ASSIGNMENT #01

BS(DS)-M

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Q1. (a) $P(t) = P_{20}\, e^{a(t-20)}$, using taylor series at $t=20$.

$$P(t) = P'(20)(t-20) + \frac{P''(20)}{2!}(t-20)^2$$

since $P'(t) = a e^{(t-20)}$

$P''(t) = a^2 e^{(t-20)}$

$$\therefore \quad P(t) = P(20) + a(t-20) + \frac{a^2}{2!}(t-20)^2$$

$$P(t) = 1 + a(t-20) + \frac{a^2}{2!}(t-20)^2$$

For linear approximation $\longrightarrow$ $P(t) = 1 + a(t-20)$

For quadratic approximation $\longrightarrow$ $P(t) = 1 + a(t-20) + \frac{a^2}{2}(t-20)^2$

(b) Q1 $a = 0.0039/°C$ $P_{20} = 1.7 \times 10^{-8}\,\Omega\text{-m}$.

For linear approximation $= 1.7 \times 10^{-8}\left[1 + 0.0039(t-20)\right]$

For quadratic approximation $= 1.7 \times 10^{-8}\left[1 + 0.0039(t-20) + \frac{(0.0039)^2}{2}(t-20)^2\right]$

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Q2. (a)

$L \to$ length

$d \to$ depth.

when $x \to \infty$,

$\tanh x \to 1$

So $\tanh \frac{2\pi d}{L} \approx 1$

$$\therefore v^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L} \approx \frac{gL}{2\pi}$$

$$\sqrt{v^2} \approx \sqrt{\frac{gL}{2\pi}} \Rightarrow v \approx \sqrt{\frac{gL}{2\pi}}$$

(b) $\boxed{Q2}$ $f(x) = \tanh x$, $f'(x) = \operatorname{sech}^2 x$, $f''(x) = -2\tanh x \operatorname{sech}^2 x$

$$f'''(x) = -2(-2\tanh^2 x \cdot \operatorname{sech}^2 x + \operatorname{sech}^4 x)$$

$$= 2\operatorname{sech}^2 x \,(3\tanh^2 x - 1)$$

If $x = 0$ then $f(x) = f(0) = \tanh 0 \to 0$

then $f'(0) = 1$, $f''(0) = 0$ & $f'''(0) = -2$

thus the corresponding Maclaurin series will be

$$\tanh x \approx 0 + x + 0 - \frac{2}{3!} x^2 \approx x - \frac{2x^2}{3!}$$

$\boxed{\text{here } x = \frac{2\pi d}{L}}$

$$\tanh \frac{2\pi d}{L} \approx \frac{2\pi d}{L} - \frac{2}{3!}\left(\frac{2\pi d}{L}\right)^2$$

$$\approx \frac{2\pi d}{L} - \frac{1}{3}\left(\frac{2\pi d}{L}\right)^3$$

$$\approx \frac{2\pi}{L}\left[d - \frac{1}{3}\left(\frac{2\pi d}{L}\right)^2\right]$$

when water is shallow,

d is small.

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Q2 (b)

Substitute the Maclaurin series in equation

$$V^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L}$$

$$\frac{g\cancel{L}}{\cancel{2\pi}}\left(\frac{\cancel{2\pi}}{\cancel{L}}\left[d - \frac{1}{3}\left(\frac{2\pi d}{L}\right)^2\right]\right)$$

$$= g\left[d - \frac{1}{3}\left(\frac{2\pi d}{L}\right)^2\right]$$

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as d is small so d^2 will be smaller

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thus $V^2 \approx g\left[d - \frac{1}{3}\left(\frac{2\pi d}{L}\right)^2\right]$

$$V^2 \approx g\left[d - 0\right]$$

$$\cancel{\sqrt{V^2}} \approx \sqrt{gd} \Rightarrow V \approx \sqrt{gd}$$

thus velocity $= \sqrt{gd}$ if water is shallow.

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Q3 As $e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$$\therefore e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \cdots$$

$$= 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$$

Applying integration

$$\int e^{-x^2}\,dx \Rightarrow \int \left[1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} \cdots \right] dx$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} + \cdots dx$$

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Q4. (a) $$a_{n+1} = \frac{(-1)^{n+1} x^{2(n+1)+1}}{(n+1)!(n+2)!2^{2(n+1)+1}}$$

$$= \frac{(-1)^{n+1} x^{2n+3}}{(n+1)!(n+2)!2^{2n+3}}$$

$$a_n = \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$$

using ratio test since $\frac{(-1)^n}{(-1)^{n+1}} = 1$

$$\therefore \frac{a_{n+1}}{a_n} = \frac{\frac{x^{2n+3}}{(n+1)!(n+2)!2^{2n+3}}}{\frac{x^{2n+1}}{n!(n+1)!2^{2n+1}}}$$

$$\lim_{n\to\infty} \left| \frac{x^{2n+3}}{(n+1)!(n+2)!2^{2n+3}} \times \frac{n!(n+1)!2^{2n+1}}{x^{2n+1}} \right|$$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{\cancel{x^{2n}} \cdot x^3}{\cancel{(n+1)}!(n+2)!\cancel{2^{2n}} \cdot 2^3} \times \frac{n!\cancel{(n+1)}!\cancel{2^{2n}} \cdot 2}{\cancel{x^{2n}} \cdot \cancel{x}} \right|$$

$$\lim_{n\to\infty} \left| \frac{x^2}{4} \times \frac{1}{(n+1)(n+2)} \right| \Rightarrow \left| \frac{x^2}{4} \times \frac{1}{(\infty+1)(\infty+2)} \right| = \frac{x^2}{4} \times \frac{1}{\infty}$$

$$= \frac{x^2}{4}(0) \to 0 < 1$$

$\therefore$, this function is convergent for any real value of x

$\boxed{\text{Domain } (-\infty, \infty)}$

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Q5. $\sum_{n=0}^{\infty} \frac{4^{1+2n}}{5^{n-1}}(x+3)^n$

$$a_{n+1} = \frac{4^{1+2(n+1)}}{5^{(n+1)+1}}(x+3)^{n+1}$$

$$= \frac{4^{2n+3}}{5^{n+2}}(x+3)^{n+1}$$

$$a_n = \frac{4^{1+2n}}{5^{n+1}}(x+3)^n$$

$$\frac{a_{n+1}}{a_n}$$

$$\downarrow$$

$$\frac{\frac{4^{2n+3}}{5^{n+2}}(x+3)^{n+1}}{\frac{4^{1+2n}}{5^{n+1}}(x+3)^n}$$

$$\lim_{n\to\infty}\left|\frac{\cancel{4^{2n}}\cdot 4^3}{\cancel{5^n}\cdot 5^2}\cancel{(x+3)^n}\cdot(x+3) \times \frac{\cancel{5^n}\cdot \cancel{5}}{\cancel{4^{2n}}\cdot \cancel{4}\cancel{(x+3)^n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{4^2}{5}(x+3)\right| \Longrightarrow \frac{16}{5}(x+3)$$

As series is convergent so

$$-1 < \frac{16}{5}(x+3) < 1 \Rightarrow -\frac{5}{16} < x+3 < \frac{5}{16}$$

$$-\frac{5}{16}-3 < x < \frac{5}{16}-3$$

$$-\frac{53}{16} < x < -\frac{43}{16}$$

Radius of convergence = $5/16$ $\boxed{\text{Interval} = \left(\frac{-53}{16}, \frac{-43}{16}\right)}$

