$\theta' = v$

$v' = \frac{1}{J}(K.i - b.v) => Vs = \frac{1}{J}(K.I - b.V)$ (1)

$i' = \frac{1}{L}.(e - K.V - R.i)$

Động cơ

$J.V(s)s + b.V(s) = K.I(s) \quad => I(s) = V(s).\frac{(Js+b)}{K}$ (1)

$L.I(s)s + R.I(s) = E(s) - KV(s) \quad => E(s) = I(s)(Ls+R) + KV(s)$ (2)

Thế (1) vào (2)

$$E(s) = V(s).\left(\frac{(Js+b).(Ls+R)}{K} + K\right)$$

$$=> \frac{V(s)}{E(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} = \frac{K}{JLs^2 + (JR+bL)s + bR + K^2}$$

$$JL\ddot{v} + (JR+bL)\dot{v} + (bR+K^2)v = K.e$$

$$\ddot{v} = \left(\frac{v_k - v_{k-1}}{T_s}\right)' = \frac{\frac{v_k - v_{k-1}}{T_s} - \frac{v_{k-1} - v_{k-2}}{T_s}}{T_s} = \frac{v_k - 2v_{k-1} + v_{k-2}}{T_s^2}$$

$$JL\frac{v_k - 2v_{k-1} + v_{k-2}}{T_s^2} + (JR+bL)\frac{v_k - v_{k-1}}{T_s} + (bR+K^2)v(k) = K.e(k)$$

$$JL.v_k - JL\frac{2v_{k-1} - v_{k-2}}{T_s^2} + (JR+bL)\frac{v_k}{T_s} - v_{k-1}\frac{JR+bL}{T_s} + (bR+K^2)v(k) = K.e(k)$$

$$v_k\left(JL + \frac{JR+bL}{T_s} + bR + K^2\right) - JL\frac{2v_{k-1} - v_{k-2}}{T_s^2} - v_{k-1}\frac{JR+bL}{T_s} = K.e(k)$$

$$v_k = \frac{K.e(k) + JL\frac{2v_{k-1} - v_{k-2}}{T_s^2} + v_{k-1}\frac{JR+bL}{T_s}}{JL + \frac{JR+bL}{T_s} + bR + K^2}$$

$$v(k)=\frac{K.e(k)+v(k-1).\left(\frac{2JL}{T_s^2}+\frac{JR+bL}{T_s}\right)-v(k-2).\frac{JL}{T_s^2}}{JL+\frac{JR+bL}{T_s}+bR+K^2}$$

$$a=\frac{2JL}{T_s^2}+\frac{JR+bL}{T_s}$$

$$b=JL+\frac{JR+bL}{T_s}+bR+K^2$$

$$v(k)=\frac{K.e(k)+v(k-1).a-v(k-2).\frac{JL}{T_s^2}}{b},\quad e(k)=SP-v(k-1)$$

PID

$$U(s)=E(s)\left(K_p+\frac{K_i}{s}+K_d s\right)$$

$$s.U(s)=E(s)s.K_p+E(s).K_i+E(s).K_d.s^2$$

$$\dot{u}=K_p.\dot{e}+K_i.e+K_d.\ddot{e}$$

$$\frac{u(k)-u(k-1)}{T_s}=K_p.\frac{e(k)-e(k-1)}{T_s}+K_i.e(k)+K_d.\frac{\dot{e}(k)-\dot{e}(k-1)}{T_s}\quad(3)$$

Ta có:

$$\frac{\dot{e}(k)-\dot{e}(k-1)}{T_s}=\frac{\frac{e(k)-e(k-1)}{T_s}-\frac{e(k-1)-e(k-2)}{T_s}}{T_s}=\frac{e(k)-2e(k-1)+e(k-2)}{T_s^2}\quad(4)$$

Thay (4) vào (3) ta được:

$$u(k)-u(k-1)=K_p.\big(e(k)-e(k-1)\big)+T_s.K_i.e(k)+K_d.\frac{e(k)-2e(k-1)+e(k-2)}{T_s}$$

$$u(k)=u(k-1)+\left(K_p+\frac{K_d}{T_s}+T_sK_i\right).e(k)-\left(K_p+\frac{2K_D}{T_s}\right).e(k-1)+\frac{K_d}{T_s}.e(k-2)$$

Nháp:

$$s.U(s) = E(s)s.K_p + E(s).K_i + E(s).K_d.s^2$$

$$\dot{u} = \dot{e}K_p + eK_i + \ddot{e}K_d$$

Euler Backward

$$\frac{u(k) - u(k-1)}{T_s} = \frac{e(k) - e(k-1)}{T_s}K_p + e(k).K_i + \frac{\dot{e}(k) - \dot{e}(k-1)}{T_s}K_d$$

$$u(k) = u(k-1) + (e(k) - e(k-1))K_p + e(k).K_i.T_s + \frac{e(k) - 2e(k-1) + e(k-2)}{T_s}K_d$$

$$u(k) = u(k-1) + \left(K_p + K_iT_s + \frac{K_d}{T_s}\right).e(k) + \left(-K_p - \frac{2K_d}{T_s}\right)e(k-1) + \frac{K_d}{T_s}e(k-2)$$