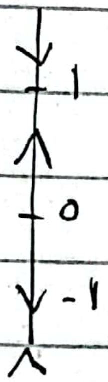
HAMMAD JAVAID

i21-1661 DS-M

$$\frac{dy}{dx} = y - y^3$$

Q1 $y - y^3 = 0 \rightarrow y(1-y^2) = 0$

$y = 0$ | $y = \pm 1$



(a) $y_0 > 1$

$(1, \infty)$ decreasing

(b) $0 < y_0 < 1$

$(0, 1)$ increasing

(c) $-1 < y_0 < 0$

$(-1, 0)$ decreasing

(d) $y_0 < -1$

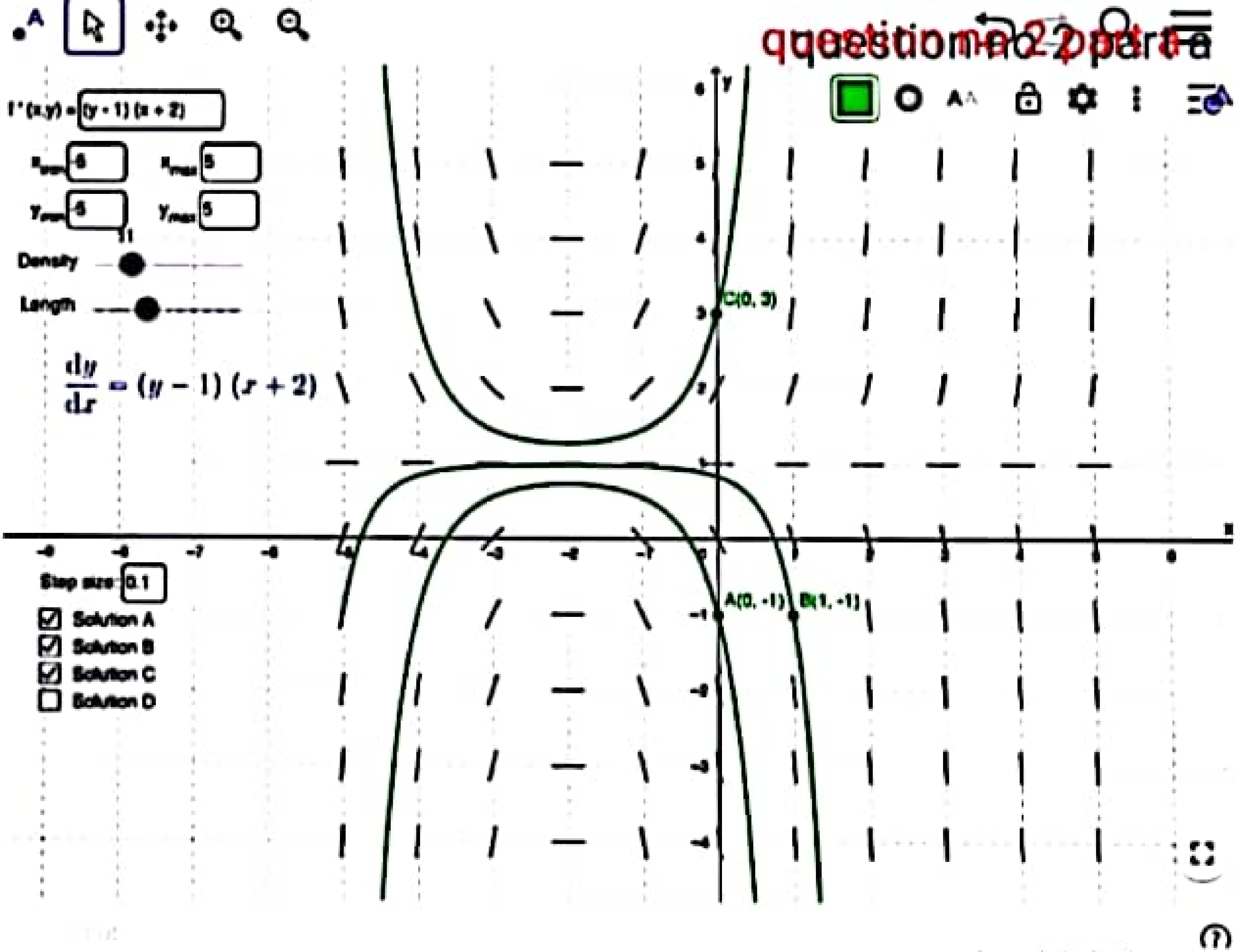
$(-\infty, -1)$ increasing