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Q6

$f(x) = \ln(3+4x)$, $f(0) = \ln 3$

$f'(x) = \frac{4}{3+4x}$, $f'(0) = 4/3$

$f''(x) = -16(3+4x)^{-2}$, $f''(0) = -16/9$

$f'''(x) = 128(3+4x)^{-3}$, $f'''(0) = 128/27$

$f''''(x) = -1536(3+4x)^{-4}$, $f''''(0) = -1536/81$

$f'''''(x) = 24576(3+4x)^{-5}$, $f'''''(0) = 24576/243$

$$T_3(x) = f(0) + f'(0)(x) + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}$$

$$= \ln 3 + 4/3\, x - \frac{16x^2}{18} + \frac{128x^3}{182}$$

$$T_4(x) = \ln 3 + 4/3\, x - \frac{16x^2}{18} + \frac{128x^3}{182} - \frac{1536}{1944}x^4$$

$$T_5(x) = \ln 3 + 4/3\, x - \frac{16x^2}{18} + \frac{128x^2}{182} - \frac{1536x^4}{1944} + \frac{24576x^5}{29160}$$

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Q7 $\lim_{x \to 0} \dfrac{1 - \cos x}{1 + x - e^x}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\frac{1 - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right)}{1 + x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right)}$$

$$= \frac{\frac{x^2}{2!} \cancel{- \frac{x^4}{4!} - \frac{x^6}{6!}} + \cdots}{\frac{x^2}{2!} \cancel{- \frac{x^3}{3!} - \frac{x^4}{4!} - \frac{x^5}{5!} - \frac{x^6}{6!}} - \cdots}$$

$$\frac{\frac{1}{2} - 0}{-\frac{1}{2} - 0} \Longrightarrow -1$$

$$\therefore \lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x} = -1$$

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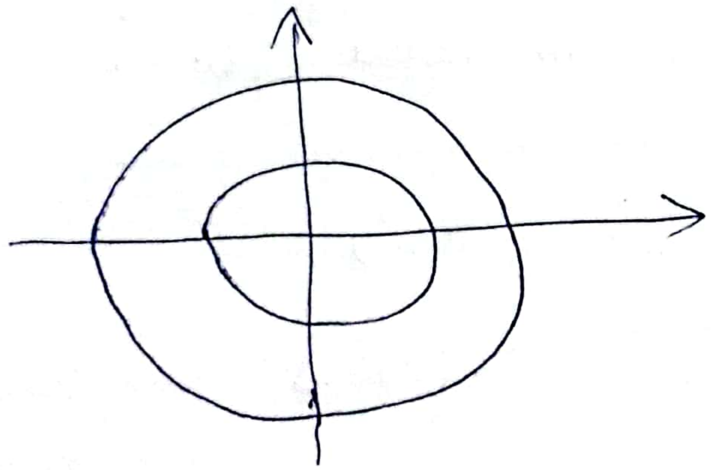
(Q8) $T(x,y) = \dfrac{100}{1+x^2+2y^2}$

since $\dfrac{100}{1+x^2+2y^2} = c$

$1+x^2+2y^2 = \dfrac{100}{c}$, $x^2+2y^2 = \dfrac{100}{c} - 1$

$\dfrac{x^2}{\frac{100}{c}-1} + \dfrac{y^2}{\frac{50}{c}-\frac{1}{2}} = 1 \leftarrow$ $\boxed{\text{equation of ellipse}}$

$a^2 = \sqrt{\frac{100}{c}-1}$ & $b^2 = \sqrt{\frac{50}{c}-\frac{1}{2}}$



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Q9 $f(x,y) = 16 - (x-3)^2 - (y-2)^2$

$$16 - (x-3)^2 - (y-2)^2 = c$$

$$16 - c = (x-3)^2 + (y-2)^2$$

at $c = 0$, $16 = (x-3)^2 + (y-2)^2$

$$(x-3)^2 + (y-2)^2 = 4^2$$

$\Downarrow$ form of

$$(x-h)^2 + (y-2) = r^2$$

this equation describes it is a circle centred at point $(3,2)$ with radius $\sqrt{16-c}$

Q10 $V(x,y) = \pi x^2 y$

$V(2,5) = \pi \cdot (2)^2 \cdot 5 \Rightarrow 20\pi$

$= 62.8$

This means when radius is 2 & height = 5, volume of right circular cylinder = 62.8

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QIV

L_1: $x = 3+2t$, $y = -6t$, $z = 1-2t$

L_2: $x = 1+2t$, $y = -1-t$, $z = 3t$

L_3: $x = -1+2t$, $y = 3-10t$, $z = 1-4t$

L_4: $x = 5+2t$, $y = 1-t$, $z = 8+3t$

symmetric equation of line is given by:

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

L_1: $\frac{x-3}{2} = \frac{y}{-6} = \frac{z-1}{-2}$ & is parallel to $2\hat{i} - 6\hat{j} - 2\hat{k}$

L_2: $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ & is parallel to $2\hat{i} - \hat{j} + 3\hat{k}$

L_3: $\frac{x+1}{2} = \frac{y-3}{-10} = \frac{z-1}{-4}$ & is parallel to $2\hat{i} - 10\hat{j} - 4\hat{k}$

L_4: $\frac{x-5}{2} = \frac{y-1}{-1} = \frac{z-8}{3}$ & is parallel to $2i - j + 3k$

Since the direction of L_2 & L_4 are proportional. ∴ Lines L_2 & L_4 are parallel.

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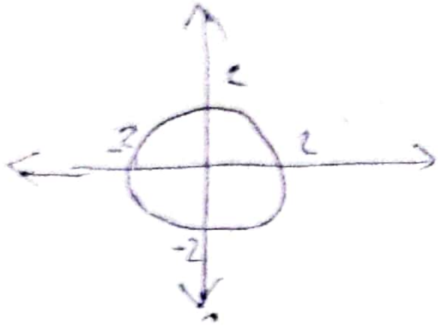
Q12 (a) $f(x,y) = \sqrt{4-x^2-y^2}$



$4-x^2-y^2 \geq 0$

$4 \geq x^2+y^2$

$x^2+y^2 \leq 2^2$

Domain = $\{(x,y) \in R^2 \mid x^2+y^2 \leq 4\}$

Range = $x^2=0$ & $y^2=0$

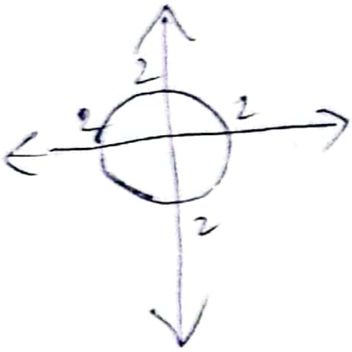
$\therefore f(x,y) = \sqrt{4} = 2$

$0 \leq$ Range ≤ 2

(b) Q12 $f(x,y) = \sqrt{4-x^2-4y^2}$

$4-x^2-4y^2 \geq 0 \rightarrow 4 \geq x^2+4y^2$

$x^2+(2y)^2 \leq 2^2$



Domain $\Rightarrow \{(x,y) \in R^2 \mid x^2+(2y)^2 \leq 4\}$

Range $\Rightarrow x^2=0$ & $y^2=0$ $\therefore f(x,y) = \sqrt{4} = 2$

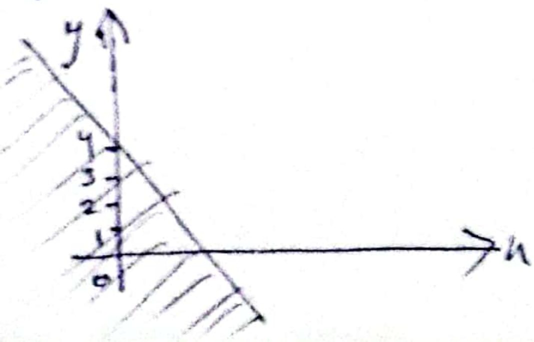
$0 \leq$ Range ≤ 2

(c) Q12 $f(x,y) = \ln(4-x-y)$

$4-x-y > 0 \rightarrow 4 > x+y \Rightarrow y < 4-x$

Domain $\Rightarrow \{(x,y) \in R^2 \mid -\infty < x < \infty,\ y < 4-x\}$

Range $\rightarrow f(x,y)$ is set of all numbers ie R^2



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Q13 (a) $f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

$1 - x^2 - y^2 - z^2 \geq 0$

$1 \geq x^2 + y^2 + z^2 \Rightarrow x^2 + y^2 + z^2 \leq 1$

hence function is sphere of radius 1.

