$.

Starting with a payoff function composed of absolute accuracy and optimism, Table 8 presents the effect on information quality when the reward for relative accuracy is added. The reward for relative accuracy substantially decreases the forecast error by 33.37% and increases forecast variance by 3.74% in the sample. The decrease in forecast error shows that the disciplinary effect dominates in the overall market. However, the distortionary effect is still present as the forecast variance increases. In fact, the reward for optimism enhances the distortionary effect by bring down the cost of making an optimistic distortion, so the increase in forecast variance is higher here with optimism (3.74%) than previously without optimism (1.64%).

The effect of competition on information quality is more nuanced with the optimism incentive in place. Overall, competition, created by the reward for the most accurate analyst, is extremely effective in disciplining optimism. It reduces forecast error dramatically at only a small cost of making the forecasts more noisy. However, when I break down the improvement by level of competition, I find that the biggest improvement is actually generated with 2 analysts. With more rivals, analysts distort their forecasts to differentiate themselves, just as in the case without optimism. This implies that while some competition is desirable in the financial market to address the optimism incentive, a lot of competition impairs the quality of information by making it less accurate and more noisy.

8 Conclusion

In this paper, I estimate the incentives of security analysts to study how competition affects the quality of information in the financial market.

Analysts are rewarded for relative accuracy, that is, issuing forecasts that are closer to the truth than those of their rivals. This reward generates competition, which leads to two opposing effects. On the one hand, analysts overweigh their private signals in forecasts in order to differentiate and outperform their rivals, resulting in higher forecast error and variance. I call this the *distortionary effect*. On the other hand, if analysts are also rewarded for optimism, they will forecast less optimistically to differentiate, so competition will have *disciplinary effect* on optimism and reduce forecast error at a cost of higher forecast variance.

I find that the disciplinary effect dominates in the current market, implying a positive effect of competition on information quality overall. The estimated analyst payoff suggests a huge reward for outforming rivals in covering one security, as well as a significant reward for optimism. Counterfactual experiments show that the reward for relative accuracy contributes to a 33.37% decrease in analysts' forecast errors at a cost of a 3.74% increase in forecast variance. However, while competition improves information quality, the improvement is not necessarily greater with more rivals. As more analysts start to cover a security, the *distortionary effect* becomes more pronounced and forecasts become noisier and further from the truth.

This result provides a new perspective to regulating optimism in the financial industry, which has been the focus of most regulations on sell-side research. Existing measures try to restrict optimism for each analyst *independently* by mandating disclosure of conflict of interest and prohibiting ties between their compensation and investment banking revenues. This paper shows that the competition for better relative accuracy has contained optimism far more effectively than the independent reward for absolute accuracy. Therefore, perhaps a new and more effective way to regulate optimism would be to optimize the competition environment in sell-side research, encouraging competition for securities covered by only one analyst and discouraging over-coverage of popular securities.

More broadly, experts in many markets face incentive to outperform each other as well as to bias their information in a systematic direction. For example, media outlets may benefit both from providing more accurate news reports and telling stories that accomodate their readers' political preference. When approchaed by a home seller, real estate brokers may want to make not only an accurate evaluation that will be achiveable in the market, but also a high evaluation that will be attractive to the seller. The notion that competition has both a distortionary and a disciplinary effect is widely applicable and more analyses need to be done to understand the nuanced effect of competition in these markets.

Methodologically, I contribute to the identification and estimation of common value models. I structurally estimate a contest model where analysts with asymmetric information simultaneously make forecasts on a firm's earnings. Because analysts' payoff depends on relative accuracy, this model shares a defining characteristic of common value auctions: rivals' signals are relevant to players' assessment of their own payoff. An important challenge for this class of models is to identify between signals and payoffs. To tackle the challenge, I use the variation in the number of analysts in a contest and the resulting variation in forecast strategy to identify payoff. And being able to observe firms' true earnings also helps with identification as it allows me to compute the trade-offs with different forecast strategies. This identification method is similar to using variation in the number of bidders and ex post valuation to identify common value auctions. With this paper, I extend the current knowledge about common value auctions to a broader class of common value models, an exciting area for future research.

