Q1

untitled9.m

```
syms A(t)
ode= (diff(A,t))==-0.0004332*A;
cond = A(1)==0.005;
Asol (t)=dsolve(ode,cond)
```

Command Window

```
Asol(t) =

(exp(-(3995564766365489*t)/9223372036854775808)*exp(3995564766365489/9223372036854775808))/200

>> Asol(-0.002)

ans =

exp(2001777947949109989/4611686018427387904000)/200

>>
```

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DIFFERENTIAL EQUATIONS ASSIGNMENT #4

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i21-1661 — DS-M

Q1 $\rightarrow$ MATLAB

Q2 $\rightarrow$ $f(t)$ = Pizzas temperature
$T(t)$ = Oven temperature

$$\frac{df}{dt} \propto (f(t) - T(t))$$

T

Ta

$$\therefore \frac{df}{dt} = k\left[f(t) - T(t)\right]$$

objects temperature

Ambient temperature

Q3 $\frac{dy}{dx} + y = f(x)$, where $f(x) = \begin{cases} e^{-x} & 0 \leq x \leq 2 \\ e^{x} & x \geq 2 \end{cases}$

$y(0) = 1$

$P(x) = 1$

Integrating factor $= e^{\int P(x)dx} \rightarrow e^{\int 1\, dx}$
$= e^{x}$

$$\left(\frac{dy}{dx} + y\right) x e^{x} = e^{-x} \cdot x e^{x}$$

$$\int \frac{d}{dx}(y \cdot e^{x}) = \int 1\, dx \Rightarrow y \cdot e^{x} = x + c$$

$$y \cdot e^{x} = x + c$$

$$y \cdot 1 = 0 + c \rightarrow \boxed{c = 1}$$

$$y(2) = y \cdot e^{2} \Rightarrow 3, \quad y = 3e^{-2}$$

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Q3 $\frac{dy}{du} + y = e^u$

$$\int \frac{d}{du}(y \cdot e^u) = \int e^{2u}$$

$$y = \frac{ce^{2u}}{2e^u} \Rightarrow y = \frac{ce^u}{2}$$

$$3e^{-2} = \frac{ce^2}{2}$$

$$3e^{-4} \times 2 = c$$

$$c = 6e^{-4}$$

$$y = 6e^{-4} \times \frac{e^u}{2}$$

$$= 3e^{-4} \cdot e^u$$

$$\therefore \quad y = 3e^{u-4}$$

Q4 Mass $= m$, Initial velocity $= V_0$

$$v(0) = V_0$$

$$\frac{m\,dv}{dt} = -\frac{mgR^2}{(R+u)^2}, \quad \frac{dV}{dt} = \frac{dV}{du} \times \frac{du}{dt}$$

$$\frac{V\,dV}{du}$$

$$\int V \frac{dV}{du} = \int \frac{-gR^2}{(R+u)^2} \Rightarrow \frac{V^2}{2} = \frac{gR^2}{(R+u)} + c$$

$t = 0$, $v = V_0$ & $u = 0$

$$\frac{v^2}{2} - \frac{gR^2}{(R+u)} = c \Rightarrow c = \frac{V_0^2}{2} - gR$$

$$\frac{v^2}{2} = \frac{gR^2}{R+u} + \frac{V_0^2}{2} - gR$$

$$\therefore \quad v = \pm\sqrt{V_0^2 - 2gR + \frac{2gR^2}{(R+u)}}$$

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Q4 (b) Initial velocity when $x = 3$, $v = 0$

$$0 = \frac{gR^2}{R+3} + \frac{1}{2}v_0^2 - gR$$

$$gR - \frac{1}{2}v_0^2 = \frac{gR^2}{R+3}$$

$$\frac{2gR - v_0^2}{2} = \frac{gR^2}{R+3} \Rightarrow (R+3)(2gR - v_0^2) = 2gR^2$$