Domain $\rightarrow \{ f(x,y,z) \in R^2 \mid x^2 + y^2 + z^2 \leq 1 \}$

Range $\rightarrow$ $0 \leq$ Range ≤ 1

(b) $f(x,y,z) = \ln(16 - 4x^2 - 4y^2 - z^2)$

$16 - 4x^2 - 4y^2 - z^2 > 0$

$16 > 4x^2 + 4y^2 + z^2$

Domain $\Rightarrow \{ f(x,y,z) \in R^3 \mid 4x^2 + 4y^2 + z^2 < 16 \}$

Range $\rightarrow (-\infty, \ln 16)$

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Q14 (a) $f(x,y) = e^{xy/2}$, $c = 2, 3, 4, 1/2, 1/3, 1/4$

$c = e^{xy/2}$

$\ln c = xy/2 \ln e$

for $c=2$, $\ln 2 = xy/2$ $y = \frac{2\ln 2}{x}$	for $c=3$, $\ln 3 = xy/3$ $y = \frac{(\ln 3)\,2}{x}$	for $c=4$ $\ln 4 = xy/2$ $y = \frac{2\ln 4}{x}$
for $c = 1/2$, $\ln 1/2 = xy/2$ $y = \frac{2\ln 1/2}{x}$	for $c = 1/3$, $\ln\frac{1}{3} = \frac{xy}{2}$, $y = \frac{2\ln 1/3}{x}$	for $c = 1/4$, $\ln\frac{1}{4} = xy/2$ $y = \frac{2\ln\frac{1}{4}}{x}$

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Q14 (b) $f(x,y) = \frac{x}{x^2+y^2}$

$c = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$

$c = \frac{x}{x^2+y^2}$,

$cx^2 + cy^2 = x$

$y = \sqrt{\frac{x - cx^2}{c}}$

for $c = \frac{1}{2}$,

$y = \sqrt{\frac{x - \frac{1}{2}x^2}{1/2}}$

for $c = -\frac{1}{2}$,

$y = \sqrt{\frac{x + \frac{1}{2}x^2}{-\frac{1}{2}}}$

for $c = 3/2$,

$y = \sqrt{\frac{x - 3/2\, x^2}{3/2}}$

for $c = -3/2$,

$y = \sqrt{\frac{x + 3/2\, x^2}{-\frac{3}{2}}}$

for $c = 1$,

$y = \sqrt{\frac{x - x^2}{1}}$

for $c = -1$,

$y = \sqrt{\frac{x + x^2}{-1}}$

for $c = 2$,

$y = \sqrt{\frac{x - 2x^2}{2}}$

for $c = -2$

$y = \sqrt{\frac{x + 2x^2}{-2}}$

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i21-1661 Q14 (c)

$f(x,y) = \ln(x-y)$,

$c = 0, \pm 1/2, \pm 1, \pm 3/2, \pm 2$

$c = \ln(x-y) \longrightarrow e^c = (x-y)$ & $y = x - e^c$

for $c = 0$, $y = x - e^0$, $y = x - 1$	for $c = \frac{1}{2}$, $y = x - \sqrt{e}$	for $c = -\frac{1}{2}$, $y = x - \frac{1}{\sqrt{e}}$	for $c = 1$, $y = x - e$
for $c = -1$, $y = x - \frac{1}{e}$	for $c = 3/2$, $y = x - e^{3/2}$, $y = x - (\sqrt{e})^3$	for $c = -\frac{3}{2}$, $y = x - e^{-3/2}$, $y = x - \frac{1}{(\sqrt{e})^3}$	
for $c = 2$, $y = x - e^2$	for $c = -2$, $y = x - \frac{1}{e^2}$		

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Q16. $T = 600 - 0.75x^2 - 0.75y^2$

At 0, $0.75x^2 + 0.75y^2 = 600$

At 100, $0.75x^2 + 0.75y^2 = 500$

At 200, $0.75x^2 + 0.75y^2 = 400$

At 300, $0.75x^2 + 0.75y^2 = 300$

At 400, $0.75x^2 + 0.75y^2 = 200$

$$\boxed{x^2 + y^2 = r^2}$$

This equation is of a circle so it will be formed.

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Q17 $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{xy}$

Let $(x,y)\to(0,0)$ along the line $y=ax$,

then $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{x\to 0} f(x, ax)$

$= \lim\limits_{x\to 0} \dfrac{x^2+a^2x^2}{ax^2} = \dfrac{1+a^2}{a}$

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since $f(x, xa)$ is path dependent so

the limit of $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{xy}$

DOES NOT EXIST

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(Q18) (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

let $x = r\cos\theta$ & $y = r\sin\theta$

$x^2+y^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$= r^2(\cos^2\theta + \sin^2\theta)$

$= r^2(1) \longrightarrow r^2$

$\therefore \lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2} = \lim\limits_{r\to 0} \dfrac{\sin r^2}{r^2}$

now applying L'Hopitals rule.

$\lim\limits_{r\to 0} \dfrac{d/dr \sin r^2}{d/dr\ r^2} = \lim\limits_{r\to 0} = \dfrac{\cos r^2 . \cancel{2r}}{\cancel{2r}}$

$= \lim\limits_{r\to 0} = \cos r^2 \longrightarrow \cos(0) = 1$

$\therefore \lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2} = 1$

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Q 18. (b) $\lim_{(x,y)\to(0,0)} \dfrac{1-\cos(x^2+y^2)}{x^2+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = \lim_{r\to 0} = \frac{1-\cos r^2}{r^2}$$

$\boxed{\text{now applying L'Hopitals rule :}}$

$$\lim_{r\to 0} \frac{d/dr\ 1-\cos r^2}{d/dr\ \ r^2} = \lim_{r\to 0} \frac{0-(-\sin r^2)\cdot \cancel{2r}}{\cancel{2r}}$$

$$\lim_{r\to 0} = \sin r^2 \quad \& \quad \sin(0) = 0$$

$$\therefore \lim_{(x,y)\to(0,0)} \cdot \frac{1-\cos(x^2+y^2)}{x^2+y^2} = 0$$

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$\boxed{Q19}$

$$f(x,y) = \begin{cases} \dfrac{\sin xy}{xy} & , \; xy \neq 0 \\ 1 & , \; xy = 0 \end{cases}$$

