



## MT1006 – Differential Equations (Cal-II)

**Assignment No:** 04

**Individual Assignment**


**Section:** BS CS, BS AI, BS DS, BS CySec

**Semester:** Spring 2022



**Due date:** As per GCR

**Marks:** 10\*10=100

### Instructions:

1. **Plagiarized work will result in zero marks.**
2. **No retake or late submission will be accepted.**
3. Attach complete code, results, and screenshot for questions that require programming solution. Programs/codes should not be handwritten.
4. Questions that show the icon  require partial or complete solution using the approved programming tool.
5. The assignment is to be submitted in softcopy as well as in hardcopy.
6. The softcopy should be a single PDF file of your complete assignment including programming and non-programming questions.
7. The PDF file should be according to the following **format: id\_section\_A1** e.g. i21123456\_A\_A1. A1 in the end denotes Assignment 1.
8. The images of by-hand solution should be properly scanned. You can use any mobile application such as Cam Scanner or Adobe Scan for scanning. Each of these applications allow you to export pdf or image files which you can use to combine with your programming solutions. Do not attach direct images from the camera application of your mobile phone, or screenshots.

## Questions:

1.  Suppose that  $\frac{dA}{dt} = -0.0004332 A(t)$  represents a mathematical model for the radioactive decay of radium – 226, where  $A(t)$  is the amount of radium (measured in grams) remaining at time  $t$  (measured in years). How much of the radium sample remains at the time  $t = -0.002$  with initial condition  $A(1) = 0.005$ . (Solve the question using MATLAB).
2. A hungry college student in a rush to eat, turns the oven on and puts a frozen pizza in it without preheating the oven. Let  $f(t)$  denote the temperature of the pizza and  $T(t)$  the oven's temperature  $t$  minutes after the oven was turned on. According to Newton's Law of cooling, the rate of change of  $f(t)$  is proportional to the difference between the oven's temperature and temperature of the pizza. Find a differential equation that is satisfied by  $f(t)$ .
3. Consider and solve the initial value problem of a new form of life discovered on a distant planet. Outside its habitable zone, the rate of change of population of the life form is governed by the following data:
 
$$\frac{dy}{dx} + y = f(x), \text{ where } f(x) = \begin{cases} e^{-x}, & 0 \leq x < 2 \\ e^x, & x \geq 2 \end{cases} ; y(0) = 1$$
4. A body of constant mass  $m$  is projected away from the earth in a direction perpendicular to the earth's surface with an initial velocity  $v_0$ . Assuming that there is no air resistance but taking into account the variation of the earth's gravitational field with distance, find an expression for the velocity during the ensuing motion. Also find the initial velocity that is required to lift the body to a given minimum altitude above the surface of the earth, and find the least initial velocity for which the body will not return to the earth; the latter is the escape velocity.
5.  Draw a direction plot using given differential equation. Also find the equilibrium solution and plot.
  - a)  $y' = -2 + t - y$
  - b)  $y' = 3\sin t + 1 + y$
6. Consider a flask that contain 3 liters of salt water. Suppose that water containing 25 grams per liters of salt is pumped into the flask at the rate of 2 liters per hour, and the mixture, being steadily stirred, is pumped out of the flask at the same rate. Find a differential equation satisfied by the amount of salt  $f(t)$  in the flask at time  $t$ .
7. A pond on a fish farm has a carrying capacity of 1000 fish. The pond was originally stocked with 100 fish. Let  $N(t)$  denote the number of fish in the pond after  $t$  months.
  - a) Set up a logistic differential equation satisfied by  $N(t)$ , and plot an approximate graph of a fish population.
  - b) Find the size of the population of fish with the highest rate of growth. Find this rate given that the intrinsic rate of growth is 3.

8. A 12 volt battery is connected to a series circuit in which the inductance is  $\frac{1}{2}$  henry and the resistance is 10 ohms. Determine the current  $i$  if the initial current is zero.
9. In a chemical manufacturing plant, a certain type of chemical B is produced through reactions in chemical A. Through observation the officers on duty have noted that the rate of conversion from chemical A to B is proportion to the amount of chemical A present at any time, further investigation has revealed that 10% amount has been converted in first five minutes.
- a) The plant manager wishes to know the percentage of chemical A that will be converted in 20 minutes.
  - b) The plant manager wishes to know the time it will take to convert 60 minutes of chemical A.
10. Consider the differential equation:
- $$(4x + 3y^2)dx + 2xydy = 0$$
- a) Show that this equation is not exact.
  - b) Find an integrating factor of the form  $x^n$ , where n is a positive integer.
  - c) Multiply the given equation through by the integrating factor found in step (b) and solve the resulting exact equation.