A Supplementary Tables

B Prestige Measure Example

Table B2 provides a hypothetical example of how the prestige measure is computed. Suppose in a given year, A is from brokerage house α with 5 analysts and B is from brokerage house β with 8 analysts. Then B has higher prestige than A.

Equation (7) summarizes the computation procedure for any given analyst i, employed by brokerage house b, year t.

$$
\begin{aligned}
Prestige_{bt} = & \quad Quantile(\#\text{Employed Analysts in Brokerage House } \mathbf{b} \mid \text{Year } \mathbf{t}) \\
Prestige_{it} = & \begin{cases} High & \text{if } Prestige_{bt} \geq 0.9 \\ Low & \text{if } Prestige_{bt} < 0.9 \end{cases}
\end{aligned} \tag{7}
$$

C Equilibrium Existence with Finite State Space

I extend Proposition 3 of Ottaviani and Sørensen (2005) to incorporate absolute accuracy and optimism in the utility function.

Proposition. If Assumption 1 holds and $\mathcal{Z}$ and $\mathcal{S}$ are finite., there exists a pure strategy equilibrium.

Proof. Consider an arbitrary N. Following Ottaviani and Sørensen (2005), I apply results from Milgrom and Weber (1985). Their state space T_0 is $\mathcal{Z}$ and their action space A_i is $\mathcal{Z}$ in my model. As $\mathcal{Z}^2$ is finite and $-(X_i - Z)^2 + \gamma_o X_i$ is continuous in X_i and Z, by Extreme Value Theorem, a maxmimum and a minimum exist and can be attained at least once in $\mathcal{Z}^2$. Let $\overline{M} = \max_{X_i,Z} -(X_i - Z)^2 + \gamma_o X_i$ and $\underline{M} = \min_{X_i,Z} -(X_i - Z)^2 + \gamma_o X_i$. Then my payoff function

Table A1: Distribution of Analysts

Year	Number of Houses	Mean	Pctl(25)	Median	Pctl(75)
1984	113	12.593	3	8	16
1985	122	11.984	3	6.500	17
1986	121	12.769	3	7	17
1987	131	12.679	3	8	15.500
1988	143	10.748	2	7	12
1989	157	11.561	3	6	13
1990	163	10.479	2	5	14
1991	156	9.641	2	5	12.250
1992	167	8.641	2	5	10
1993	172	9.343	2	5	11
1994	171	10.246	2	5	13
1995	170	10.447	2	5	11.750
1996	180	10.239	2	5	12
1997	207	9.657	1	4	10.500
1998	216	9.903	2	4	10
1999	206	11.252	2	4	11.750
2000	204	11.833	2	4.500	12
2001	200	12.540	2	4.500	13
2002	197	13.310	2	5	12
2003	229	11.450	1	4	11
2004	247	10.506	1	3	11
2005	255	10.443	1	3	11
2006	239	11.021	1	3	12
2007	228	11.351	1	3	13
2008	224	11.188	1	4	13
2009	244	9.971	1	4	11
2010	236	10.869	1	5	12
2011	229	11.511	1	5	14
2012	219	11.607	1	4	13
2013	204	12.064	2	5	15
2014	198	12.141	1	5	14.750
2015	193	11.731	1	4	14
2016	198	11.187	1	4	12.750

Table B2: Hypothetical Example: Prestige Measure

Analyst	Brokerage House	Number of Analysts	Prestige
A	α	5	
B	β	8	Higher

is bounded between

$$[-\sum_{k=1}^{N}|\gamma_k(N)|+\underline{M},\sum_{k=1}^{N}|\gamma_k(N)|+\overline{M}].$$