Since $f(0,0) = 1$ & as $\dfrac{\sin\theta}{\theta} = 1$

$\therefore$ we can say that $f(x,y)$ = continuous

$\boxed{Q20}$ $A = 0.885t - 22.4h + 1.2th - 0.544$

(a) $\dfrac{\partial A}{\partial t} = ?$, $\dfrac{\partial A}{\partial h} = ?$ when $t = 30^\circ$, $h = 0.8$

$$\frac{\partial A}{\partial t} = 0.885 + 1.2h \rightarrow 0.885 + 1.2(0.8)$$
$$= 1.845$$

$$\frac{\partial A}{\partial h} = -22.4 + 1.2t$$
$$-22.4 + 1.2(30) = 13.6$$

(b) As $\dfrac{\partial A}{\partial h} > \dfrac{\partial A}{\partial t}$ $\therefore$ humidity has greater effect on A

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Q21 $(1,-3,1)$, $(3,-4,2)$, $x-y+z=2$

since $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}$

$$\frac{x-1}{3-1} = \frac{y-(-3)}{-4-(-3)} = \frac{z-1}{2-1}$$

$$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-1}{1} = t$$

$\therefore$ $x-1 = 2t \longrightarrow \boxed{x = 2t+1}$

$y+3 = -t$, $\boxed{y = -t-3}$

$z-1 = t$, $\boxed{z = t+1}$

Plane is given by $x-y+z=2$

substituting value of x,y,z :

$$2t+1-(-t-3)+t+1 = 2$$

$$2t+1+t+3+t+1 = 2$$

$$4t+5 = 2 \longrightarrow t = -3/4$$

substitute value of t in x, y, z.

$x = 2t + 1$

$= 2\left(-\frac{3}{4}\right) + 1$

$x = -\frac{1}{2}$

$y = -t - 3$

$y = -\left(-\frac{3}{4}\right) - 3$

$y = -\frac{9}{4}$

$z = t + 1$

$z = -\frac{3}{4} + 1$

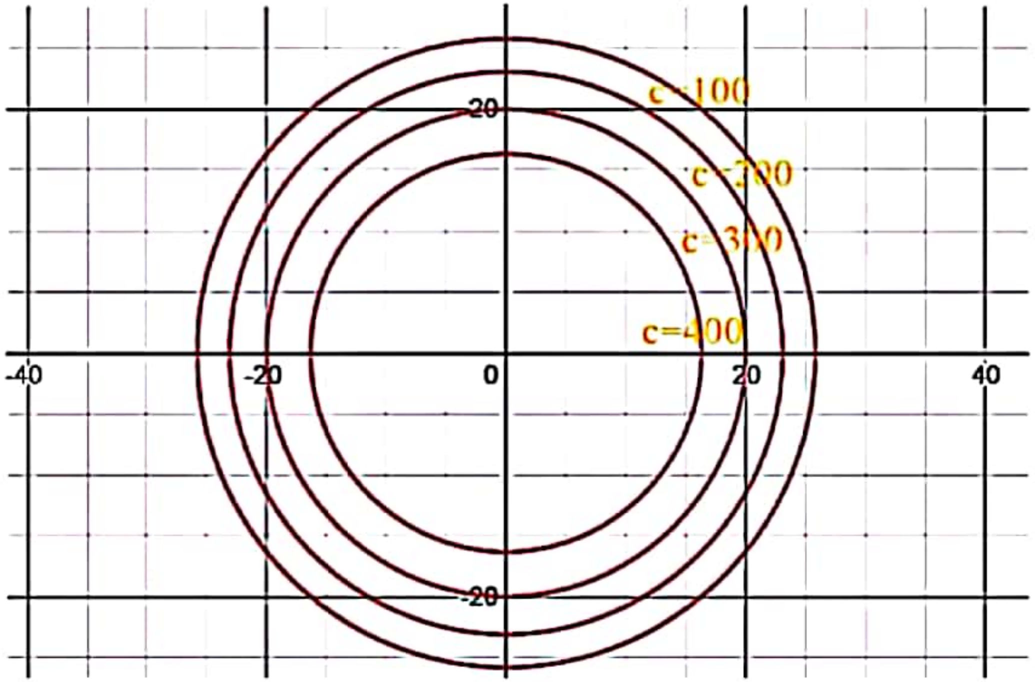
$z = \frac{1}{4}$

Thus point of intersection of line (x, y, z) is

$$\left[-\frac{1}{2}, -\frac{9}{4}, \frac{1}{4}\right]$$

$$x^2 + y^2 = \frac{2400-4c}{3} \rightarrow x^2 + y^2 = \left(\sqrt{\frac{2400-4c}{3}}\right)^2 \text{ Circle with radius } r = \frac{2400-4c}{3} \text{ and center } (0,0)$$

Now for plotting we used value of $c = 100, 200, 300, 400$



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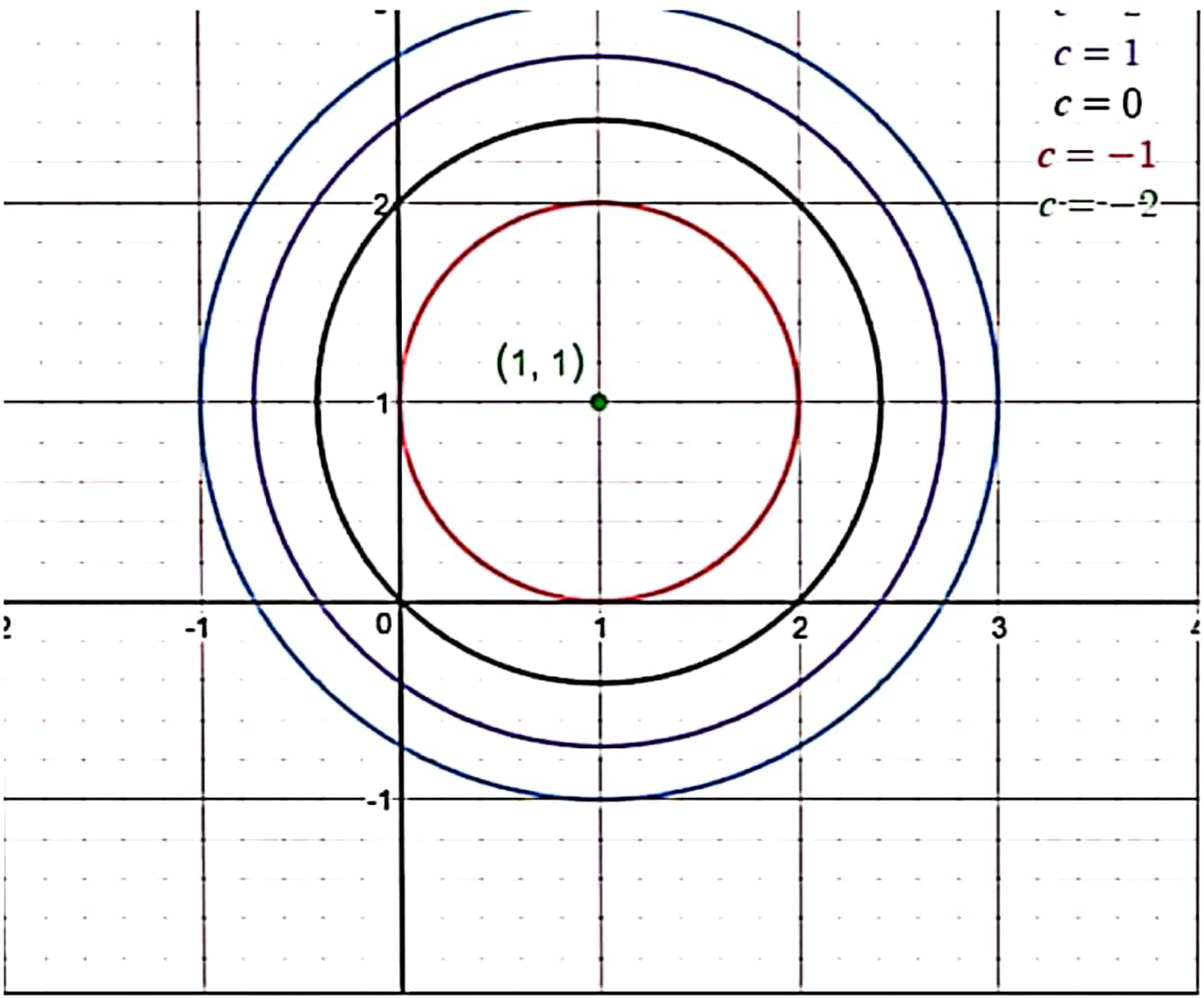

