Section:- DS-M

Roll no:- i21-1661

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Calculus & Analytical Geometry

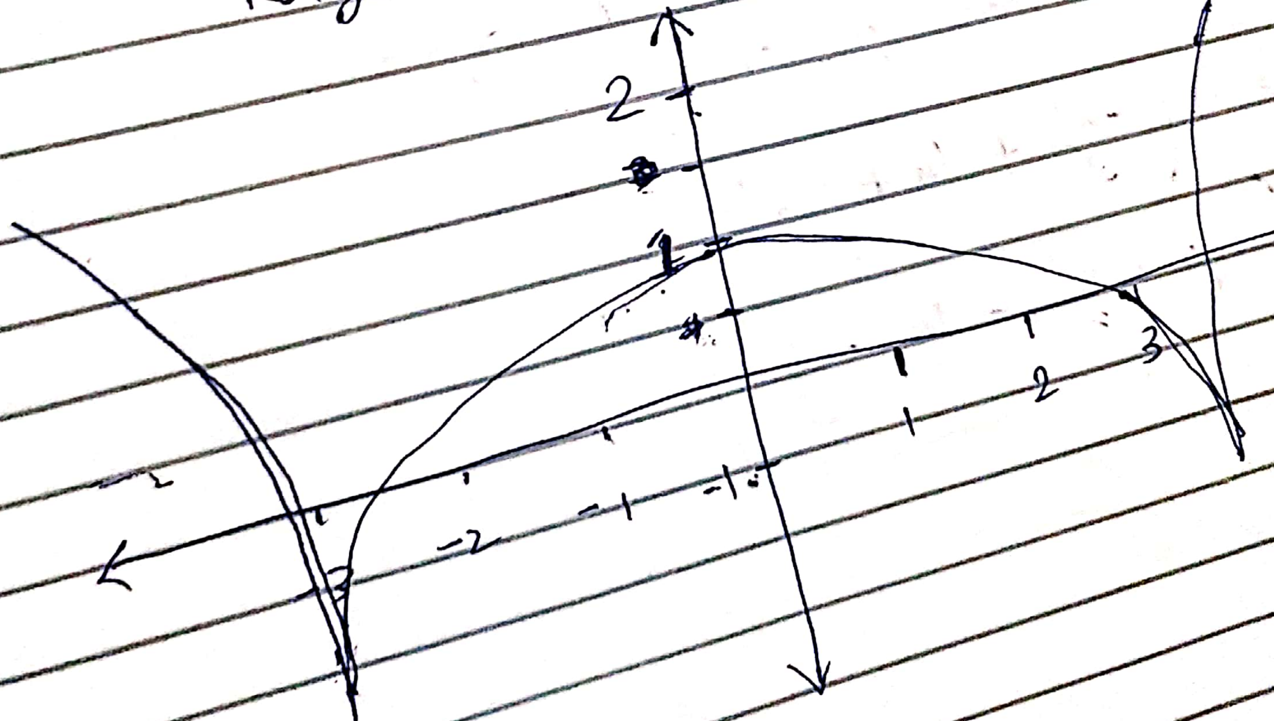
Assignment # 1.

Graph the function & find zeroes & domain, range

$$f(x) = \sqrt[3]{|9 - x^2|} - 1$$

Domain = $(-\infty, \infty)$

Range = $(-1, \infty)$



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Q2

$$x^2 - 12x + 3y = 1$$

$$3y = -x^2 + 12x + 1$$

$$y = \frac{-x^2}{3} + \frac{12x}{3} + \frac{1}{3} \rightarrow \frac{-x^2}{3} + 4x + 1/3$$

~~y~~ Curve reflected in origin
therefore $y \times -1$

$$y = -1\left(\frac{-x^2}{3} + 4x + 1/3\right)$$

$$\therefore \boxed{y = \frac{x^2}{3} - 4x - \frac{1}{3}}$$

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Q3. $f(x) = \dfrac{1}{x}$

4 units $\rightarrow$ right $\quad f(x-4)$

$\frac{1}{2}$ units up $\longrightarrow f(x) + \dfrac{1}{2}$

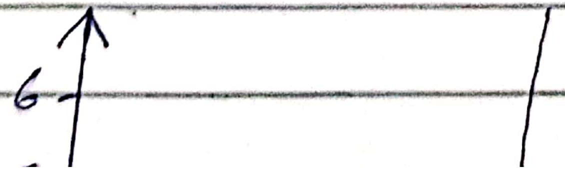
Compressed vertically $\frac{3}{2}$ units $\longrightarrow \dfrac{1}{3/2} f(x) \Rightarrow \dfrac{2}{3} f(x)$

new function

$\downarrow$

$\cancel{f(x) = \frac{2}{3}\left(\frac{1}{x-4}\right) + \frac{1}{2} \quad f(x) = \left(\frac{2}{3}\right)\frac{1}{x-4} + \frac{1}{2}}$

