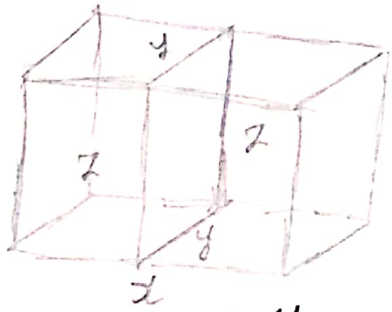
Q1



Let x = length, y = width and z = height

perimeter to cross section $= 2y + 2z$.

length and ~~width~~ girth $= x + 2y + 2z = 108$

$2z = 108 - x - 2y$.

$z = 54 - \frac{x}{2} - y$.

volume of box = xyz and as $z = \uparrow$ therefore

$V(x,y) = xy(54 - \frac{x}{2} - y)$

$V(x,y) = 54xy - \frac{x^2y}{2} - xy^2$

$V_x = 54y - xy - y^2$

for critical point $V_x = 0$

$54y - xy - y^2 = 0$

$y(54 - x - y) = 0$

$y \neq 0$, $54 - x - y = 0$

$54 - y = x \rightarrow$ eq 1

$V_y = 54x - \frac{x^2}{2} - 2xy$

& $V_y = 0$ $x(54 - \frac{x}{2} - 2y) = 0$

$x \neq 0$, $54 - \frac{(54-y)}{2} - 2y = 0$ from eq1

$\times 2$ on both sides

$108 - 54 + y - 4y = 0$

$54 - 3y = 0$, $3y = 54$

$y = 18 \rightarrow$ put in eq 1

$54 - 18 = x$,

$x = 36$.

Therefore critical points are $(36, 18)$

Next to show maximum values we do second derivative test

$V_{xx} = -y$

$V_{yy} = -2x$

$V_{xy} = 54 - x - 2y$

$D(x,y) = V_{xx} \cdot V_{yy} - (V_{xy})^2$

$D(36,18) = 2 \times 36 \times 18 - (54 - 36 - 2 \times 18)^2$

$= 1296 - 324$

$= 972$ which is greater than 0

$V_{xx}(36,18) = -18$ which is less than 0

therefore at $(36,18)$ volume is minimum

$z = 54 - \frac{36}{2} - 18$

$z = 18$

therefore dimensions of box is

length $x = 36$

width $y = 18$

height $z = 18$

volume $= xyz = 36 \times 18 \times 18 = 11664 m^3$.

Q2 $T(x,y) = x^2 + xy + y^2 - 6x + 2$ for $0 \le x \le 5$ & $-3 \le y \le 3$.
Absolute maxima & minima

$f_x = 2x + y - 6$ — ① $f_y = x + 2y = 0$ — ②

as critical points so $f_x = 0$ & $f_y = 0$

$f_x: 2x + y = 6$ $f_y: x + 2y = 0$ solve them simultaneously

$x + 2y = 0$
$4x + 2y = 12$ → × by 2 for solving simultaneously
$- \quad - \quad -$

$-3x = -12$

$x = 4$ put in eq(2) $4 + 2y = 0$, $y = -2$

so critical points are $(4, -2)$

$f_{xx} = 2$ $f_{yy} = 2$ $f_{xy} = 1$

therefore $D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4 - 1 = 3$ as $D > 0$

$f_{xx} > 0$ therefore $(4, -2)$ is minimum

for absolute maximum

$0 \le x \le 5$ $-3 \le y \le 3$

x	y	$f(x,y)$
0	0	2
5	0	-5
4	-2	-10
3	-3	-13

therefor Absolute maxima
$f(0,0) = 2$.

6

Q4) $z = \frac{y}{2}$ $\quad R: 0 \leq x \leq 4, \; 0 \leq y \leq 2$

$$V = \int_0^4 \int_0^2 \frac{y}{2} \, dy dx$$

$$V = \int_0^4 \frac{y^2}{4} \Big|_0^2 = \int_0^4 \frac{(2)^2}{4} - \frac{(0)^2}{2} = \int_0^4 \frac{4}{4}$$