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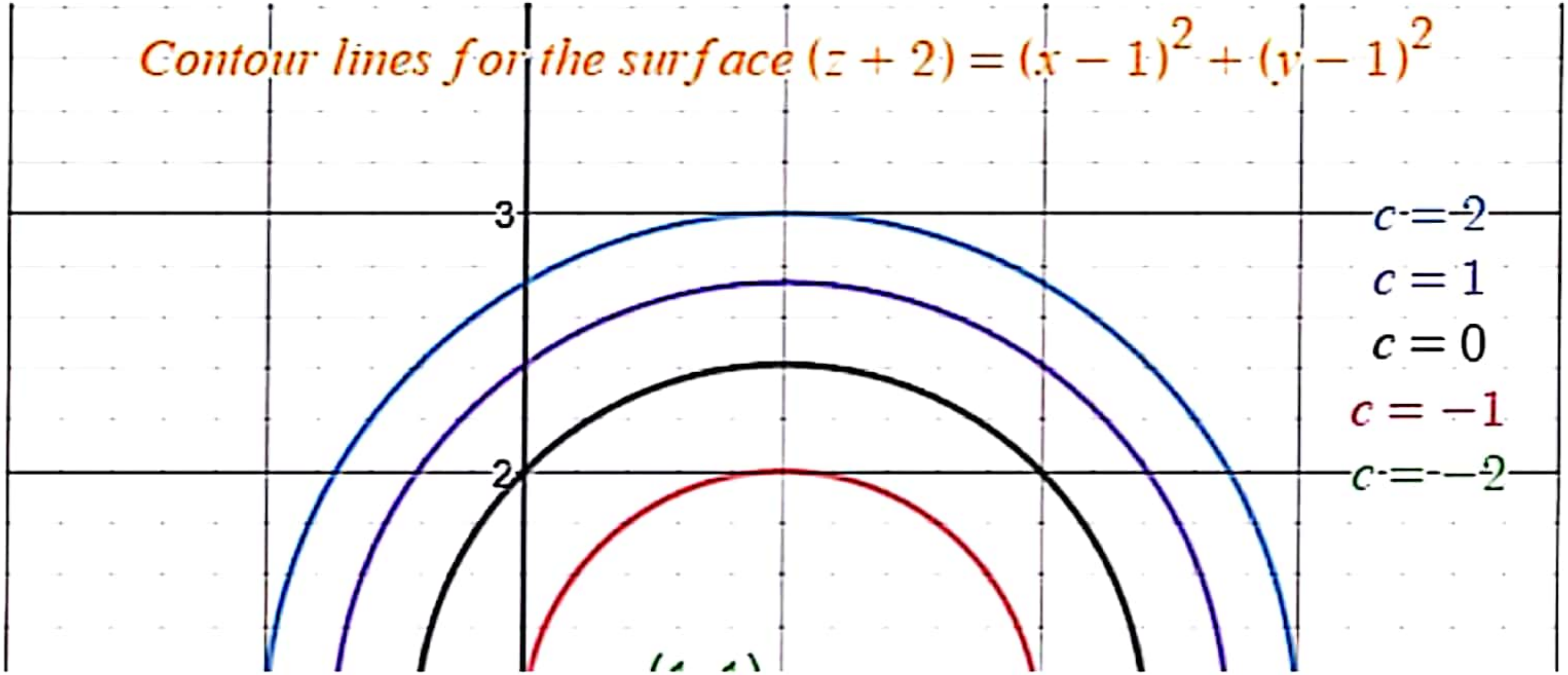

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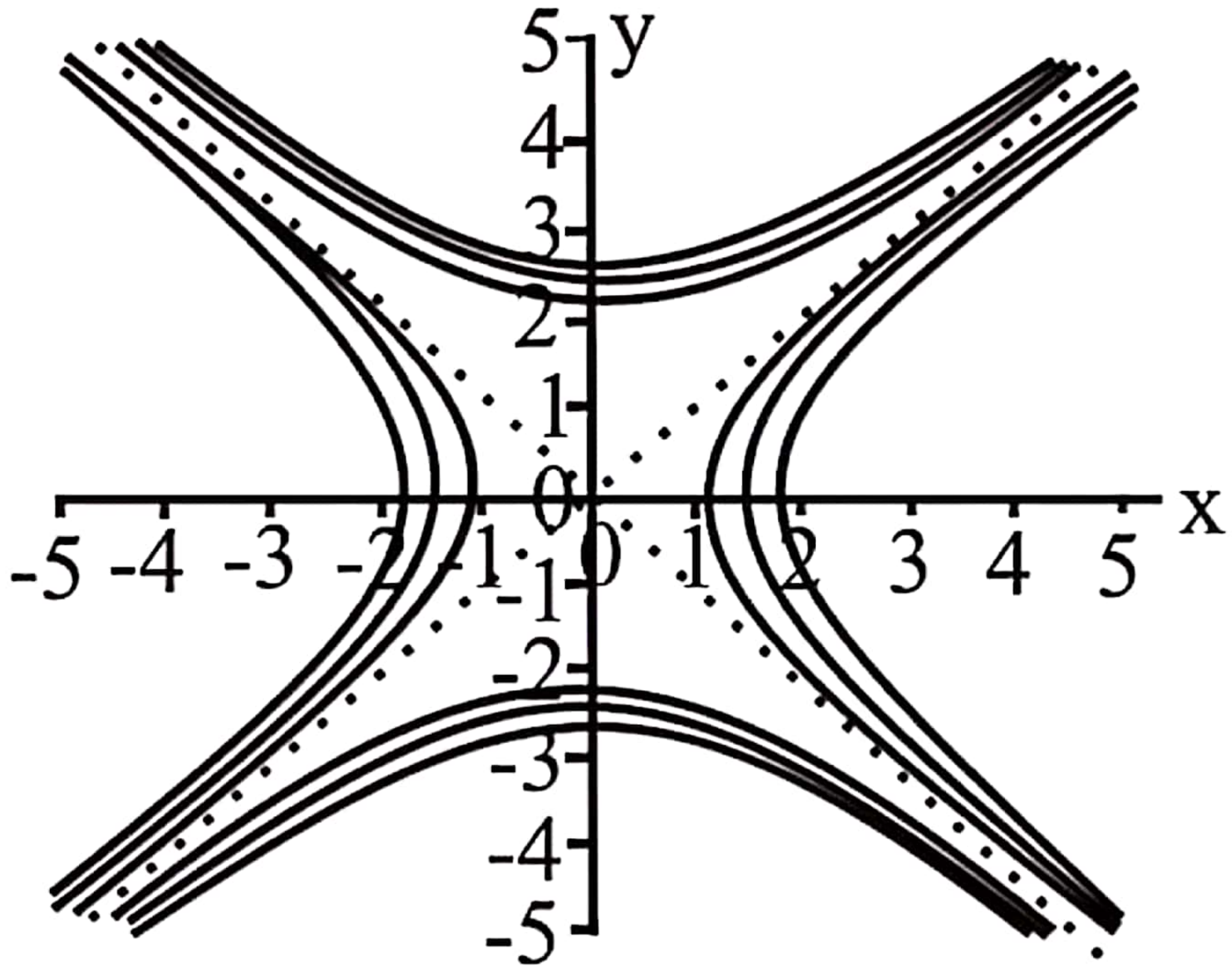

