CALCULUS Assignment # 2

Name :- HAMMAD JAVAID Section :- DS-M Roll number :- i21-1661

Q#1 (a) $\lim_{x \to -7} f(x)$

Ans:- ~~limit exists~~ limit doesnot exist as LHL $\neq$ RHL

(b) $\lim_{x \to -3} f(x)$

Ans:- $\lim_{x \to -3^+} f(x) \Rightarrow +\infty$

$\lim_{x \to -3^-} f(x) \Rightarrow -\infty$

(c) $\lim_{x \to 0} f(x)$

Ans:- limit exists $\longrightarrow 0$

(d) $\lim_{x \to 6^-} f(x)$

Ans:- limit exists $\longrightarrow 1$

(e) $\lim_{x \to 6^+} f(x)$

Ans:- limit exists $\longrightarrow 1$

(f) Vertical asymptotes exist at $x = -3$,

$x = 2$,

$x = 5$

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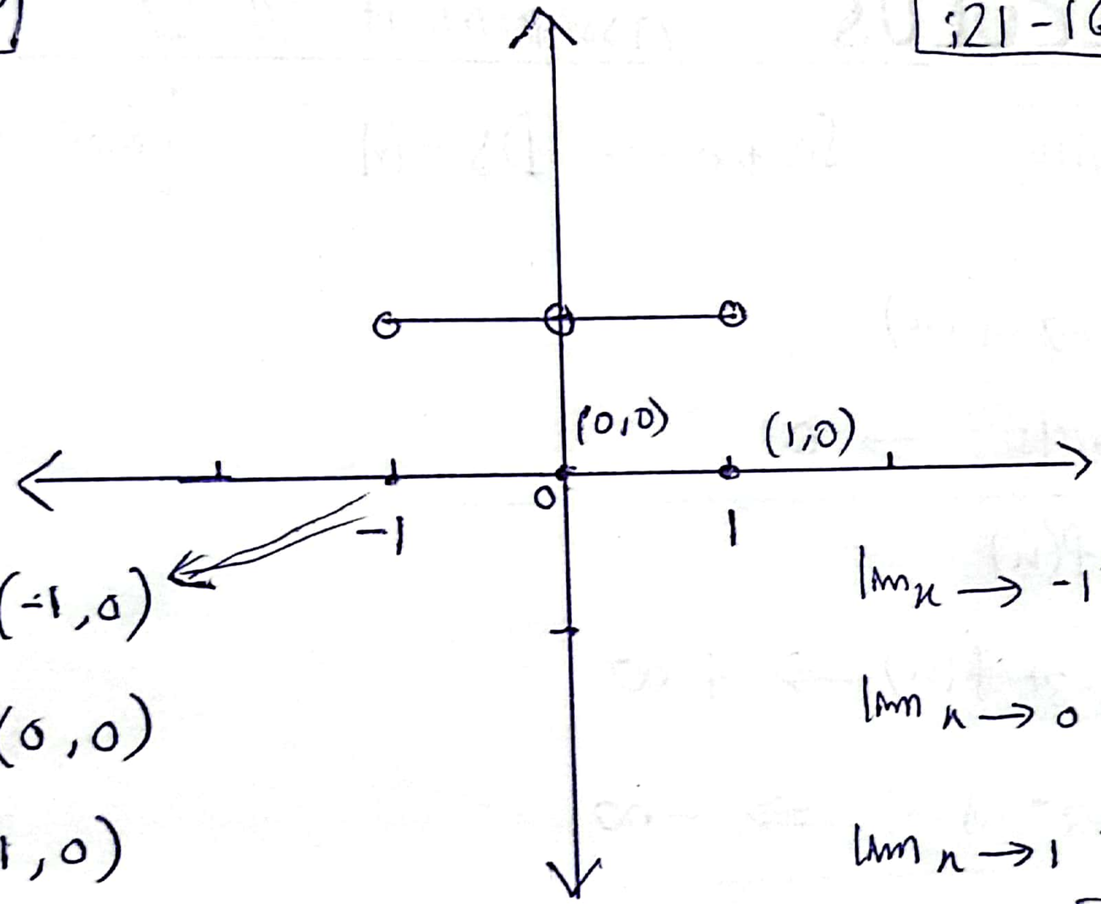
Q2



$f(-1) = 0 \Rightarrow (-1, 0)$

$f(0) = 0 \Rightarrow (0, 0)$

$f(1) = 0 \Rightarrow (1, 0)$

$\lim_{x \to -1^+} f(x) = 1$

$\lim_{x \to 0} f(x) = 1$

$\lim_{x \to 1} f(x) = 1$

represented with hollow circles.