$$V = \int_0^4 1 \, dx = \int_0^4 x = (4) - (0)$$

$$V = 4 .$$

Q5 $\iiint 6z^2$ below $4x+2y+2z=10$ in first octant

$4x+y+2z=10$

$z=\dfrac{10-4x-y}{2}$ so $0 \le z \le \dfrac{10-4x-y}{2}$

Projection on xy plane is $4x+y=10$

so $0 \le x \le \dfrac{5}{2}$ $\quad 0 \le y \le 10-4x.$

$$\int_0^{5/2}\int_0^{10-4x}\int_0^{\frac{10-4x-y}{2}} 6z^2 \, dz\, dy\, dx. \quad = \int_0^{5/2}\int_0^{10-4x} \frac{\cancel{6}^{2} z^3}{\cancel{3}} \Big|_0^{\frac{10-4x-y}{2}}$$

$$\int_0^{5/2}\int_0^{10-4x} 2z^3 \Big|_0^{\frac{10-4x-y}{2}} dy\,dx \Rightarrow \frac{1}{4}\int_0^{5/2}\int_0^{10-4x} (10-4x-y)^3\, dy\,dx.$$

$\dfrac{2(10-4x-y)}{48}$

$$\frac{1}{4}\int \frac{(10-4x-y)^4}{4}\Big|_0^{10-4x}$$

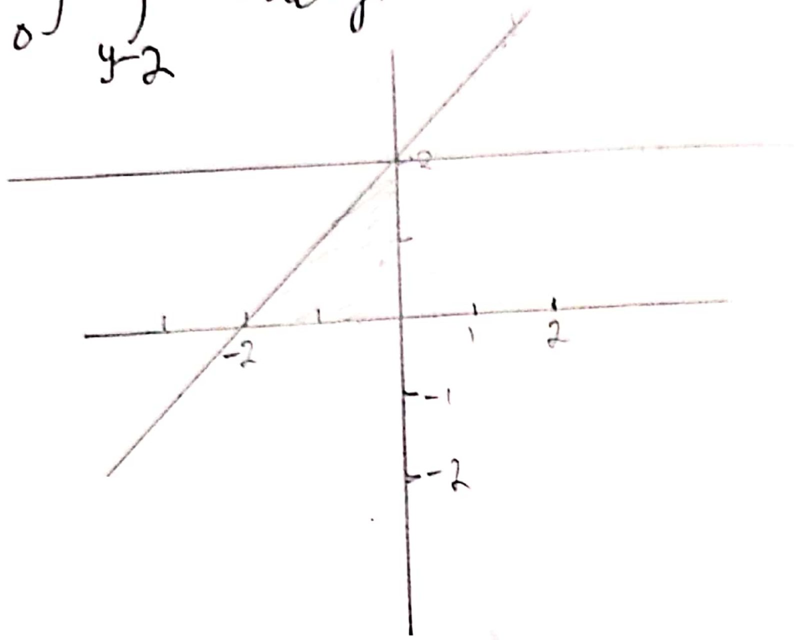
$$\frac{1}{16}\int_0^{5/2} (10-4x)^5\, dx.$$

$$= -\frac{1}{64}\frac{(10-4x)^5}{5}\Big|_0^{5/2} = \frac{1}{64}\cdot\frac{(10)^5}{5} = \frac{625}{2}$$

$$= 312.5.$$

Q6) Sketch the region of integration

$$\int_0^2 \int_{y-2}^{0} dx\,dy$$



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Q 7 Average value of $F(x,y,z) = xyz$, $x=2, y=2, z=2$.

$$\text{Average} = \frac{1}{\text{volume}} \int_0^2\int_0^2\int_0^2 f(x,y,z)\, dz\,dy\,dx \quad \text{where } f(x,y,z) = xyz$$

$$\text{avg} = \frac{1}{2\cdot2\cdot2}\int_0^2\int_0^2\int_0^2 xyz\, dz\,dy\,dx$$

$$\text{avg} = \frac{1}{8}\int_0^2\int_0^2 \frac{xyz^2}{2}\Big|_0^2 dy\,dx = \frac{1}{8}\int_0^2\int_0^2 2xy\, dx\,dy$$

$$= \frac{1}{8}\int_0^2 \frac{\cancel{2}xy^2}{\cancel{2}}\Big|_0^2 dx = \frac{1}{8}\int_0^2 x(2)^2 - x(0)\,\Big| dx.$$

$$\frac{1}{8}\int_0^2 4x\, dx = \frac{1}{8}\int_0 \frac{4^2x^2}{\cancel{2}} = \frac{1}{8}\left(2x^2\Big|_0^2\right.$$

$$= \frac{1}{8}\left(2(2)^2 - 2(0)\right)$$

$$= \frac{1}{\cancel{8}}(\cancel{8})$$

$= 1 \rightarrow$ average volume.

