Machine Learning Guest Lecture
A Glimpse of Quantum Machine Learning

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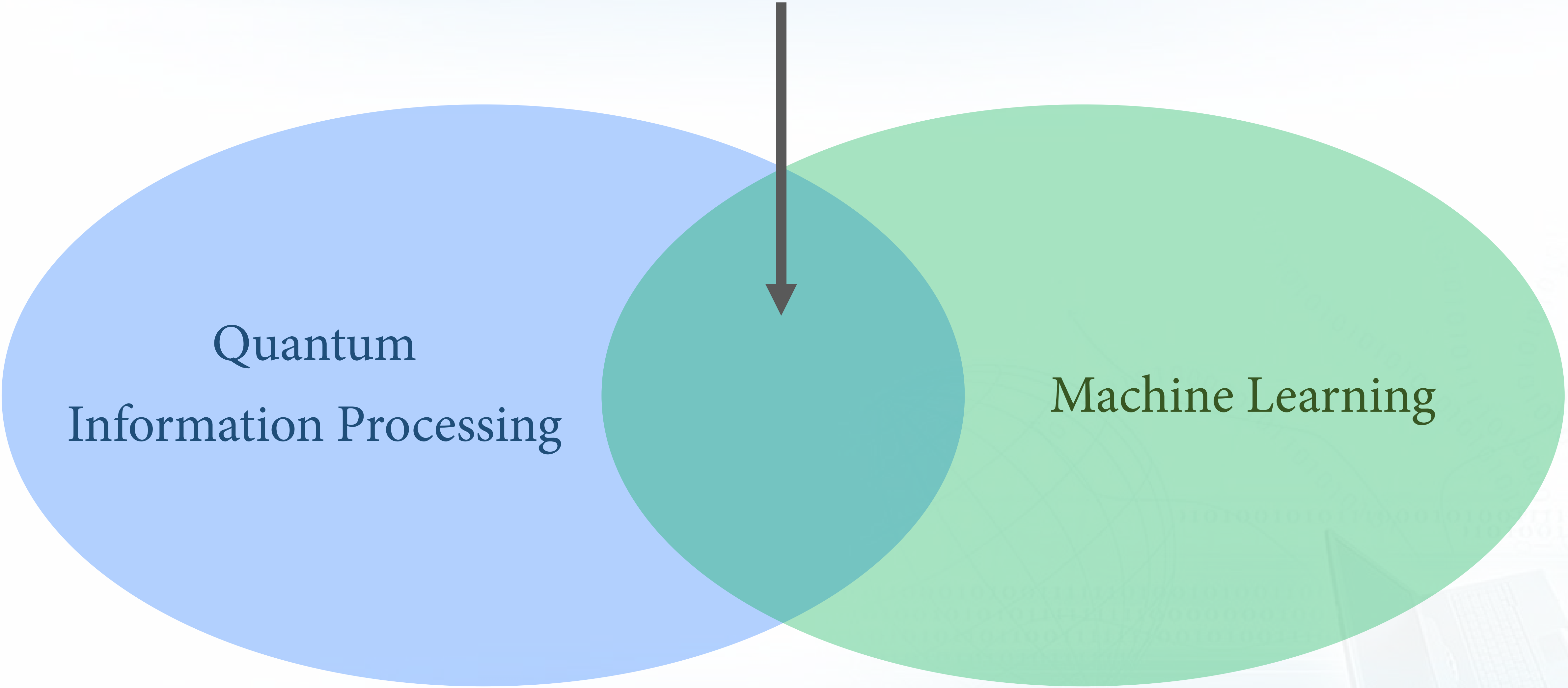
Outline

Quantum Machine Learning?

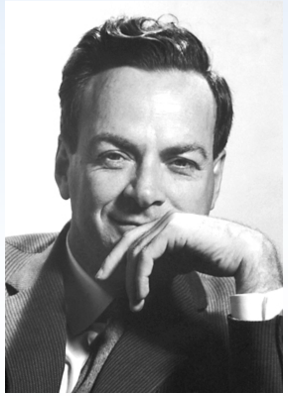


Prologue

- *Quantum information science* have demonstrated advantages in several fields:
  - Computing: speed-ups certain computational tasks where no classical methods can do. e.g. 質因數分解
  - Communication: entanglement-assisted communication increases channel capacity.
  - Cryptography: extending classical key with information-theoretic security.
  - Sensing: more accurate estimation, positioning, and synchronization.
  - Simulation: simulating complex reactions that are formidable for classical computers.
- Any other applications/advantages of quantum information technologies?
- Artificial intelligence and machine learning tasks are essentially implemented on a physical device. How about doing the job on a quantum computational device?
- Can quantum information science revolutionize the way of learning from data?

Part I – Basics of Quantum Information Processing

Brief History of Quantum Computation (1/2)



Richard Feynman
(1918-1988)

- Paul Benioff (1979):
  "The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines.
- Feynman (1981): "Why don't we store information on individual particles that already follow the very rules of quantum mechanics that we try to simulate?

"Nature Isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical."

- David Deutsch (1985) described what a quantum algorithm would look like, and Richard Jozsa (1992) demonstrated a *deterministic* quantum advantage.
- Umesh Vazirani and Ethan Bernstein (1993) pushed it forward (bounded error).
- Daniel Simon (1994) demonstrated an exponential speedup.
  如果有問題的難度是exponential的話，那moore's law也追趕不上
  但有些exponential的問題用quantum algorithm卻可以變成polynomial time

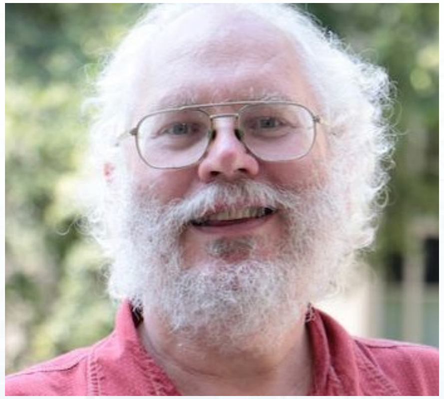
Brief History of Quantum Computation (2/2)

- Seth Lloyd (1993) described a method of building a working quantum computer.
- Peter Shor (1994) invented a polynomial-time quantum algorithm for factoring.
- David DiVincenzo (1996) outlined the key criteria of a quantum computer.
- Isaac Chuang *et al.* (2001) implemented Shor's algorithm on a nuclear magnetic resonance (NMR) system to factor the number 15 as a demonstration.

⋮

- → A variety of interdisciplinary fields such as Quantum Computation, Quantum Communication, Quantum Simulation, Quantum Sensing, Quantum Chemistry, etc.

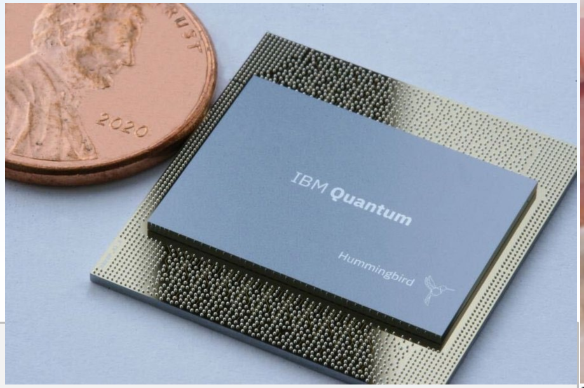
Quantum Information Science

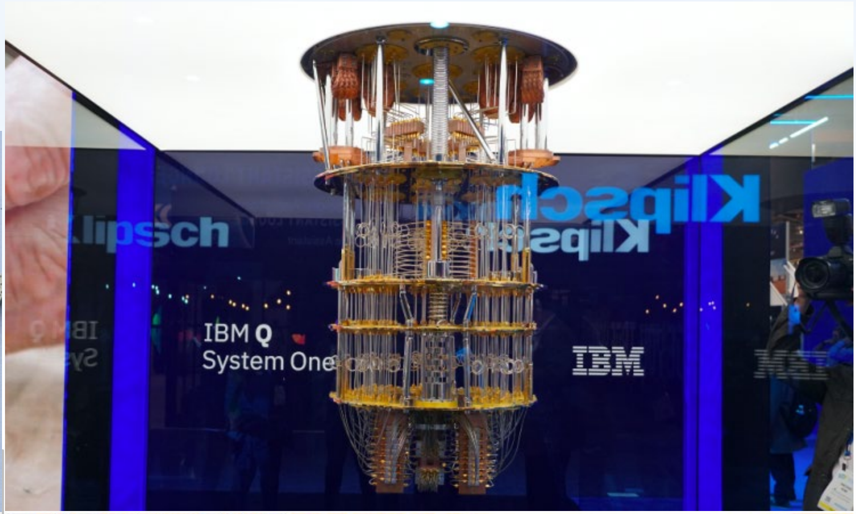


Peter Shor (1959 -)

Industry – Tech Giants

- IBM
  - IBM Q System One Computer Center
  - 53, 65-qubit processor for IBM Q Network





- Google AI
  - 54-qubit processor “Sycamore”
  - 72-qubit processor “Bristlecone”

nature

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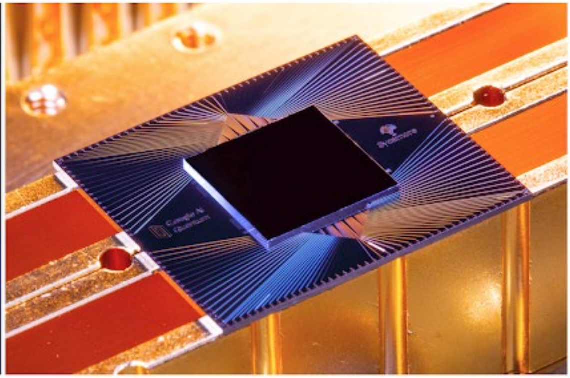
Article | Published: 23 October 2019

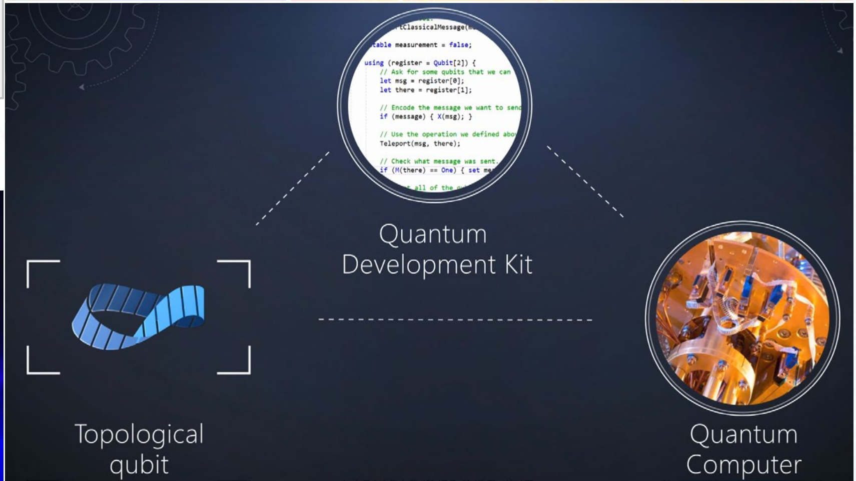
Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, […] John M. Martinis

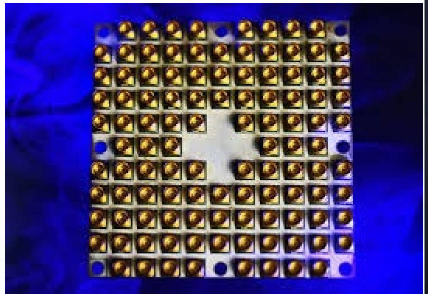
Nature **574**, 505–510(2019) | Cite this article



- Microsoft
  - Quantum Development Kit
  - Q# Programming Language
  - Azure Quantum – cloud service

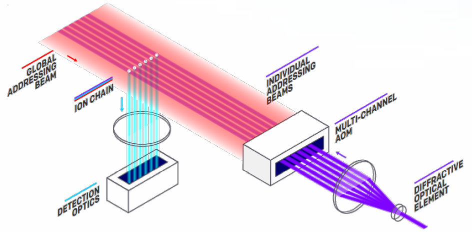


- intel
  - 49-qubit processor

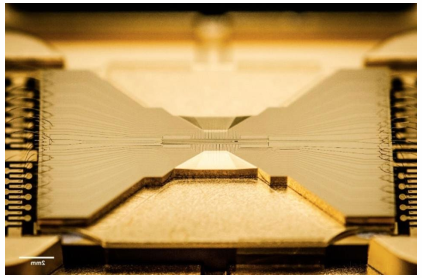


Other Quantum Processors

Trapped Ion



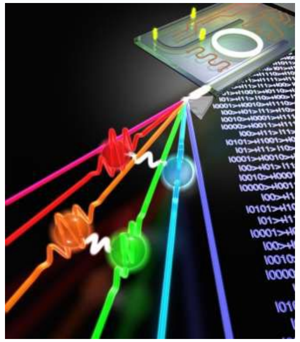
IONQ

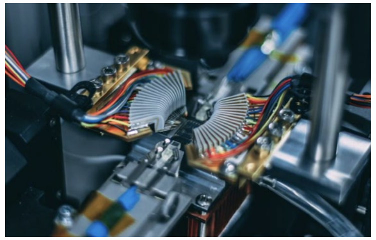


(32 qubits)

