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# HW2: Histogram and Spatial Filtering Report

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## 1 Exercises

Please answer the following questions in the report.

### 1.1 Histogram Equalization (15 Points)

Suppose that you have performed histogram equalization on a digital image. Will a second pass of histogram equalization (on the histogram-equalized image) produce exactly the same result as the first pass? Prove your answer.

**My answer:**

一般情况下，第二趟直方图均衡化得到的结果不会跟第一趟的一样。但有例外，比如图像每一点得像素值都一样的时候，无论多少趟均衡化，结果都是一样的。

证明如下：

书本已经证明，在连续的情况下，原图像的像素值，在经过均衡化变化，也就是积分变换后，得到的输出图像的像素值的pdf是均衡化的，等于  $1/(L - 1)$ 。得到的直方图，也是理想均衡的。但是，实际数字图像，处理的是离散的值，而且是采样一些点进行处理，因此，对输入图像的离散值，得到的输出图像的pdf也是近似均衡的pdf。

对近似均衡的输出图像再次进行均衡化处理，第二次得到的输出图像，那就是更均衡的输出图像了。

因此，一般情况下，第二趟直方图均衡化得到的结果不会跟第一趟的一样。

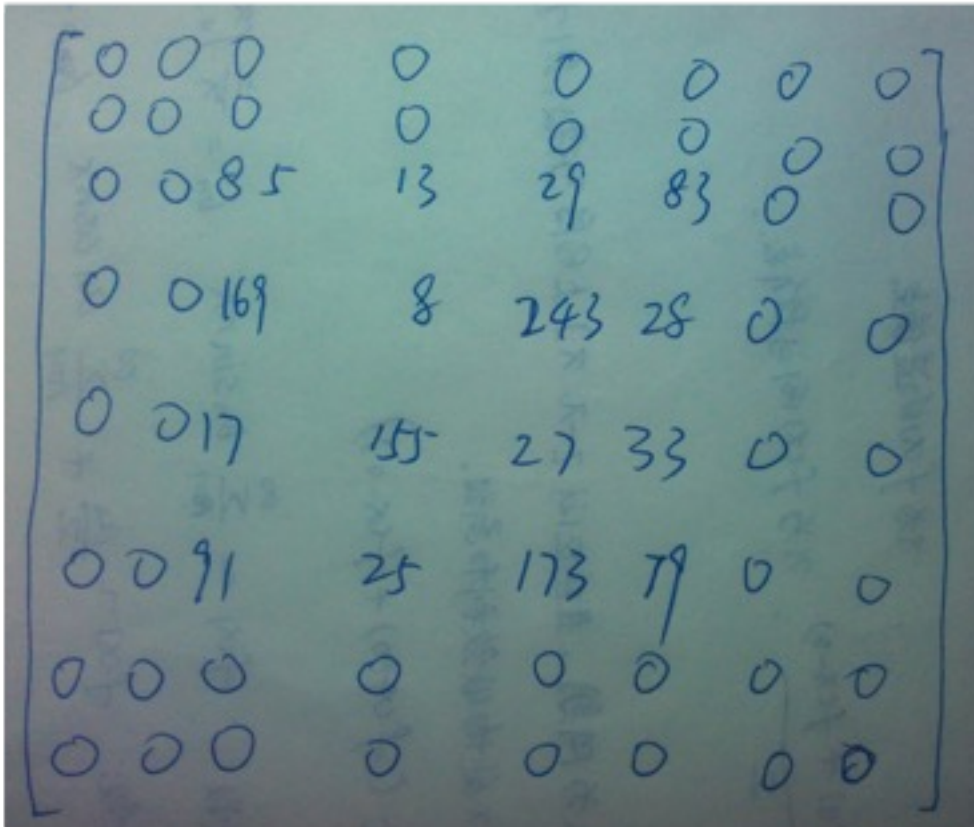
## 1.2 Spatial Filtering (20 Points)

$$\text{Image : } \begin{bmatrix} 85 & 13 & 29 & 83 \\ 169 & 8 & 243 & 28 \\ 17 & 155 & 27 & 33 \\ 91 & 25 & 173 & 79 \end{bmatrix} \quad \text{Filter : } \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Consider a 4×4 gray image and a 3×3 filter:

