Contents

1 Notations

Frames:

$$\text{body frame: B} \tag{1}$$
$$\text{robot frame: R} \tag{2}$$
$$\text{world frame: W} \tag{3}$$
$$\text{global frame: G} \tag{4}$$

Translation:

$$\mathbf{t} \tag{5}$$
$$_{\mathrm{G}}\mathbf{t}_{\mathrm{GB}} \tag{6}$$

Position:

$$\mathbf{p} \tag{7}$$
$$_{\mathrm{G}}\mathbf{p}_{\mathrm{GB}} \tag{8}$$

Velocity:

$$\mathbf{v} \tag{9}$$
$$_{\mathrm{G}}\mathbf{v}_{\mathrm{GB}} \tag{10}$$

Angular Velocity:

$$\boldsymbol{\omega} \tag{11}$$
$$_{\mathrm{G}}\boldsymbol{\omega}_{\mathrm{GB}} \tag{12}$$

Acceleration:

$$\mathbf{a} \tag{13}$$
$$_{\mathrm{G}}\mathbf{a}_{\mathrm{GB}} \tag{14}$$

Rotation:

$$\mathbf{R} \tag{15}$$
$$\mathbf{R}_{\mathrm{BG}} \tag{16}$$

Transforms:

$$\mathbf{T} \tag{17}$$
$$\mathbf{T}_{\mathrm{BG}} \tag{18}$$

2 Rotation Matrix

Z-Y-X rotation sequence:

$$\mathbf{R}_{zyx} = \begin{bmatrix} c(\psi)c(\theta) & c(\psi)s(\theta)s(\phi) - s(\psi)c(\phi) & c(\psi)s(\theta)c(\phi) + s(\psi)s(\phi) \\ s(\psi)c(\theta) & s(\psi)s(\theta)s(\phi) + c(\psi)c(\phi) & s(\psi)s(\theta)c(\phi) - c(\psi)s(\phi) \\ -s(\theta) & c(\theta)s(\phi) & c(\theta)c(\phi) \end{bmatrix} \tag{19}$$

3 Quaternions

A quaternion, $\mathbf{q} \in \mathbb{R}^4$, generally has the following form

$$\mathbf{q} = q_w + q_x\mathbf{i} + q_y\mathbf{j} + q_z\mathbf{k}, \tag{20}$$

where $\{q_w, q_x, q_y, q_z\} \in \mathbb{R}$ and $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ are the imaginary numbers satisfying

$$\begin{gathered} \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1 \\ \mathbf{ij} = -\mathbf{ji} = \mathbf{k}, \;\; \mathbf{jk} = -\mathbf{kj} = \mathbf{i}, \;\; \mathbf{ki} = -\mathbf{ik} = \mathbf{j} \end{gathered} \tag{21}$$

corresponding to the Hamiltonian convention. The quaternion can be written as a 4 element vector consisting of a *real* (*scalar*) part, q_w, and *imaginary* (*vector*) part $\mathbf{q}_v$ as,

$$\mathbf{q} = \begin{bmatrix} q_w \\ \mathbf{q}_v \end{bmatrix} = \begin{bmatrix} q_w \\ q_x \\ q_y \\ q_z \end{bmatrix} \tag{22}$$

3.1 Some Quaternion Properties

3.1.1 Sum

Let $\mathbf{p}$ and $\mathbf{q}$ be two quaternions, the sum of both quaternions is,

$$\mathbf{p} \pm \mathbf{q} = \begin{bmatrix} p_w \\ \mathbf{p}_v \end{bmatrix} \pm \begin{bmatrix} q_w \\ \mathbf{q}_v \end{bmatrix} = \begin{bmatrix} p_w \pm q_w \\ \mathbf{p}_v \pm \mathbf{q}_v \end{bmatrix}. \tag{23}$$

The sum between two quaternions $\mathbf{p}$ and $\mathbf{q}$ is commutative and associative.

$$\mathbf{p} + \mathbf{q} = \mathbf{q} + \mathbf{p} \tag{24}$$

$$\mathbf{p} + (\mathbf{q} + \mathbf{r}) = (\mathbf{p} + \mathbf{q}) + \mathbf{r} \tag{25}$$

3.1.2 Product

The quaternion multiplication (or product) of two quaternions $\mathbf{p}$ and $\mathbf{q}$, denoted by $\otimes$ is defined as

$$\mathbf{p} \otimes \mathbf{q} = (p_w + p_x\mathbf{i} + p_y\mathbf{j} + p_z\mathbf{k})(q_w + q_x\mathbf{i} + q_y\mathbf{j} + q_z\mathbf{k}) \tag{26}$$

$$= \begin{matrix} (p_w q_w - p_x q_x - p_y q_y - p_z q_z) & \\ (p_w q_x + p_x q_w + p_y q_z - p_z q_y) & \mathbf{i} \\ (p_w q_y - p_y q_w + p_z q_x + p_x q_z) & \mathbf{j} \\ (p_w q_z + p_z q_w - p_x q_y + p_y q_x) & \mathbf{k} \end{matrix} \tag{27}$$

$$= \begin{bmatrix} p_w q_w - p_x q_x - p_y q_y - p_z q_z \\ p_w q_x + q_x p_w + p_y q_z - p_z q_y \\ p_w q_y - p_y q_w + p_z q_x + p_x q_z \\ p_w q_z + p_z q_w - p_x q_y + p_y q_x \end{bmatrix} \tag{28}$$

$$= \begin{bmatrix} p_w q_w - \mathbf{p}_v^\top \mathbf{q}_v \\ p_w \mathbf{q}_v + q_w \mathbf{p}_v + \mathbf{p}_v \times \mathbf{q}_v \end{bmatrix}. \tag{29}$$

4 Computer Vision

4.1 Pinhole Camera Model

$$K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{30}$$

4.2 Radial Tangential Distortion

$$\begin{aligned} k_{\text{radial}} &= 1 + (k_1 r^2) + (k_2 r^4) \\ x' &= x \cdot k_{\text{radial}} \\ y' &= y \cdot k_{\text{radial}} \\ x'' &= x' + (2 p_1 x y + p_2 * (r^2 + 2x^2)) \\ y'' &= y' + (p_1 (r^2 + 2y^2) + 2 p_2 x y) \end{aligned} \tag{31}$$

4.3 Equi-distant Distortion

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ \theta &= \arctan(r) \\ \theta_d &= \theta(1 + k_1 \theta^2 + k_2 \theta^4 + k_3 \theta^6 + k_4 \theta^8) \\ x' &= (\theta_d / r) \cdot x \\ y' &= (\theta_d / r) \cdot y \end{aligned} \tag{32}$$