# Neuroprothetics Exercise 2

Chutong Ren

10. November 2023

## 1 Slope fields

Linear Ordinary Differential Equations are analytically solvable, that means they have a function. In contrast, most nonlinear Ordinary Differential Equations are not easy to be expressed using simple algebraic and functional operations.

### 1.1 Plotting slope fields

Plotting slope fields offers a geometric perspective on differential equations. These fields visually represent the values of the derivatives of the equation at each point in the plane. For instance, equation 1 exemplifies a linear ordinary differential equation, while equation 2 represents a nonlinear ordinary differential equation.
In this report plt.quiver() in python was used to plot slope fields. Figures 1 illustrate the slope fields corresponding to equations 1 and 2 respectively.

$$\frac{dV}{dt} = -10 - V - t \tag{1}$$

$$\frac{dV}{dt} = \cos(t) - \frac{1}{2}V + 20 \tag{2}$$

### 1.2 Isoclines

Isoclines are essential in understanding differential equation solutions. They are lines or curves in a slope field where the solution's slope is constant. This constant slope means that along an isocline, the differential equation's rate of change does not vary.
Isoclines help in visualizing the behavior of solutions, allowing us to intuitively grasp their trends. In slope fields, they signify specific rates, for example, each point on a red line in Figure 1 indicates a steady rate of -3V/s.