By Proposition 1(a) of Milgrom and Weber (1985), their assumption R1 is satisfied. Conditional independence of signals and the finiteness of the signal space ensure that their assumption R2 is satisfied. Finally, a player's payoff function depends only on the state, i.e., the truth Z, and the vector of forecasts. By Theorem 4 of Milgrom and Weber (1985), there exists an equilibrium in pure strategies. ■

D Proof of Lemma 1

Assume without loss of generality that $Z \sim N(0,1)$, $S_i|Z \sim N(Z,1/\tau)$, and that analyst i has received a positive signal $S_i > 0$. In a winner-takes-all game, the reward for issuing a forecast with the lowest forecast error is a positive constant, so the first order condition at the posterior simplifies to,

$$\int_{\frac{\tau}{1+\tau}S_i}^{\infty}\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)dZ-\int_{-\infty}^{\frac{\tau}{1+\tau}S_i}\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)dZ=0$$

where $\Xi(\frac{\tau}{1+\tau}S_i|Z)=\frac{1+\tau}{\tau}[g(S_i|Z)+g(\frac{1+\tau}{\tau}(2Z-\frac{\tau}{1+\tau}S_i)|Z)]$.

$\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)$ can be simplified using normality

$$\begin{aligned}\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)=&\frac{1+\tau}{\tau}[g(S_i|Z)+g(\frac{1+\tau}{\tau}(2Z-\frac{\tau}{1+\tau}S_i)|Z)]f(Z|S_i)\\=&\frac{1+\tau}{\tau}\sqrt{\tau(1+\tau)}[\phi(\frac{Z-S_i}{\frac{1}{\sqrt{\tau}}})+\phi(\frac{\frac{2+\tau}{\tau}Z-S_i}{\frac{1}{\sqrt{\tau}}})]\phi(\frac{Z-\frac{\tau}{1+\tau}S_i}{\frac{1}{\sqrt{1+\tau}}})\\=&\frac{(1+\tau)^{\frac{3}{2}}}{2\tau^{\frac{1}{2}}\pi}\Big[\exp\big(-\frac{1}{2}[\tau(Z-S_i)^2+(1+\tau)(Z-\frac{\tau}{1+\tau}S_i)^2]\big)+\\&\exp\big(-\frac{1}{2}[\tau(\frac{2+\tau}{\tau}Z-S_i)^2+(1+\tau)(Z-\frac{\tau}{1+\tau}S_i)^2]\big)\Big]\end{aligned}$$

Let $t=Z-\frac{\tau}{1+\tau}S_i$.

$$\begin{aligned}\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i)=&\frac{(1+\tau)^{\frac{3}{2}}}{2\tau^{\frac{1}{2}}\pi}\Big[\exp\big(-\frac{1}{2}[\tau(t-\frac{1}{1+\tau}S_i)^2+(1+\tau)t^2]\big)+\\&\exp\big(-\frac{1}{2}[\tau(\frac{2+\tau}{\tau}t+\frac{1}{1+\tau}S_i)^2+(1+\tau)t^2]\big)\Big]\end{aligned}$$

Completing the squares within the exponential function, we can obtain

$$\Xi(\frac{\tau}{1+\tau}S_i|Z)f(Z|S_i) = \frac{(1+\tau)^{\frac{3}{2}}}{\sqrt{2\pi\tau}}\Big[\frac{1}{\sqrt{1+2\tau}}\exp\big(-\frac{1}{2}\frac{\tau}{(1+\tau)(1+2\tau)}S_i^2\big)\cdot\sqrt{1+2\tau}\phi(\frac{t-\frac{\tau}{(1+\tau)(1+2\tau)}S_i}{\frac{1}{\sqrt{1+2\tau}}})+$$
$$\frac{\sqrt{\tau}}{\sqrt{4+5\tau+2\tau^2}}\exp\big(-\frac{1}{2}\frac{\tau^2}{(1+\tau)(4+5\tau+2\tau^2)}S_i^2\big)\cdot\frac{\sqrt{4+5\tau+2\tau^2}}{\sqrt{\tau}}\phi(\frac{t+\frac{(2+\tau)\tau}{(1+\tau)(4+5\tau+2\tau^2)}S_i}{\sqrt{\frac{\tau}{4+5\tau+2\tau^2}}})\Big]$$