$f(x) = \dfrac{2}{3}\left(\dfrac{1}{x-4}\right) + \left(\dfrac{1}{2}\right)$

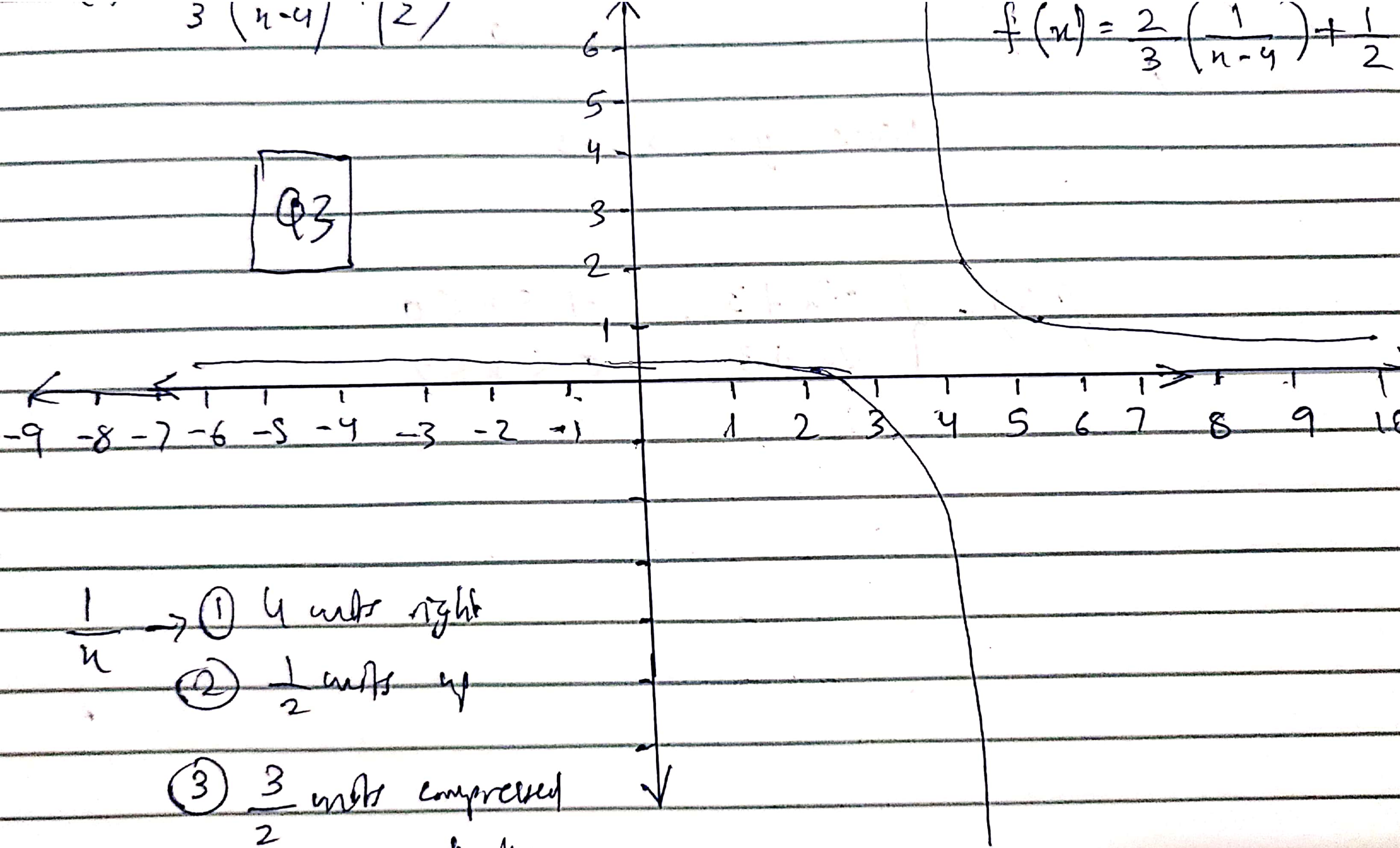


$f(x) = \dfrac{2}{3}\left(\dfrac{1}{x-4}\right) + \dfrac{1}{2}$

3 (x-4) |2/

$$f(x) = \frac{2}{3}\left(\frac{1}{x-4}\right)+\frac{1}{2}$$

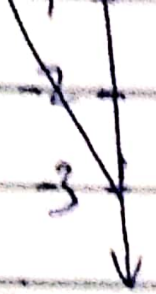
$\boxed{Q3}$



$\frac{1}{x} \to$ (1) 4 units right

(2) $\frac{1}{2}$ units up

(3) $\frac{3}{2}$ units compressed vertically



Line through $(0,3)$ & $(2,-1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-1-3}{2-0} = -2$$