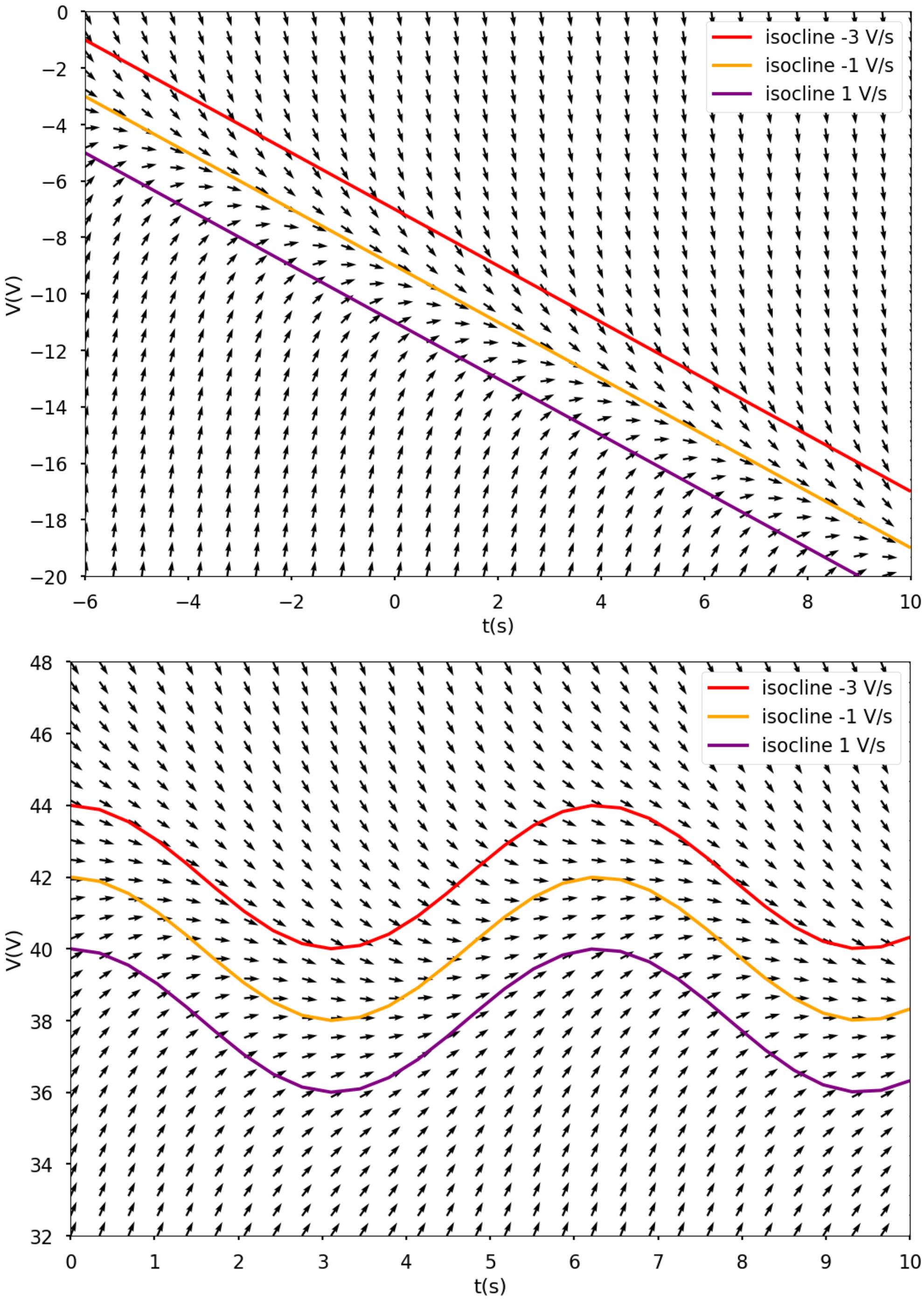


Figure 1: Slope fields and isoclines of equation 1 (top) and 2 (bottom)

## 2 Simple cell model

Figure 2 depicted a simple equivalent circuit that represents a cell model. In this circuit, $I_{\text{ex}}$ denotes the stimulating current, as defined by equation 3, $R_{\text{l}}$ represents the membrane's leak resistance, and $C_{\text{m}}$ symbolizes the membrane capacitance. For the purposes of this exercise, the cell's behavior is governed by the differential equation outlined in equation 4.

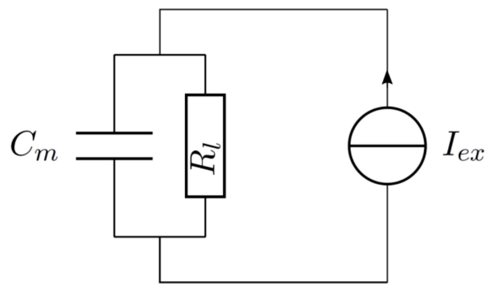


Figure 2: EQC of a simple cell model

$$I_{ex} = I_{\max} \cdot \sin(t) \tag{3}$$

$$\frac{dV}{dt} = f(V, t) \tag{4}$$

### 2.1 Deriving the Differential Equation

The derivation process employs Kirchhoff's Current Law (KCL), a fundamental principle in circuit analysis. KCL states that at any electrical junction, the total incoming current is equal to the total outgoing current. Applying KCL, we obtain the following result:

$$I_{\text{ex}} = I_C + I_R \tag{5}$$

Based on the fundamental definition of a capacitor, the current flowing through it equals the product of the capacitor's capacitance and the rate of change of voltage across its terminals over time. Therefore, we derive:

$$I_C = C_m \cdot \frac{dV}{dt} \tag{6}$$

According to Ohm's Law, the current passing through the resistor $R_{\text{l}}$ can be calculated as:

$$I_R = \frac{V}{R_l} \tag{7}$$

Based on equation (3)(5)(6)(7), we can derive the differential equation of the cell:

$$\frac{dV}{dt} = \frac{I_{\text{ex}}R_l - V}{C_m R_l} = \frac{I_{\max}\sin(t)R_l - V}{C_m R_l} \tag{8}$$

## 2.2 plotting slope fields

Axis limit: The axis limits are carefully selected to effectively visualize the cell model's behavior. The time axis (t) ranges either from -6 to 6 or 0 to 6, providing a broad view of the system's behavior over time. Meanwhile, the voltage axis (V) is set from -20 to 20, which allows us to observe the voltage variations within a significant range.
Figure 3 displays the slope fields for the equation derived from the equivalent circuit in Figure 2, under the following two scenarios.

- $R_l = 1\Omega$, $C_m = 2\text{F}$, $I_{\max} = 0\text{A}$
- $R_l = 1\Omega$, $C_m = 2\text{F}$, $I_{\max} = 10\text{A}$

## 2.3 plotting slope fields considering a constant term

When considering the addition of a constant term D=5A to the current source, both $I_{\text{ex}}$ and the differential equation of the cell will change, as follows:

$$I_{ex} = I_{\max} \cdot \sin(t) + D \tag{9}$$

$$\frac{dV}{dt} = \frac{(I_{\max} + D)\sin(t)R_l - V}{C_m R_l} \tag{10}$$

Figure 4 displays the slope fields for the equation derived from the equivalent circuit in Figure 2, with the constant term D=5A, under the two scenarios in 2.2.

