Assignment No. 01

Linear Algebra

Q1. Suppose the system

$$\begin{aligned} 2x_1 + 4x_2 + 3x_3 &= f \\ x_1 + 2x_2 - 3x_3 &= g \\ x_1 + 2x_2 + cx_3 &= h \end{aligned}$$

Find a relation (if possible) between f, g, h, c, d such that the system is inconsistent and consistent. Can we find a relation which gives a unique solution, infinite many solution? Justify your answer.

Q2. Two matrix are said to be row equivalent are Determine whether the given matrices are equivalent or not $\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 5 \\ 1 & 0 & 1 \end{bmatrix}$. Justify your answer.

Q3 Determine (if possible) the value of h such that the matrix is the augmented matrix of an inconsistent system,

$$\begin{bmatrix} 1 & -1 & 1 & 3 \\ -1 & 8 & 2 & h \\ 1 & -2 & -1 & 1 \end{bmatrix}.$$

Q4. Let $\mathbf{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$. Check the vector equation $2\mathbf{u} - 3\mathbf{v} - \mathbf{w} = \mathbf{0}$ is valid or not? If it is valid equation then without any calculations find x_1, x_2 (solution) of the following system $\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$.

Q5. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 9 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 4 \\ -2 \\ -6 \end{bmatrix}$. Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ Span $\mathbb{R}^3$? Justify your answer.

Q6. If the given matrices are augmented matrices of some system of linear equations $A\mathbf{x} = \mathbf{b}$, then conclude about the solutions of system $A\mathbf{x} = \mathbf{b}$. Write parametric equation of solution (if applicable). Write solution of corresponding homogenous system of linear equations (if applicable without any calculations).

$$(i) \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad (ii) \begin{bmatrix} 0 & -2 & 3 & 3 & 1 \\ 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (iii) \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

Q7.

Three people play a game in which there are always two winners and one loser. They have the understanding that the loser gives each winner an amount equal to what the winner already has. After three games, each has lost once and each has 24 dollars. With how much money did each begin?