Q3 $$\frac{dy}{dx} = \frac{(2y+3)^2}{(4x+5)^2} \rightarrow (4x+5)^2 dy = (2y+3)^2 dx$$

$$\int \frac{1}{(2y+3)^2} dy = \int \frac{1}{(4x+5)^2} dx$$

$$\frac{1}{2}\int \frac{2}{(2y+3)^2} dy = \frac{1}{4}\int \frac{4}{(4x+5)^2} dx$$

questione 2 parte a



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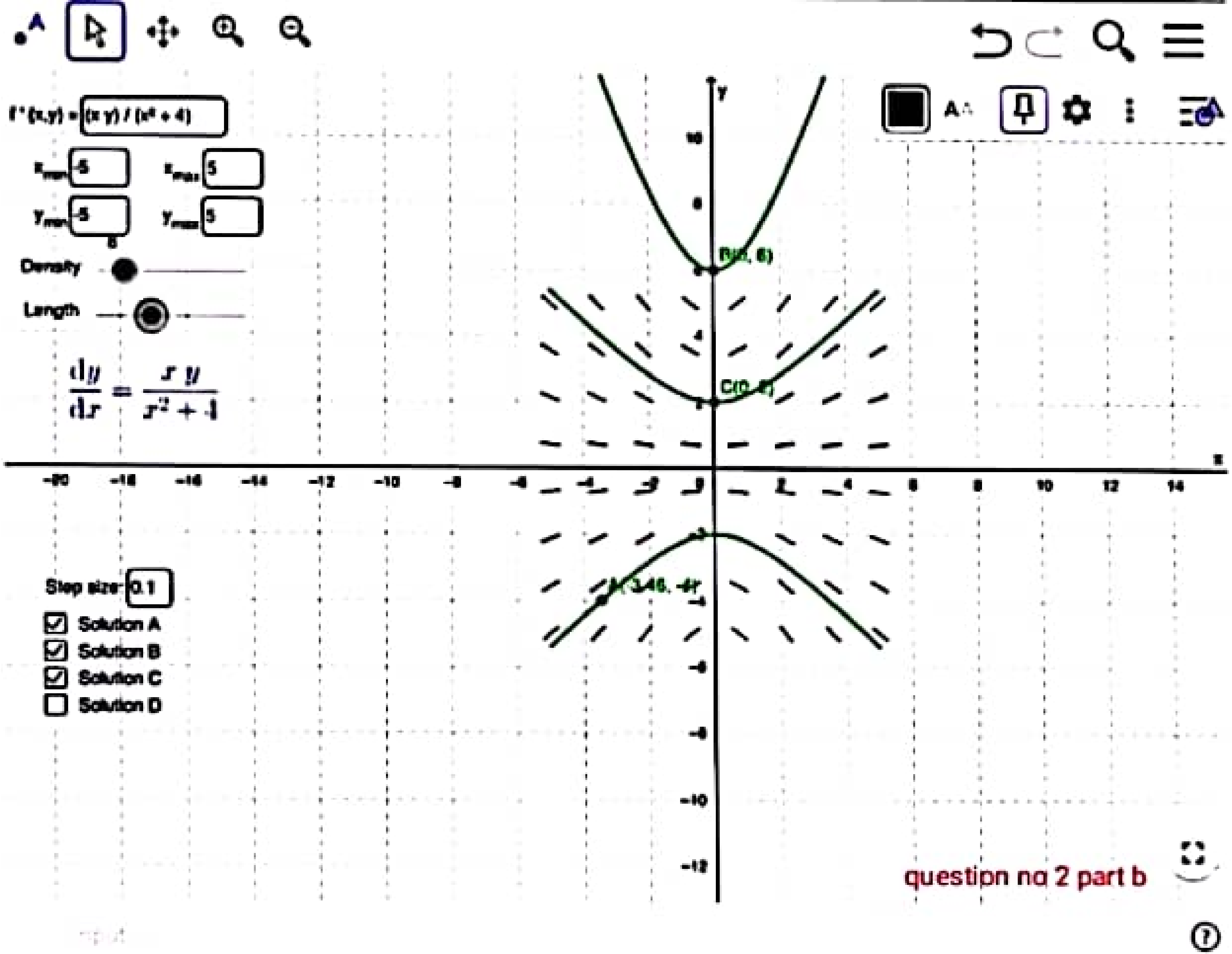

