National University
of Computer & Emerging Sciences
Islamabad

MT1006 – Differential Equations (Cal-II)

Assignment No: 05 **Individual Assignment**

Section: BS CS, BS AI, BS DS, BS CySec **Semester:** Spring 2022

Due date: 30-04-2022 **Marks:** 15*10=150

Instructions:

1. Plagiarized work will result in zero marks.
2. No retake or late submission will be accepted.
3. Attach complete code, results, and screenshot for questions that require programming solution. Programs/codes should not be handwritten.
4. Questions that show the icon </> require partial or complete solution using the approved programming tool.
5. The assignment is to be submitted in softcopy as well as in hardcopy.
6. The softcopy should be a single PDF file of your complete assignment including programming and non-programming questions.
7. The PDF file should be according to the following format: id_section_A5 e.g., i21-123456_A_A5. A5 in the end denotes Assignment 5.
8. The images of by-hand solution should be properly scanned. You can use any mobile application such as CamScanner or Adobe Scan for scanning. Each of these applications allow you to export pdf or image files which you can use to combine with your programming solutions. Do not attach direct images from the camera application of your mobile phone, or screenshots.

1. Consider two functions:

$$y_1(x) = \begin{cases} x^2, & x \leq 0 \\ 0, & x > 0 \end{cases}$$

Find the differential equation satisfied by each of the following two-parameter families of plane curves:

$$y = \cos(c_1 x + c_2)$$
$$y = ae^x + bxe^x$$

2. Verify that $y_1(x) = 1$ and $y_2(x) = x^{1/2}$ are solutions of the differential equation $yy'' + (y')^2 = 0$ for $x > 0$. Then show that $y = c_1 + c_2 x^{1/2}$ is not, in general, a solution to the equation. Explain why this does not contradict superposition principle.

3. Find an interval for which the given initial-value problem has a unique solution.

a) $(x+3)y'' + xy' + (\ln|x|)y = 0, \quad y(1) = 0, \ y'(1) = 1$

b) $(1 - x^2)y'' - 2xy' + (\alpha(\alpha+1) + \mu^2/(1 - x^2))y = 0, \quad y(0) = y_0, \ y'(0) = y_1$

4. If the Wronskian of f and g is $t\cos t - \sin t$, and if $u = f + 3g$, $v = f - g$, find the Wronskian of u and v.

5. Consider the equation $y'' - y' - 2y = 0$.

a) Show that $y_1(t) = e^{-t}$ and $y_2(t) = e^{2t}$ form a fundamental set of solutions.

b) Let $y_3(t) = -2e^{2t}$, $y_4(t) = y_1(t) + 2y_2(t)$ and $y_5(t) = 2y_1(t) - 2y_3(t)$. Are $y_3(t)$, $y_4(t)$ and $y_5(t)$ also solutions of the given differential equation?

c) Determine whether each of the following pairs form a fundamental set of solutions: $[y_1(t), y_3(t)]$, $[y_2(t), y_3(t)]$, $[y_1(t), y_4(t)]$ and $[y_4(t), y_5(t)]$.

6. An equation of the form

$$t^2 y'' + \alpha t y' + \beta y = 0, \qquad t > 0$$

where α and β are real constants, is called an Euler equation. Show that the substitution $x = \ln t$ transforms an Euler equation into an equation with constant coefficients. Using that result, find the solution of the given equation

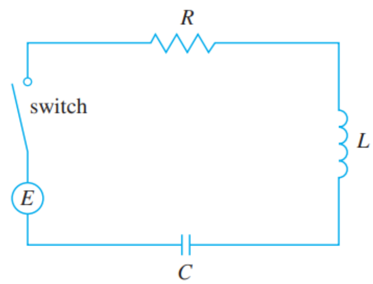
$$t^2 y'' + 3ty' + 1.25y = 0$$

7. The differential equation

$$y'' + \delta(xy' + y) = 0$$

arises in the study of the turbulent flow of a uniform stream past a circular cylinder. Verify that $y_1(x) = e^{(-\delta x^2/2)}$ is one solution and then find the general solution in the form of an integral.

8. A circuit is shown in Figure 1.



It contains an electromotive force E (supplied by a battery or generator), a resistor R, an inductor L, and a capacitor C, in series. Kirchhoff's voltage law says that the sum of the voltage drops is equal to the supplied voltage. If the charge on the capacitor at time t is $Q = Q(t)$, then the current is the rate of change of Q with respect to t is $I = \frac{dQ}{dt}$ and Kirchhoff's voltage law can be expressed as:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

If $R = 24$ ohms, $L = 2$ henry, $C = 0.005$ farad, $E(t) = 12\sin 10t$ volt and the initial charge is $Q = 0.001$ coulomb and the initial current is 0 ampere,

a) Find the charge at time t using undetermined coefficient (annihilator appraoch).
b) Use MATLAB to find the charge and to sketch the graph of the charge function.

9. The motion of a spring–mass system in addition to the restoring force $-kx$ (where k is a positive constant, called the *spring constant*), the damping force $-c\,dx/dt$ (where c is a positive constant, called the *damping constant*) and an external force $F(t)$ is governed by the differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

If $m = 1$ kg, $c = 8$ kg/s, and $k = 16$ N/m, then

a) Find the equation of motion using undetermined coefficient (superposition approach) if the mass is driven by an external force equal to $F(t) = e^{-t} \sin 4t$ with initial condition $x(0) = 0, \;\; x'(0) = 0$. Analyze the displacements for $t \to \infty$.

b) Graph the transient and steady-state solutions on the same coordinate axes.

c) Graph the equation of motion.

10. Find the general solutions of the given equations:

a) $3y''' + 10y'' + 15y' + 4y = 0$

b) $2y^{(4)} + 3y''' + 2y'' + 6y' - 4y = 0$

11. Solve the initial value problem using undetermined coefficients (superposition approach):

$y'' + y = f(t), \qquad y(0) = 0, \;\; y'(0) = 1$

where $f(t) = \begin{cases} t, & 0 \le t \le \pi \\ \pi e^{\pi - t}, & t > \pi \end{cases}$

Also graph the solution.

In Problems **12-13**:

a) Determine a suitable form for particular solution using undetermined coefficient (annihilator approach).

b) Use MATLAB to find the general solution of the given equations.

12. $y'' - 5y' + 6y = e^{t} \cos 2t + e^{2t}\,(3\mathrm{t} + 4) \sin t$

13. $y'' + 3y' + 2y = e^{t}\,(t^2 + 1)\sin 2t + 3e^{-t}\, \cos t + 4e^{t}$

In Problems **14-15**, use variation of parameters to find the general solution:

14. $y'' - 2y' + y = e^{t}/(1 + t^2)$

15. $y'' + 2y' + y = e^{-x} \ln x$