$$\cancel{2gR^2} - Rv_0^2 + 6gR - 3v_0^2 = \cancel{2gR^2}$$

$$6gR - 3v_0^2 = Rv_0^2$$

$$3(2gR - v_0^2) = Rv_0^2 \quad , \quad 3 = Rv_0^2$$

$$2gR - v_0^2 \rightarrow V_0 = \sqrt{2gR\left(\frac{3}{R+3}\right)}$$

(c) escape velocity , $3 \rightarrow \infty$

$$V_R = \sqrt{2gR} \quad , \quad V_R = 11.1 \text{ km/s}$$

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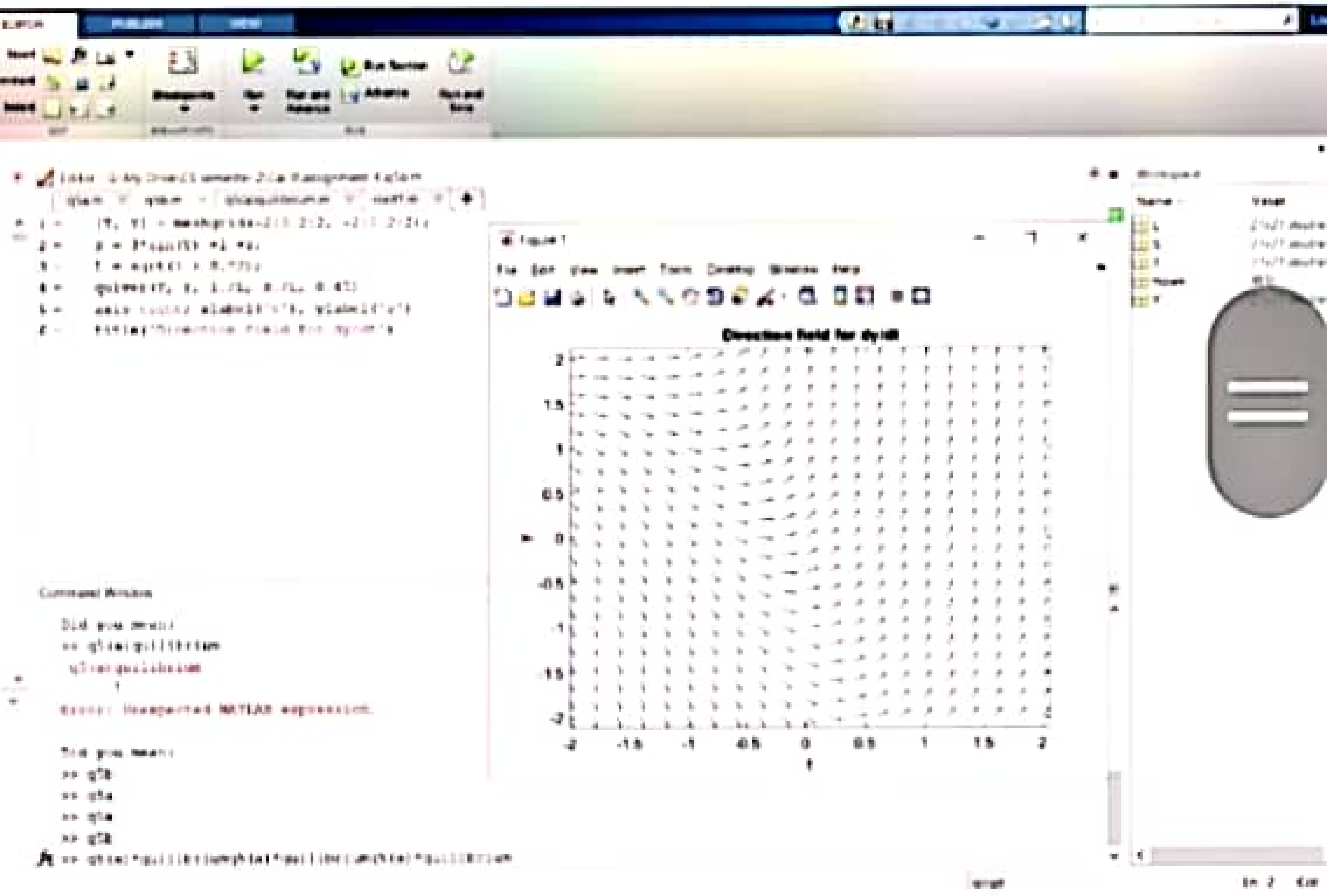

