Linear Algebra Assignment # 3

HAMMAD - JAVAID.     DS-M     $\boxed{i21-1661}$

Q1

$$\begin{bmatrix} 6 & 2 & 1 & 0 & 5 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 1 & 2 & -2 & 3 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{bmatrix} \xrightarrow{R_1 \times \frac{1}{6}} \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 1 & 2 & -2 & 3 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{bmatrix}$$

$$\xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1/3 & 2/3 & -2 & -2/3 \\ 1 & 1 & 2 & -2 & 3 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{bmatrix}$$

$\boxed{R_3 - R_1}$ $\boxed{R_4 - 3R_1}$ and $\boxed{R_5 + R_1}$

$$\begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1/3 & 2/3 & -2 & -2/3 \\ 0 & 2/3 & 11/6 & -2 & 13/6 \\ 0 & -1 & 3/2 & 3 & -7/2 \\ 0 & -2/3 & -17/6 & 4 & 17/6 \end{bmatrix}$$

$R_2 \times 3$

$$\begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 2/3 & 11/6 & -2 & 13/6 \\ 0 & -1 & 3/2 & 3 & -7/2 \\ 0 & -2/3 & -17/6 & 4 & 17/6 \end{bmatrix} \xrightarrow{R_3 - \frac{2}{3}R_2} \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1/2 & 2 & 7/2 \\ 0 & -1 & 3/2 & 3 & -7/2 \\ 0 & -2/3 & -17/6 & 4 & 17/6 \end{bmatrix} \xrightarrow{R_4 + R_2} \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1/2 & 2 & 7/2 \\ 0 & 0 & 7/2 & -3 & -11/2 \\ 0 & -2/3 & -17/6 & 4 & 17/6 \end{bmatrix}$$

$R_5 + \frac{2}{3}R_2$

$$\begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1/2 & 2 & 7/2 \\ 0 & 0 & 7/2 & -3 & -11/2 \\ 0 & 0 & -3/2 & 0 & 3/2 \end{bmatrix} \xrightarrow{R_3 \times 2} \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 7/2 & -3 & -11/2 \\ 0 & 0 & -3/2 & 0 & 3/2 \end{bmatrix}$$

$R_4 - \frac{1}{2} R_3$

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$$\begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & -17 & -30 \\ 0 & 0 & -3/2 & 0 & 3/2 \end{bmatrix} \xrightarrow[R_4 \times -\frac{1}{17}]{R_5 + \frac{3}{2}R_3} \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 1 & 30/17 \\ 0 & 0 & 0 & 6 & 12 \end{bmatrix}$$

$R_5 - 6R_4$ & $R_5 \times 17/24$

$$B = \begin{bmatrix} 1 & 1/3 & 1/6 & 0 & 5/6 \\ 0 & 1 & 2 & -6 & -2 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 1 & 30/17 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\det(B) = 1$

$$\cancel{\left[2(17) - 1(10) - (17) + 2(4) + 2(-7) + 3 - 15 + 6 - 4 + 2(-7) - 6(-3)\right]}$$

$\boxed{\det(A) = 24}$

1. $\begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & -17 & -30 \\ 0 & 0 & 0 & 24/17 \end{bmatrix}$ $\longrightarrow$ 1.1. $\begin{bmatrix} 1 & 4 & 7 \\ 0 & -17 & -30 \\ 0 & 0 & 24/17 \end{bmatrix}$