Honeywell

Photonics

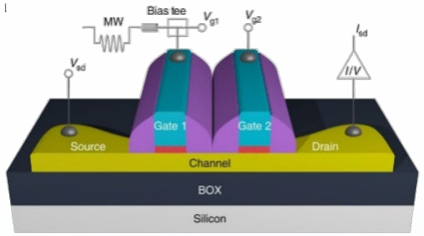




XANADU

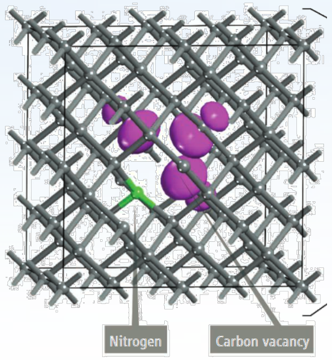
Ψ PsiQuantum

Silicon-Based Spin



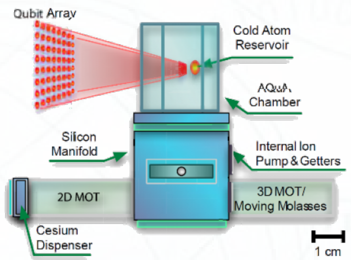
Silicon Quantum Computing

NV Center-in-Diamond

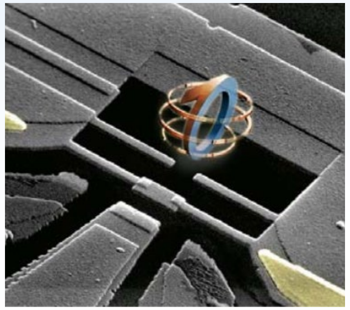


QUANTUM BRILLIANCE

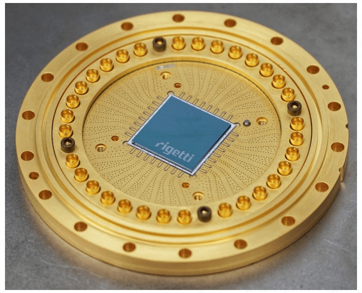
NMR



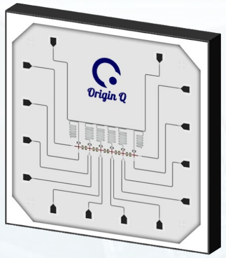
Superconducting



rigetti

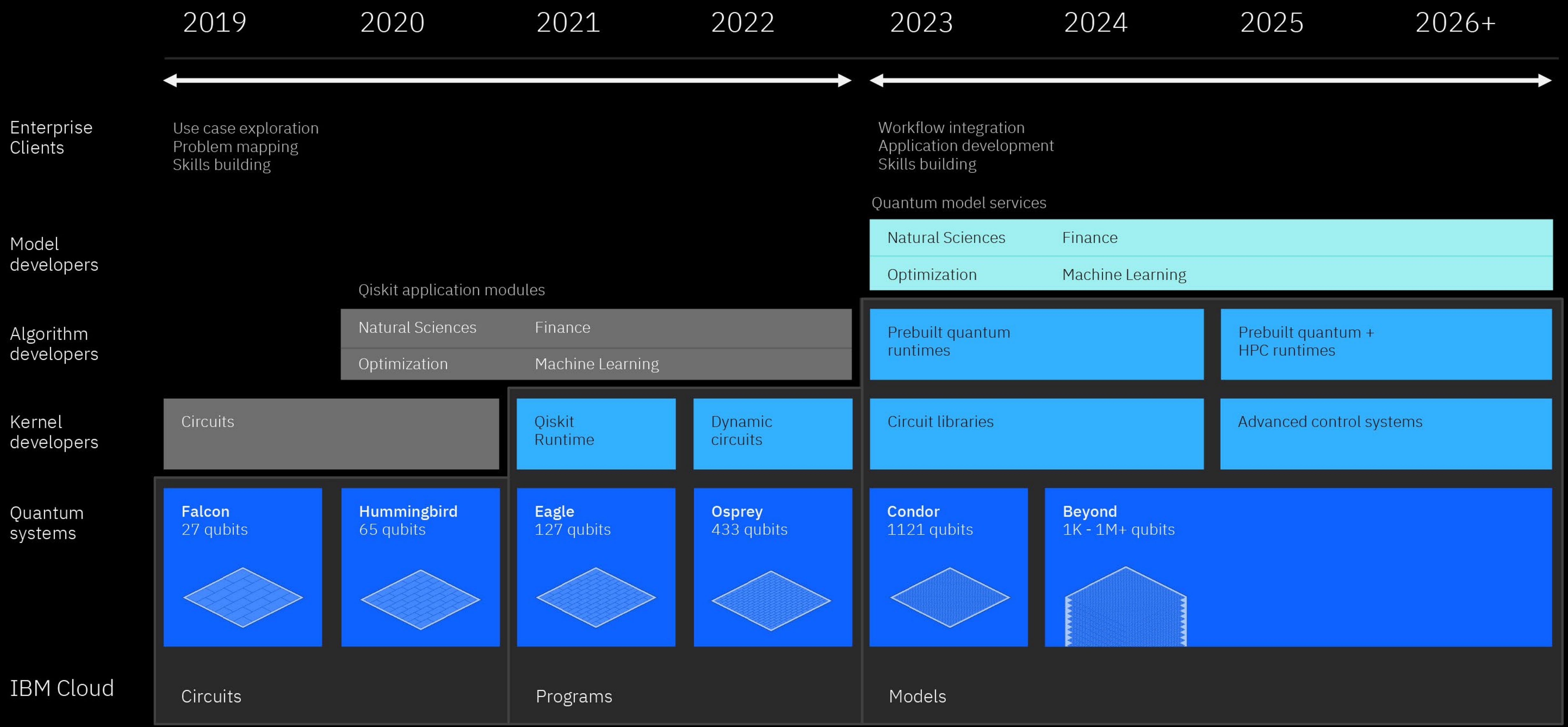


本源量子 Origin Quantum



Development Roadmap

IBM **Quantum**



Photonics is the only way to deliver

1,000,000 qubits

A useful quantum computer requires **at least a million qubits**.

Our quantum computer will be built using the **same industrial tools that produce your laptop**.

Error correction is at the centre of everything we do. It is the only known way to ensure that such a complex device can function reliably.

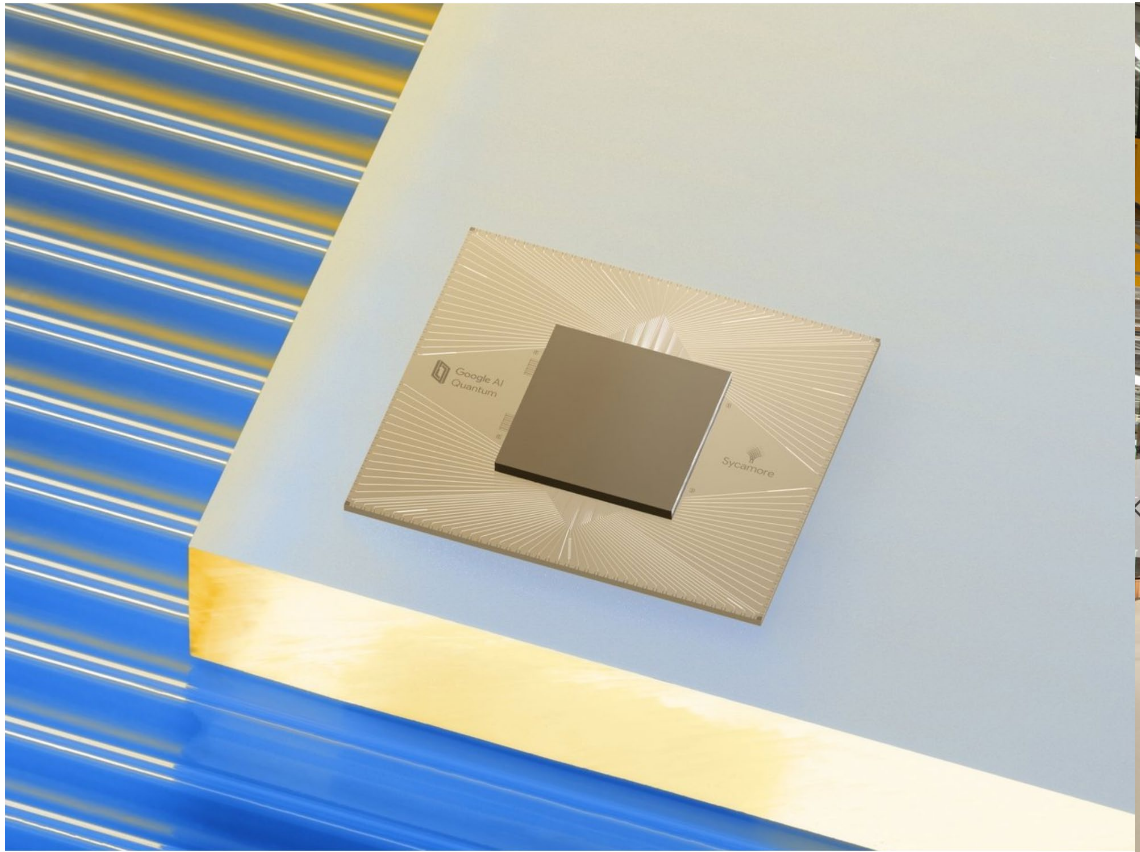
Thirty years ago photonic quantum computing was believed impossible. Twenty years ago, it was proved possible but dismissed as impractical. Today, after numerous architectural breakthroughs and advances in silicon photonics, PsiQuantum uniquely has a clear path to a useful quantum computer.

Jeremy O'Brien

CIO JOURNAL

Google Aims for Commercial-Grade Quantum Computer by 2029

Tech giant is one of many companies racing to build a business around the nascent technology