Then, the first order condition can be simplified to

$$\frac{(1+\tau)^{\frac{3}{2}}}{\sqrt{2\pi\tau}}\Big\{\frac{1}{\sqrt{1+2\tau}}\exp\big(-\frac{1}{2}\frac{\tau}{(1+\tau)(1+2\tau)}S_i^2\big)\cdot(2\Phi(\frac{\tau}{(1+\tau)\sqrt{1+2\tau}}S_i)-1)-$$
$$\frac{\sqrt{\tau}}{\sqrt{4+5\tau+2\tau^2}}\exp\big(-\frac{1}{2}\frac{\tau^2}{(1+\tau)(4+5\tau+2\tau^2)}S_i^2\big)\cdot(2\Phi(\frac{(2+\tau)\sqrt{\tau}}{(1+\tau)\sqrt{4+5\tau+2\tau^2}}S_i)-1)\Big\}.$$

Note that the first order condition is equal to 0 when $S_i = 0$ and increasing in S_i. Therefore, there exists $\bar{s}$ such that for $0 < S_i < \bar{s}$, the first order condition is positive and a deviation towards the signal, or exaggeration, increases expected utility for two players.

E Removal of Inverse Function in the First Order Condition

We can write the first order condition to be free of inverses with a change of variable, similar to the transformation of equation (5.4) to equation (5.5) in Ottaviani and Sørensen (2005). Let $Y = \beta^{*-1}(2Z - \beta^*(S_i))$. Then

$$Z = \frac{\beta^*(Y)+\beta^*(S_i)}{2}$$
$$dZ = \frac{1}{2}\beta^{*'}(Y)dY.$$

Also, note that since β^* is the equilibrium strategy, $X_i = \beta^*(S_i)$. Then we can write

$$\begin{aligned}
p(X_i, Z) =& \tilde{p}(S_i, Y) \\
=& 1 - |\int_{S_i}^{Y} g\left(S_j | \frac{\beta^*(Y)+\beta^*(S_i)}{2}\right) dS_j| \\
\Xi(X_i, Z, N, k) =& \tilde{\Xi}(S_i, Y, N, k) \\
=& [1 - 2 \cdot \mathbf{1}(S_i > Y)][\frac{g\left(S_i | \frac{\beta^*(Y)+\beta^*(S_i)}{2}\right)}{\beta^{*\prime}(S_i)} + \frac{g\left(Y | \frac{\beta^*(Y)+\beta^*(S_i)}{2}\right)}{\beta^{*\prime}(Y)}] \\
& \big[\mathbf{1}(N > k)(N-k)\tilde{p}(S_i, Y)^{N-k-1}(1 - \tilde{p}(S_i, Y))^{k-1} - \\
& \mathbf{1}(k > 1)\tilde{p}(S_i, Y)^{N-k}(k-1)(1 - \tilde{p}(S_i, Y))^{k-2}\big].
\end{aligned}$$

The first order condition can be written as

$$\begin{aligned}
&\sum_{k=1}^{N} \gamma_k(N) \binom{N-1}{N-k} \int_{-\infty}^{\infty} \tilde{\Xi}(S_i, Y, N, k) f(\frac{\beta^*(Y)+\beta^*(S_i)}{2}|S_i)\frac{1}{2}\beta^{*\prime}(Y)dY \\
&- 2[\beta^*(S_i) - \mathbb{E}(Z|S_i)] + \gamma_o = 0, \qquad (8)
\end{aligned}$$

which is free of the inverse function of the equilibrium strategy.

F Details of Equilibrium Computation

In this section, I provide the details of the equilibrium computation and the experimentation with different polynomial orders as the pre-specified function family in the projection method. I show that a linear strategy is a great approximation to the equilibrium strategy.

