Assignment 2 Linear Algebra

Q1. Write the solution set of the given homogeneous $A\mathbf{x} = \mathbf{0}$ system in parametric vector form, where

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}.$$

Write the vectors whose span gives the solution set of the homogenous system. Describe the solutions of the system $A\mathbf{x} = \mathbf{b}$ in parametric vector form, where $\mathbf{b} = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}$. Give a geometric description of the solution set and compare it to the solution of homogeneous system.

Q2. Find the value(s) of h (if possible) for which the vectors are linearly independent

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ h \\ 2 \end{bmatrix}.$$

Q3.

1. Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $\mathbf{y}_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation which maps $\mathbf{e}_1$ into $\mathbf{y}_1$ and $\mathbf{e}_2$ into $\mathbf{y}_2$, find the image of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$, and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Write the standard matrix of that transformation.
2. Show that the transformation defines by $T(x_1, x_2) = (x_1 - 2|x_2|, 2x_1 - 5x_2)$ is not linear.

Q4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x_1, x_2) = (x_1 - 2x_2, 4x_1 + 5x_2)$. Find $\mathbf{x}$ such that $T(\mathbf{x}) = (1, 3)$. Determine whether T is onto and one to one linear transformation?

Q5. **Q5**. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation which

- rotates every point about origin (anticlockwise) thorough an angle θ, then
- reflects that point about a line $y = x$, then
- rotates that point about origin (anticlockwise) to an angle ϕ.

Write the matrix of transformation A such that $T(\mathbf{x}) = A\mathbf{x}$. Write the matrix when $\theta = \pi/4$ and $\phi = \pi/6$.