Linear Algebra
Assignment # 1.

i21-1661 HAMMAD JAVAID

Q1

$$2x_1 + 4x_2 + 3x_3 = f$$
$$x_1 + 2x_2 - 3x_3 = g$$
$$x_1 + 2x_2 - cx_3 = h$$

$$\begin{bmatrix} 2 & 4 & 3 & f \\ 1 & 2 & -3 & g \\ 1 & 2 & c & h \end{bmatrix} \xrightarrow{2R_2 - R_1} \begin{bmatrix} 2 & 4 & 3 & f \\ 0 & 0 & -9 & 2g-f \\ 1 & 2 & c & h \end{bmatrix}$$

$$\xrightarrow{2R_3 - R_1} \begin{bmatrix} 2 & 4 & 3 & f \\ 0 & 0 & -6 & 2g-f \\ 0 & 0 & 2c-3 & 2h-f \end{bmatrix}$$

This system is consistent & has infinite many solution because pivot doesnot exist in each row and zero ≠ nonzero

Hammad Javaid   Q2   i21-1661

$$\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{R_1 \times \frac{1}{2}} \begin{bmatrix} 1 & 0 & 3 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 8 & 8 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 8 & 8 \\ 0 & -2 & -2 \end{bmatrix}$$

$$\xrightarrow{R_2 \times \frac{1}{8}} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{R_3 + 2R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 5 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

These matrices are not row equivalent, because their ~~[illegible]~~ row echelon forms are not same

$\boxed{Q3}$ Hammad Javaid 121-1661

$$\begin{bmatrix} 1 & -1 & 1 & 3 \\ -1 & 8 & 2 & h \\ 1 & -2 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 7 & 3 & h+3 \\ 1 & -2 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 7 & 3 & h+3 \\ 0 & -1 & -2 & -2 \end{bmatrix} \xrightarrow{7R_3 + R_2} \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 7 & 3 & h+3 \\ 0 & 0 & -11 & h-11 \end{bmatrix}$$

$$h - 11 = -11 \Rightarrow h = 0$$

$$\begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 7 & 3 & h+3 \\ 0 & 0 & -11 & -11 \end{bmatrix}$$

$2u - 3v - w = 0$ Q4

$$2\begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix} - 3\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} = 0$$

Q4

$$\begin{bmatrix} 14 \\ 4 \\ 10 \end{bmatrix} - \begin{bmatrix} 9 \\ 3 \\ 9 \end{bmatrix} - \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 14 - 9 - 5 \\ 4 - 3 - 1 \\ 10 - 9 - 1 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

therefore $2u - 3v - w = 0$ is valid

$\boxed{Q4}$

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 7x_1 + 3x_2 \\ 2x_1 + x_2 \\ 5x_1 + 3x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 3 & 5 \\ 2 & 1 & 1 \\ 5 & 3 & 1 \end{bmatrix} \xrightarrow{R_1 \times \frac{1}{7}} \begin{bmatrix} 1 & 3/7 & 5/7 \\ 2 & 1 & 1 \\ 5 & 3 & 1 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 3/7 & 5/7 \\ 0 & 1/7 & -3/7 \\ 5 & 3 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 - 5R_1} \begin{bmatrix} 1 & 3/7 & 5/7 \\ 0 & 1/7 & -3/7 \\ 0 & 6/7 & -18/7 \end{bmatrix} \xrightarrow{R_2 \times 7} \begin{bmatrix} 1 & 3/7 & 5/7 \\ 0 & 1 & -3 \\ 0 & 6/7 & -18/7 \end{bmatrix}$$

$$\xrightarrow{R_3 - \frac{6}{7}R_2} \begin{bmatrix} 1 & 3/7 & 5/7 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{x_2 = -3}$$

$$x_1 + \frac{3}{7}(-3) = \frac{5}{7}$$

$$x_1 = \frac{5}{7} + \frac{9}{7} \Rightarrow \frac{14}{7}$$

$$\boxed{x_1 = 2}$$

Scanned with CamScanner

$c_1 v_1 + c_2 v_2 + c_3 v_3$

$\boxed{Q5}$ $c_1 \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ -3 \\ 9 \end{bmatrix} + c_3 \begin{bmatrix} 4 \\ -2 \\ -6 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & -3 & -2 \\ -3 & 9 & -6 \end{bmatrix} \xrightarrow{R_3 + 3R_1} \begin{bmatrix} 1 & 0 & 4 \\ 0 & -3 & -2 \\ 0 & 9 & 6 \end{bmatrix} \xrightarrow{R_2 \times -\frac{1}{3}} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2/3 \\ 0 & 9 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2/3 \\ 0 & 9 & 6 \end{bmatrix} \xrightarrow{R_3 - 9R_2} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2/3 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix (v_1, v_2, v_3) doesnot have pivot in each row, therefore, it doesnot span R^3.

Q6 Hammad Javaid i21-1661

(i) $\begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & -1 & 0 \end{bmatrix} \Rightarrow \begin{aligned} x_1 - x_2 + x_3 &= 3 \\ 2x_3 &= 5 \\ -x_3 &= 0 \end{aligned}$

As zero = non-zero, therefore, system is inconsistent & has no solution.

(ii) $\begin{bmatrix} 0 & -2 & 3 & 3 & 1 \\ 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{aligned} -2x_1 + 3x_2 + 3x_3 &= 1 \\ 2x_2 + 5x_3 &= 0 \\ -x_3 &= 3 \end{aligned}$

$\boxed{x_3 = -3}$

$2x_2 = -5x_3 \rightarrow x_2 = \frac{-5(-3)}{2} \Rightarrow \boxed{\frac{15}{2} = x_2}$

$-2x_1 + 3\left(\frac{15}{2}\right) + 3(-3) = 1$

$-2x_1 + \frac{45}{2} = 10 \rightarrow 2x_1 = \frac{45}{2} \;\; 10$

$2x_1 = 12.5 \rightarrow \boxed{x_1 = 6.25 = \frac{25}{4}}$

System is consistent & has unique solution.

(iii) $\left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{array}\right] \implies$

$$x_1 - x_2 = 1$$

$$2x_2 = 2$$

$$x_2 = 1$$

$$\boxed{x_2 = 1}$$

system is consistent
& has unique solution

$$x_1 - 2 = 1$$

$$\boxed{x_1 = 3}$$

Q7 After 3^{rd} game, each player has \$24.

P1 looses so now he has $12 + 14 + 24 = 48$

__1st game__ P2, P3 each have $\frac{24}{2} = 12$

P2 looses, now he has $12 + 24 + 6 = 42$

__2nd game__ P1 has $\frac{48}{2} = 24$, P3 has $\frac{12}{2} = 6$

P3 looses, now he has $6 + 21 + 12 = 39$

__3rd game__ P1 has $\frac{24}{2} = 12$, P2 has $\frac{42}{2} = 21$

At the start; P1 $\longrightarrow$ 12 \$

P2 $\longrightarrow$ 21 \$

P3 $\longrightarrow$ 39 \$