1. Convolve the gray image with the given filter with zero-padding, and show your result (whose size should be 4 × 4). (7 Points)

**My answer:**

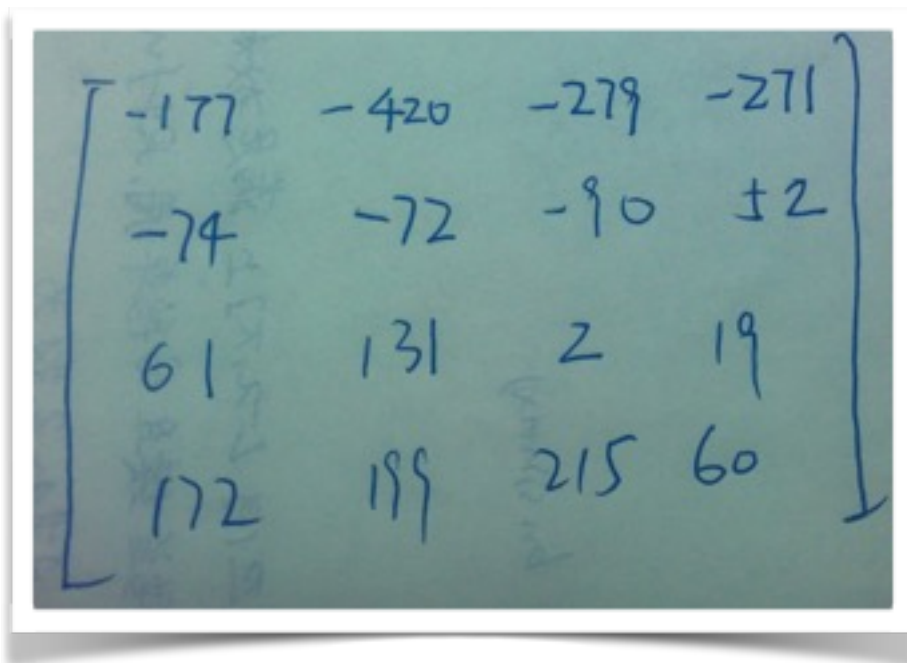


The image shows a handwritten 8x8 matrix representing the 4x4 image padded with zeros to an 8x8 size. The padding is 2 pixels on all sides. The original 4x4 image values are placed in the center 4x4 region of the 8x8 matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 85 & 13 & 29 & 83 & 0 & 0 \\ 0 & 0 & 169 & 8 & 243 & 28 & 0 & 0 \\ 0 & 0 & 17 & 155 & 27 & 33 & 0 & 0 \\ 0 & 0 & 91 & 25 & 173 & 79 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

zero-padding

计算结果：


$$\begin{bmatrix} -177 & -420 & -279 & -271 \\ -74 & -72 & -90 & 12 \\ 61 & 131 & 2 & 19 \\ 172 & 199 & 215 & 60 \end{bmatrix}$$

2. Discuss the meanings of positive values and negative values in your convolution result respectively. (8 Points)

**My answer:**

题目所给的滤波算子是x方向上的一个滤波算子。将原图像做了一遍处理后，很明显看出，结果分成了第一二行是负数，第三四行是正数，起到了预期的突出边缘的作用。该滤波器是x方向上的边缘检测滤波器。

正负数的意义只是表明整体图像的边缘方向。由于在第一二行全是负数，第三四行全是正数，因此表明在x方向上，也就是 $90^\circ$ 方向上，原图像具有明显的边缘。

3. Describe some applications of the given filter based on your own knowledge. (5 Points)

如2所诉，该滤波器可以用于x方向上的边缘检测。

## 2 Programming Tasks

Write programs to finish the following three tasks, and answer questions in your report. Don't forget to submit all relevant codes.

