EE 141 – Project
Spring 2020

Due on June 5th by 5pm

In this project we will study the spreading dynamics of COVID-19 and how to design a controller to stabilize the number of infected people. Please keep in mind the model we will use is simplified and that it is difficult to build accurate models (and their parameters) until enough data becomes available.

We will use a population model with the following five classes:

S: number of susceptible individuals;
E: number of exposed individuals (they show no symptoms and are not contagious);
I: number of infected individuals with mild or no symptoms at all (it is believed that about 80% of the infected population belongs to this class);
J: number of infected individuals that are seriously ill;
R: number of recovered individuals;
D: number of individuals that have died.

There are three different ways in which COVID-19 can progress: 1) $S \to E \to I \to R$; 2) $S \to E \to J \to R$; and 3) $S \to E \to J \to D$. The first corresponds to the asymptomatic or mildly symptomatic case and the second and third to the strongly symptomatic case. Eventually, every element of the population will settle to the class R or D. The interaction between the different classes is described by the model:

$$
\begin{aligned}
\dot{S} &= -\beta_1 SI - \beta_2 SJ && (1)\\
\dot{E} &= \beta_1 SI + \beta_2 SJ - \gamma E && (2)\\
\dot{I} &= \sigma_1 \gamma E - \rho_1 I && (3)\\
\dot{J} &= \sigma_2 \gamma E - \rho_2 J - qJ && (4)\\
\dot{R} &= \rho_1 I + \rho_2 J && (5)\\
\dot{D} &= qJ. && (6)
\end{aligned}
$$

The products SI and SJ describe the effect of susceptive elements of the population meeting with infected individuals and the parameters β_1 and β_2 model the rate at which such encounters contribute to moving individuals from the susceptible to the exposed class. The parameter

γ describes the rate at which exposed individuals become infected, i.e., they either move to class I or J. The parameters σ_1 and σ_2 are used to discriminate between classes I and J. Parameters ρ_1 and ρ_2 model the rates at which infected individuals recover. Finally, the parameter q models the rate at which elements of the J class die.

We will use the following values for the parameters:

$$\beta_1 = 0.25, \beta_2 = 0.025, \gamma = 0.2, \sigma_1 = 0.8, \sigma_2 = 0.2, \rho_1 = 1/14, \rho_2 = 1/21, q = 0.15.$$

1. Let the S, E, I, J, R, and D range between 0 and 1 so that their values can be interpreted as the fraction of the population in the respective class. Simulate the model in simulink for different initial conditions and comment on the observed behavior. Here is a suggested initial condition:

   $$S(0) = 0.8, E(0) = 0.145, I(0) = 0.05, J(0) = 0.005, R(0) = 0, D(0) = 0.$$

2. Comment on the number of hospital beds required to treat all the population in class J.

3. We now consider the effect of using masks. We introduce an input u that describes the fraction of the population using masks assuming that masks are 100% effective in preventing infected elements of the population from releasing droplets in the air (this is considered to be the main spreading mechanism).

   $$\dot{S} = -\beta_1 SI - \beta_2 SJ \quad (7)$$
   $$\dot{E} = \beta_1 SI + \beta_2 SJ - (1-u)\gamma E \quad (8)$$
   $$\dot{I} = (1-u)\sigma_1 \gamma E - \rho_1 I \quad (9)$$
   $$\dot{J} = (1-u)\sigma_2 \gamma E - \rho_2 J - qJ \quad (10)$$
   $$\dot{R} = \rho_1 I + \rho_2 J \quad (11)$$
   $$\dot{D} = qJ. \quad (12)$$

   Note that $0 \leq u < 1$ with $u = 1$ corresponding to all the population using masks at all times which is impossible to implement.

   Compute the equilibrium pairs for this system of nonlinear differential equations. Show the linearization of this model at an equilibrium pair cannot be used for the purposes of control. This illustrates the limits of the techniques you learned in this class and in this project we will use a more advanced technique (switched linear systems) for control purposes.

4. Our objective is keep the number of infected people requiring hospitalization (class J) small. Using the linearized model, compute the transfer function from $J(0)$ to J. How do the poles of this transfer function depend on u?

5. Simulate the linearized model for different values of u and for the initial condition:

$$S(0) = 0.8, E(0) = 0.145, I(0) = 0.05, J(0) = 0.005, R(0) = 0, D(0) = 0.$$

   Comment on what you observe and how it relates to the poles of the transfer function in the preceding question. Compare the simulation results of the linearized model with those of the nonlinear model. Explain the differences.

6. Using the insights you acquired in the previous two questions, regarding the effect of u, devise a controller that consists in switching the value of u at discrete instants of time while keeping it constant in between the switches. The switching instants correspond to the instants when new policies for the use of masks are announced. Different policies will result in different values of u. Your objective is to ensure that no more that 1% of the population is hospitalized while keeping the value of u as low as possible, and never greater than 0.9, so as to minimize the burden on the population. Consider the same initial condition as before for your design. Does your policy change the fraction of the population that ends in classes R or D? Note that your controller needs to be validated using the nonlinear model.

7. How does your controller perform when you change the initial conditions? How would you change it so that it works for any initial condition?