National University
of Computer & Emerging Sciences
Islamabad

MT1006 – Differential Equations (Cal-II)

Assignment No: 01 **Individual Assignment**

Section: BS CS, BS AI, BS DS, BS CySec **Semester:** Spring 2022

Due date: Thursday, 03 March 2022 **Marks:** 21*10=210

Instructions:

1. Plagiarized work will result in zero marks.
2. No retake or late submission will be accepted.
3. Attach complete code, results, and screenshot for questions that require programming solution. Programs/codes should not be handwritten.
4. Questions that show the icon  require partial or complete solution using the approved programming tool.
5. The assignment is to be submitted in softcopy as well as in hardcopy.
6. The softcopy should be a single PDF file of your complete assignment including programming and non-programming questions.
7. The PDF file should be according to the following format: id_section_A1 e.g., i21-123456_A_A1. A1 in the end denotes Assignment 1.
8. The images of by-hand solution should be properly scanned. You can use any mobile application such as CamScanner or Adobe Scan for scanning. Each of these applications allow you to export pdf or image files which you can use to combine with your programming solutions. Do not attach direct images from the camera application of your mobile phone, or screenshots.

Question 1

The resistivity ρ of a conducting wire is the reciprocal of the conductivity and is measured in units of ohm-meters $(\Omega - m)$. The resistivity of a given metal depends on the temperature according to the equation:

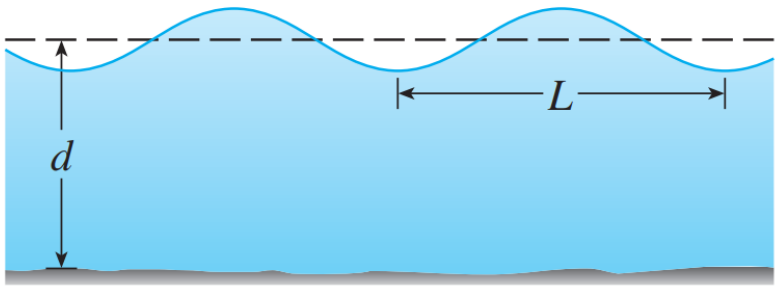
$$\rho(t) = \rho_{20} e^{\alpha(t-20)},$$

where *t* is the temperature in ^{0}C. There are tables that list the values of α (called the temperature coefficient) and ρ_{20} (the resistivity at 20^0C) for various metals. Except at very low temperatures, the resistivity varies almost linearly with temperature and so it is common to approximate the expression for $\rho(t)$ by its first- or second-degree Taylor polynomial at *t=20.*

a) Find the expressions for these linear and quadratic approximations.
b) For copper, the tables give $\alpha = 0.0039/^0$C and $\rho_{20} = 1.7 * 10^{-8}\ \Omega - m$. Graph the resistivity of copper and the linear and quadratic approximations for $-250^0\text{C} \leq t \leq 1000^0$C using programming tool.

Question 2

If a water wave with length *L* moves with a velocity v across a body of water with depth *d*, as in figure below,



then

$$v^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L},$$

a) If the water is deep, show that $v \approx \sqrt{\frac{gL}{2\pi}}$.
b) If the water is shallow, use the Maclaurin series for tanh to show that $v \approx \sqrt{gd}$. (Thus, in shallow water the velocity of a wave tends to be independent of the length of the wave.)

Question 3

Evaluate $\int e^{-x^2}\, dx$ as an infinite series.

Question 4

The function J_1 defined by

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!\,(n+1)!\,2^{2n+1}},$$

is called the Bessel function of order 1.

a) Find its domain.
b) Graph the first several partial sums on a common screen using programming software.

Question 5

For the following power series determine the interval and radius of convergence.

$$\sum_{n=0}^{\infty} \frac{4^{1+2n}}{5^{n+1}} (x+3)^n.$$

Question 6

Write down $T_3(x), T_4(x), and\ T_5(x)$ for the Taylor series of $f(x) = \ln(3+4x)$ about $x = 0$. Graph all three of the Taylor polynomials and $f(x)$ on the same graph for the interval $[-\frac{1}{2}, 2]$ using programming tool.