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Q8) Evaluate

$$\int_0^\pi \int_0^\pi \int_0^\pi \cos(u+v+w)\, du\, dv\, dw$$

$$\int_0^\pi \int_0^\pi \sin(u+v+w) \Big|_0^\pi dv\, dw$$

$$\int_0^\pi \int_0^\pi \sin(\pi+v+w) - \sin(0+v+w)\, dv\, dw$$

$$\downarrow$$

$$\int_0^\pi \int_0^\pi \sin(\pi+(v+w)) - \sin(v+w)\, dv\, dw$$

as $\sin(\pi+\theta) = -\sin\theta$

$$\int_0^\pi \int_0^\pi -2\sin(v+w)\, dv\, dw = -2\int_0^\pi \int_0^\pi \sin(v+w)\, dv\, dw$$

$$+2\int_0^\pi + \cos(v+w) \Big|_0^\pi dw \rightarrow +2\int_0^\pi +\cos(\pi+w) - \cos(0+w)$$

since $\cos(\pi+\theta) = -\cos\theta$ therefore

$$+2\int_0^\pi -\cos w - \cos w\, dw = 2\int_0^\pi -2\cos w\, dw = -4\int_0^\pi \cos w\, dw$$

$$-4\left(\sin w \Big|_0^\pi\right) = -4(\sin(\pi) - \sin(0))$$

$$= -4(0-0)$$

triple integral $= 0$.

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Q9.
Find function avg and value of c for which $f(c) = f_{avg}$

$f(x) = 9 - 2e^{4x+1}$ on $[2, 6]$.

f_{avg} can be $= \frac{1}{b-a}\int_a^b f(x)\,dx$ for $[a, b]$

$f_{avg} = \frac{1}{6-2}\int_2^6 (9 - 2e^{4x+1})$

$= \frac{1}{4}\left(9x - \frac{2e^{4x+1}}{4}\right)\Big|_2^6$

$= \frac{1}{4}\left[\left(\frac{9(6)(4) - 2e^{25}}{4}\right) - \left(\frac{9(2)(4) - 2e^{9}}{4}\right)\right]$

$\frac{1}{4}\left(\frac{216 - 2e^{25}}{4} - \frac{72 + 2e^{9}}{4}\right)$

f_{avg} $\frac{1}{4}\left(\frac{216 - 72 - 2e^{25} + 2e^{9}}{4}\right) = \underline{-9.00 \times 10^{9}}$

$f(c) = f_{avg}$, $9 - 2e^{4c+1} = -9.00 \times 10^{9}$.

$e^{4c+1} = 4.50 \times 10^{9}$

$4c+1 \ln e = \ln(-4.50 \times 10^{9})$

$4c+1 = 22.227$.

$4c = 23.227$

$\underline{c = 5.80}$.

(b) $f_{avg} = \frac{1}{4\pi}\int_0^{4\pi} 8 - \cos\frac{x}{4}$

$f_{avg} = \frac{1}{4\pi}\left(8x - \frac{\sin(x/4)}{1/4}\right)\Big|_0^{4\pi} = \frac{1}{4\pi}[32\pi - 4\sin\pi - 0]$

$f_{avg} = 8$.

Q11 $\iint 12x^3 - 3\,dA$ over region in the graph

Q9 as $f_{avg} = f_c$ $= 8 = 8 - \cos(\frac{c}{4})$

$\cos(\frac{c}{4}) = 0$, $\frac{c}{4} = \cos^{-1}(0)$

$\cancel{4} \times \frac{c}{\cancel{4}} = \frac{\pi}{\cancel{2}} \times \cancel{4}^2$, $\boxed{c = 2\pi}$

Q10 $v(t) = t^3 - 6t^2$ for $0 \le t \le 10$

(a) furthest to left (time).

first find critical points

$t^3 - 6t^2 = 0$

$\frac{t^3}{t^2} = \frac{6t^2}{t^2}$, $t = 6$. or

particle to left when $v(t) < 0$

$t^3 - 6t^2 < 0$

$t^2(t-6) < 0$ so $t = 0$, $t = +6 \Rightarrow$ critical points.