### 2.1 Pre-requirement

**Input** Please download the archive “hw2.zip”, unzip it and choose the image corresponding to the last two digits of your student ID. This image is the initial input of your programming tasks in HW2. For example, if your student ID is “13110349”, then you should take “49.png” as your input. You can convert the image format (to BMP, JPEG, ...) via Photoshop if necessary.

Make sure that you have selected the correct image. Misusing images may result in zero scores.

**Language** Any language is allowed.

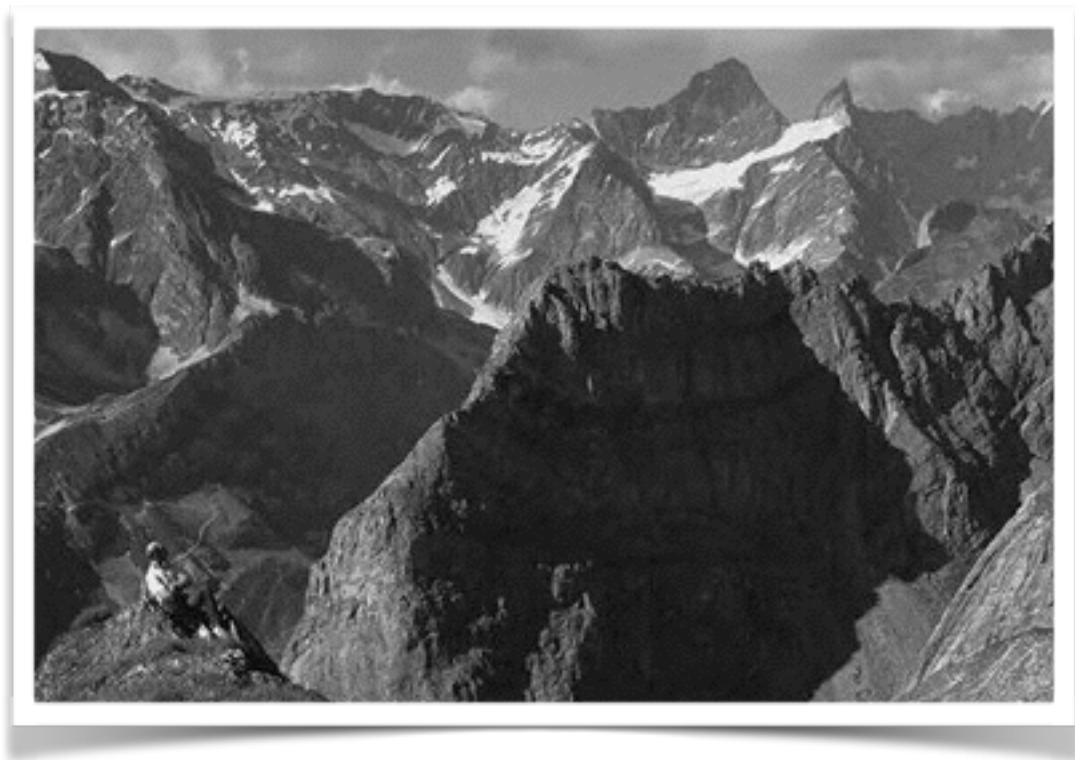
**Others** There remain some issues that you should pay attention to:

1. You can use third-party packages for operating images. But you should manually implement your programming tasks. For example, though you can use “imread” of Matlab to load an image, you cannot invoke “conv2” or “filter2” or so forth of Matlab for spatial filtering.
2. Good UX (User Experience) is encouraged, but will only bring you negligible bonuses. Please don't spend too much time on it, since this is not an HCI course.
3. Keep your codes clean and well-documented. Bad coding styles will result in 20% penalty at most.

## 2.2 Histogram Equalization (35 Points)

Write a function that applies histogram equalization on a gray scale image. The function prototype is “equalize hist(input img) → output img”, returning a gray scale image whose histogram is approximately flat. You can modify the prototype if necessary.

