LINEAR ALGEBRA

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Q1 $A = \begin{bmatrix} 1 & -12 & 0 \\ -4 & 8 & 2 \end{bmatrix}$ $B = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$ $2\times3 - 3\times2$

$AB = 0 \rightarrow \begin{bmatrix} 1 & -12 & 0 \\ -4 & 8 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

① $a - 12b = 0$ $d - 12e = 0$ ③

② $-4a + 8b + 2c = 0$ $-4d + 8e + 2f = 0$ ④

consider $c = -1$ take $f = -1$

Adding ① and ②   Adding ③ and ④

$a = -0.6$ $d = -0.6$

$b = -0.05$ $e = -0.05$

$$B = \begin{bmatrix} -0.6 & -0.6 \\ -0.05 & -0.05 \\ -1 & -1 \end{bmatrix}$$

and $AB = 0$

so we can have construct a 3×2 matrix such that $AB = 0$

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Q2 $T(p) = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}$ [Consider p & g two polynomials]

(1) $T(p+g) = \begin{bmatrix} (p+g)(0) \\ (p+g)(1) \end{bmatrix} \rightarrow \begin{bmatrix} p(0) + g(0) \\ p(1) + g(1) \end{bmatrix}$

$$= \begin{bmatrix} p(0) \\ p(1) \end{bmatrix} + \begin{bmatrix} g(0) \\ g(1) \end{bmatrix}$$

$$\therefore = T(p) + T(g)$$

hence T is a linear transformation.

(2) $K = \{ p(t) \in P_3 : T(p(t)) = (0) \}$

$T(p(t)) = (0) \Rightarrow T \begin{bmatrix} p(0) \\ p(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$p(0) = 0$ & $p(1) = 0$

0 and 1 must be the roots of polynomial

$K = \{ x(x-1)\, q(x) : \deg q(x) < 2 \}$

$= \{ x(x-1)(ax + c) \}$

$\{ +a(x-1),\ ax^2(x-1) \}$

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Q3

$$C = \begin{bmatrix} 0 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 6 & -5 & 0 \\ 0 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & -3 & 4 \\ -2 & 2 & 9 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 6 & 6 & -5 & 0 \\ 0 & 4 & 8 & -3 & -7 \\ 0 & 12 & 27 & -20 & 12 \\ 0 & 18 & 39 & 5 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 6 & -3 & 0 \\ 0 & 4 & 8 & -3 & -7 \\ 0 & 0 & 3 & -11 & 33 \\ 0 & 0 & 6 & -17 & 93 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 6 & 6 & -5 & 0 \\ 0 & 4 & 8 & -3 & 7 \\ 0 & 0 & 3 & 11 & 33 \\ 0 & 0 & 0 & 5 & 27 \end{bmatrix}$$

Each column here has a pivot

$\therefore$ c_1, c_2, c_3 & c_4 are column space of given matrix.

Consider $A = (c_1, c_2\ c_3\ c_4)$ → linearly independent

& $B = (c_1\ c_2\ c_3\ c_5)$

A is invertible so $Ax = 0$ has a unique $\therefore$ trivial solution.

$$A = \begin{bmatrix} 3 & 6 & 6 & 0 \\ 0 & 4 & 8 & -7 \\ 0 & 0 & 3 & 33 \\ 0 & 0 & 0 & 27 \end{bmatrix}$$

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as $Bu = 0$

B is invertible, it has a unique solution.

Q4 ① Let $H = \{(x,y) \mid x^2 + y^2 \leq 1\} \leq R^2$

To check whether H is subspace of R^2 or not

For this, let $v = (1,0) \in H$ since $1^2 + 0^2 \leq 1$

$u = (0,1) \in H$ since $0^2 + 1^2 \leq 1$

but $u + v = (0,1) + (1,0) = (1,1) \leq H$ ($1^2 + 1^2 = 2$ but not 1)

Hence proved that H doesnot belong to subspace of R^2

② $H = \left\{ \begin{bmatrix} 2s + 3t \\ -s + t \\ 2t + s \end{bmatrix} : s \text{ and } t \text{ are real} \right\}$

Subspace is $H = \text{span}\{v_1, v_2\}$

$v_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

since v_1 & v_2 are not multiples of each other

$\therefore \{v_1, v_2\}$ is linearly independent

$\boxed{\text{Hence dimension of } H = 2}$

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$T: R^2 \to R^3$, $T(x) = Ax$

Q5

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 1 \\ 2 & 6 & -2 \end{bmatrix} \quad \text{for } p = \begin{bmatrix} 7 \\ 6 \\ 3 \end{bmatrix}$$

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \right\}$$

To show that
$T(p+s) = T(p) + T(s)$

let $s \in S$ be any vector
then
$$s = \alpha \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}^t + \beta \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}^t$$

so we have to show that
$T(p+s) = T(p) + T(s)$

$A(p+s) = Ap + As$

By matrix multiplication & addition rule
$T(p) + T(s) = T(p) + T(s)$

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Q6

1) $P_1(t) = 1+t^2$, $P_2(t) = 1-t^2$

let $ap_1(t) + bp_2(t) = 0$ for some scalars.

a and b, $a(1+t^2) + b(1-t^2) = 0$ implies:

$(a+b) + (a-b)t^2 = 0$ so

$a+b = 0$ & $a-b = 0$

$\therefore$ $a = 0$ and $b = 0$.

Thus, the ~~solution~~ scalars involved in this linear combination of p_1 & p_2 are equal to zero.

so this implies that p_1 p_2 are linearly independent.

2)

$P_1(t) = 1+t$

$P_2(t) = 1-t$

$P_3(t) = 2$

here

$P_1(t) + P_2(t)$

$(1+t) + (1-t)$

$= 2 = P_3(t)$