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$$\frac{1}{2}\frac{(2y+3)^{-2+1}}{-2+1} = \frac{1}{4}\frac{(4x+5)^{-2+1}}{-2+1} + c$$

$$-\frac{1}{2}(2y+3)^{-1} = -\frac{1}{4}(4x+5)^{-1} + c$$

$$c = \frac{1}{4(4x+5)} - \frac{1}{2(2y+3)}$$

$\boxed{Q4}$

$$\frac{dy}{dx} = y^2 \sin x^2 \qquad y(-2) = \frac{1}{3}$$

$$\int \frac{1}{y^2}\, dy = \int \sin x^2\, dx$$

$$\frac{y^{-1}}{-1} = \int \sin x^2\, dx$$

Integral is not possible

$$y(-2) = \frac{1}{3}$$

$$-\frac{1}{y} = \int \sin x^2\, dx$$

let x & -2 be the interval

$$\int_{-2}^{x} \left| -\frac{1}{y} \right| = \int_{-2}^{x} \sin x^2\, dx \rightarrow -\frac{1}{y(x)} + \frac{1}{y(-2)}$$

$$= \int_{-2}^{x} \sin t^2\, dx$$

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$$-\frac{1}{y(x)} + \frac{1}{1/3} = \int_{-2}^{x} sm^{-t}\, dx$$

$$\frac{-1}{y} + 3 = \int_{-2}^{x} sm^{-t}\, dx$$

$\boxed{Q5}$ $$(x+1)\frac{dy}{dx} + (x+2)y = 2xe^{-x}$$

$$\frac{dy}{dx} \cancel{+} \frac{x+2}{x+1} y = \frac{2xe^{-x}}{x+1}$$

$$P(x) = e^{\int \frac{x+2}{x+1} dx}$$

$$P(x) = e^{\int (1 + 1/x+1) dx}.$$

$$P(x) = e^{x + \ln|x+1|} \Rightarrow e^x \cdot \cancel{e}^{\cancel{l}(x+1)}$$

$$P(x) = e^{x}(x+1)$$

$$(x+1)e^x \frac{dy}{dx} + \cancel{(x+1)} e^x \frac{x+2}{\cancel{x+1}} y = \frac{2xe^{\cancel{-x}}}{\cancel{(x+1)}} \cancel{e^x}\cancel{(x+1)}$$