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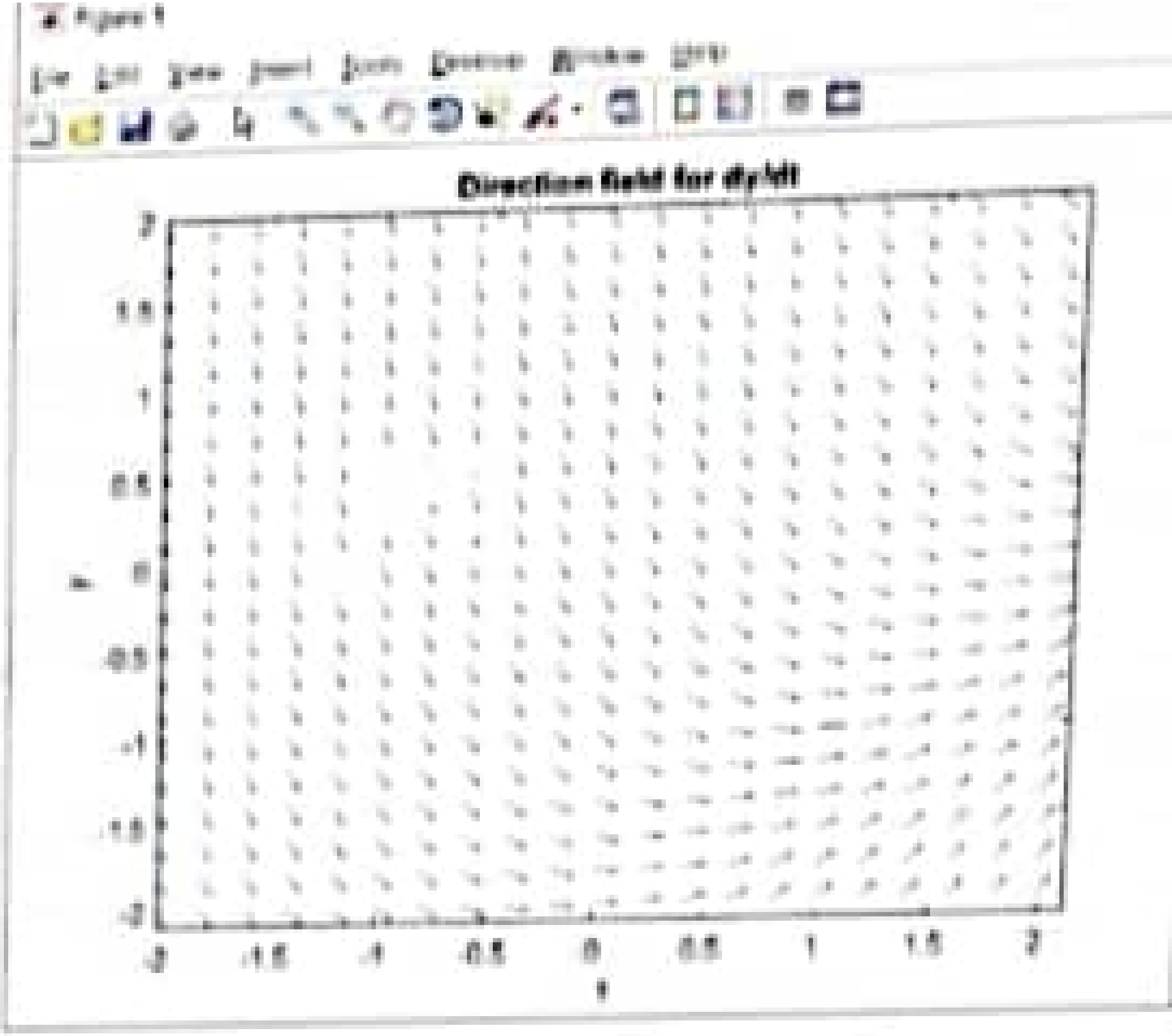
```
[T, Y] = meshgrid(-2:0.2:2, -2:0.2:2);
S = [illegible] + Y - Y;
L = sqrt(1 + S.^2);
quiver(T, Y, 1./L, S./L, 0.45)
axis tight, xlabel('t'), ylabel('y')
title('Direction field for dy/dt')
```