IQM

Qnami

QUANTUM SYSTEMS ACCELERATOR
Catalyzing the Quantum Ecosystem

ENTROPICA LABS

ColdQuanta
The Quantum Atomics Company

Aliyun
Alibaba Cloud Computing

riverlane

ATOM COMPUTING

HUAWEI

Baidu 百度

ISARA

JoS QUANTUM

QCWARE

QUANTUMXCHANGE

qci

kiutra

QUANTUM MOTION

PHASECRAFT

QUANTUM MACHINES

GTN LTD

Q^X Branch

Crypta Labs

QUANDELA

Tencent 腾讯

id Quantique

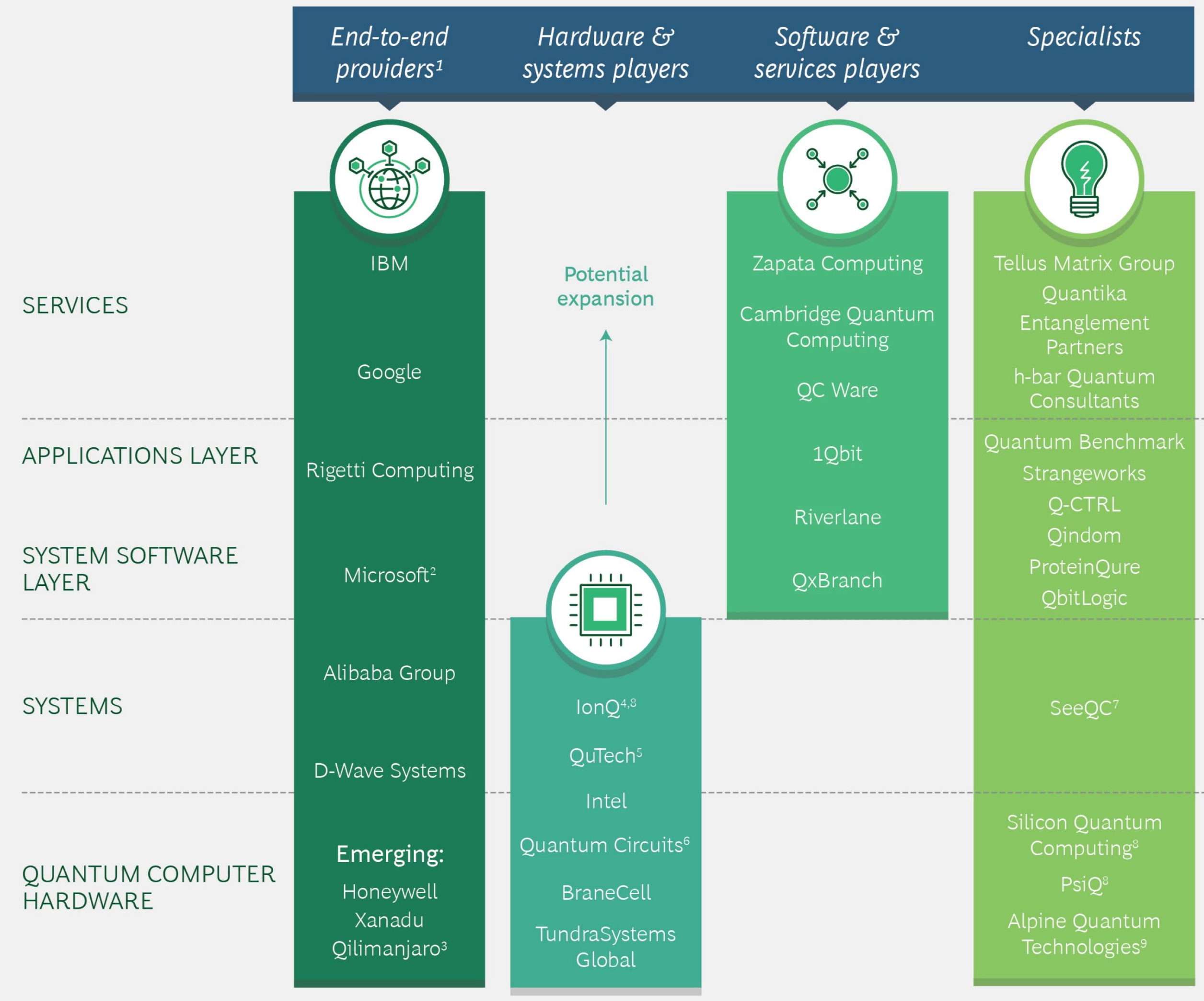
Q-LION

|KETS⟩ QUANTUM SECURITY

Sparrow Quantum

QINDOM

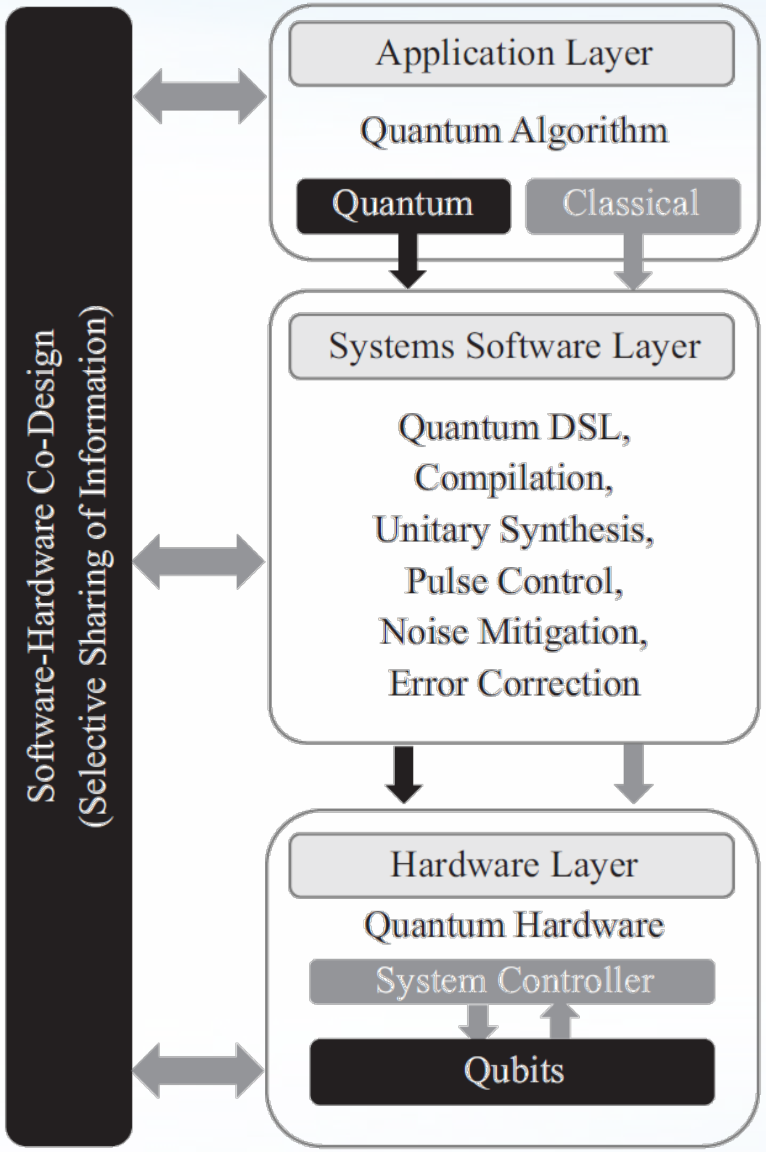
EXHIBIT 1 | Companies Assume Four Roles Across Layers of the Stack in the Quantum Computing Ecosystem

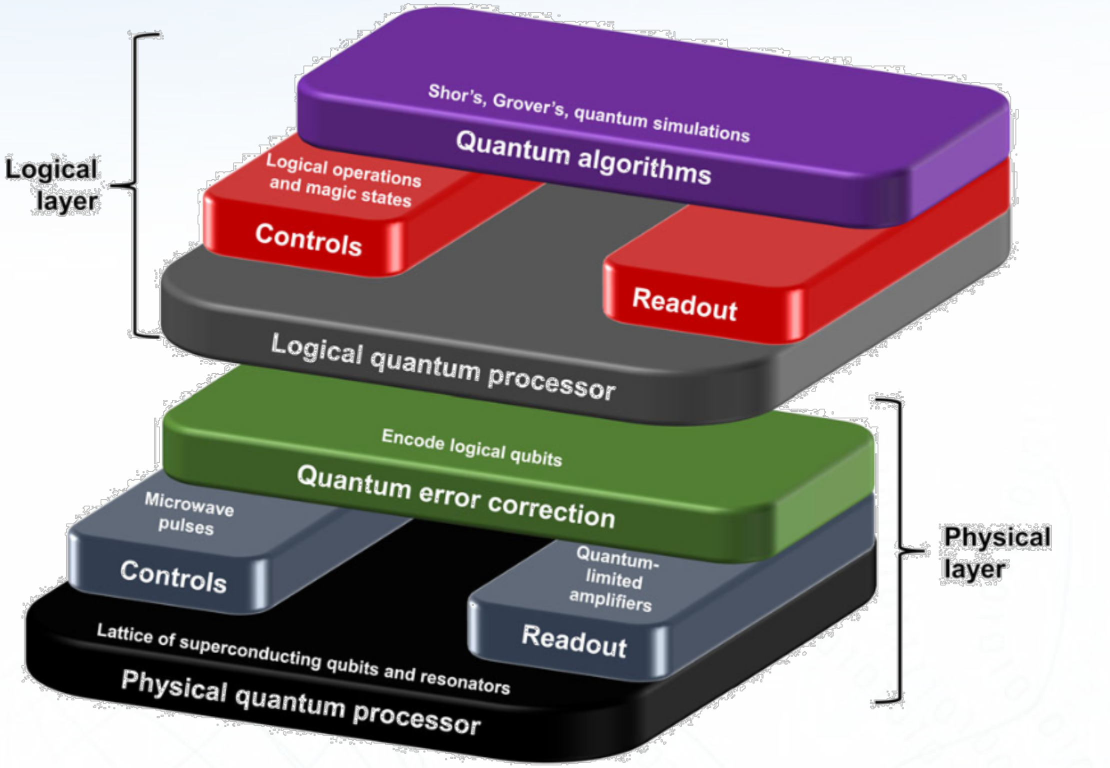


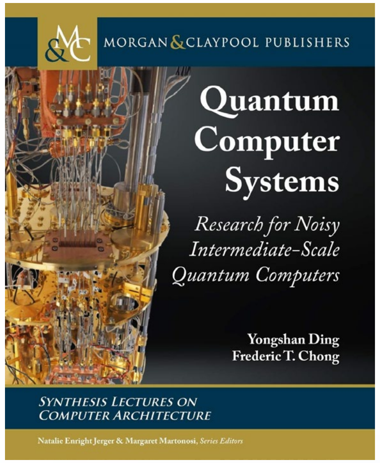
Sources: Quantum Computing Report (quantumcomputingreport.com); BCG analysis.
[1]Based on player's ambition with varying levels of maturity and service activities.
[2]Multiple technologies in the labs with focus on topological qubits.
[3]Qilimanjaro is a spinoff from the University of Barcelona.
[4]AWS is invested in IonQ.
[5]QuTech was founded by TU Delft and TNO, and has collaborations with Intel and Microsoft.
[6]Quantum Circuits (qci) is a spinoff from Yale University.
[7]SeeQC is a subsidiary of Hypres.
[8]Vision to become end-to-end provider.
[9]Alpine Quantum Technologies (AQT) is a spinoff from University of Innsbruck.

BOSTON CONSULTING GROUP

Quantum Computer System Stacks





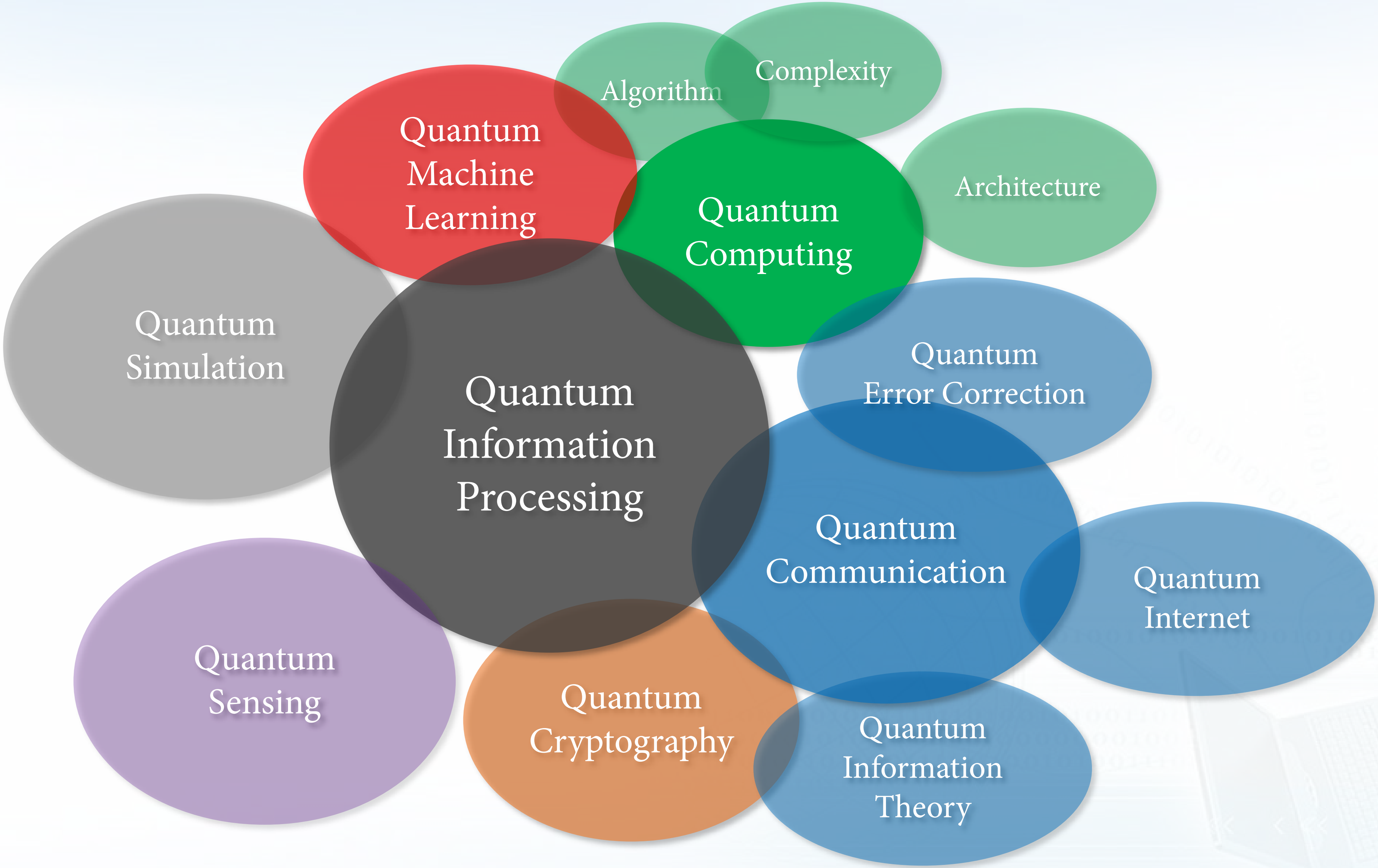


Review Article | Open Access | Published: 13 January 2017

Building logical qubits in a superconducting quantum computing system

Jay M. Gambetta, Jerry M. Chow & Matthias Steffen

npj Quantum Information **3**, Article number: 2 (2017) | Cite this article


Algorithm
Complexity
Quantum Machine Learning
Quantum Computing
Architecture
Quantum Simulation
Quantum Information Processing
Quantum Error Correction
Quantum Communication
Quantum Internet
Quantum Sensing
Quantum Cryptography
Quantum Information Theory

A Statistical Framework of Quantum Theory

- Preparation: A preparation procedure determines the ***state*** of a system.
- Evolution: How is a quantum state ***evolving***?
- Measurement: A measurement procedure produces some random outcomes.