$$(x+1)e^x \frac{dy}{dx} + e^x(x+2)y = 2x$$

$$\int [e^x(x+1)y] = \int 2x$$

$$e^x(x+1)y = x^2 + C$$

$$y = \frac{x^2}{e^x(x+1)} + \frac{C}{e^x(x+1)}$$

$\boxed{Q6}$ $\frac{dy}{dt} = t + y \qquad y(0) = 3$

$$\frac{dy}{dt} - y = t$$

$$P(u) = e^{\int -1\, dt}$$

$$P(u) = e^{-t}$$

$$e^{-t}\frac{dy}{dt} - e^{-t}y = e^{-t}.t$$

$$\int (e^{-t}.y) = \int e^{-t}.t$$

$$e^{-t}.y = \frac{te^{-t}}{-1} - \int \frac{d}{dt}(t)\int e^{-t}$$

$$e^{-t}.y = -te^{-t} + \int e^{-t}$$

$$e^{-t}.y = -te^{-t} + e^{-t} + c$$

$$y = \frac{-t\cancel{e^{-t}}}{\cancel{e^{-t}}} - \frac{\cancel{e^{-t}}}{\cancel{e^{-t}}} + \frac{c}{e^{-t}}$$

$$y = -1 - t + \frac{c}{e^{-t}}$$

$y(0) = 3$

$$3 = 0 - 1 + \frac{c}{e^{-(0)}}$$

$$3 = -1 + c \rightarrow c = 4$$

$$y = -t - 1 + \frac{4}{e^{-t}}$$

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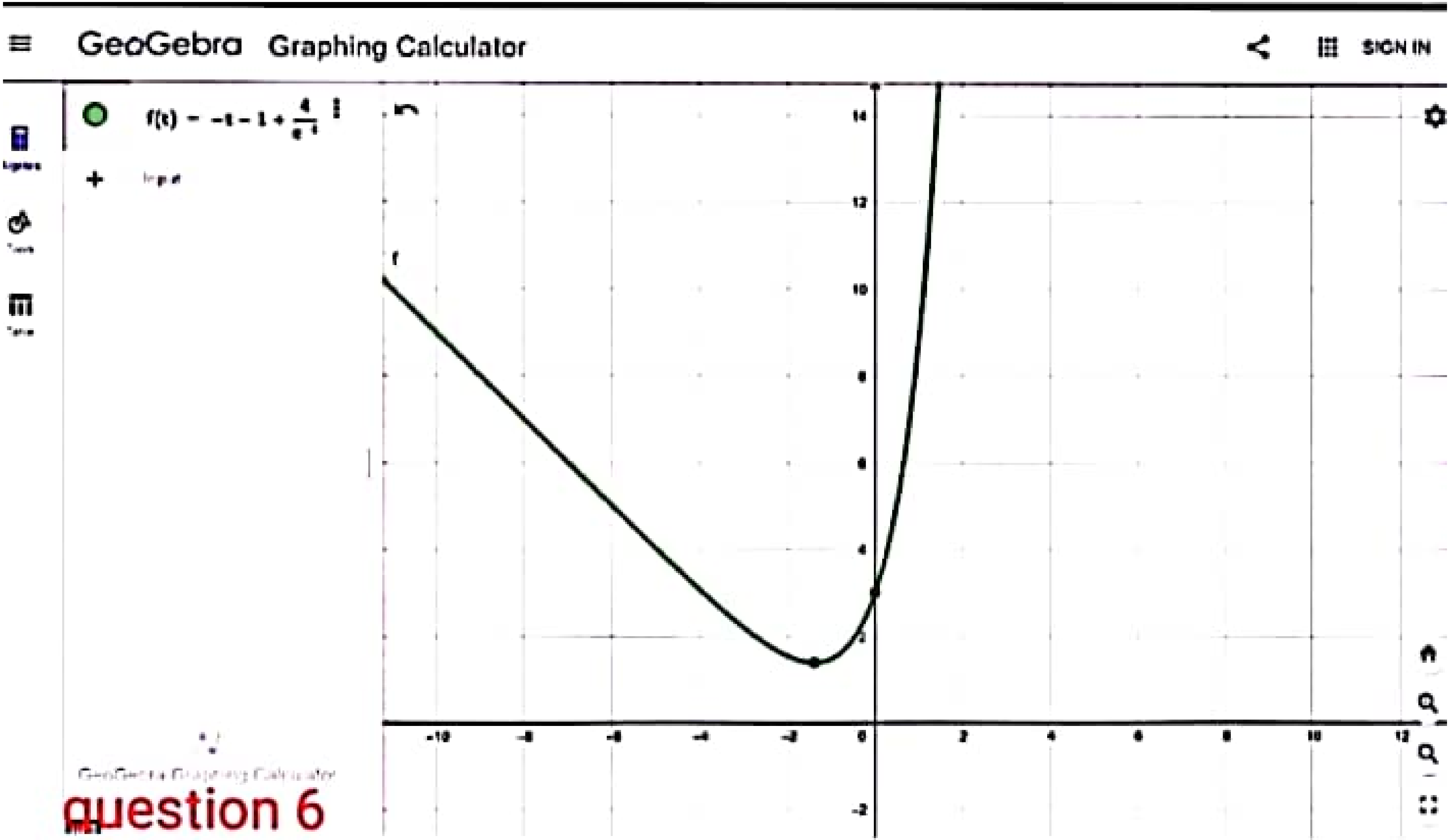

