

National University of Computer & Emerging
Sciences
Assignment 3

November 15, 2021

Question 1 Find the determinant of the following matrix.

$$\begin{bmatrix} 6 & 2 & 1 & 0 & 5 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 1 & 2 & -2 & 3 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{bmatrix}$$

By using cofactor expansion and row operations.

Question 2 Find the inverse of given matrix using determinant.

$$\begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

Question 3 Find the volume $V(S)$ of the parallelopiped S in R^3 determined by the vectors $v_1 = (1, 1, 0)$, $v_2 = (1, 1, 1)$, $v_3 = (0, 2, 3)$.

Question 4 Use row operation to show that

$$\det T = 0$$
$$\begin{bmatrix} x^2 & 2x + 1 & 4x + 4 & 6x + 9 \\ y^2 & 2y + 1 & 4y + 4 & 6y + 9 \\ z^2 & 2z + 1 & 4z + 4 & 6z + 9 \\ w^2 & 2w + 1 & 4w + 4 & 6w + 9 \end{bmatrix}$$

5. Let $T: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be the linear transformation determined by the matrix

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}, \text{ where } a, b, c \text{ are positive numbers. Let } S \text{ be the unit ball, whose}$$

bounding surface has the

$$\text{equation } x_1^2 + x_2^2 + x_3^2 = 1.$$

a. Show that $T(S)$ is bounded by the ellipsoid with the equation $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$.

b. Use the fact that the volume of the unit ball is $4\pi/3$ to determine the volume of the region bounded by the ellipsoid in part (a).