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Q1)

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & 8 \\ 0 & -3 & -3 \end{bmatrix}, \quad Ax = 0 \implies \begin{bmatrix} 2 & 2 & 4 & 0 \\ -4 & -4 & -8 & 0 \\ 0 & -3 & -3 & 0 \end{bmatrix}$$

$R_1 \times \frac{1}{2}$ &

then $R_2 + 4R_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 0 \end{bmatrix}$$

Interchange R_2 & R_3

then $R_3 \times \frac{1}{3}$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Downarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \implies \underbrace{\begin{bmatrix} -x_3 \\ -x_3 \\ 0 \end{bmatrix} \implies x_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}}_{\text{parametric vector form}}$$

$x_1 = -x_3$

$x_2 = -x_3$

x_3 = free variable

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$Ax = b \longrightarrow \begin{bmatrix} 2 & 2 & 4 & 8 \\ -4 & -4 & -8 & -16 \\ 0 & -3 & -3 & 12 \end{bmatrix}$ $R_1 \times \frac{1}{2}$ & then $R_2 + 4R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & -3 & 12 \end{bmatrix} \xrightarrow[\text{then } R_2 \times -\frac{1}{3}]{\text{Interchange } R_2 \,\&\, R_3} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 1 & 8 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_3 = 8 \rightarrow \boxed{x_1 = 8 - x_3}$

$x_2 + x_3 = 4 \longrightarrow \boxed{x_2 = 4 - x_3}$

$x_3 \Rightarrow$ free variable

$$x = x_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$$

The solution of $Ax = b$ & $Ax = 0$ both contain free variable x_3 with $\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ but $Ax = b$ has $\begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$

so the 2 are parallel.

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Q2. $\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ h \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ -2 & -6 & h & 0 \\ 4 & 7 & 2 & 0 \end{bmatrix} \xrightarrow[R_3 - 2R_1]{R_2 + R_1} \begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -2 & h-2 & 0 \\ 0 & -1 & 6 & 0 \end{bmatrix}$$

$R_1 \times \frac{1}{2}$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -2 & h-2 & 0 \\ 0 & -1 & 6 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -2 & h-2 & 0 \\ 0 & 0 & 14-h & 0 \end{bmatrix}$$

$$\Downarrow$$

$$h = 14$$

This vectors are linearly independent.

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Q3 $e_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\end{bmatrix}$ & $y_1 = \begin{bmatrix}2\\5\end{bmatrix}$, $y_2 = \begin{bmatrix}-1\\6\end{bmatrix}$

★ Image of $\begin{bmatrix}5\\-3\end{bmatrix} \Rightarrow x = 5\begin{bmatrix}1\\0\end{bmatrix} - 3\begin{bmatrix}0\\1\end{bmatrix}$

1

$$x = 5e_1 - 3e_2$$

$$T(x) = T(5e_1 - 3e_2)$$

$$T(x) = 5T(e_1) - 3T(e_2)$$

As e_1 maps into y_1 & e_2 maps into y_2.

$$T(x) = 5y_1 - 3y_2.$$

STANDARD MATRIX OF TRANSFORMATION

$$Ax = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

$$5\begin{bmatrix}2\\5\end{bmatrix} - 3\begin{bmatrix}-1\\6\end{bmatrix} \Rightarrow \begin{bmatrix}10+13\\25-18\end{bmatrix}$$

$$= \begin{bmatrix}13\\7\end{bmatrix}$$

★ Image of $\begin{bmatrix}x_1\\x_2\end{bmatrix}$:- we know that $x = x_1\begin{bmatrix}1\\0\end{bmatrix} + x_2\begin{bmatrix}0\\1\end{bmatrix}$

i.e $x_1e_1 + x_2e_2 = x$.

thus $T(x) = T(x_1e_1 + x_2e_2) \Rightarrow T(x) = x_1T(e_1) + x_2T(e_2)$

$$T(x) = x_1y_1 + x_2y_2$$

1

$$x_1\begin{bmatrix}2\\5\end{bmatrix} + x_2\begin{bmatrix}-1\\6\end{bmatrix} \Rightarrow \begin{bmatrix}2x_1 - x_2\\5x_1 + 6x_2\end{bmatrix} \rightarrow A$$

$$A = \begin{bmatrix} x_1\begin{pmatrix}2\\5\end{pmatrix} + x_2\begin{pmatrix}-1\\6\end{pmatrix} \end{bmatrix}$$