~~By squeeze theorem~~

$\cancel{f(x) \leq g(x) \leq h(x)}$

Q3 (i) $\lim_{t \to 9} \frac{9-t}{\sqrt{3}-t} \Rightarrow \frac{9-9}{\sqrt{3}-9} \Rightarrow \frac{0}{\sqrt{3}-9} = 0$

(ii) $\lim_{y \to -\infty} \frac{2-y}{\sqrt{7+6y^2}} \Rightarrow \frac{\frac{2}{y} - \frac{y}{y}}{\sqrt{\frac{7}{y^2} + \frac{6y^2}{y^2}}} \Rightarrow \frac{\frac{2}{y} - 1}{\sqrt{\frac{7}{y^2} + 6}}$

$\lim_{y \to -\infty} \left(\frac{2}{y} - 1\right) \Rightarrow \frac{2}{-\infty} - 1 = 0 - 1 = -1$

$\lim_{y \to -\infty} \left(\sqrt{\frac{7}{y^2} + 6}\right) \Rightarrow \sqrt{\frac{7}{-\infty^2} + 6} \Rightarrow \sqrt{0+6} = \sqrt{6}$

Ans $\Rightarrow \frac{-1}{\sqrt{6}}$

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Q3 (iii) $\lim_{x\to\infty} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)$

$$\left[\frac{e^x + \frac{1}{e^x}}{e^x}\right] \div \left[\frac{e^x - \frac{1}{e^x}}{e^x}\right]$$

$$(1 + e^{-2x}) \div (1 - e^{-2x}) \Rightarrow \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

$$\lim_{x\to\infty} 1 + \frac{1}{e^{2\infty}} \Rightarrow 1 + 0$$

$$\Rightarrow \frac{1}{1} = 1$$

$$\lim_{x\to\infty} 1 - \frac{1}{e^{2\infty}} \Rightarrow 1 - 0$$

(iv) $\lim_{x\to\pi} \cos^2(x - \tan x)$

$\boxed{\text{Put Limit :}}$ $\cos^2(\pi - \tan\pi) \Rightarrow \cos^2(\pi - 0)$

As $\cos\pi = -1$ then

$(-1)^2 = 1$

$$\boxed{\therefore \lim_{x\to\pi} \cos^2(x - \tan x) = 1}$$

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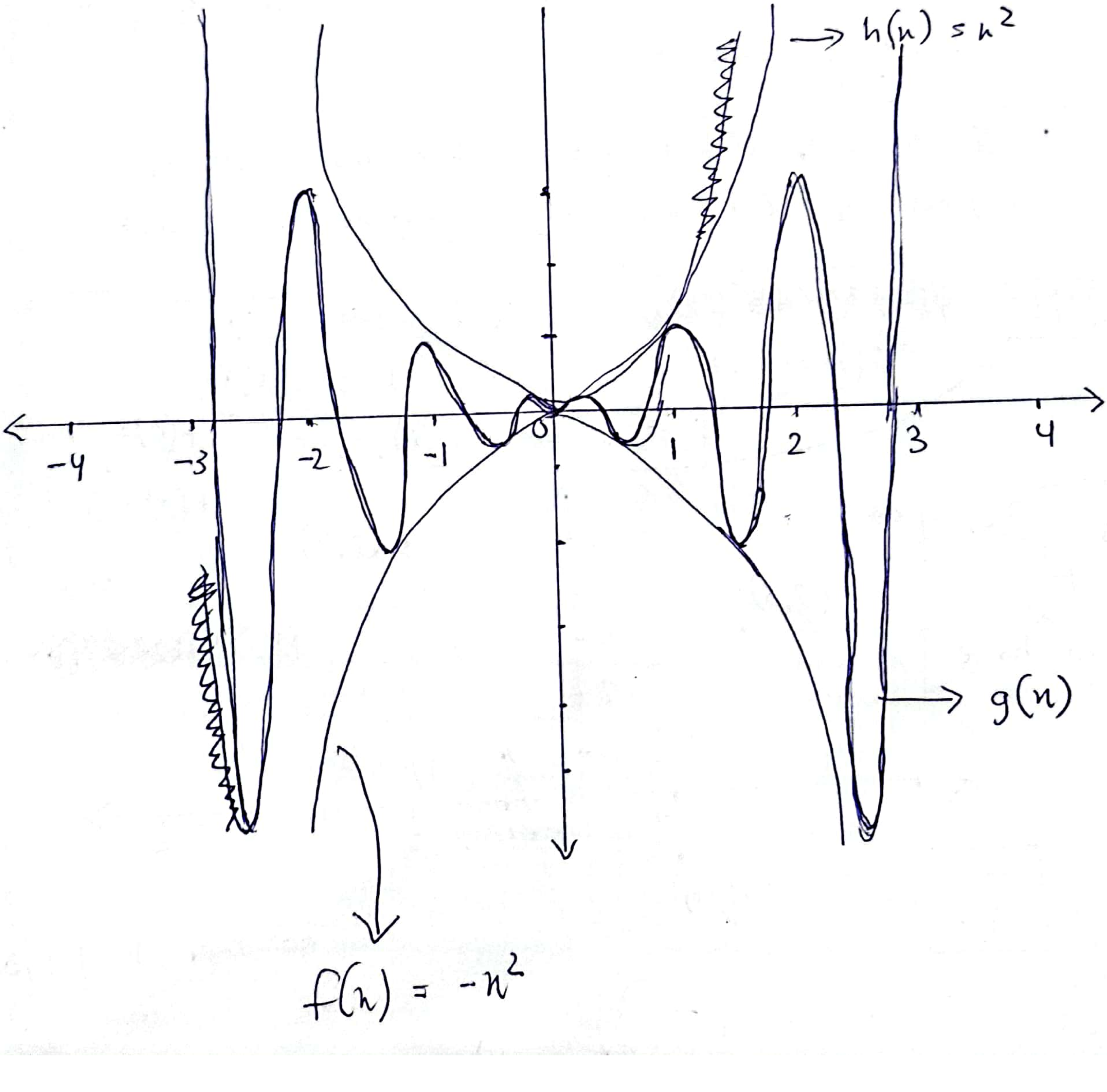
Q4 $\lim_{x \to 0} x^2 \cos 2\pi x = 0$