Question 7

Use series to evaluate the limit

$$\lim_{x \to 0} \frac{1 - cosx}{1 + x - e^x}.$$

Question 8

A thin metal plate, located in the xy-plane, has temperature $T(x,y)$ at the point (x,y). The level curves of *T*are called isothermals because at all points on such a curve the temperature is the same. Sketch the isothermals if the temperature is given by

$$T(x,y) = \frac{100}{1 + x^2 + 2y^2}.$$

Question 9

A profit function for a hardware manufacturer is given by

$$f(x,y) = 16 - (x-3)^2 - (y-2)^2,$$

where x is the number of nuts sold per month (measured in thousands) and y represents the number of bolts sold per month (measured in thousands). Profit is measured in thousands of dollars. Simplify the expression and then use programming tool to sketch a graph of the simplified function.

Question 10

The volume of a right circular cylinder is calculated by a function of two variables, $V(x,y) = \pi x^2 y$, where x is the radius of the right circular cylinder and y represents the height of the cylinder. Evaluate $V(2,5)$ and explain what this means.

Question 11

Determine whether any of the lines are parallel or identical.

$$L_1: x = 3 + 2t, \quad y = -6t, \quad z = 1 - 2t,$$

$$L_2: x = 1 + 2t, \quad y = -1 - t, \quad z = 3t,$$

$$L_3: x = -1 + 2t, \quad y = 3 - 10t, \quad z = 1 - 4t,$$

$$L_4: x = 5 + 2t, \quad y = 1 - t, \quad z = 8 + 3t.$$

Question 12

Find the domain and range of the given functions. Also sketch the domain of the function by-hand.

a) $f(x,y) = \sqrt{4 - x^2 - y^2}$.
b) $f(x,y) = \sqrt{4 - x^2 - 4y^2}$.
c) $f(x,y) = \ln(4 - x - y)$.

Question 13 </>

Find and sketch the domain of the following functions using programming tool:

a) $f(x,y,z) = \sqrt{1 - x^2 - y^2 - z^2}$.
b) $f(x,y,z) = \ln(16 - 4x^2 - 4y^2 - z^2)$.

Question 14 </>

Describe the level curves of the function. First, sketch a contour map of the surface using level curves for the given c-values.

a) $f(x,y) = e^{\frac{xy}{2}}, \quad c = 2, 3, 4, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.
b) $f(x,y) = \frac{x}{x^2+y^2}, \quad c = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$.
c) $f(x,y) = \ln(x - y), \quad c = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$.

Question 15

Describe the contour lines for several values of c for the following function using the programming tool.

$$z = x^2 + y^2 - 2x - 2y.$$

Question 16

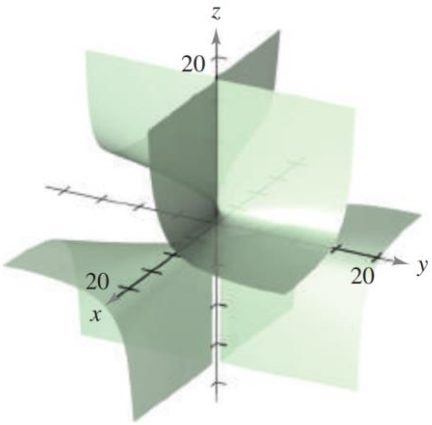
The temperature T (in degrees Celsius) at any point (x, y) in a circular steel plate of radius 10 meters is:

$$T = 600 - 0.75x^2 - 0.75y^2.$$

where $x \; and \; y$ are measured in meters. Sketch some of the isothermal curves using programming tool.

Question 17

Consider $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy}$ (see figure).



Determine the limit along any line of the form $y = ax$.

Question 18

Use polar coordinates and L'H$\hat{o}$pital's to find the limits:

a) $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$.

b) $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$.

Question 19

Discuss the continuity of the function:

$$f(x,y) = \begin{cases} \frac{sinxy}{xy}, & xy \neq 0 \\ 1, & xy = 0 \end{cases}.$$

Question 20

A measure of how hot weather feels to an average person is the Apparent Temperature Index. A model for this index is:

$$A = 0.885t - 22.4h + 1.20th - 0.544,$$

where A is the apparent temperature in degrees Celsius, t is the air temperature, and h is the relative humidity in decimal form.

a) Find $\frac{\partial A}{\partial t}$ and $\frac{\partial A}{\partial h}$ when $t = 30^0$ and $h = 0.80$.
b) Which has a greater effect on A, air temperature or humidity? Explain.

Question 21

Find the point of intersection of the line through $(1, -3, 1)\ and\ (3, -4, 2)$ and the plane given by $x - y + z = 2$.