question 6

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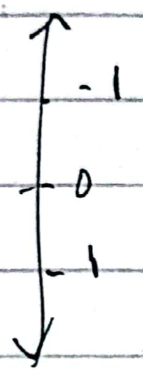
(Q7) $x' = x(x+1)(x+2)$

$x = 0, \quad x = +1, \quad x = -2$



$(-\infty, -2)$	$(-2, 0)$	$(0, 1)$	$(1, \infty)$
$-$ decreasing	$+$ increasing	$(-)$ decreasing	$+$ increasing

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Q8 Autonomous, since the RHS only depends on 1 factor which is $P(t)$ & it

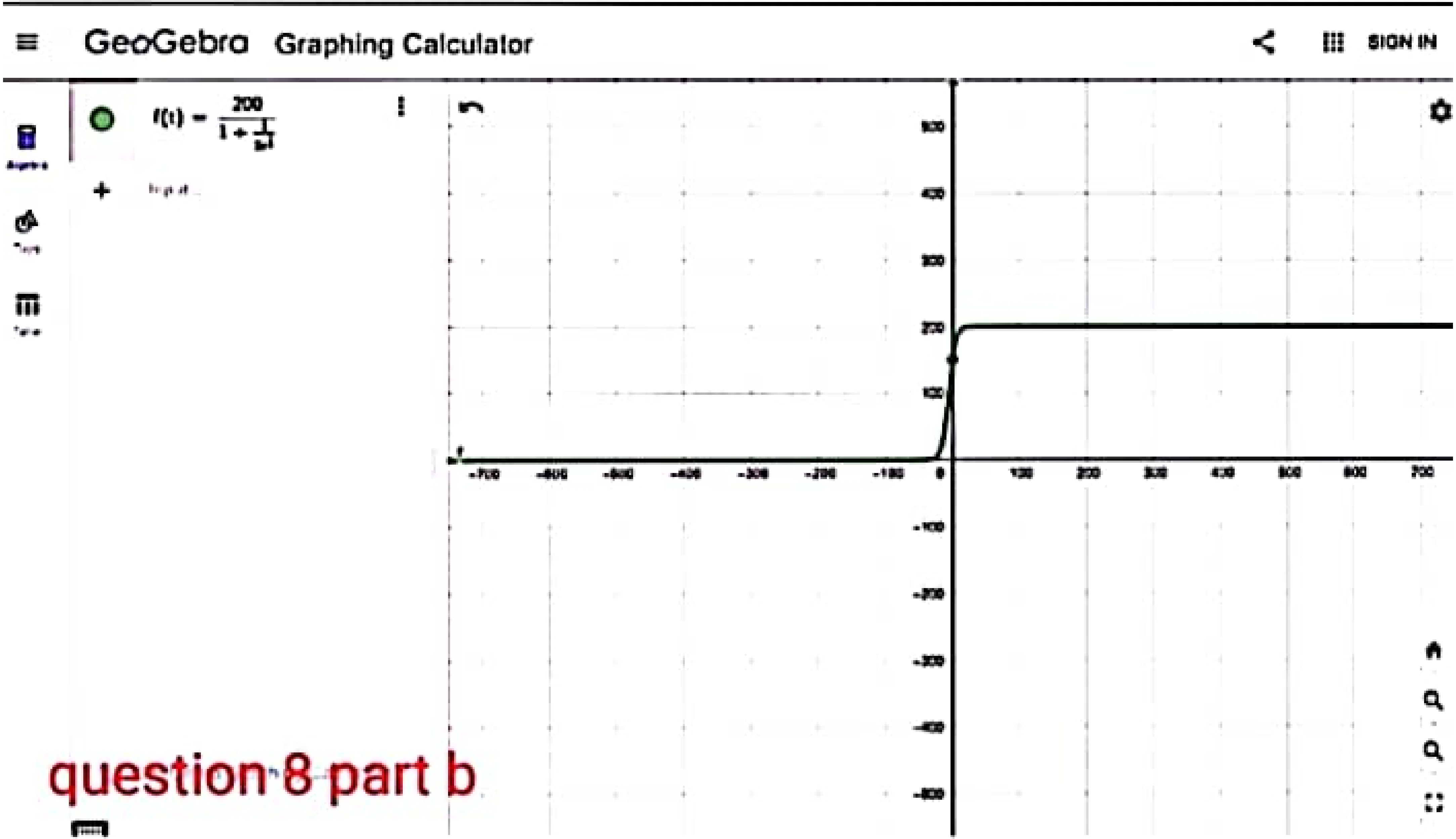
(a) doesnot depend on t.