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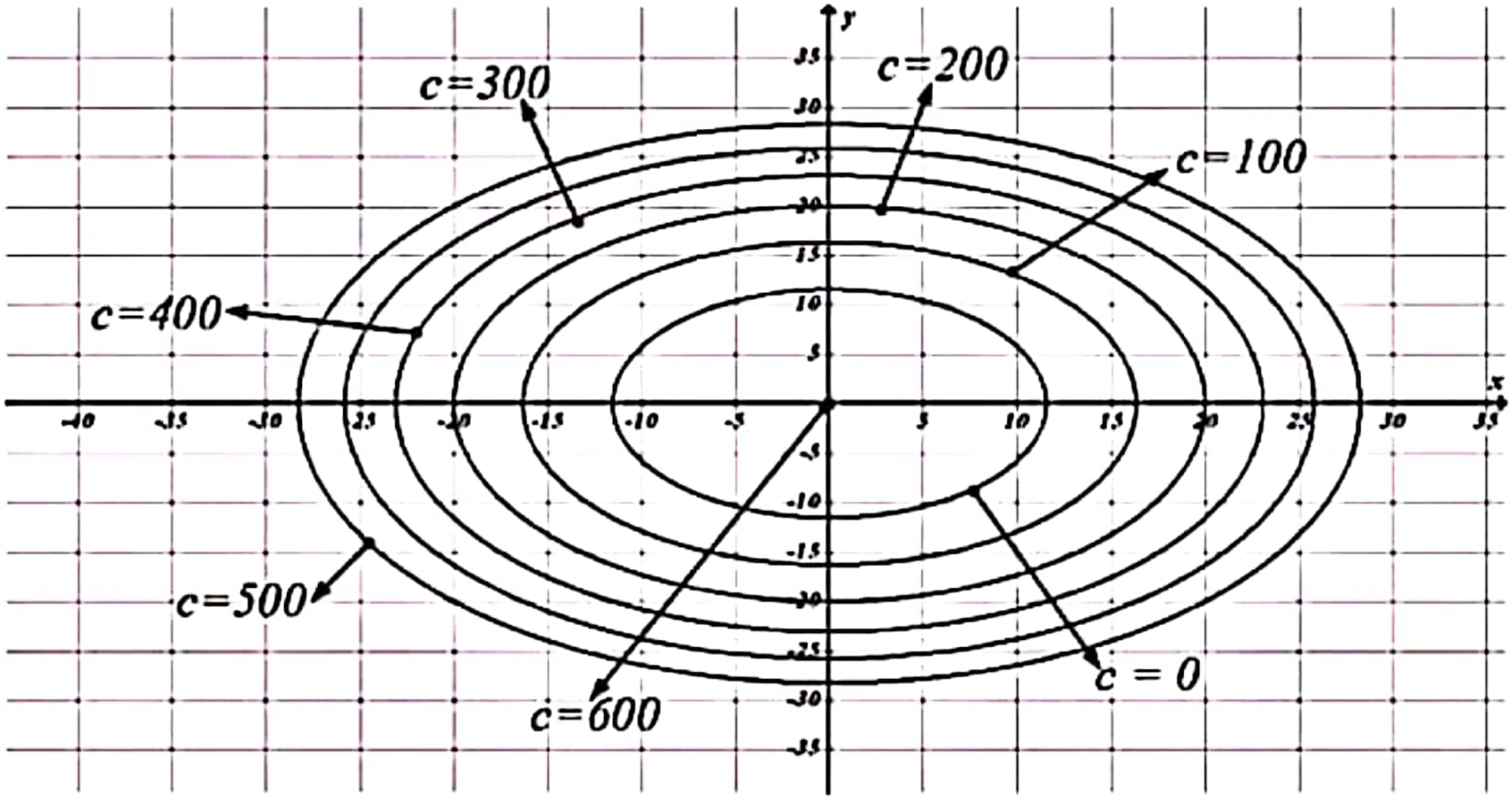
Plot these graphs on a coordinate plane. These are the level curves, which represent the isothermals.



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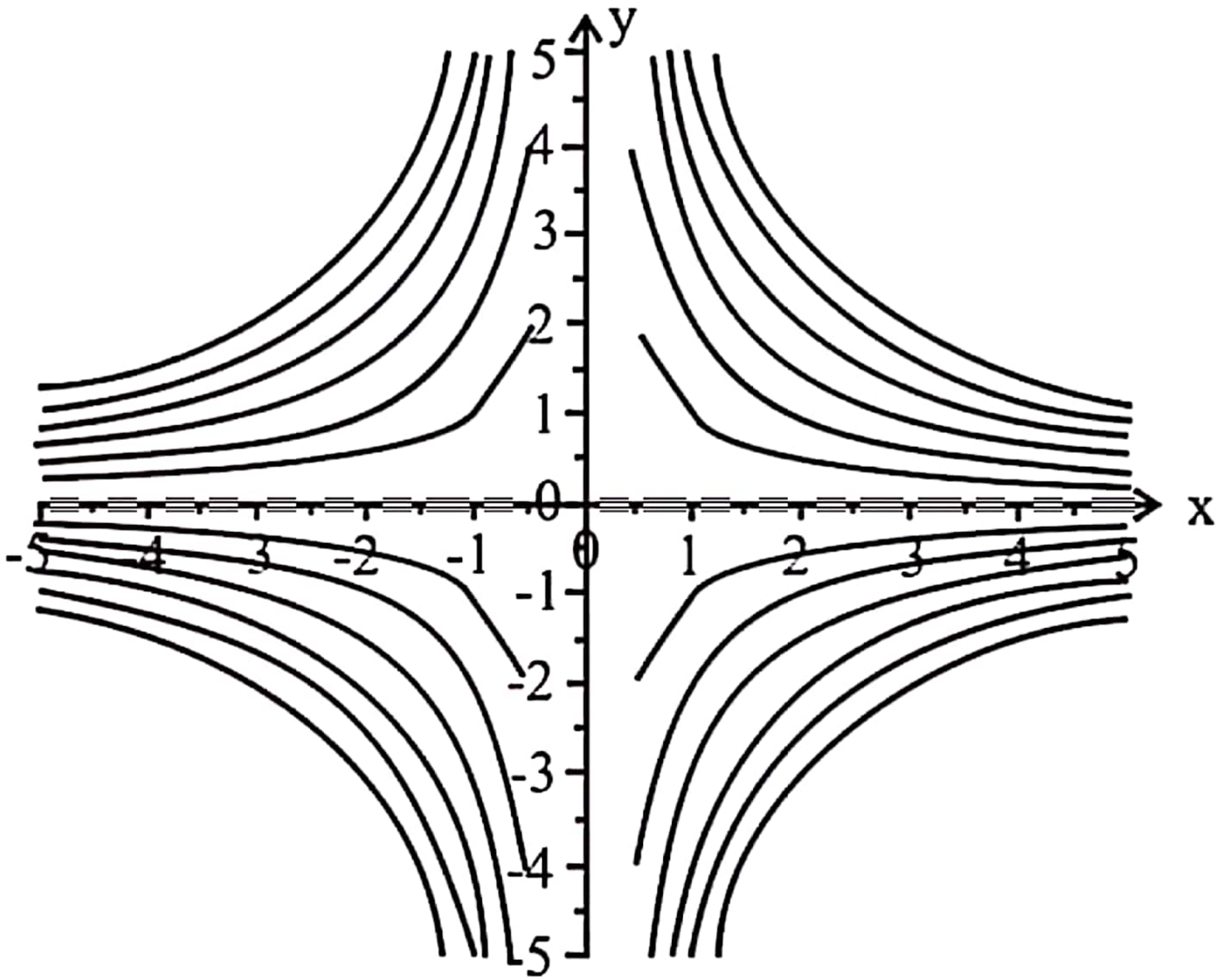