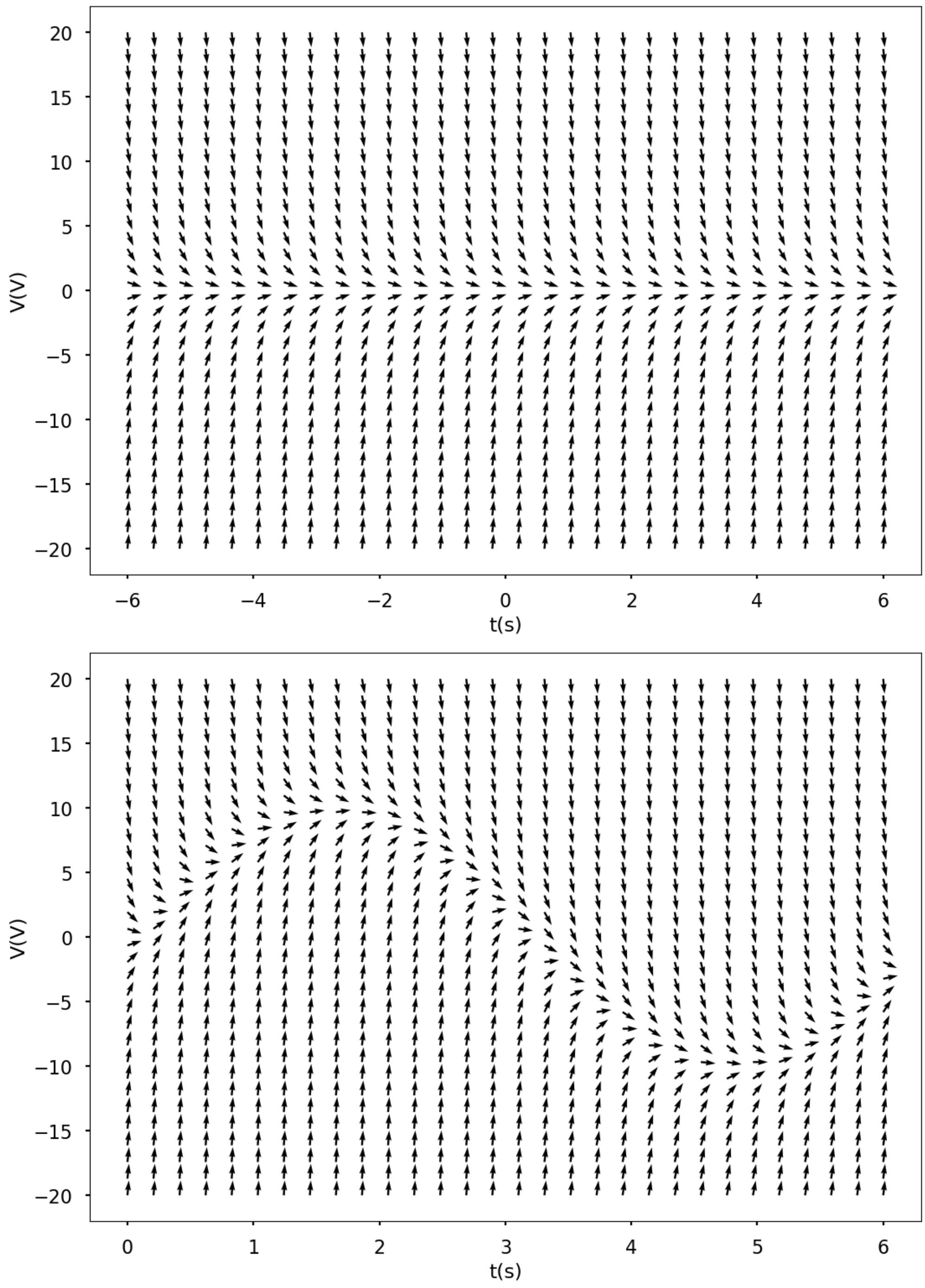


Figure 3: Slope fields for the equation derived from the equivalent circuit in Figure 2. Top: $R_l = 1\Omega$; $C_m = 2\text{F}$; $I_{\max} = 0\text{A}$. Bottom: $R_l = 1\Omega$; $C_m = 2\text{F}$; $I_{\max} = 10\text{A}$