(b) $$\frac{dP}{dt} = 0.2\, P(t)\left(1 - \frac{P(t)}{200}\right)$$

$$y(0) = 150 \longrightarrow \frac{1000\, dP}{P(P-200)} = dt$$

$$5\int \frac{1}{P-200} - \int \frac{1}{P}\, dP = dt$$

$$5 \ln\left|\frac{P-200}{P}\right| = t + C \Rightarrow \frac{P-200}{P} = Ce^{t/5}$$



question 8 part b

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$$P(t) = \frac{200}{1 - c^{e^{t/15}}} \Rightarrow P(0) = 150 = \frac{200}{1 - c^{e^{t/15}}}$$

$$c = \frac{1}{3} \text{ so } P(t) \frac{200}{1 + \frac{1}{3} e^{\cdot t/15}}$$

$\boxed{Q9}$ $$\frac{dy}{dx} + 3x^2 y = 6x^2$$

$$P(x) = e^{\int 3x^2}$$

$$P(x) = e^{x^3}$$

$$e^{x^3} \frac{dy}{dx} + e^{x^3} 3x^2 y = e^{x^3} 6x^2$$

$$\int \cancel{DE} \left[e^{x^3} y \right] = \int 6x^2 e^{x^3}$$

$$\cancel{\phi} \; e^{x^3} \cdot y = \int 6x^2 e^{x^3}$$

$$\frac{x^2}{2x} \cdot \frac{e^{x^3}}{e^{x^3}/3x^2}$$

$$e^{x^3} \cdot y = \frac{e^{x^3}}{3} + c$$

$$y = \frac{e^{x^3}}{3(e^{x^3})} + \frac{c}{e^{x^3}} \Rightarrow y = \frac{1}{3} + \frac{c}{e^{x^3}}$$

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$\boxed{Q10}$ (a) $(2x-1)dx + (3y+7)dy = 0$