now for find furthest to the left when we have $0 \le t \le 10$ endpoints and c.p

$0 \le t \le 6$ therefore $\int v(t) = \int_6^{10} x(t)$ & $\int_0^6 x(t)$

$x(t) = \int_6^{10} \frac{t^4}{4} - 3t^3 = 2200m$

$x(t) = \int_6^6 \frac{t^4}{4} - 3t^3 = 216m$

As $x(t) = 216$ is lowest therefore furthest to left is at time $\underline{0 \le t \le 6}$

(b) Velocity of the particle fastest
velocity increasing when $v'(t) > 0$
therefore
$v'(t) = 3t^2 - 12t > 0$.
$3t(t-4) > 0$
$0 \leq t \leq 4$ as total interval is
$0 \leq t \leq 10$ therefore the answer is
particle is fastest between $\underline{0 \leq t \leq 4}$.

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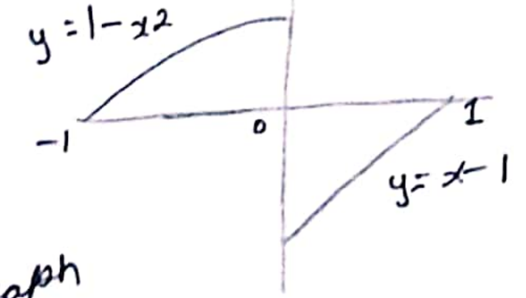
Q11 $\iint_D 12x^3 - 3\,dA$ over region in the graph



from graph
the bounds for D are

x -1 to 0 & x 0 to 1
y 0 to $1-x^2$ & y $x-1$ to 0

therefore

$$= \iint (12x^3-3)\,da = \int_{-1}^{0}\int_{0}^{1-x^2} (12x^3-3)\,dy\,dx + \int_{0}^{1}\int_{x-1}^{0} (12x^3-3)\,dy\,dx$$

$$= \int_{-1}^{0} (12x^3-3)y\Big|_{0}^{1-x^2} + \int_{0}^{1} (12x^3-3)y\Big|_{x-1}^{0}$$

$0 - (x-1)$
$-x+1$

$$= \int_{-1}^{0} (12x^3-3)(1-x^2-0)\,dx + \int_{0}^{1} (12x^3-3)(0-x+1)\,dx$$

$$= \int_{-1}^{0} (12x^3-12x^5-3+3x^2)\,dx + \int_{0}^{1} (-12x^4+12x^3+3x-3)$$

$$= \frac{12x^4}{4} - \frac{12x^6}{6} - 3x + \frac{3x^3}{3}\Big|_{-1}^{0} + \frac{-12x^5}{5} + \frac{12x^4}{4} + \frac{3x^2}{2} - 3x\Big|_{0}^{1}$$

$$[3(0)^4 - 2(0)^6 - 3(0) + 3(0)] - [3(-1)^4 - 2(-1)^6 - 3(-1) + 3(-1)] -$$

$$\left[\frac{-12(1)^5}{5} + 3(1)^4 + \frac{3(1)}{2} - 3(1)\right]$$

$$= -3 + \frac{12}{5} - \frac{3}{2}$$

$$= -2.1.$$

Q14 $T = xyz$ at $(0,0,0)$ & $(2,2,2)$

volume = $l \times w \times h$, volume $= 2 \times 2 \times 2 = 8$

$$\text{Average temprature} = \frac{1}{8} \iiint_V xyz \, dv$$

$$= \frac{1}{8} \int_0^2 \int_0^2 \int_0^2 xyz \, dz\,dy\,dx$$

$$= \frac{1}{8} \int_0^2 \int_0^2 \frac{xyz^2}{2} \Big|_0^2 \, dy\,dx$$

$$= \frac{1}{8} \int_0^2 \int_0^2 xy2 \, dy\,dx$$

$$= \frac{1}{8} \int_0^2 \frac{xy^2 \cancel{2}}{\cancel{2}} \Big|_0^2 dx = \frac{1}{8} \int_0^2 4x \, dx$$

$$= \frac{1}{8} \left(\frac{\cancel{4}x^2}{\cancel{2}} \Big|_0^2 \right)$$

$$= \frac{1}{8} \left(2(2)^2 - 2(0) \right)$$

$$= \frac{1}{8}(2(4)) \;, = \frac{1}{\cancel{8}}(\cancel{8})$$

averge temprature $= 1$