I choose the representative signals (collocation points) to be 20 finite and evenly distributed quantiles between 0 and 1. Then, under the parametric assumption that $S_i = Z + \epsilon_i$ where $Z \sim N(0,1)$ and $\epsilon_i \sim N(0, 1/\tau)$, these 20 collocation points are

$$\left\{\sqrt{1+\frac{1}{\tau}}\Phi^{-1}(\frac{1}{21}), \sqrt{1+\frac{1}{\tau}}\Phi^{-1}(\frac{2}{21}), \cdots, \sqrt{1+\frac{1}{\tau}}\Phi^{-1}(\frac{20}{21})\right\}.$$

The equilibria are computed following the the algorithm described in Section 5.2. I experiment with polynomials up to the 7th order as the pre-specified function at various model parameters and compare the performance of these different polynomial orders using the residual function, i.e., the sum of squared first order conditions. To illustrate, Figure F plots the approximated equilibria with 10 players using polynomials up to the 7th order at the parameter estimate 6.3. Table F3 presents their corresponding residual functions and computation time with Matlab on a Windows 10 machine with 8GM RAM and Intel Core i5 @ 2.6GHz.

Figure 3: Equilibrium Strategy Approximated With Polynomials up to the 7th Order

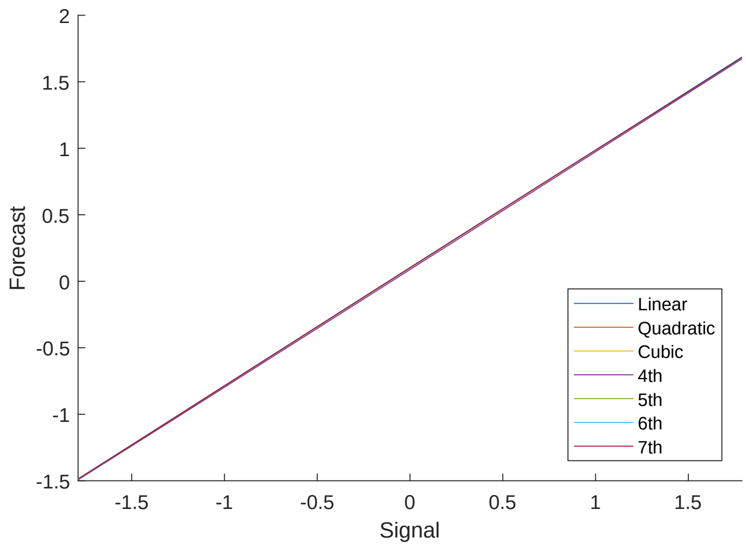


Table F3: Equilibrium Approximation Performance of Polynomials up to the 7th Order

Polynomial Order	Residual Function	Computation Time
1	1.53E-02	1.77
2	3.02E-04	4.60
3	1.61E-04	6.83
4	2.98E-01	23.24
5	3.58E-10	24.98
6	1.28E-05	17.34
7	7.64E-05	42.04

As we can see, higher polynomial orders generally achieve smaller residual functions, but the estimated polynomials are very close to the linear function and cost much more computation time. This is observed consistently at different model parameters. Therefore, I use a linear function to approximate the equilibrium in both estimation and counterfactual analysis.

G Interpolation of Equilibrium Strategy

I interpolate the equilibrium forecast strategy to alleviate the computation burden as the following. First, I assume that the coefficients in the forecasts are a linear function of $\log(N)$, i.e., $X_i = (a_0 + a_1 \log(N))S_i, +(b_0 + b_1 \log(N))$. I initialize the values of (a_0, a_1, b_0, b_1). Then, for a given set of parameter values, I compute the first order conditions at the 5th percentiles of the number of analysts, with strategies computed from this equation. After that, I minimize the sum of squared first order conditions to find the equilibrium strategy. These interpolated equilibrium strategies demonstrate similar properties as the equilibrium strategies computed individually for each number of analysts.

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