$M(x,y) = 2x-1$

$\frac{dM}{dx} = 0 \rightarrow \frac{dN}{dy} = \frac{d}{dy}(3y+7)$

here $\frac{dM}{dx} = \frac{dN}{dy}$ ~~it~~ ~~it~~ exact D.E

(b) $(\sin y - y\sin x)dx + (\cos x - x\cos y - 7)dy = 0$

$\frac{dM}{dx} = -\sin x - \cos y$

$\frac{dM}{dx} \neq \frac{dN}{dy}$ so not exact

(c) $(5x + 4y)dx + (4x - 8y)dy = 0$

$\frac{dM}{dx} = 4$ $\qquad \frac{dM}{dx} = \frac{dN}{dy}$

so exact

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$\boxed{Q11}$ $A\frac{dy}{dA} = A^2 + 3y$ $\boxed{A>0}$

$$\frac{dy}{dA} = A + \frac{3y}{A}$$

$$dy = A(1+3y)dA$$

$$\frac{dy}{dA} - \frac{3y}{A} = A'$$

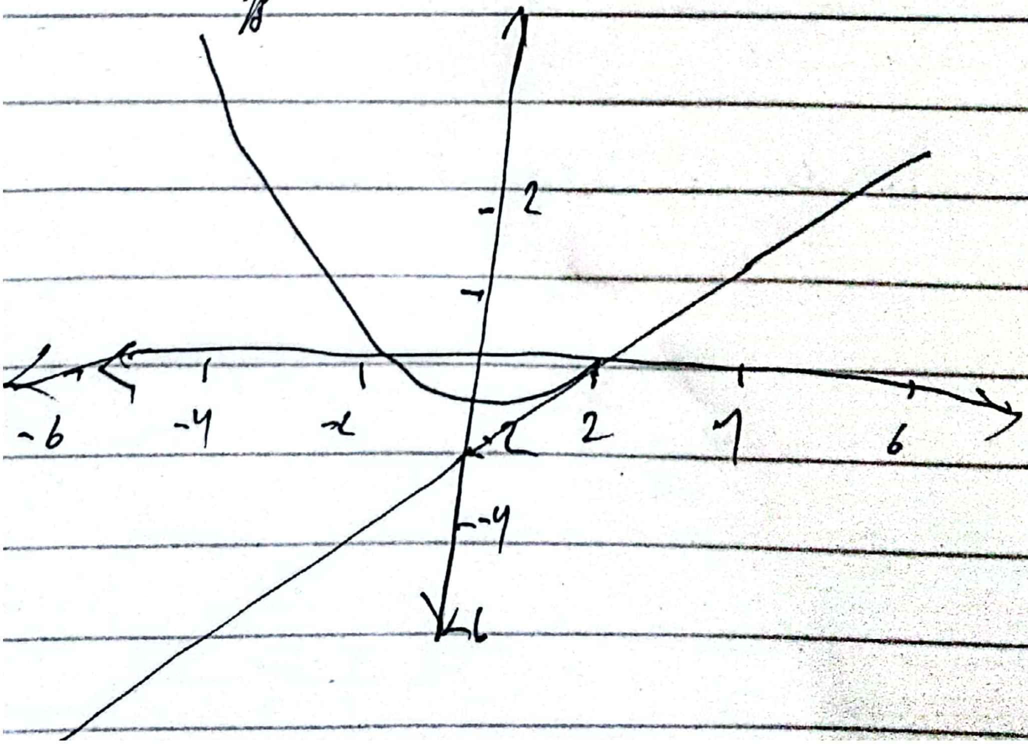
It is standard linear form

Q12 (a) $y' = -2+t-y$

$$y' + 2 + y = t$$

$$\frac{dy}{dt} + 2 + y = t$$

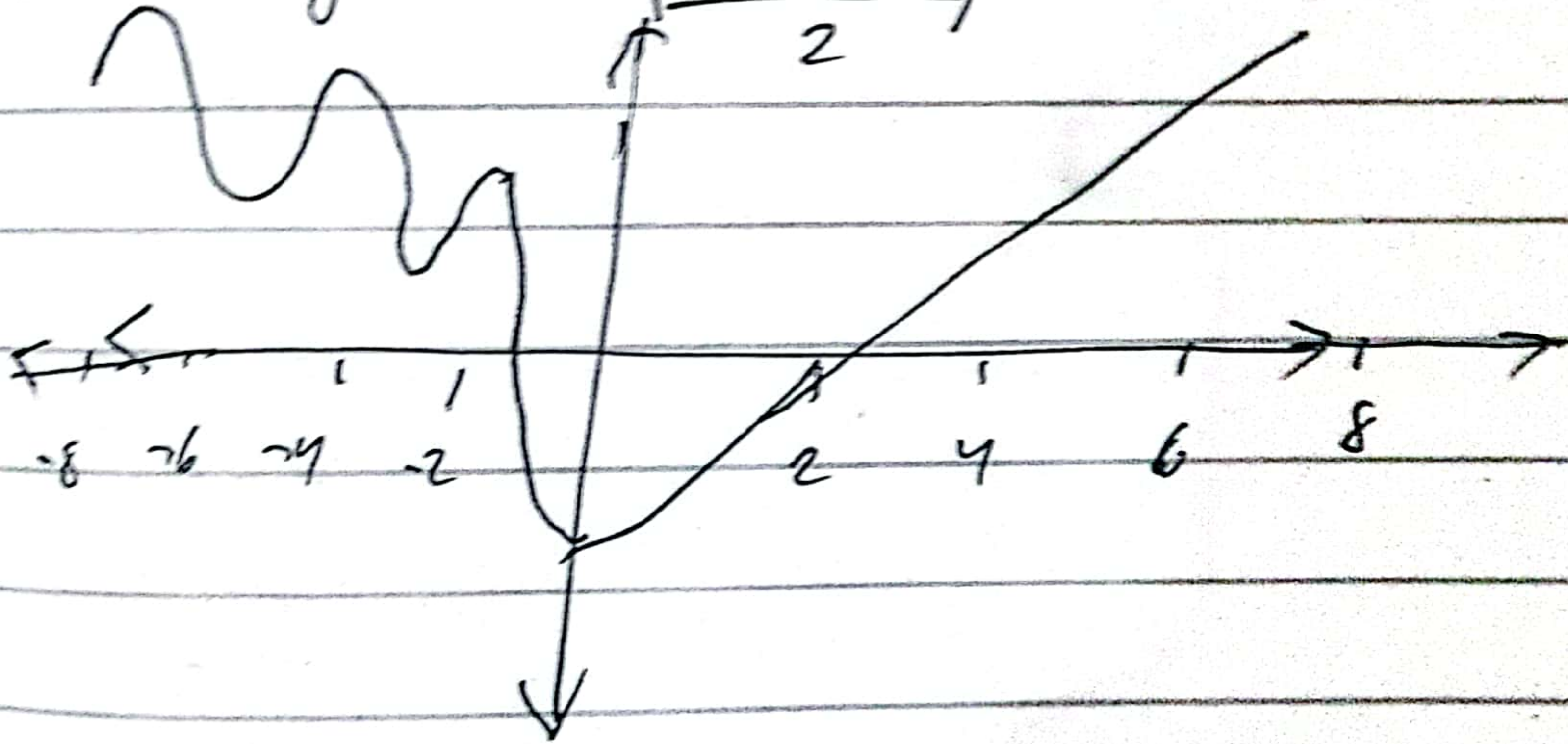
$$y = t^{-3} + \frac{c}{e^t}$$



(b) $y' = 3\sin t + t + y$

$$\frac{dy}{dt} = 3\sin t + t + y$$

$$y = 3e^{t}\left(\frac{-e^{-t}\cos t}{2}\right) - e^{-t}\sin t - t + 1 + ce^{t}$$



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