Command Window

```
Error: [illegible]

Did you mean:
>> q1(a)equilibrium
 q1(a)equilibrium
      |
Error: Unexpected MATLAB expression.

Did you mean:
>> q1b
>> q1a
>> q1a
```



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$\boxed{Q6}$ $\frac{dA}{dt} = R_{in} - R_{out}$, $R_{in} = C_{in} \times F_{in}$

$R_{in} = 25(2) = 50$

$R_{out} = C_{out} \times F_{out} \Rightarrow R_{out} = \frac{A}{3}(2)$

$R_{out} = \frac{A}{V}$ → Amount of salt

↙ volume of total capacity

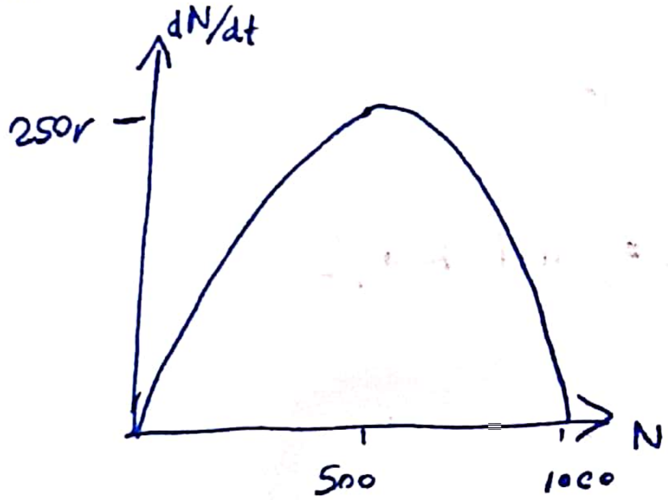
$$\frac{dA}{dt} = 50 - \frac{2A}{3}$$

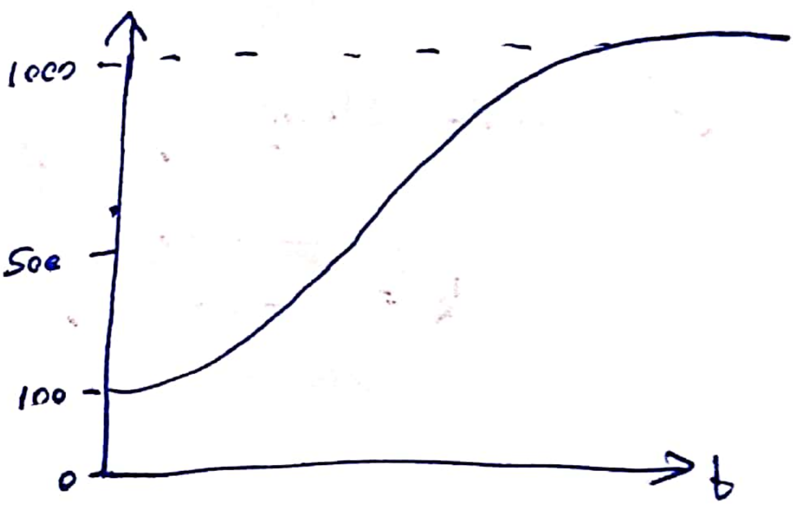
∴, as function of salt A is in form of $\frac{df}{dt} = 50 - \frac{2f}{3}$

$\boxed{Q7}$ $k = 1000$, $\frac{dN}{dt} = rN\left(\frac{1000 - N}{1000}\right)$

$$= \frac{r}{1000} \cdot N(1000 - N)$$

Initial condition is $N(0) = 1000$





$$\frac{dN}{dt} = \frac{r}{1000} N(1000 - N)$$

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Q7) The size of population with highest rate of growth = 500.
To find numerical value of fastest growth rate, given that $r = 0.3$

substitute $r = 0.3$ & $N = 500$

$$\left.\frac{dN}{dt}\right|_{N=500} = \frac{0.3}{1000}(500)(1000-500)$$

$$= 75$$

→ Max. number is attained at 500 fish in pond.