1.1.1 $\begin{bmatrix} -17 & -30 \\ 0 & 24/17 \end{bmatrix}$

$$-\cancel{17}\left(\frac{24}{\cancel{17}}\right) - 30(0)$$

$\boxed{\text{determinant} = -24}$

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**Q2**

$$\begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix} \Rightarrow 3 \cdot \begin{bmatrix} 6 & 3 \\ -4 & 0 \end{bmatrix} \cdot 2 \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} \cdot 1 \begin{bmatrix} 1 & 6 \\ 2 & -4 \end{bmatrix}$$

$$3 \begin{bmatrix} 0-(-12) \end{bmatrix} \cdot 2 \begin{bmatrix} 0-(6) \end{bmatrix} \cdot -1 \begin{bmatrix} -4-12 \end{bmatrix}$$

$$= 64$$

$$\begin{bmatrix} + \begin{bmatrix} 6 & 3 \\ -4 & 0 \end{bmatrix} & - \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} & + \begin{vmatrix} 1 & 6 \\ 2 & -4 \end{vmatrix} \\ - \begin{vmatrix} 2 & -1 \\ -4 & 0 \end{vmatrix} & + \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} & - \begin{vmatrix} 3 & 2 \\ 2 & -4 \end{vmatrix} \\ + \begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} & - \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} & + \begin{vmatrix} 3 & 2 \\ 1 & 6 \end{vmatrix} \end{bmatrix}$$

$$\begin{bmatrix} +(+12) & +(6) & (-16) \\ +(4) & +(-2) & (16) \\ 12 & -10 & 16 \end{bmatrix} \quad \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & 12 \\ -16 & 2 & -10 \end{bmatrix}$$

$$\text{Inverse} \Rightarrow \begin{bmatrix} 3/16 & 1/16 & 3/16 \\ 3/32 & 1/32 & -5/32 \\ -1/4 & 1/4 & 1/4 \end{bmatrix}$$

$\boxed{Q3}$ $v_1 = (1,1,0)$ $v_2 = (1,1,1)$ $v_3 = (0,2,3)$

$\downarrow$ a $\quad$ $\downarrow$ b $\quad$ $\downarrow$ c

$$\text{volume} = |\bar{a} \cdot (b \times c)|$$

$$\vec{b} \times \vec{c} \quad \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix}$$

$$i \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} - j \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} - k \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$$

$$(3-2) \qquad (3-0) \qquad (2-0)$$

$$\langle 1, -3, 2 \rangle = b \times c$$

as $\vec{a} = (1,1,0)$

$$1 + (-3) + 0$$

$$-2$$

absolute value

$$|-2| \rightarrow 2$$

$\boxed{\text{volume}}$

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Q4

$$\begin{bmatrix} x^2 & 2x & 4x & 6x \\ y^2 & 2y & 4y & 6y \\ z^2 & 2z & 4z & 6z \\ w^2 & 2w & 4w & 6w \end{bmatrix} + \begin{bmatrix} 0 & 1 & 4 & 9 \\ 0 & 1 & 4 & 9 \\ 0 & 1 & 4 & 9 \\ 0 & 1 & 4 & 9 \end{bmatrix}$$

now $|T| = \begin{vmatrix} x^2 & 2x & 4x & 6x \\ y^2 & 2y & 4y & 6y \\ z^2 & 2z & 4z & 6z \\ w^2 & 2w & 4w & 6w \end{vmatrix} + 0$

As in second determinant 2 rows become same i.e zero {property of determinant applied}

Take 2 from Column II & 4 from column III while 6 from IV

$$2.4.6 \begin{vmatrix} x^2 & x & x & x \\ y^2 & y & y & y \\ z^2 & z & z & z \\ w^2 & w & w & w \end{vmatrix}$$

as 2 rows are again same so determinant = zero

therefore $48 \times 0 = 0$

$\therefore \det(T) = 0$

$|T| = 0$

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Q5 HAMMAD - JAVAID [21-1661] DS-M

(a) $T(S)$ is bounded by ellipsoid with equation $\frac{x_1^2}{a^2} + \frac{x_2^2}{b} + \frac{x_3^2}{c} = 1$

Consider $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ & $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = Au$.

$u_1 = x_1/a$ , $u_2 = x_2/b$ & $u_3 = x_3/c$

$u$ lies inside $S$ or $u_1^2 + u_2^2 + u_3^2 \leq 1$

if and only and ONLY if $x$ lies inside $T(S)$

OR

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} \leq 1$$