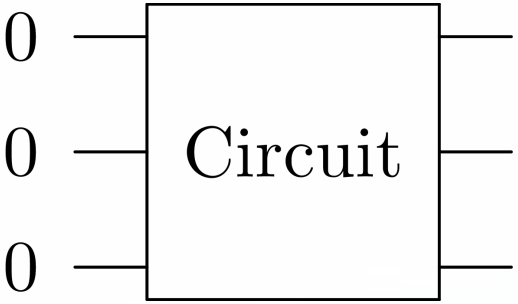
The Quantum Bit (Qubit)

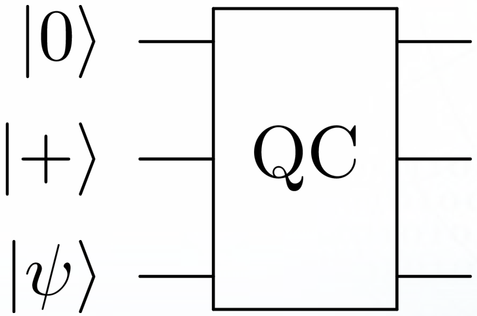
- Definition: A **qubit** is the fundamental unit of quantum information.
  It is a *superposition* state represented by a linear combination of $|0\rangle$ and $|1\rangle$ in $\mathbb{C}^2$:

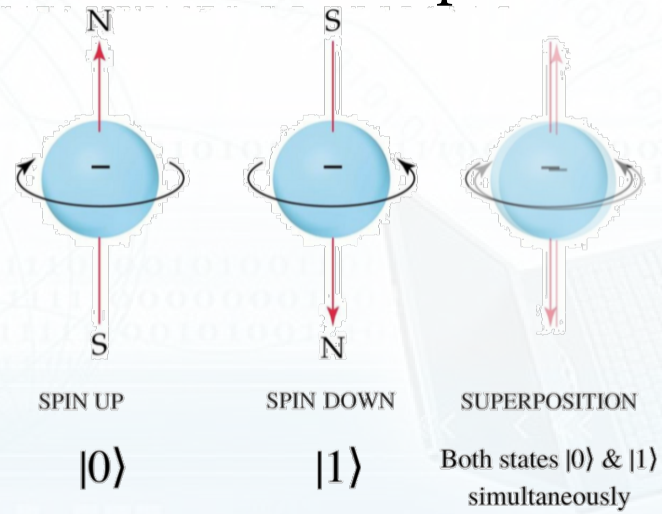
$$|\psi\rangle = a|0\rangle + b|1\rangle,\ a, b \in \mathbb{C},\ |a|^2 + |b|^2 = 1$$

- Physically, a qubit can be a realized by a two-state (two-level) quantum-mechanical system e.g. a spinning electron or polarized light.

- A *quantum register* (a quantum system) is a collection of qubits we use for computation.

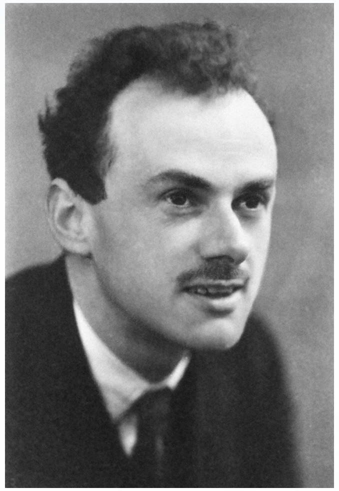






Vector Representation (Dirac Notation) for a Qubit

- $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \quad \Rightarrow |\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix}a\\b\end{bmatrix} \in \mathbb{C}^2, \ |a|^2 + |b|^2 = 1$



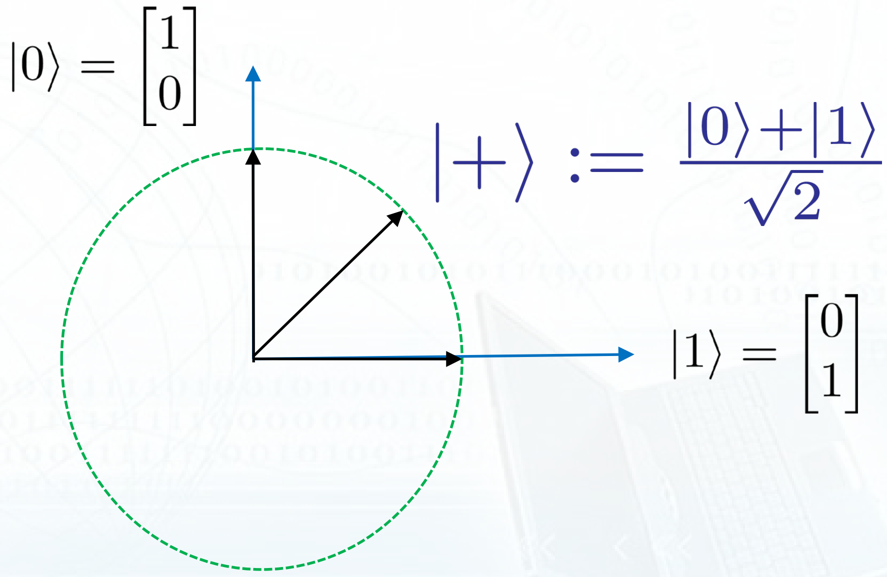
Paul Dirac (1902-1984)

- Bra–Ket notation: the Ket vectors $|\psi\rangle = \begin{bmatrix}a\\b\end{bmatrix}, |\phi\rangle = \begin{bmatrix}c\\d\end{bmatrix}$

- Inner product: $\langle\phi| := |\phi\rangle^\dagger = \begin{bmatrix}c^*, & d^*\end{bmatrix}$

$$\Rightarrow \langle\phi|\psi\rangle = \begin{bmatrix}c^*, & d^*\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}$$

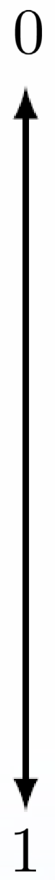
$$= c^* a + d^* b \in \mathbb{C}$$

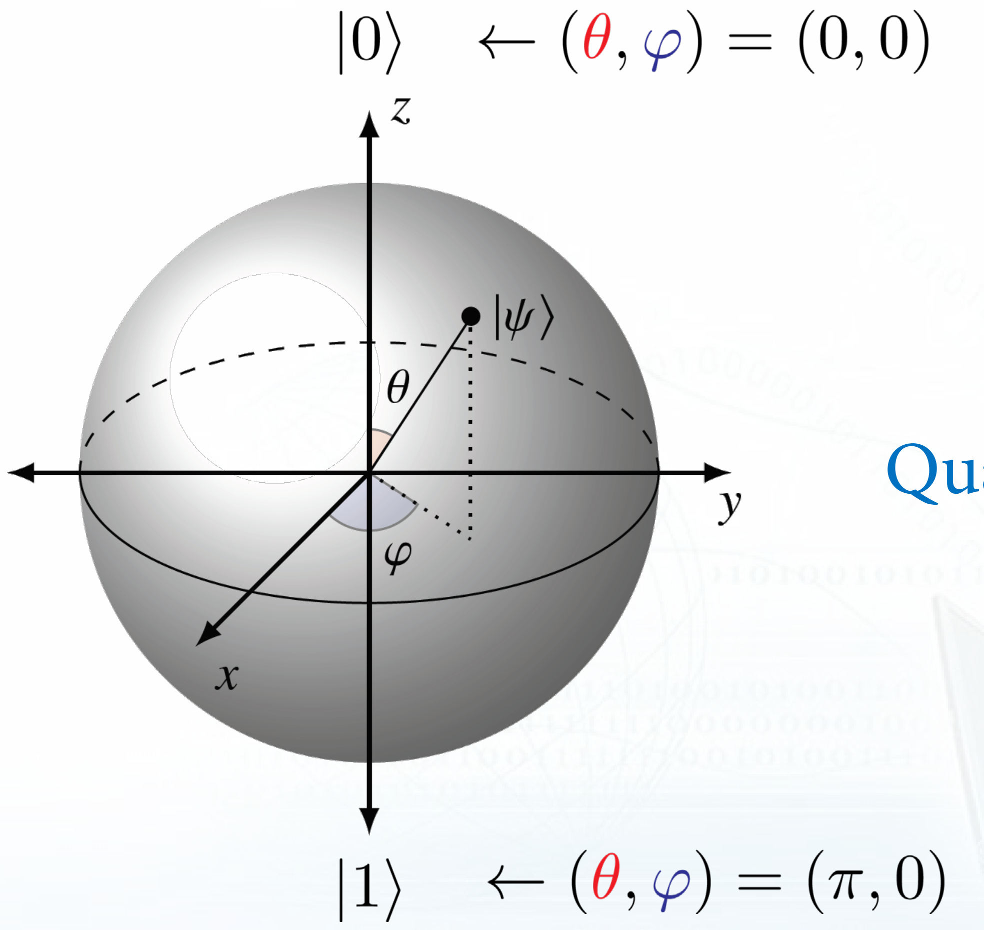


Bloch-Sphere Representation for a Qubit

$$\Rightarrow |\psi\rangle = \cos(\tfrac{\theta}{2})|0\rangle + \sin(\tfrac{\theta}{2})e^{i\varphi}|1\rangle,\ \theta \in [0,\pi],\ \varphi \in [0,2\pi]$$

Classical Bit





Quantum Bit

Quantum Measurement

- A **quantum measurement** (with respect to the computational basis $\{|0\rangle, |1\rangle\}$) gives you the readout of ‘0’ or ‘1’ with certain probability

- The Born rule: $|\psi\rangle = a|0\rangle + b|1\rangle$

$$\Pr(0) = |\langle 0|\psi\rangle|^2 = \left| \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right|^2 = |a|^2$$

$$\Pr(1) = |\langle 1|\psi\rangle|^2 = \left| \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right|^2 = |b|^2$$

Max Born (1882-1970)

- After measurement, a qubit is forced to *collapse* (irreversibly) through projection to one of the basis

Single-Qubit Gates (1/2)

- In gate-based quantum computers, a quantum operation is a *unitary operation*.

- The quantum X gate is given by the Pauli X matrix $|0\rangle \;—\boxed{X}—\; |1\rangle$
  → It is called the *NOT* gate or the "*bit flip*" gate since it rotates π around the x-axis.

Matrix representation

$$X := |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \Rightarrow X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

- The quantum Z gate rotates π around the z-axis $Z := |0\rangle\langle 0| - |1\rangle\langle 1|$
  → It is called the "*phase flip*" gate $\Rightarrow Z|\psi\rangle = r_1|0\rangle + r_2 e^{i(\pi+\psi)}|1\rangle$

- The quantum Y gate rotates π around the y-axis.
  → It does the bit flip and phase flip at the same time.

Wolfgang Pauli (1900-1958)

Single-Qubit Gates (2/2)

$$H := \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- The Hadamard gate changes the basis from $\{|0\rangle, |1\rangle\}$ to $\{|+\rangle, |-\rangle\}$
  → It creates *superposition*; and it is self-inverse: $HH = I$

$$|b\rangle \;—\boxed{H}—\; \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^b|1\rangle\right), \; b \in \{0,1\}$$

Jacques Hadamard (1865-1963)

- The quantum R^Z_φ gate rotates φ around the z-axis $\quad R^Z_\varphi := \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\varphi} \end{pmatrix}$
- The quantum R^X_φ gate rotates φ around the x-axis $\quad R^X_\varphi := \begin{pmatrix} \cos(\frac{\varphi}{2}) & -\mathrm{i}\sin(\frac{\varphi}{2}) \\ -\mathrm{i}\sin(\frac{\varphi}{2}) & \cos(\frac{\varphi}{2}) \end{pmatrix}$
- The quantum R^Y_φ gate rotates φ around the y-axis $\quad R^Y_\varphi := \begin{pmatrix} \cos(\frac{\varphi}{2}) & -\sin(\frac{\varphi}{2}) \\ \sin(\frac{\varphi}{2}) & \cos(\frac{\varphi}{2}) \end{pmatrix}$

Just a bit math...

- Definition. *Tensor product* of matrices $A := \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$, $B := \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$

$$A \otimes B = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \otimes B$$

$$:= \begin{pmatrix} a_{1,1}B & a_{1,2}B \\ a_{2,1}B & a_{2,2}B \end{pmatrix} = \begin{pmatrix} a_{1,1}\begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} & a_{1,2}\begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} \\ a_{2,1}\begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} & a_{2,2}\begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} a_{1,1}b_{1,1} & a_{1,1}b_{1,2} & a_{1,2}b_{1,1} & a_{1,2}b_{1,2} \\ a_{1,1}b_{2,1} & a_{1,1}b_{2,2} & a_{1,2}b_{2,1} & a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} & a_{2,1}b_{1,2} & a_{2,2}b_{1,1} & a_{2,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,1}b_{2,2} & a_{2,2}b_{2,1} & a_{2,2}b_{2,2} \end{pmatrix}$$

More Qubits – Quantum Entanglement (1/2)

- General 2-qubit state:

Probability amplitude

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle \in \mathbb{C}^4 \qquad \sum_{i,j\in\{0,1\}} |a_{ij}|^2 = 1$$

- A two-qubit is represented by a unit vector in *four*-dimensional linear space $\mathbb{C}^{2\times 2}$

  → The computational basis:

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

- Tensor product: $|01\rangle \equiv |0\rangle \otimes |1\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} := \begin{pmatrix}1\begin{pmatrix}0\\1\end{pmatrix}\\0\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1\times 0\\1\times 1\\0\times 0\\0\times 1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

More Qubits – Quantum Entanglement (1/2)

- Entangled state: there are states that cannot be expressed as the product form, i.e.

$$\nexists |q_1\rangle, |q_2\rangle \in \mathbb{C}^2, \text{ s.t. } |\psi\rangle = |q_1\rangle \otimes |q_2\rangle$$

$$\begin{cases} \text{with } \frac{1}{2} \text{ probability,} & \uparrow \; \uparrow \\ \text{with } \frac{1}{2} \text{ probability,} & \downarrow \; \downarrow \end{cases} \quad \text{or} \quad \begin{cases} \text{with } \frac{1}{2} \text{ probability,} & \nearrow \; \nearrow \\ \text{with } \frac{1}{2} \text{ probability,} & \swarrow \; \swarrow \end{cases}$$

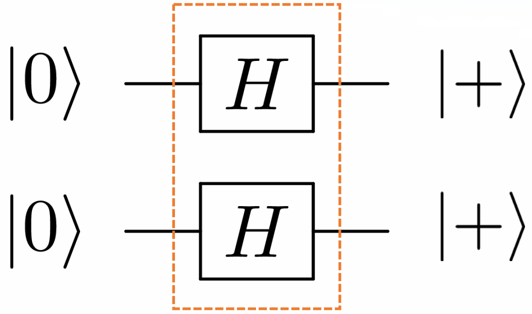
Einstein, Podolsky, and Rosen

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|++\rangle + |--\rangle}{\sqrt{2}}$$

John S. Bell (1928-1990)

Multi-Qubit Gates (1/3)

- A qubit gate has a 2 by 2 unitary matrix in a given basis.
  → For an n-qubit gate, the matrix is 2^n by 2^n (tensor product of matrices).