$f(x) = -x^2,$
$g(x) = x^2 \cos 2\pi x,$
$h(x) = x^2$

Put limit $0^2 \cdot \cos 2\pi \cdot 0 = 0$



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Q5

(1) As $g(x) \neq 0$, the vertical asymptotes of $\frac{f(x)}{g(x)}$ donot exist.

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(2) If we apply the limit, $\lim_{x \to +\infty}$ & $\lim_{x \to -\infty}$ and the Right hand side limit becomes equal to left hand side limit with some constant value. Then a horizontal asymptote of $\frac{f(x)}{g(x)}$ will exist.

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(3) If degree of $f(x)$ is greater than the degree of polynomial of $g(x)$, then oblique asymptote will exist.

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~~$f(2) = 5 \Rightarrow (2,5)$~~

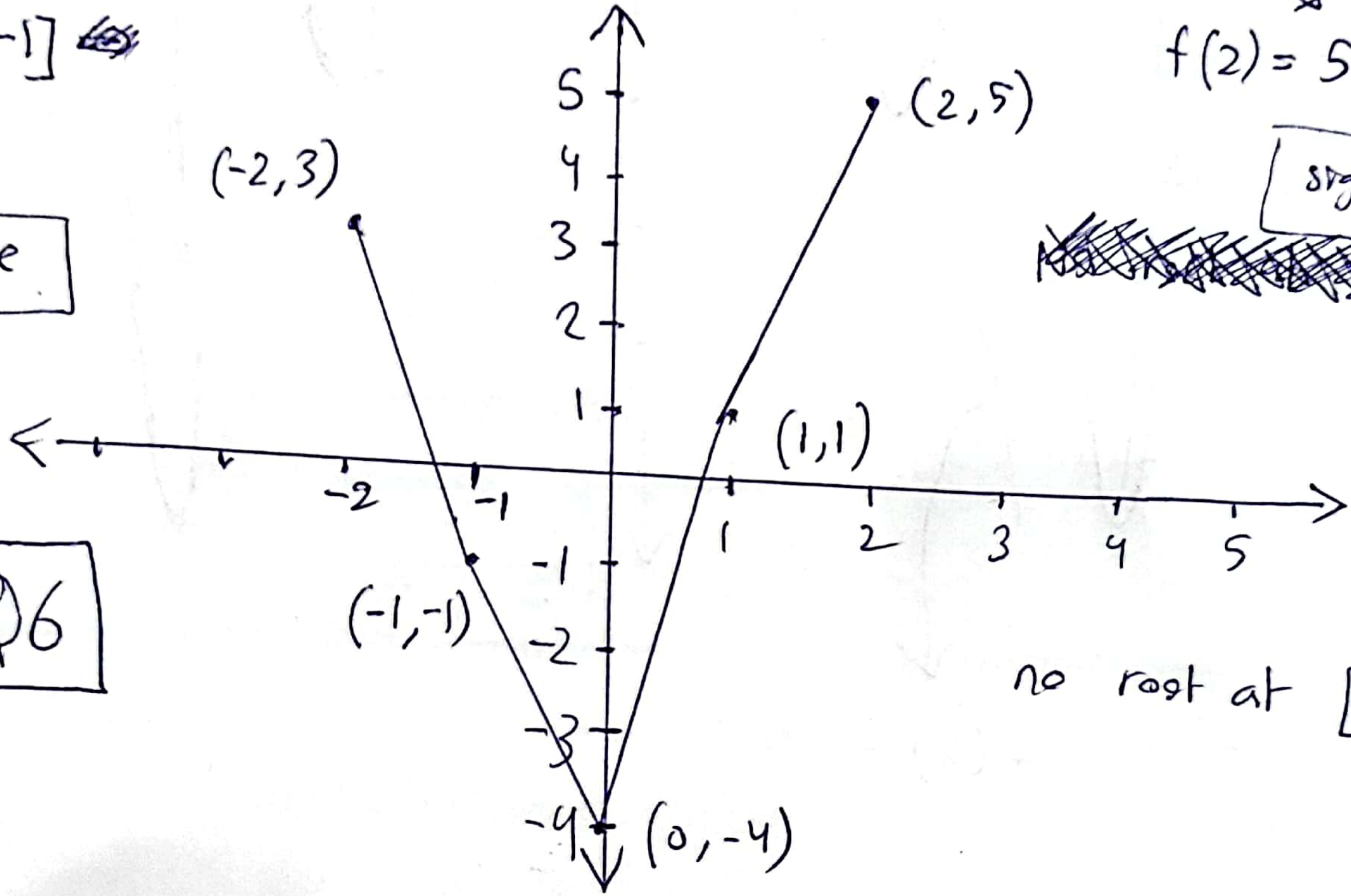
$f(-2) = 3$ & $f(-1) = -1$ $[-2, -1]$

$\boxed{\text{sign change}}$

$f(0) = -4$ & $f(2) = 5$

$\boxed{\text{sign change}}$

$\boxed{Q6}$



no root at $[-1, 0]$

ANSWER

f has root on only $[-2, -1]$ and $[0, 2]$ because now the intermediate value theorem guarantees that there two endpoints must have different signs.