logistic equation → $\frac{dP}{dt} = kP\left(1 - \frac{P}{c}\right)$

P = population, c = carrying capacity

k = constant of proportionality

Q8 $\frac{1}{2}\frac{di}{dt} + 10i = 12$, multiply DE with 2

$$\frac{d}{dt}\left[e^{20t} i\right] = 24e^{20t}$$

→ integrating each side gives: $i(t) = \frac{6}{5} + ce^{-20t}$

$i(0) = 0 \rightarrow 0 = \frac{6}{5} + c$ hence $\boxed{c = -\frac{6}{5}}$

$i(t) = \frac{6}{5} - \frac{6}{5}e^{-20t}$, $i(t) = \frac{e^{-(R/2)t}}{L}\int e^{(R/2)}E(t)\,dt + ce^{-(R/2)t}$

$E(t) = E_0$, $i(t) = \frac{E_0}{R} + ce^{-(R/2)t}$

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$\boxed{Q9}$ $t = 5$, decay conversion = 10 %

10 % conversion = $0.9A_0$ Left

$A = A_0 e^{kt}$

$0.9 A_0 = A_0 e^{k(5)} \rightarrow \ln 0.9 = 5k$

$k = \frac{\ln 0.9}{5} \Rightarrow -0.02$

Q9 (a) $t = 20$, xA_0 left

$xA_0 = A_0 e^{k(20)} \Rightarrow x = e^{(-0.02)(20)}$

$x = 0.67$

therefore 67 % left

percentage converted = 100 − 67 = 33 %

(b) $t = ?$, decay = 60 %

60 % conversion ⇒ $0.4\ A_0$ left

$0.4 A_0 = A_0 e^{-0.02t}$

$t = 45.8$ min $\leftarrow$ $\ln 0.4 = -0.02\ (t)$

$t \approx 46$ min

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$\boxed{Q10}$ $(4x + 3y^2)\,dx + 2xy\,dy = 0$

(a) $M = 4x + 3y^2$ $\quad N = 2xy$

$= \frac{\partial F}{\partial x}$ $\quad = \frac{\partial F}{\partial y}$

$F_{xy} = F_{yx}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial N}{\partial x} = 2y$

$\frac{\partial M}{\partial y} = 6y$

since $M_y \neq N_x$ or $6y \neq 2y$

the equation is not exact!

(b) $u = e^{\int \left(\frac{M_y - N_x}{N}\right) dx}$

$u = e^{\int \left[\frac{6y - 2y}{2xy}\right] dx}$

$u = e^{\int \frac{\cancel{4y}^{2}}{\cancel{2}xy} dx} = u = e^{\int 2/x}$

$= e^{2 \ln x} \Rightarrow u = x^2$

$u = e^{\int \left(\frac{N_x - M_y}{M}\right) dy}$

$= e^{\int \frac{2y - 6y}{4x + 3y^2} dy}$

Not a good choice for u as it contains both x & y

IF $= x^2$ where $n = 2$.

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Q10 (c) $x^2\left[(4x + 3y^2)dx + 2xy\,dy\right] = 0$

$$(4x^3 + 3x^2y^2)\,dx + 2x^3y\,dy = 0$$

$N_x = 6x^2y$

$M_y = 6x^2y$

$M_y = N_x \quad \therefore$ exact

$$F(x,y) = \int N\,dy \longrightarrow \int 2x^3y\,dy$$

$$= \frac{\cancel{2}x^3y^2}{\cancel{2}} + n(x)$$

$$= x^3y^2 + n(x)$$

$$\frac{\partial F}{\partial x} = 3x^2y^2 + n'(x)$$

$$4x^3 + \cancel{3x^2y^2} = \cancel{3x^2y^2} + n'(x)$$

$$4x^3 = n'(x) \longrightarrow \int n'(x)\,dx = \int 4x^3\,dx$$

$$n(x) = \frac{\cancel{4}x^4}{\cancel{4}} = x^4 + c$$

$$F(x,y) = x^3y^2 + x^4 + c$$

& $c = x^3y^2 + x^4$

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