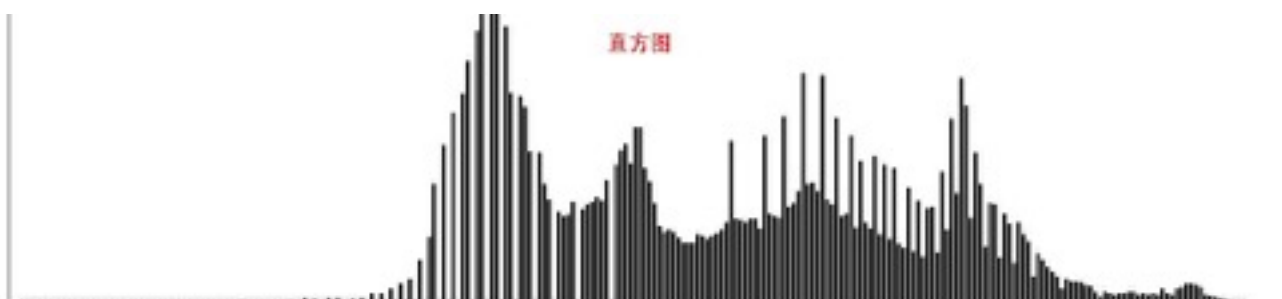
我的学号是13331270，因此选择70.png图片。



For the report, please load your input image and use your program to:

1. Compute and display its histogram. Manually paste the histogram on your report. Note: You must compute the histogram by yourself, but existing APIs can be used for display. (5 Points)

计算得到直方图如下：

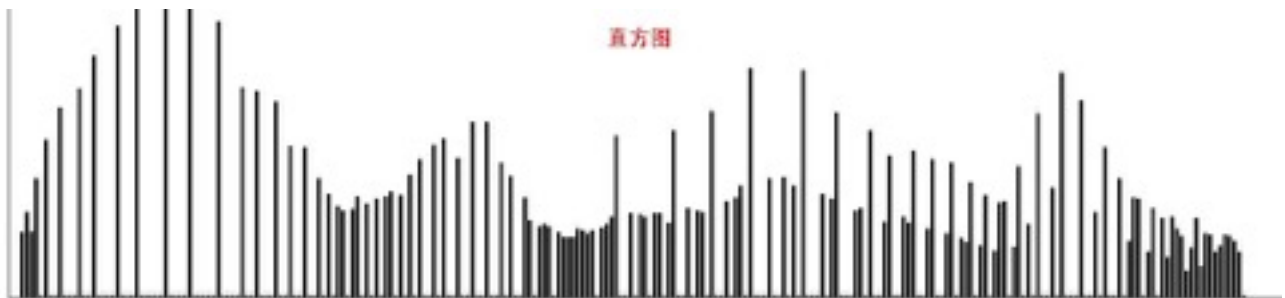


2. Equalize its histogram. Paste the histogram-equalized result and the corresponding histogram on your report. (10 Points)

直方图均衡化后的结果：



对应的直方图是：



3. Analyze your histogram-equalized result in less than 1 page. (8 Points)

在原图像的直方图中可以看出，原图像大部分的像素值处于80-200区间，对比度相对较小。原图像看起来比较灰暗。

执行了直方图均衡化后，对比度明显变大了。得到的直方图也分散在大部分的区域。因此图像得到了增强，视觉效果变好。

4. Detailedly discuss how you implement the histogram equalization operation, i.e., the “equalize\_hist” function, in less than 2 pages. Please focus on the algorithm part. Don’t widely copy/paste your codes in the report, since your codes are also submitted. (12 Points)

以下是直方图均衡的核心代码：

```
public int[] getEqualizedArray(int width, int height, int[] grayArray) {
    int[] equalizedArray = new int[width * height];
    int[] histogramArray = getHistogramArray(width, height, grayArray);
    int[] hash = new int[256];

    for (int i = 0; i < 256; i++) {
        double rate = 0;
        for (int j = 0; j < i; j++) {
            rate += (double)(histogramArray[j]) / (double)(width * height);
        }
        hash[i] = (int)(255 * rate);
    }

    for (int i = 0; i < grayArray.length; i++) {
        equalizedArray[i] = hash[grayArray[i]];
    }

    return equalizedArray;
}
```