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Q7

For k

$\boxed{\text{put } x = -1}$

$$m(x+1) + k = 2x^3 + x + 7$$

$$m(-1) + m + k = 2(-1)^3 + (-1) + 7$$

$$\cancel{-m} + \cancel{m} + k = -2 - 1 + 7$$

$$\boxed{k = 4}$$

For m

$$m(x+1) + k = x^2 + 5$$

$\boxed{\text{put } x = 2}$

$$m(2) + m + 4 = (2)^2 + 5$$

$$3m + \cancel{4} = \cancel{4} + 5$$

$$\boxed{m = \frac{3}{5}}$$

Q8 (i) $\dfrac{2x^3 + 7}{x^3 - x^2 + x + 7}$

① As denominator $\neq 0$, the vertical asymptote of this function doesnot EXIST because domain is $R(-\infty, +\infty)$

② $$\frac{\frac{2x^3}{x^3} + \frac{7}{x^3}}{\frac{x^3}{x^3} - \frac{x^2}{x^3} + \frac{x}{x^3} + \frac{7}{x^3}} \Rightarrow \lim_{x \to \pm\infty} \frac{2 + \frac{7}{x^3}}{1 - \frac{1}{x} + \frac{1}{x^2} + \frac{7}{x^3}}$$

$$\Downarrow$$

the horizontal asymptote $= \boxed{2} \leftarrow \dfrac{2+0}{1}$

③ The oblique asymptote doesnot exist because the degree of numerator is not greater than the degree of the denominator but they are equal.