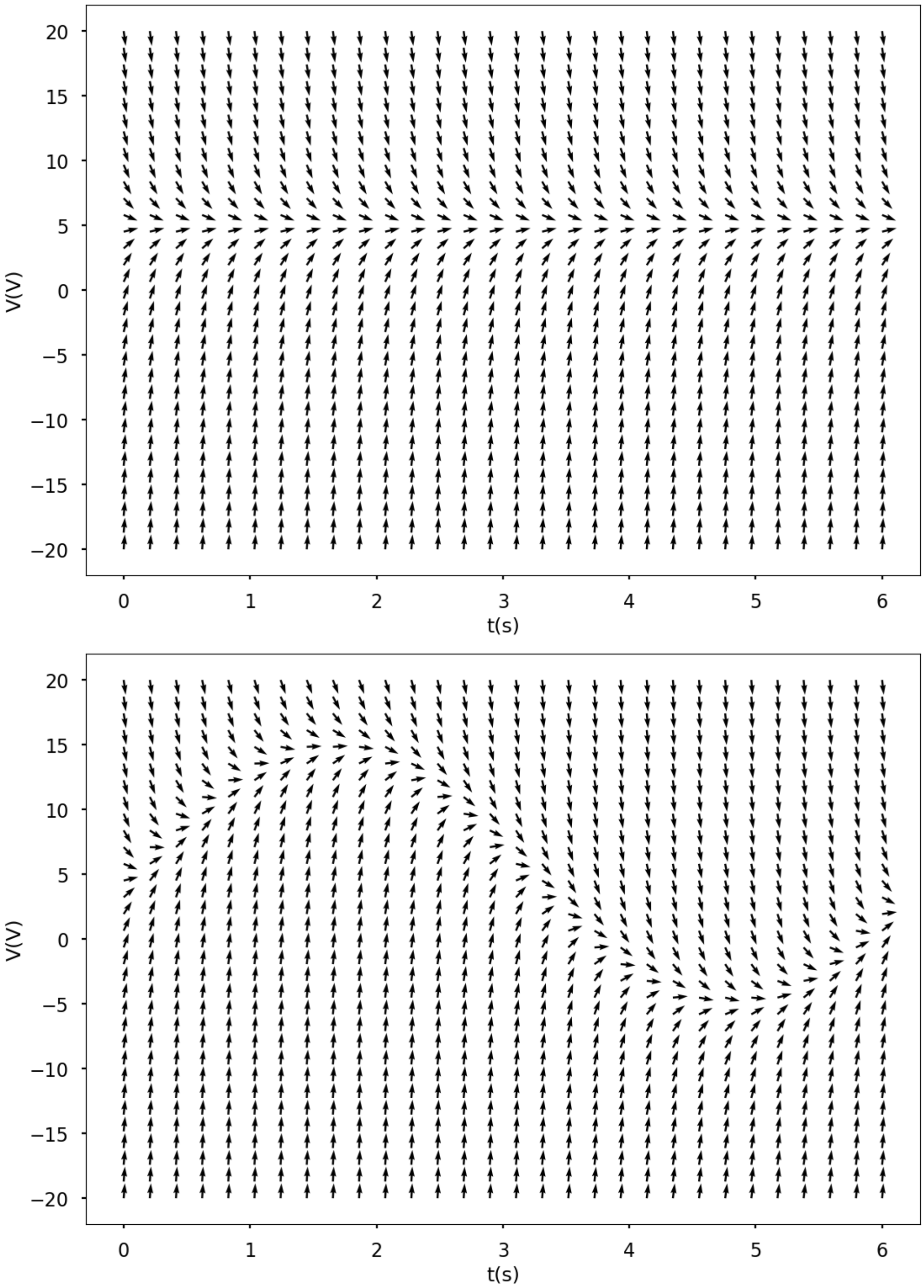


Figure 4: Slope fields for the equation derived from the equivalent circuit in Figure 2, with the constant term $D = 5\text{A}$. Top: $R_l = 1\Omega$; $C_m = 2\text{F}$; $I_{\max} = 0\text{A}$. Bottom: $R_l = 1\Omega$; $C_m = 2\text{F}$; $I_{\max} = 10\text{A}$