首先，取得整个图像每个像素点的灰度值数组。然后统计得到直方图数据，直方图数据是，每一个像素值所占的比例。

接着，利用一个hash数组来存储每个点得一一对应的关系。

hash数组是通过0-255范围的每一个像素值，计算累积概率然后乘以255得到的。

然后根据hash数组处理每一个原图像的像素值，最后得到均衡化的目的图像。



## 2.3 Spatial Filtering (30 Points)

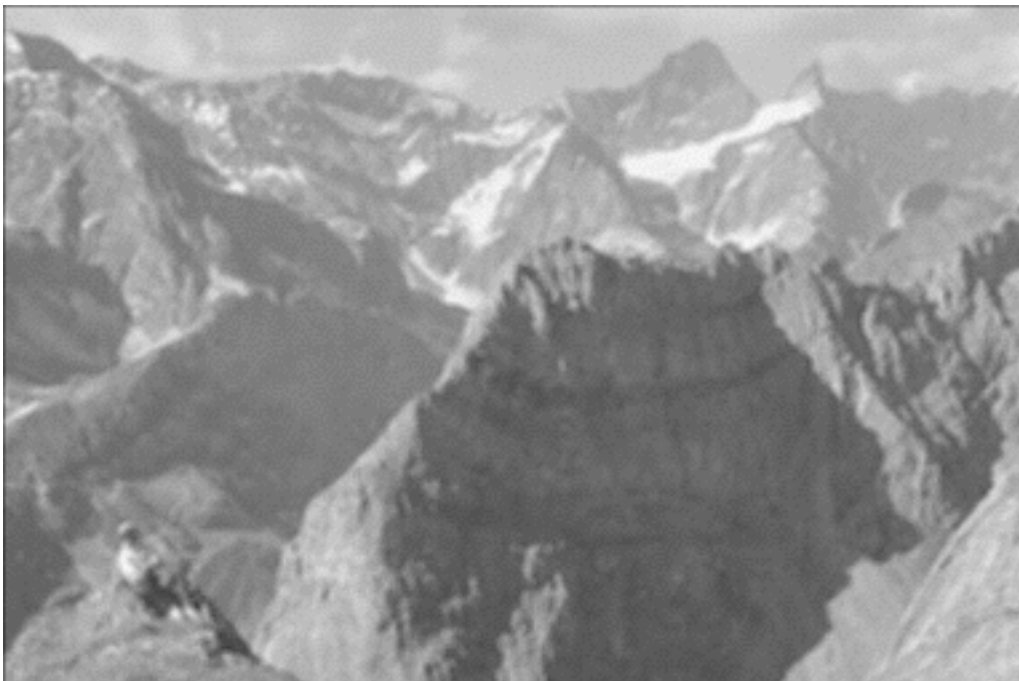
Write a function that performs spatial filtering on a gray scale image. The function prototype is “`filter2d(input img, filter) → output img`”, where “filter” is the given filter. Modify the prototype if necessary.

For the report, please load your input image and use your “`filter2d`” function to:

1. Smooth your input image with  $3 \times 3$ ,  $7 \times 7$  and  $11 \times 11$  averaging filters respectively. Paste your three results on the report. (9 Points)