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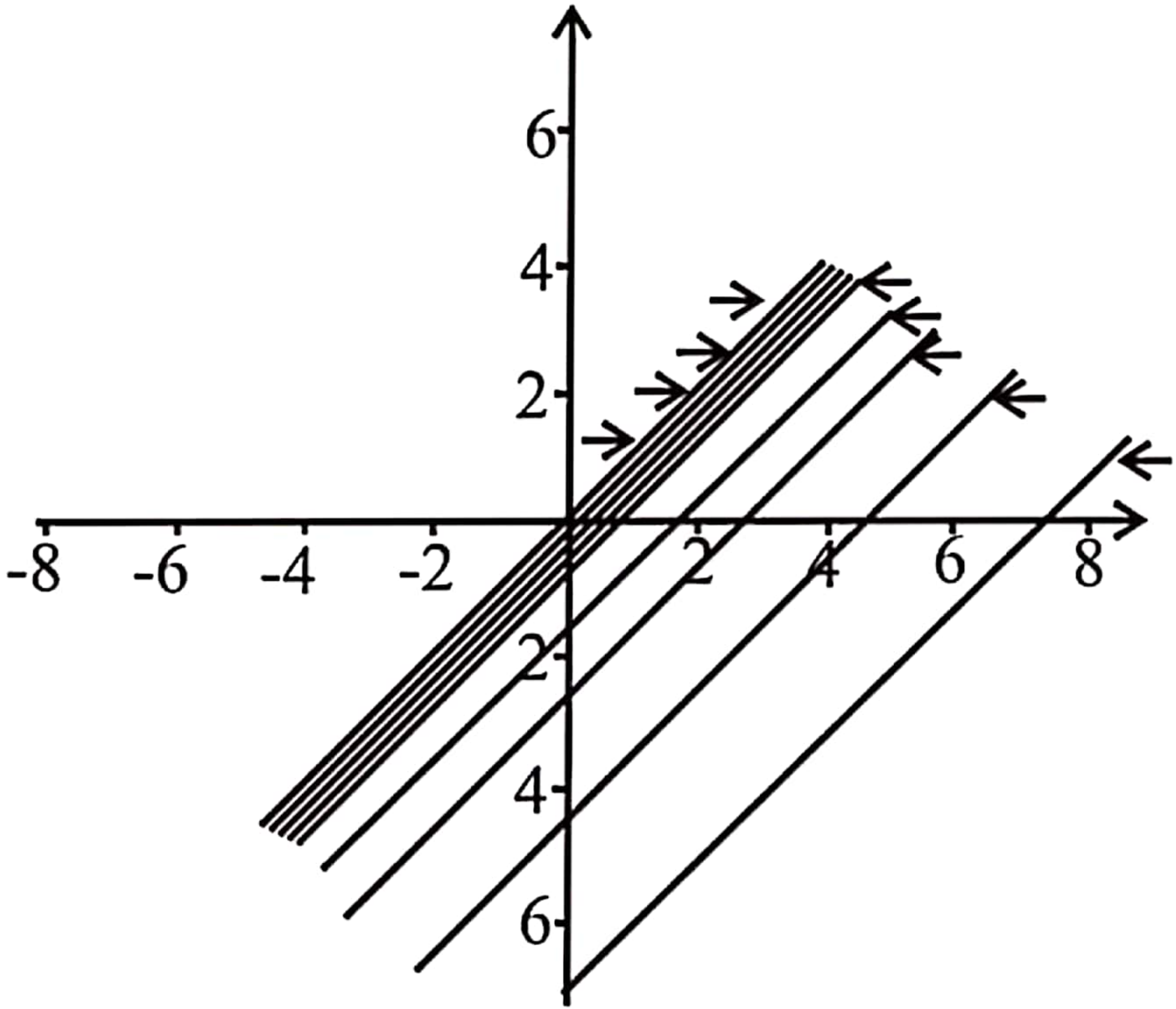

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Chapter 13.1, Problem 56E Bookmark Show all steps:

Problem

Sketching a Contour Map In Exercise, describe the level curves of the function. Sketch a contour map of the surface using level curves for the given c-values.

$f(x, y) = e^{xy/2}$, $c = 2, 3, 4, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

Step-by-step solution

Step 1 of 2

$f(x, y) = ln(x - y)$

Let $f(x, y) = c$

Then $c = ln(x - y)$

$e^c = x - y$

Which is a straight line

Thus for each value of c, the level curve in xy-plane is a straight line.

The level curves for $c = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$ are given below