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Q3 (2) $T(x_1, x_2) = (x_1 - 2|x_2|, 2x_1 - 5x_2)$

let $x_1 = 0$, $x_2 = 1$
& $c = -1$

$T(cx) = cT(x)$

$T(0,1) = [(0-2, 0-5)]$

$T(0,1) = (0-2, -5)$

$T(0,1) \neq (-2, -5)$

$\Longrightarrow$ This doesnot satisfy the $T(cx) = cT(x)$ property so this TRANSFORMATION IS NOT LINEAR.

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Q3 $e_1 = \begin{bmatrix} 1 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \end{bmatrix}$ ... $\begin{bmatrix} 2 \end{bmatrix}$ ... $\begin{bmatrix} - \\ 6 \end{bmatrix}$

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Q4 $T(x_1, x_2) = (x_1 - 2x_2, 4x_1 + 5x_2)$ & $T(x) = (1, -)$

$$Ax = \begin{bmatrix} x_1 - 2x_2 \\ 4x_1 + 5x_2 \end{bmatrix} \Rightarrow x_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & 5 & 3 \end{bmatrix} \xrightarrow{R_2 - 4R_1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 13 & -1 \end{bmatrix}$$

$$\Big\downarrow R_2 \times +\frac{1}{13}$$

$$\begin{bmatrix} 1 & 0 & 11/13 \\ 0 & 1 & -1/13 \end{bmatrix} \xleftarrow{R_1 + 2R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -\frac{1}{13} \end{bmatrix}$$

$$\boxed{x_1 = 11/13}$$

$$\boxed{x_2 = -1/13} \quad \therefore \quad x = \begin{pmatrix} 11/13 \\ -1/13 \end{pmatrix}$$

T is one-to-one transformation as the columns of A are linearly independent.

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Q5

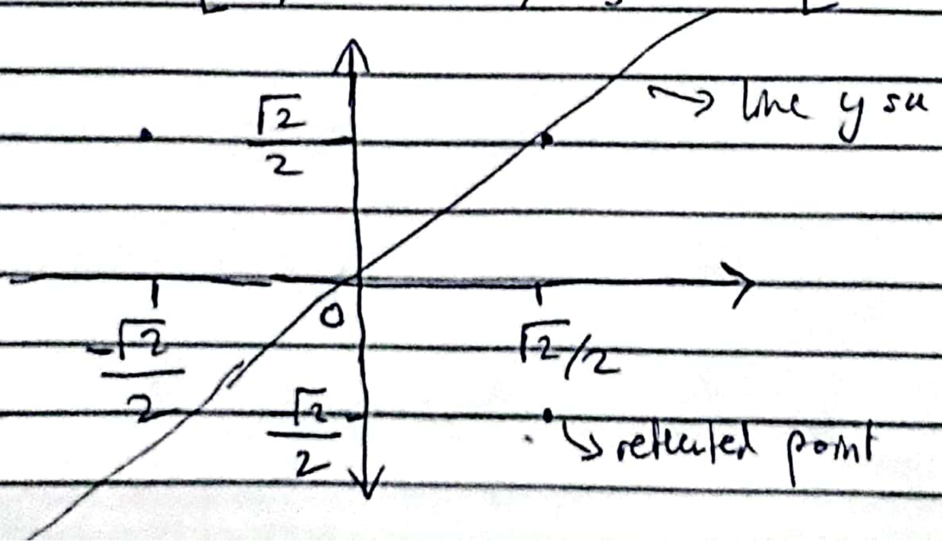
$$T(x) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

① rotates every point about origin anticlockwise through $\theta = \pi/4$.

$$\begin{bmatrix} \cos \pi/4 & -\sin \pi/4 \\ \sin \pi/4 & \cos \pi/4 \end{bmatrix} \longrightarrow \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

② Reflecting all points about line $y = x$.

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}$$

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③ Rotation of points about origin anticlockwise to angle $\pi/6$

$$\begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix} \Longrightarrow \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \Longrightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} - \frac{1}{2} \\ \frac{\sqrt{2}}{2} + \frac{1}{2} & \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \end{bmatrix}$$