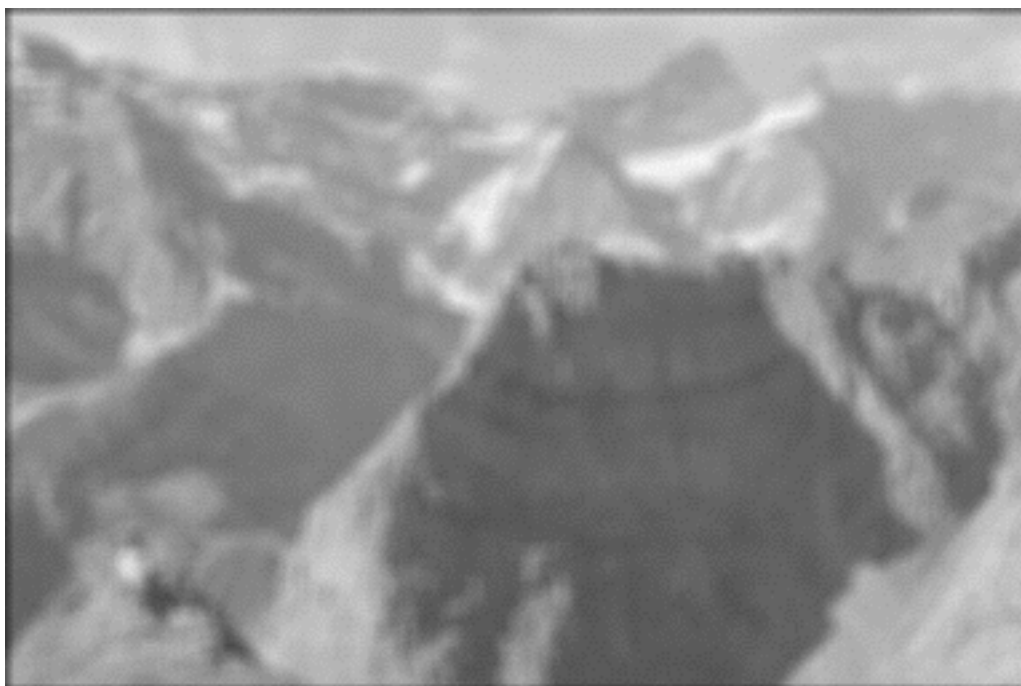
My answer:

3\*3

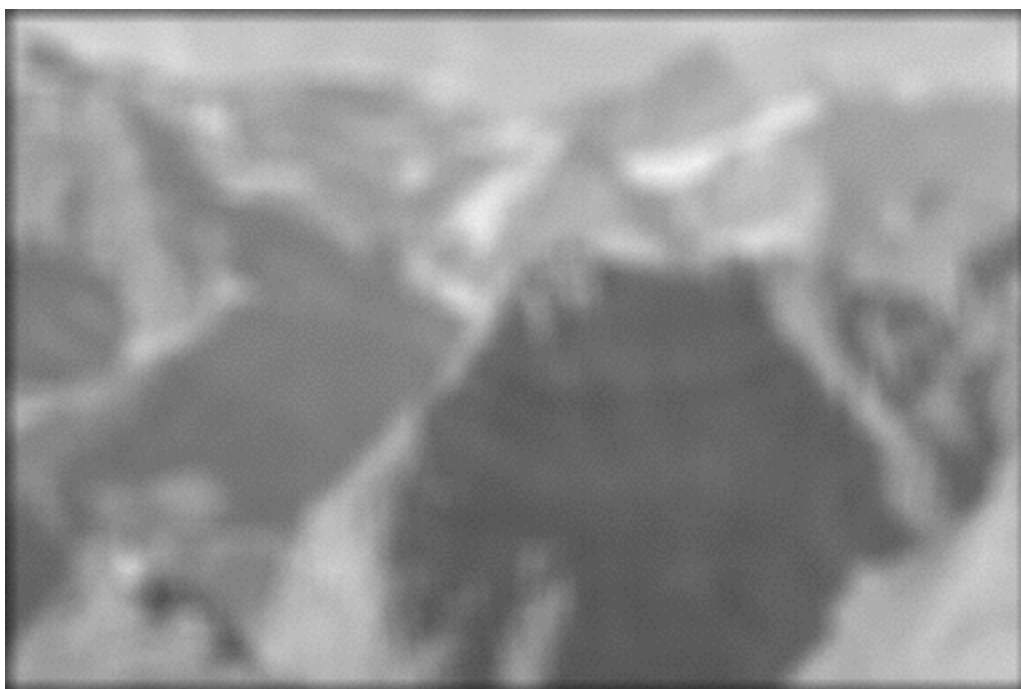




7\*7

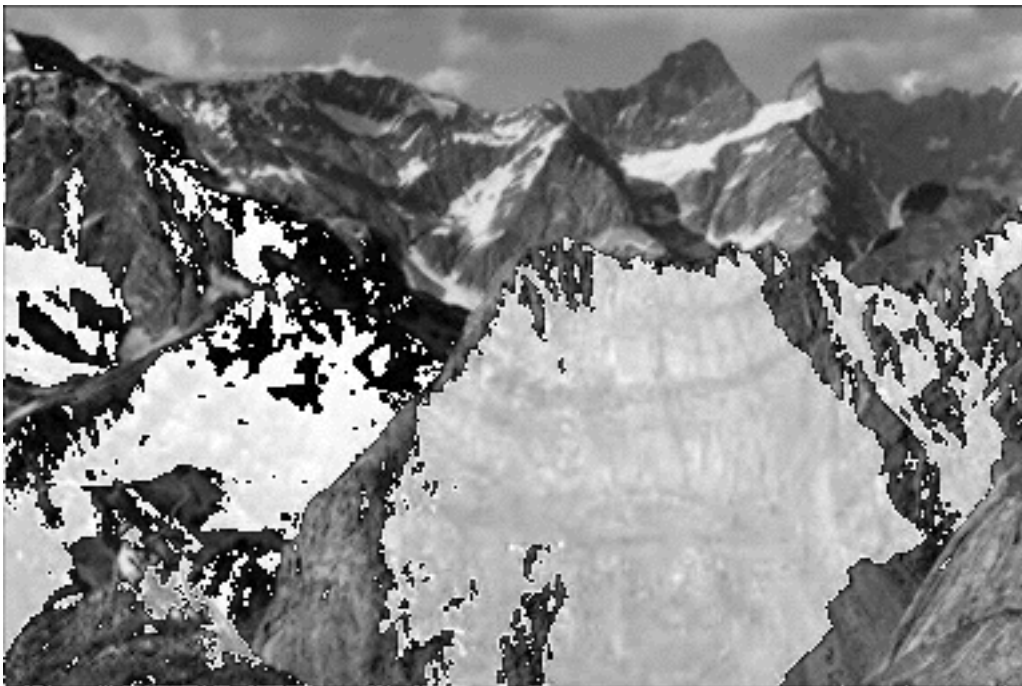


11\*11



2. Sharpen your input image with a  $3 \times 3$  Laplacian filter (There are four variants of Laplacian in Fig. 3.37 of the textbook. Pick any one you like.) and then paste the result. In addition, briefly discuss why Laplacian filter can be used for sharpening. (6 Points)

**My answer:**



由于拉普拉斯是一种微分算子，因此其应用强调的是图像中灰度的突变，并不强调灰度级缓慢变化的区域。这将产生把浅灰色边线和突变点叠加到暗色背景中的图像。将原图像和拉普拉斯图喜爱昂叠加在一起的方法，可以复原背景特性并保持拉普拉斯锐化处理的效果。

3. Perform high-boost-filtering (i.e.,  $g(x,y)=f(x,y)+k * g_{\text{mask}}(x,y)$ , see Eq.(3.6-9) of the textbook for other details) on your input image. The averaging part of the process should be done using the filter in Fig. 3.32(a) of the textbook. Choose a  $k$  (the weight in Eq. (3.6-9)) as you see fit. Write down the selected  $k$  and paste your result on the report. (5 Points)

**My answer:**

**$k = 3$ .**



4. Detailedly discuss how you implement the spatial filtering operation, i.e., the “filter2d” function, in less than 2 pages. (10 Points)

以上的全部功能已经写成了一个java应用，如果TA或者老师有时间不妨看看🍷。该应用可以处理任意的灰度图片。

界面如下：



平滑空间滤波器：

方法是，利用平滑滤波器模板（ $3 \times 3$ ， $7 \times 7$ ， $11 \times 11$ ）处理每一个像素，求和取平均。如果是边界值，则以0代替。

拉普拉斯算子锐化：

方法是，利用拉普拉斯滤波器模板，处理每一个像素值，然后权值求和，把处理得到的数据存在数组中，然后再用原图像加上处理后的数组。

高提升滤波：

1. 模糊原图像， 2. 从原图像中减去模糊图像， 3. 将模板加到源图像上。