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Q8

(ii) $\dfrac{3x^7 + 5x^2 - 1}{6x^3 - 7x + 3}$

① The vertical asymptote doesnot exist because the denominator $\neq 0$ & domain, range of denominator is $(-\infty, \infty)$

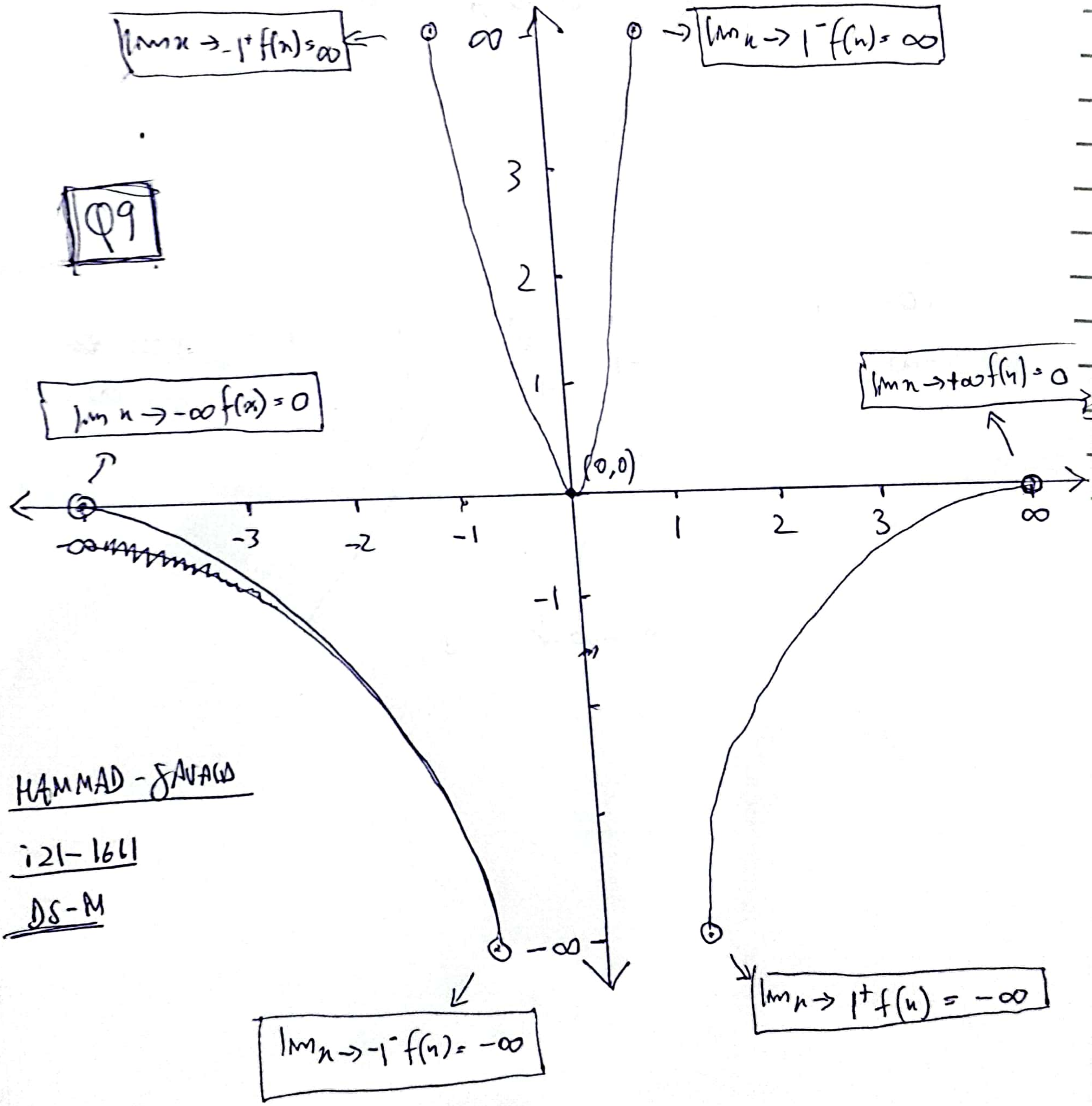
② $$\frac{\frac{3x^7}{x^7} + \frac{5x^2}{x^7} - \frac{1}{x^7}}{\frac{6x^3}{x^7} - \frac{7x}{x^7} + \frac{3}{x^7}} \Rightarrow \lim_{x \to \pm\infty} \frac{3 + \frac{5}{x^5} - \frac{7}{x^7}}{\frac{6}{x^4} - \frac{7}{x^6} + \frac{3}{x^7}}$$

$$\frac{3+0-0}{0+0+0} \Rightarrow \infty$$

∴, no horizontal asymptote exists because by applying limit, we are getting infinity / ∞.

③ No oblique asymptote exists because the degree of numerator should only be one degree greater than the degree of denominator.

Q9



HAMMAD - SAVAGD

i21-1661

DS-M