$\boxed{Q4}$

$$\boxed{y = -2x + 3}$$

line through $(-1,0)$ & $(0,-3)$, $m = \dfrac{-3-0}{0-(-1)} = -3$

hence $y = -3x - 3$

$$f(x) = \begin{cases} -3x + 3, & -1 < x \leq 0 \\ -2x + 3, & 0 < x \leq 2 \end{cases}$$

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h = height of triangle

As triangle is also isoceles

$(AB)^2 + (AB)^2 = (2)^2$

Q5 (a)

$2(AB)^2 = 4 \longrightarrow \sqrt{AB^2} = \sqrt{\frac{4}{2}}$

$\sqrt{(AB)^2} = \sqrt{2}$. $\boxed{AB = \sqrt{2}}$

$h^2 + 1^2 = (\sqrt{2})^2 \Rightarrow h^2 = 2-1 \longrightarrow \sqrt{h^2} = \sqrt{1}$

$h = 1$ $B = (0,1)$

equation of AB $\rightarrow -x+1$ or $1-x$

$P(x, 1-x)$

Area = length $\times$ width $\longrightarrow 2x(1-x)$

Q5 (b)

$2x - 2x^2$

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Q6

$$f(x) = \cos^2(x+9)$$

$$F = f \circ t \circ h$$

lets consider $h = (x+9)$

$t = (x)$

$\therefore f = (x^2)$

$$t(h(x)) = \cos(x+9)$$

$$f(t \circ h) = [\cos(x+9)]^2 \Rightarrow f \circ t \circ h = \cos^2(x+9)$$

hence $F = f \circ t \circ h$

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Q7(i)

$$y = \frac{4x - 1}{2x + 3}$$

Interchange x and y → $x = \frac{4y - 1}{2y + 3}$

$x(2y + 3) = 4y - 1 \rightarrow 2xy + 3x = 4y - 1$

$2xy - 4y = -1 - 3x$

$y(2x - 4) = -3x - 1 \rightarrow y = \frac{-3x - 1}{2x - 4}$

$f(x)$	$f^{-1}(x)$
$x = 1/4, \; y = 0$	$x = 0, \; y = 1/4$

Domain $(-\infty, 2) \cup (2, \infty)$

As 'x' is equal to y in both so it is one-to-one function

Range $(-\infty, \infty)$

$\boxed{Q7. (ii)}$ $y = \frac{1}{1+e^{x}} \longrightarrow y + ye^{-x} = 1 \longrightarrow ye^{-x} = 1 - y$

$\ln(ye^{-x}) = \ln(1-y)$

$\ln e = 1 \therefore y(-x)\ln e = \ln(1-y)$

$x = \frac{\ln(1-y)}{-y} \longrightarrow f^{-1}(x) = \frac{\ln(-1+x)}{x}$

$f^{-1}(x)$ Domain : $x \neq 0$

Range $(\ln(-1), \infty)$

it is one-to-one function

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ONOMETRIC FUNCTIONS. Q8

Function f(x)	Domain	Range
$\sin x$	$-\infty < x < \infty \ (-\infty, \infty)$	$-1 \leq y < 1$
$\cos x$	$-\infty < x < \infty \ (-\infty, \infty)$	$-1 \leq y < 1$
$\tan x$	All real numbers except $\pi/2 + k\pi$ where k = integer	$-\infty < y < \infty$
$\sec x$	same as $\tan x$ Domain	$(-\infty, -1] \cup [1, \infty)$
$\operatorname{cosec} x$	$x \neq 0, \pm\pi, 2\pi$ All real numbers except $n\pi$	same as $\sec x$ $(-\infty, -1] \cup [1, \infty)$
	" (SAME)	
$\cot x$	All real numbers except $n\pi$	$(-\infty, \infty)$