Comment

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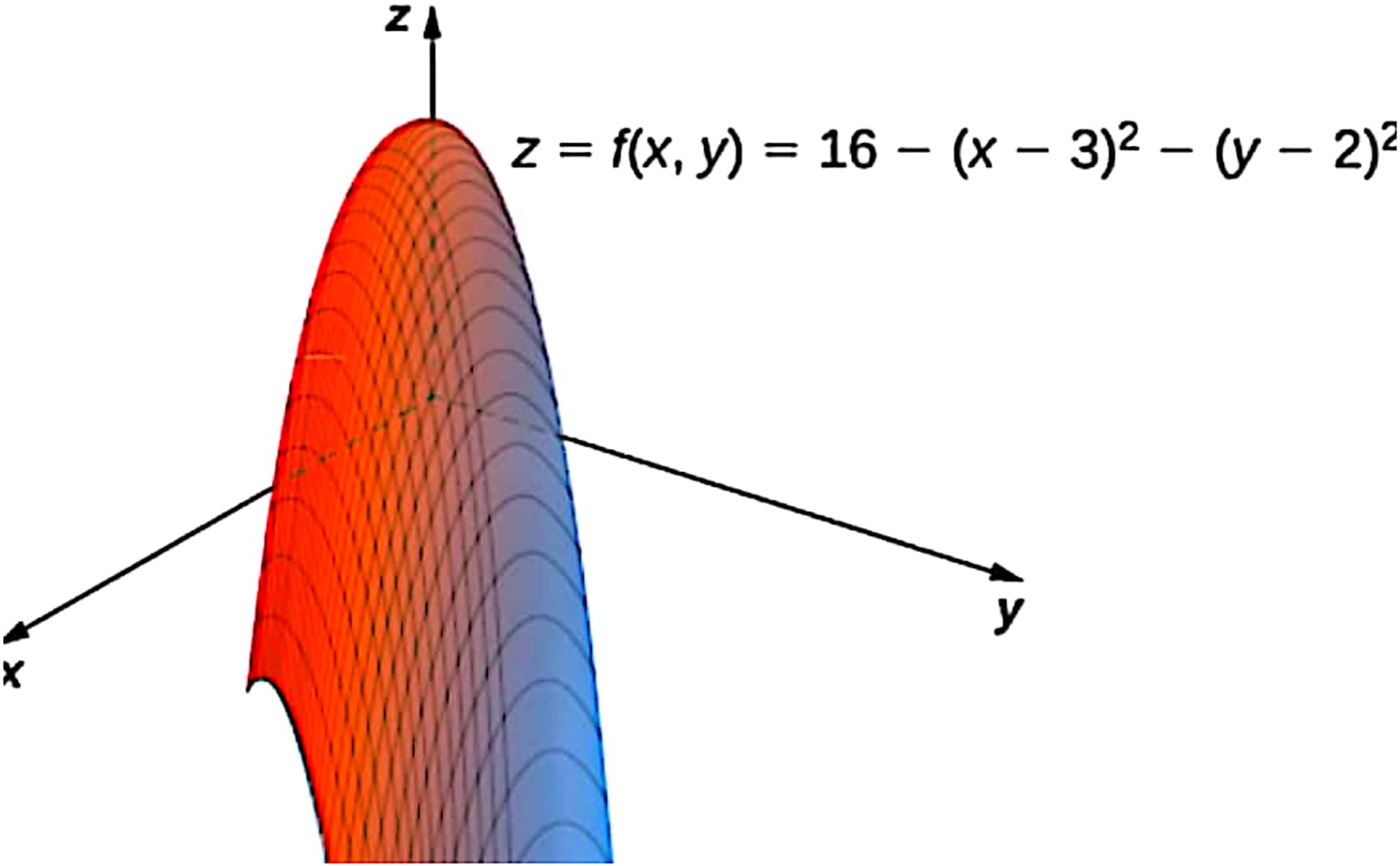

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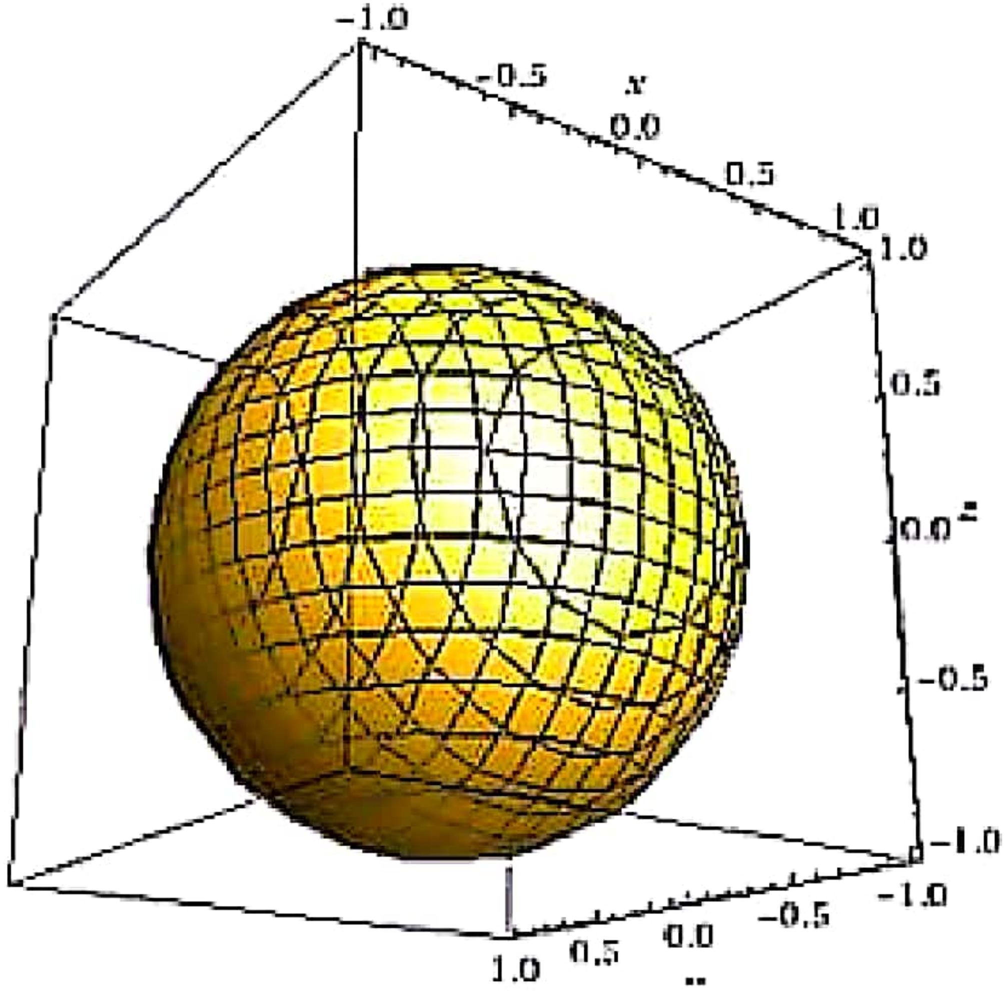
Input interpretation:

plot	$x^2 + y^2 + z^2 \leq 1$

Surface plot:



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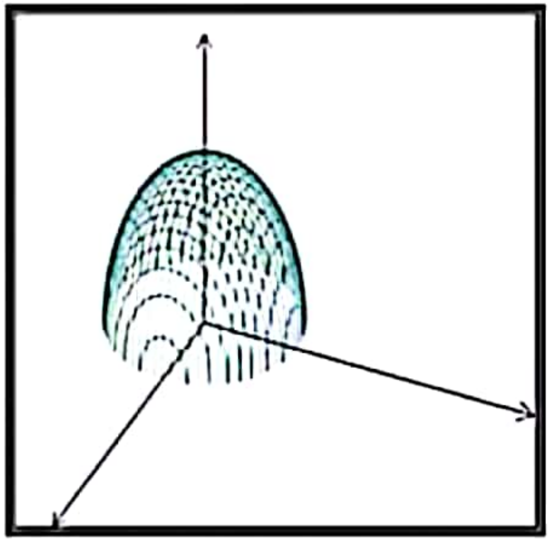
$16 - 4x^2 - 4y^2 - z^2 > 0$

Domain $= \{(x, y, z) \in \mathbb{R}^3 : 16 > 4x^2 + 4y^2 + z^2\}$

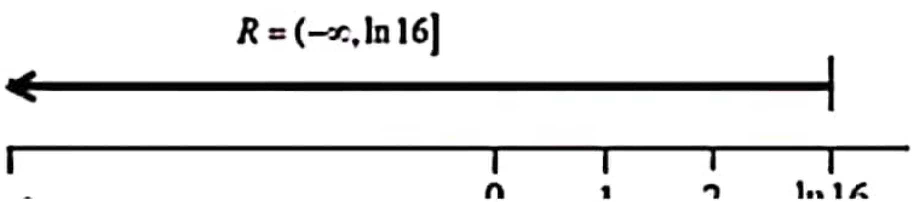
Graph of the domain:



The range of the function is

Range $= (-\infty, \ln 16]$

Graph of the range:



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