A two-qubit gate $H \otimes H$

$$H^{\otimes 2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

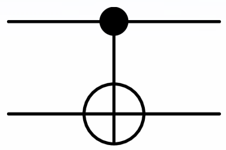
$$\Rightarrow H^{\otimes 2}|00\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = |+\rangle \otimes |+\rangle$$

$$\boxed{(A \otimes B)\mathbf{a} \otimes \mathbf{b} = (A\mathbf{a}) \otimes (B\mathbf{b})}$$

Multi-Qubit Gates (2/3)

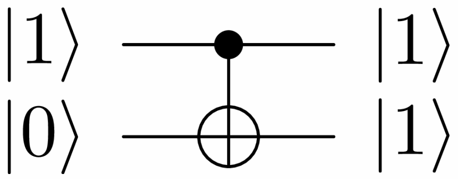
- The previous example is a 2-qubit gate of the *product form*.

- Controlled-NOT gate:

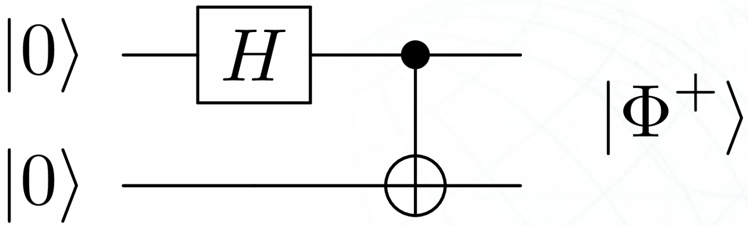


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \left(\begin{array}{c|c} I_{2,2} & 0_{2,2} \\ \hline 0_{2,2} & X \end{array}\right)$$

Ex.



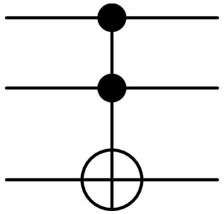
- Used to prepare the Bell state:



$$|00\rangle \overset{H\otimes I}{\mapsto} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \overset{CNOT}{\mapsto} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Multi-Qubit Gates (3/3)

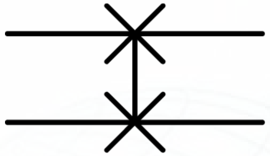
- Toffoli gate (CCNOT gate)
  → *universal* classically



Truth Table

$$
\begin{aligned}
|000\rangle &\mapsto |000\rangle \\
|001\rangle &\mapsto |001\rangle \\
|010\rangle &\mapsto |010\rangle \\
|011\rangle &\mapsto |011\rangle \\
|100\rangle &\mapsto |100\rangle \\
|101\rangle &\mapsto |101\rangle \\
|110\rangle &\mapsto |111\rangle \\
|111\rangle &\mapsto |110\rangle
\end{aligned}
$$

- Swap gate



$$
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

- *Universal* quantum gates sets: $\{CNOT, T, H\}$ or $\{CCNOT, H\}$

Elementary Quantum Gates

- Pauli gates $X := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $Y := \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$ $Z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Hadamard gates

$$H := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- Rotation gates

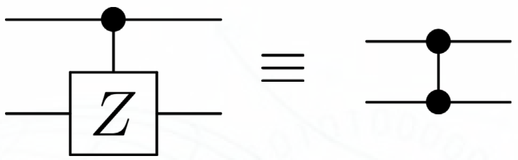
$$R_x(\phi) := \mathrm{e}^{-i\frac{\phi}{2}X} = \begin{pmatrix} \cos(\frac{\phi}{2}) & -\mathrm{i}\sin(\frac{\phi}{2}) \\ -\mathrm{i}\sin(\frac{\phi}{2}) & \cos(\frac{\phi}{2}) \end{pmatrix}$$

$$R_y(\phi) := \mathrm{e}^{-i\frac{\phi}{2}Y} = \begin{pmatrix} \cos(\frac{\phi}{2}) & -\sin(\frac{\phi}{2}) \\ \sin(\frac{\phi}{2}) & \cos(\frac{\phi}{2}) \end{pmatrix}$$

Phase gate

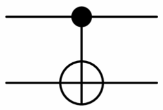
$$R_z(\phi) := \mathrm{e}^{-i\frac{\phi}{2}Z} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\phi} \end{pmatrix} \quad S := R_z(\tfrac{\pi}{2}) \quad T := R_z(\tfrac{\pi}{4})$$

- Controlled-Z gates

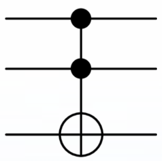


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

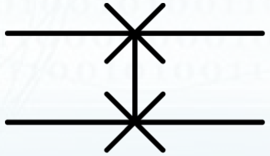
- CNOT gate

$|c\rangle$ $|t\rangle$  $|c\rangle$ $|c \oplus t\rangle$ $\begin{pmatrix} I_2 & 0 \\ 0 & X \end{pmatrix}$

- Toffoli (CCNOT) gate

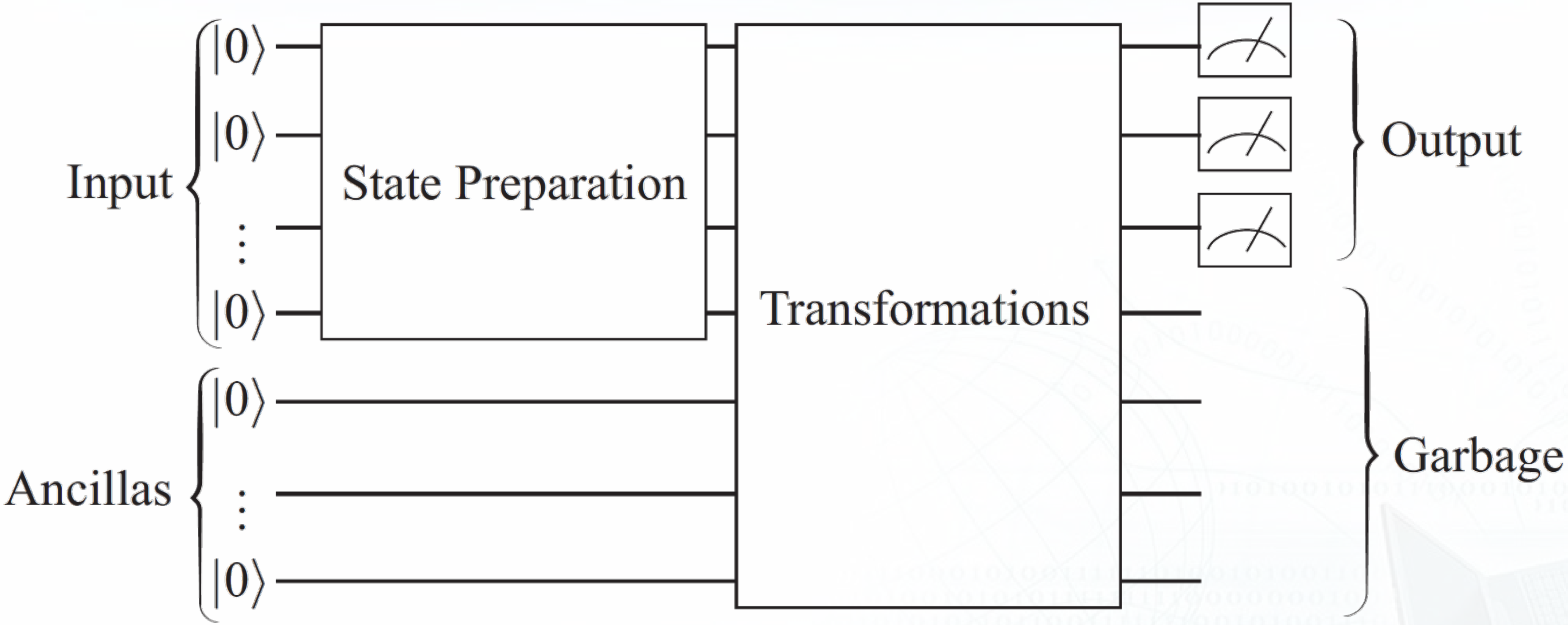
$|c_1\rangle$ $|c_2\rangle$ $|t\rangle$  $|c_1\rangle$ $|c_2\rangle$ $|(c_1 \wedge c_2) \oplus t\rangle$ $\begin{pmatrix} I_3 & 0 & 0 \\ 0 & I_3 & 0 \\ 0 & 0 & X \end{pmatrix}$

- Swap gate $|\psi\rangle|\phi\rangle \mapsto |\phi\rangle|\psi\rangle$



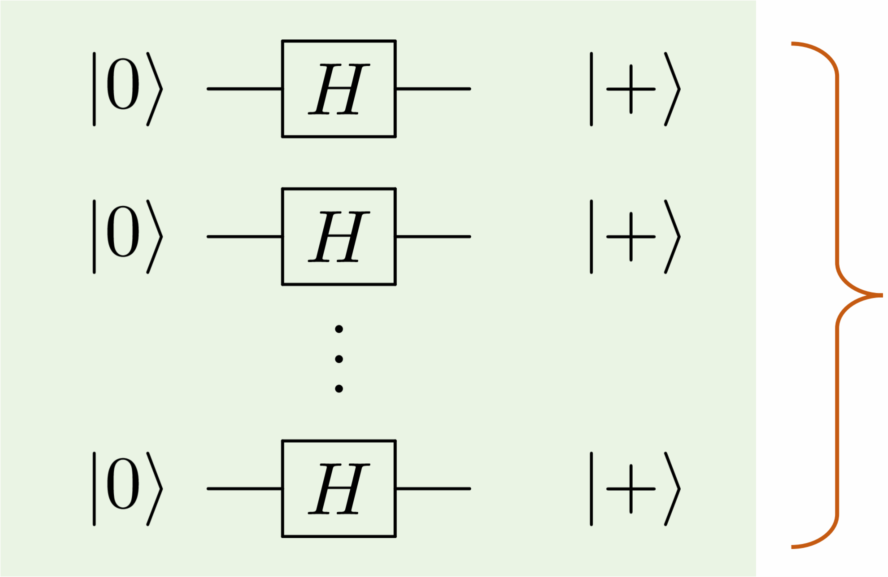
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Gate-Based Quantum Computation



Efficiently Preparing Superposition States

- We can create a superposition of *exponentially many* terms with only a *linear* number of the Hadamard gates.



$$\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)^{\otimes n}$$

$$H^{\otimes n}|0\rangle^{\otimes n} = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle$$

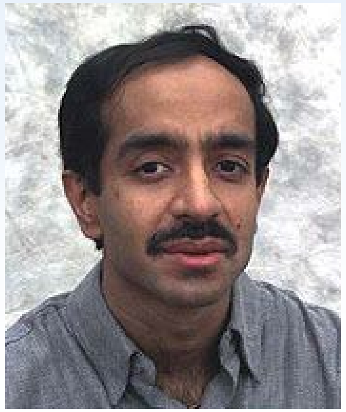
The Quantum Oracle & Quantum Parallelism

- The *quantum oracle* for any Boolean function $f: \{0,1\}^n \to \{0,1\}^m$ is given by the quantum gate $U_f: |x\rangle|y\rangle \mapsto |x\rangle|y \oplus f(x)\rangle, \forall x \in \{0,1\}^n, \forall y \in \{0,1\}^m$.
- We set the input register to an equal superposition of all 2^n possible n-bit strings:

$$U_f : \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle|0\rangle \mapsto \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle|f(x)\rangle$$

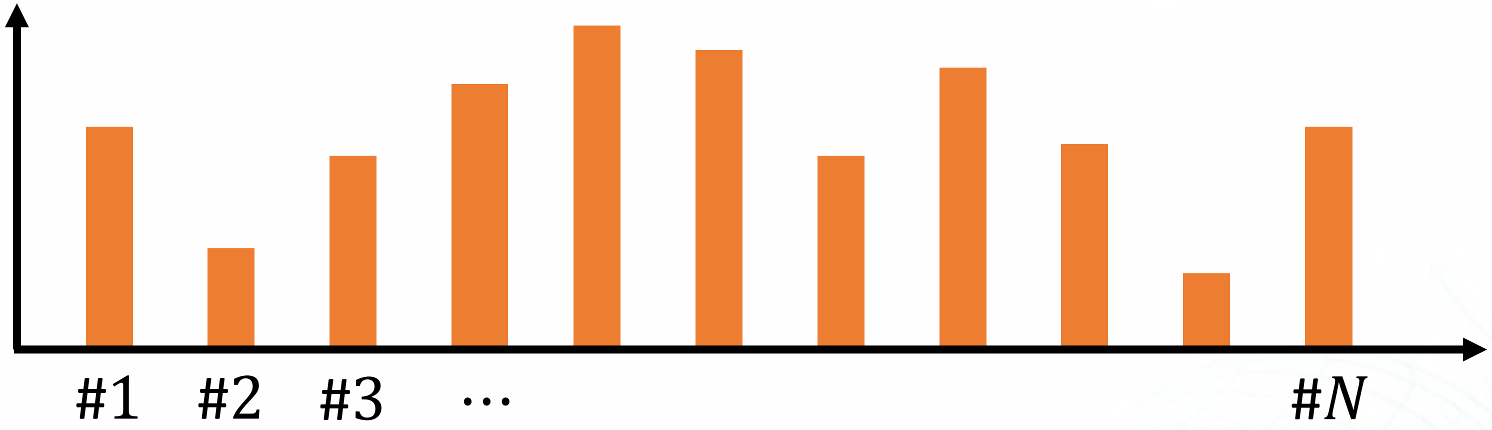
- In *one* run of the protocol, we obtain a final state with depends on all of the function values. On the other hand, we would need *exponentially* many queries to have full access to the function f.