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$\boxed{Q9}$ $\ln(y-1) - \ln 2 = x + \ln x$

$$\ln(y-1) + \ln 2 - \ln x = x$$

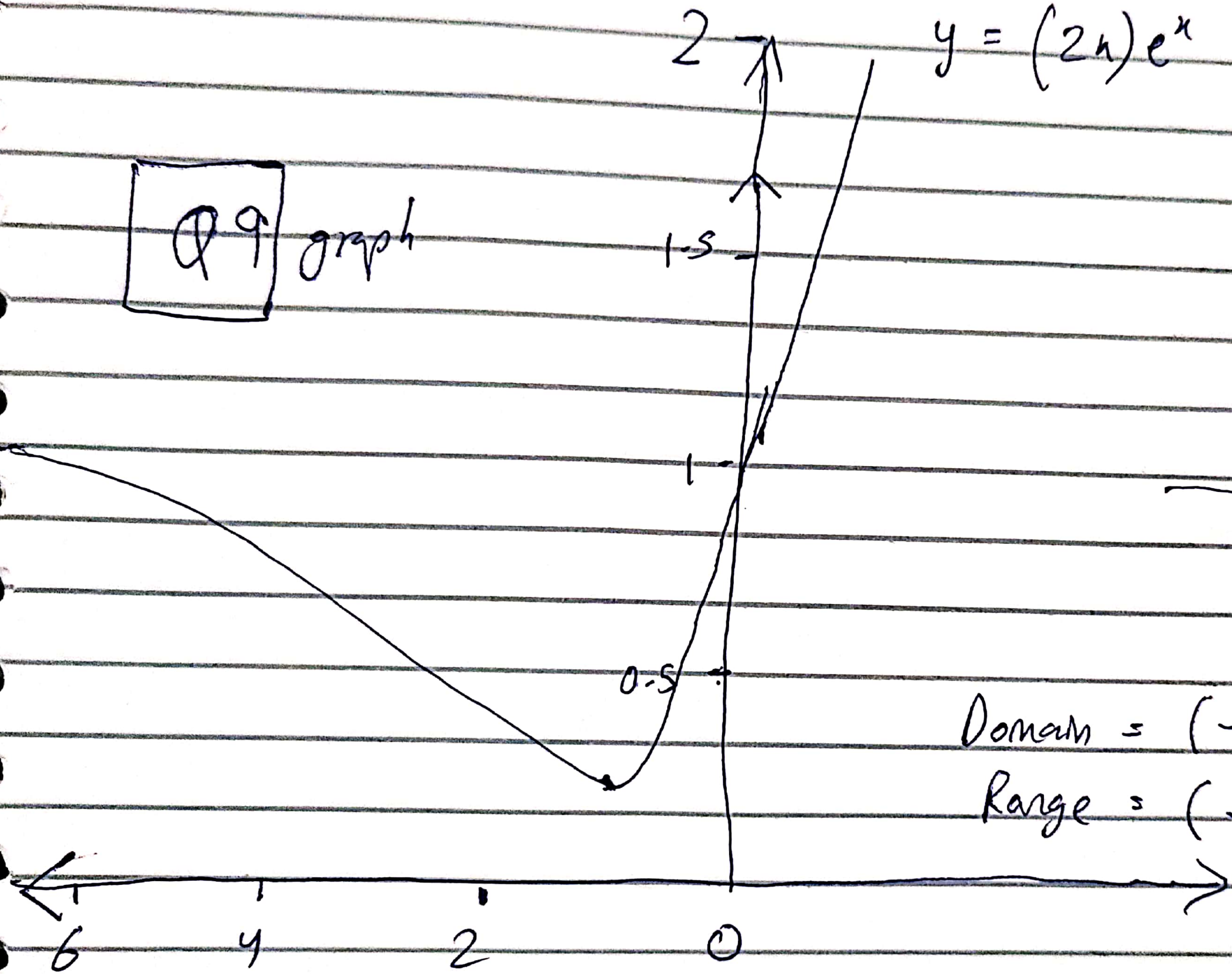
$$\ln\left(\frac{y-1}{2x}\right) = x$$

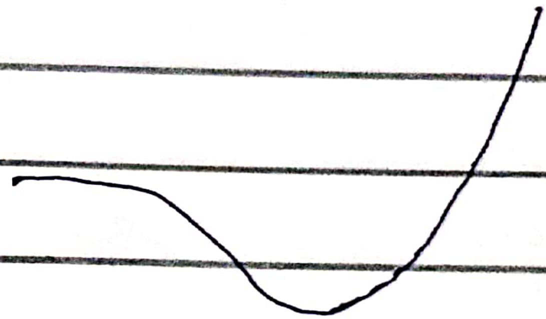
$$e^{\ln\left(\frac{y-1}{2x}\right)} = e^{x} \longrightarrow \frac{y-1}{2x} = e^{x}$$

$$y = (2x)e^{x} + 1$$

$\boxed{\text{Q9}}$ graph

$y = (2x)e^{x} + 1$





Domain $= (-\infty, \infty)$

Range $= \left(-\frac{2}{e} + 1, \infty\right)$

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Q10