Quantum Computing – Unstructured Search



Lov Grover (1961 –)

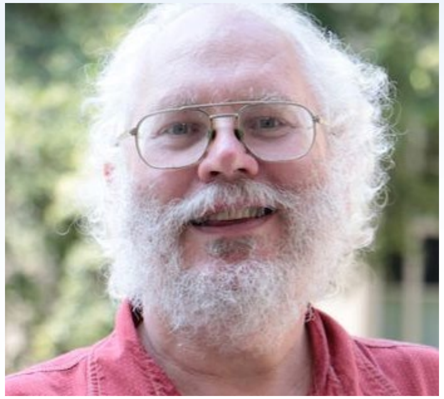
- Searching in an unstructured list with size N



The best classical algorithm requires number of queries proportional to N

→ Lov Grover (1996) proposed a quantum algorithm requires $\approx \sqrt{N}$ queries

Quantum Computing – Factorization


Peter Shor (1959 -)

- Integer Factorization used in the RSA cryptography system

  Example: $\underbrace{463570199875051}_{\approx 2^{49}} = 27644437 \times 16769023$

  The computational complexity of the best known classical algorithm scales *exponentially* in the number of bits of the integer.

  → Peter Shor (1994) invented a *polynomial-time* quantum algorithm

- Other cryptosystem such as the *Diffie–Hellman key exchange security* (based on the hardness of the *discrete logarithm problem*) and the *Elliptic curve cryptography* can be broke in polytime by applying Shor's idea.

Relations – A Glimpse of The Complexity Zoo

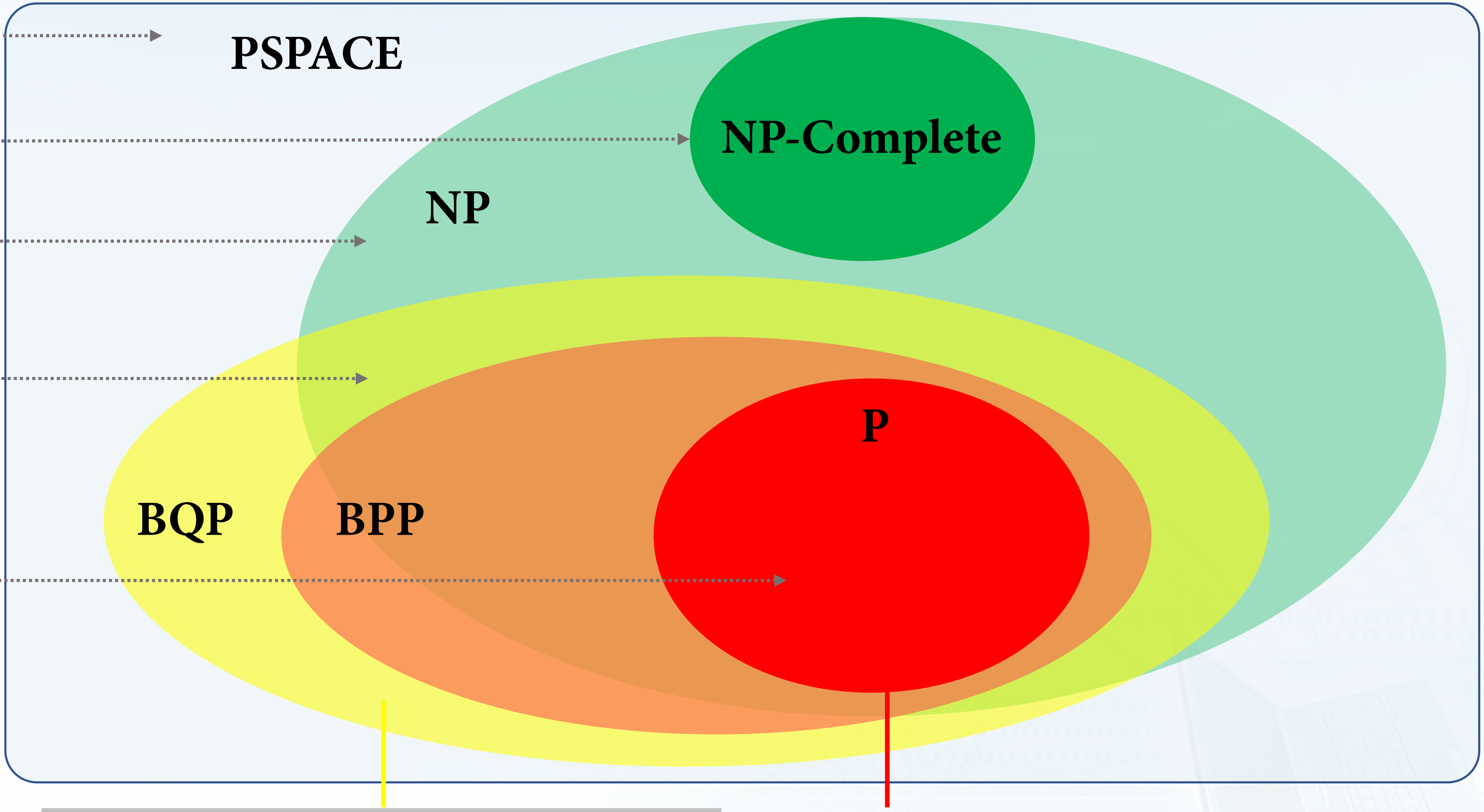
$n \times n$ Chess/Go

SAT, $n \times n$ Sudoku
Travelling salseman

Graph Isomorphism

Integer factorization,
Discrete logarithm

Multiplication,
Primality, etc.



Efficiently solvable by *quantum* computer

Efficiently solvable by *classical* computer nature

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Satellite-based entanglement distribution over 1200 kilometers

Juan Yin[1,2], Yuan Cao[1,2], Yu-Huai Li[1,2], Sheng-Kai Liao[1,2], Liang Zhang[2,3], Ji-Gang Ren[1,2], Wen-Qi Cai[1,2], Wei-Yue Liu[1]...

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Science 16 Jun 2017:
Vol. 356, Issue 6343, pp. 1140-1144
DOI: 10.1126/science.aan3211

PHYSICAL REVIEW LETTERS **120,** 030501 (2018)

Editors' Suggestion Featured in Physics

Satellite-Relayed Intercontinental Quantum Network

Sheng-Kai Liao,[1,2] Wen-Qi Cai,[1,2] Johannes Handsteiner,[3,4] Bo Liu,[4,5] Juan Yin,[1,2] Liang Zhang,[2,6] Dominik Rauch,[3,4] Matthias Fink,[4] Ji-Gang Ren,[1,2] Wei-Yue Liu,[1,2] Yang Li,[1,2] Qi Shen,[1,2] Yuan Cao,[1,2] Feng-Zhi Li,[1,2] Jian-Feng Wang,[7] Yong-Mei Huang,[8] Lei Deng,[9] Tao Xi,[10] Lu Ma,[11] Tai Hu,[12] Li Li,[1,2] Nai-Le Liu,[1,2] Franz Koidl,[13] Peiyuan Wang,[13] Yu-Ao Chen,[1,2] Xiang-Bin Wang,[2] Michael Steindorfer,[13] Georg Kirchner,[13] Chao-Yang Lu,[1,2] Rong Shu,[2,6] Rupert Ursin,[3,4] Thomas Scheidl,[3,4] Cheng-Zhi Peng,[1,2] Jian-Yu Wang,[2,6] Anton Zeilinger,[3,4] and Jian-Wei Pan[1,2]

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Entanglement-based secure quantum cryptography over 1,120 kilometres

Juan Yin, Yu-Huai Li, Sheng-Kai Liao, Meng Yang, Yuan Cao, Liang Zhang, Ji-Gang Ren, Wen-Qi Cai, Wei-Yue Liu, Shuang-Lin Li, Rong Shu, Yong-Mei Huang, Lei Deng, Li Li, Qiang Zhang, Nai-Le Liu, Yu-Ao Chen, Chao-Yang Lu, Xiang-Bin Wang, Feihu Xu, Jian-Yu Wang, Cheng-Zhi Peng, Artur K. Ekert & Jian-Wei Pan

Nature **582**, 501–505(2020) | Cite this article

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REPORT

Quantum computational advantage using photons

Han-Sen Zhong[1,2,*], Hui Wang[1,2,*], Yu-Hao Deng[1,2,*], Ming-Cheng Chen[1,2,*], Li-Chao Peng[1,2], Yi-Han Luo[1,2], Jian Qin[1,2], Dian Wu[1,2], Xing Ding[1,2], Yi Hu[1,2], Peng Hu[3], Xiao-Yan Yang[3], Wei-Jun Zhang[3], Hao Li[3], Yuxuan Li[4], Xiao Jiang[1,2], Lin Gan[4], Guangwen Yang[4], Lixing You[3], Zhen Wang[3], Li Li[1,2], Nai-Le Liu[1,2], Chao-Yang Lu[1,2], Jian-Wei Pan[1,2,†]

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[2]*CAS Centre for Excellence and Synergetic Innovation Centre in Quantum Information and Quantum Physics, University of Science and Technology of China, Shanghai 201315, China.*

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↵* These authors contributed equally to this work.

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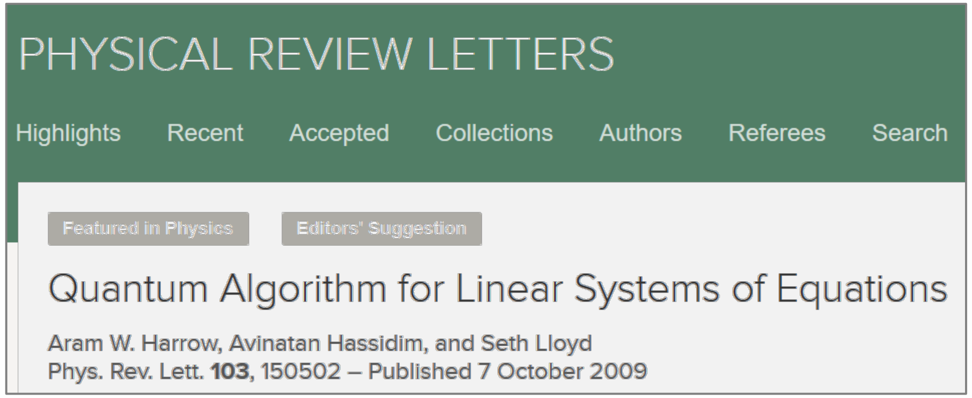
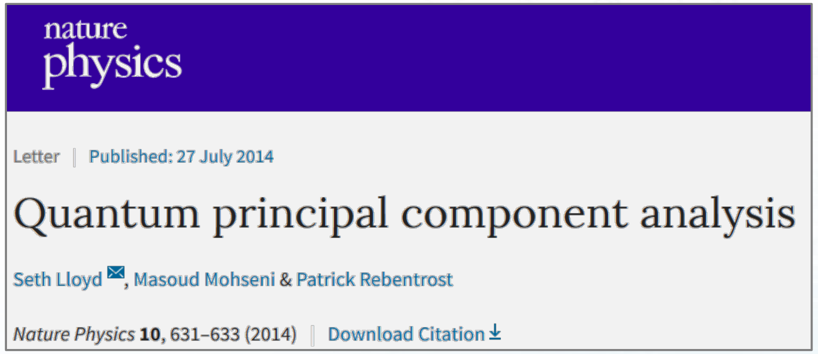
Science 03 Dec 2020:
eabe8770
DOI: 10.1126/science.abe8770

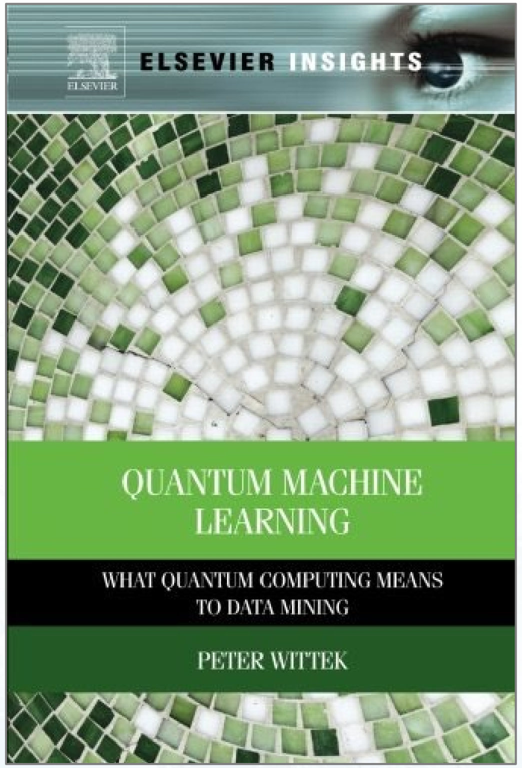
Part II – Various Models of Quantum ML

Brief History of Quantum Machine Learning (1/4)

- In 2000's – early explorations; mostly on detection & estimation instead of learning.
- The term 'quantum machine learning' was coined around 2013 (arXiv:1307.0411).
  → *HHL algorithm* for approximately solve linear equations with a *quantum RAM*.
  此時的quantum machine learning是quantum computing去加速classical的演算法，e.g. 搜尋可以加速到sqrt(N)







Brief History of Quantum Machine Learning (2/4)

- Some critiques about quantum speeding-up ML since 2015.
  → Not just about practically building a quantum compute but caveats of using it.

nature physics

Published: 02 April 2015

Read the fine print

Scott Aaronson

Nature Physics **11**, 291–293 (2015) | Cite this article

PROCEEDINGS OF THE ROYAL SOCIETY A

MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

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這篇批判quantum speed up
雖然有加速
但是把整個過程拿來看就不一定了

Quantum machine learning: a classical perspective

Carlo Ciliberto, Mark Herbster, Alessandro Davide Ialongo, Massimiliano Pontil, Andrea Rocchetto, Simone Severini, Leonard Wossnig

Published 17 January 2018. DOI: 10.1098/rspa.2017.0551

- Two types of quantum neural networks (QNNs) in 2018.
  - Unitary feed-forward network and Quantum Boltzmann machine

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Quantum Boltzmann Machine

Mohammad H. Amin, Evgeny Andriyash, Jason Rolfe, Bohdan Kulchytskyy, and Roger Melko
Phys. Rev. X **8**, 021050 – Published 23 May 2018

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Quantum circuit learning

K. Mitarai, M. Negoro, M. Kitagawa, and K. Fujii
Phys. Rev. A **98**, 032309 – Published 10 September 2018

Brief History of Quantum Machine Learning (3/4)

- Finding applications, advantages, and preliminary analysis of QNNs.

nature

Letter | Published: 13 March 2019

Supervised learning with quantum-enhanced feature spaces

Vojtěch Havlíček, Antonio D. Córcoles, Kristan Temme, Aram W. Harrow, Abhinav Kandala, Jerry M. Chow & Jay M. Gambetta

Nature **567**, 209–212 (2019) | Cite this article

COLUMN

Guest Column: A Survey of Quantum Learning Theory

Authors: Srinivasan Arunachalam, Ronald de Wolf Authors Info & Affiliations

Publication: ACM SIGACT News • June 2017 • https://doi.org/10.1145/3106700.3106710

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Sukin Sim, Peter D. Johnson, Alán Aspuru-Guzik

First published: 14 October 2019 | https://doi.org/10.1002/qute.201900070 | Citations: 24

nature communications

Article | Open Access | Published: 11 May 2021

Power of data in quantum machine learning

Hsin-Yuan Huang, Michael Broughton, Masoud Mohseni, Ryan Babbush, Sergio Boixo, Hartmut Neven & Jarrod R. McClean

Nature Communications **12**, Article number: 2631 (2021) | Cite this article

PHYSICAL REVIEW LETTERS

Editors' Suggestion

Access by

Information-Theoretic Bounds on Quantum Advantage in Machine Learning

Hsin-Yuan Huang, Richard Kueng, and John Preskill

Phys. Rev. Lett. **126**, 190505 – Published 14 May 2021

Brief History of Quantum Machine Learning (4/4)

- Barren plateaus (vanishing gradients), quantum No-Free-Lunch theorem, etc…

nature communications

Article | Open Access | Published: 16 November 2018

Barren plateaus in quantum neural network training landscapes

Jarrod R. McClean, Sergio Boixo, Vadim N. Smelyanskiy, Ryan Babbush & Hartmut Neven

Nature Communications **9**, Article number: 4812 (2018) | Cite this article

arXiv.org > quant-ph > arXiv:2010.15968

Quantum Physics

[Submitted on 29 Oct 2020 (v1), last revised 10 Mar 2021 (this version, v2)]

Entanglement Induced Barren Plateaus

Carlos Ortiz Marrero, Mária Kieferová, Nathan Wiebe

nature communications

Article | Open Access | Published: 19 March 2021

Cost function dependent barren plateaus in shallow parametrized quantum circuits

M. Cerezo, Akira Sone, Tyler Volkoff, Lukasz Cincio & Patrick J. Coles

Nature Communications **12**, Article number: 1791 (2021) | Cite this article

arXiv.org > quant-ph > arXiv:2007.04900

Quantum Physics

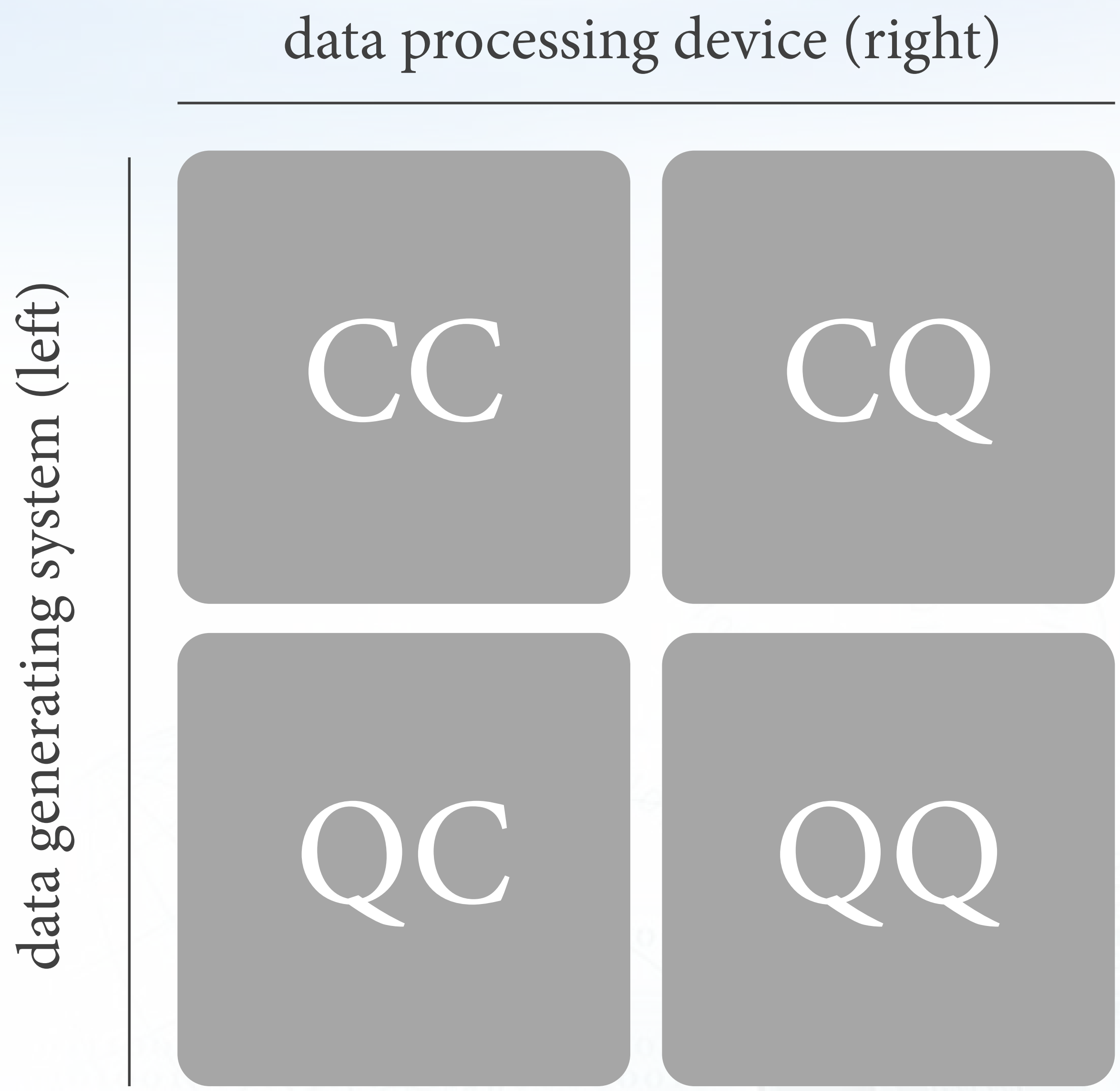
[Submitted on 9 Jul 2020]

Reformulation of the No-Free-Lunch Theorem for Entangled Data Sets

Kunal Sharma, M. Cerezo, Zoë Holmes, Lukasz Cincio, Andrew Sornborger, Patrick J. Coles

Four Categories

- CC: Learning classical data with classical machine – Classical ML.
- QC: Learning quantum objects with classical ML (e.g. matrix concentrations).
  - S. Aaronson'07 – Learning quantum states.
  - H.-C. Cheng, M.-H. Hsieh, P.-C. Yeh'15 – Learning quantum measurements.
- CQ: Processing classical datasets using quantum computing – Lloyd *et al.*
- QQ: Processing "quantum data" using quantum machine – largely open.

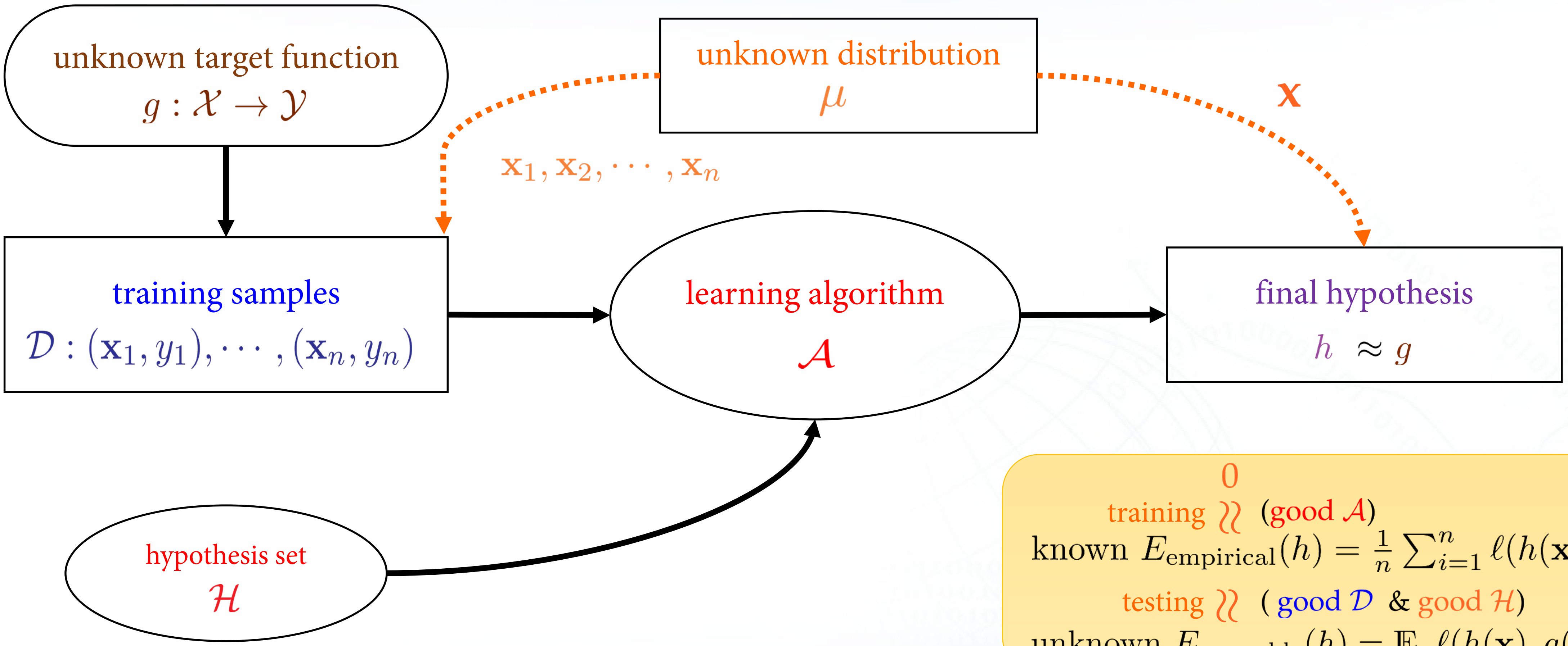


[E. Aïmeur, G. Brassard, S. Gambs, "Machine learning in a quantum world," *Advances in Artificial Intelligence*, 2006.]

Statistical Learning Framework

Different Output Space

- binary classification: $\mathcal{Y} = \{-1, +1\}$
- multiclass classification: $\mathcal{Y} = \{1, 2, \cdots, K\}$
- regression: $\mathcal{Y} = \mathbb{R}$
- unsupervised: $\mathcal{Y} = \emptyset$



0

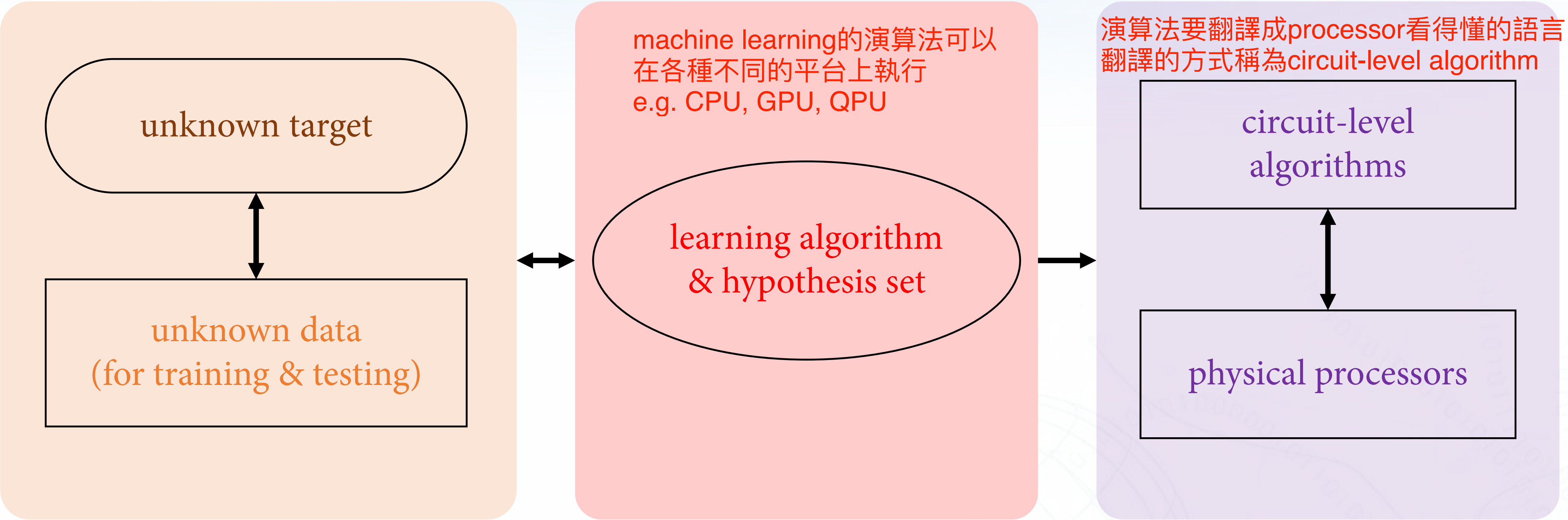
training $\wr\wr$ (good $\mathcal{A}$)

known $E_{\text{empirical}}(h) = \frac{1}{n}\sum_{i=1}^{n} \ell(h(\mathbf{x}_i), y_i)$

testing $\wr\wr$ (good $\mathcal{D}$ & good $\mathcal{H}$)

unknown $E_{\text{ensemble}}(h) = \mathbb{E}_{\mu}\ell(h(\mathbf{x}), g(\mathbf{x}))$

Categories from The Learning Framework (1/2)



- Data & Goal
  - classical or quantum
  - hybrid

    也可以data是classical
    target是quantum
- Learning Mechanism
  - classical ML algorithms
  - quantum variational circuits
- Physical device for implementation
  - classical CPU/GPU/TPU
  - quantum processors

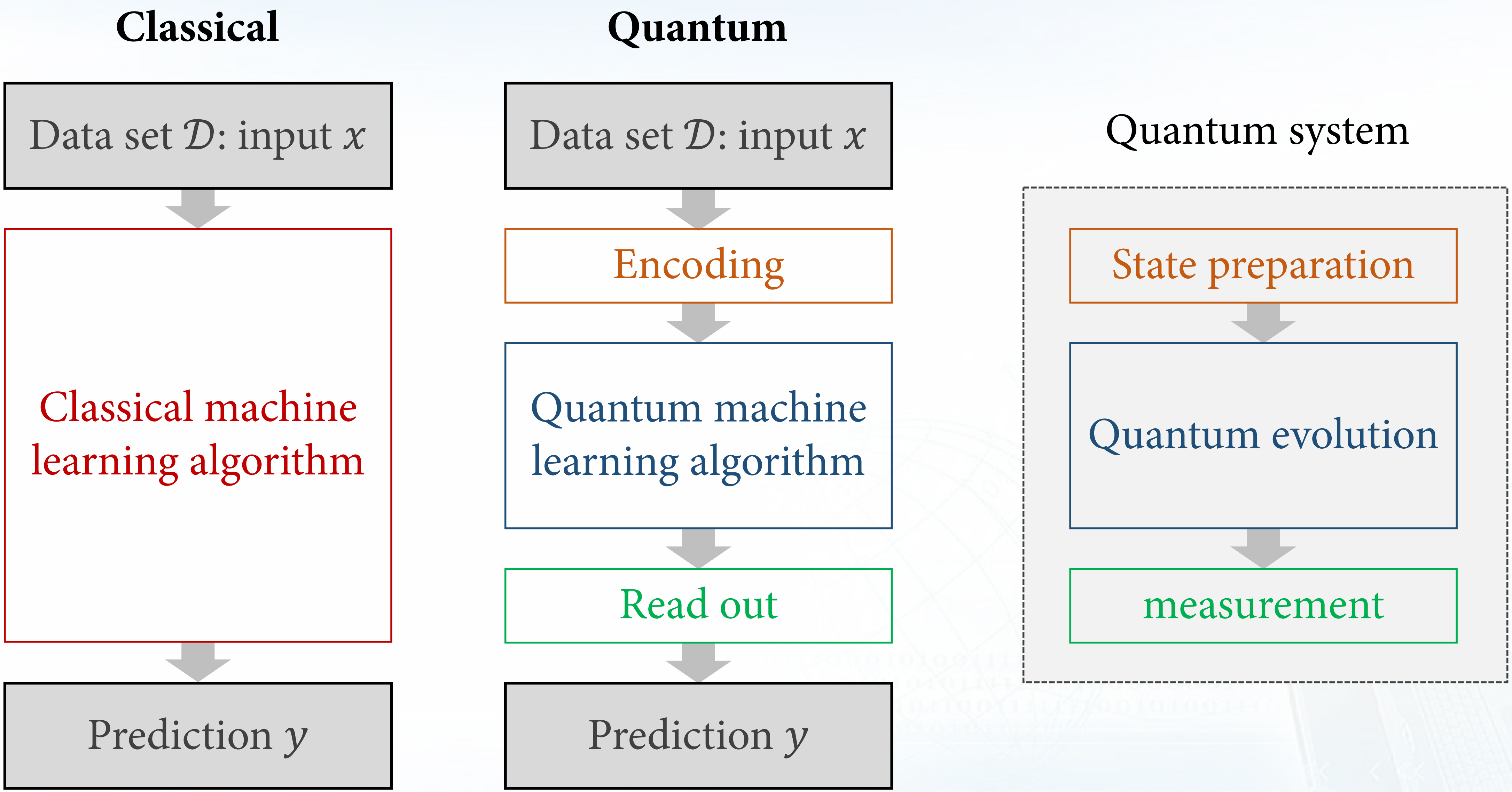
Categories from The Learning Framework (2/2)

	Data & goal	Learning mechanism	Physical device	Examples
	classical	classical	classical	classical ML
Today	classical	classical	quantum	speedup running time
Today	classical	quantum	classical	VQC & simulator
	classical	quantum	quantum	VQC & QPU
	quantum	classical	classical	tomography & simulator
	quantum	classical	quantum	tomography & QPU
	quantum	quantum	classical	quantum simulation VQC? & simulator
	quantum	quantum	quantum	quantum Boltzmann machine?

Part III – ML with Quantum Algorithmic Speeding-up

Data & goal	Learning mechanism	Physical device
classical	classical	quantum

Flowchart of Classical ML with Quantum Algorithm



Information Encoding

- To learn from classical data, we need to load data from classical memory into the quantum computer; this process is called *state preparation* in QML.
- In the training phase, we consider data set $\mathcal{D} = \{x_1, \ldots, x_m\}$ of N–dimensional real feature vectors.

不同的encoding方式，可以適用的algorithm方式不一樣

Encoding	Number of qubits	Runtime of state preparation	Input feature
Basis	N	$O(MN)$	Binary
Amplitude	$\log N$	$O(MN)/O(\log MN)$	Continuous
Qsample	N	$O(MN)/O(\log MN)$	Binary
Hamiltonian	$\log N$	$O(MN)/O(\log MN)$	Continuous

Basis Encoding (1/2)

- Assume each data is N-dimensional bit string, i.e. $x_m \in \{0,1\}^N$ for $x_m \in \mathcal{D}$:

$$x_m \mapsto |x_m\rangle \qquad \mathcal{D} \mapsto |\mathcal{D}\rangle := \frac{1}{\sqrt{M}} \sum_{m=1}^{M} |x_m\rangle$$

- For example, given $\mathcal{D} = \{x_1, x_2\} = \{0101, 1110\}$, then $|\mathcal{D}\rangle = \frac{1}{\sqrt{2}}(|0101\rangle + |1110\rangle)$.

$$|0101\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = (0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0)^T$$

$$|1110\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0)^T$$

$$\Rightarrow |\mathcal{D}\rangle = \left(0,0,0,0,0,1/\sqrt{2}\,,0,0,0,0,0,0,0,0,1/\sqrt{2}\,,0\right)^T$$

Basis Encoding (2/2)

- Preparation in time $O(MN)$ by Ventura–Martinez, and Trugenberger.
- Approach by the Quantum random access memory, in time $O(\log N)$.

$$\textbf{QRAM: } \frac{1}{\sqrt{M}} \sum_{m=1}^{M} |m\rangle|0\cdots 0\rangle \mapsto \frac{1}{\sqrt{M}} \sum_{m=1}^{M} |m\rangle|x_m\rangle$$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Quantum Random Access Memory

Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone

Phys. Rev. Lett. **100**, 160501 – Published 21 April 2008

- However, an efficient hardware implementation is still an open challenge.

Computing in Basis Encoding

- Suppose we have a Boolean logic gate $f: \{0,1\}^N \to \{0,1\}$ giving binary label to each data x_m. Then, this can be implemented by (universal) quantum Toffoli gates. This is called a *quantum oracle* $U_f: |x\rangle|0\rangle \mapsto |x\rangle|f(x)\rangle$.
- *Quantum parallelism*:

$$U_f: \frac{1}{\sqrt{M}} \sum_{m=1}^{M} |x_m\rangle|0 \cdots 0\rangle \mapsto \frac{1}{\sqrt{M}} \sum_{m=1}^{M} |x_m\rangle|f(x_m)\rangle$$

QRAM要怎麼去實踐是一個很大的問題

- To read-out the state of the qubits, we have to measure it (quantum tomography).
  → *Frequentist estimator* via multiple shots of measurements.
  最後要去重複測量多次，去看每個不同分量的振幅，當M很大時（一次處理夠多data時），測量的次數就要夠多次
- Caveats: (i) large amounts of qubits needed; (ii) realizing a quantum RAM.

Amplitude Encoding

- For simplicity, suppose each feature vector $x_m = (x_{m,1}, \ldots, x_{m,N})$ is normalized, i.e. $\sum_{i=1}^{N} x_{m,i}^2 = 1$. (This condition can be removed by non-expansive manipulations.)
- Then, we can prepare the quantum state $|x_m\rangle$ by using $n = \log N$ quantum bits:

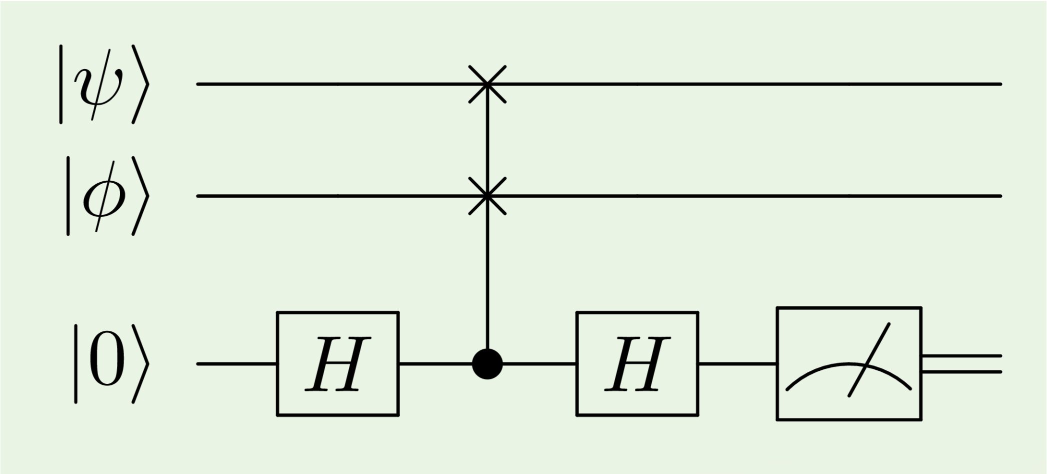
$$x_m \mapsto |x_m\rangle := \sum_{i=1}^{N} x_{m,i} |i\rangle \qquad \mathcal{D} \mapsto |\mathcal{D}\rangle := \frac{1}{\sqrt{M}} \sum_{m=1}^{M} \sum_{i=1}^{N} x_{m,i} |m\rangle |i\rangle$$

- State preparation in linear time: $O(MN)/O(\log MN)$.
- For sufficiently *uniform* vectors, the preparation may be efficiently done via qRAM.

Computing in Amplitude Encoding (1/2)

- Clustering: Assigning a vector $\vec{u} \in \mathbb{C}^N$ to two groups: $\{\vec{v}_m\}_{m=1}^M$ and $\{\vec{w}_m\}_{m=1}^M$.
- Classical approach: to compare the distance $\left|\vec{u} - \frac{1}{M}\sum_m \vec{v}_m\right|^2$.

  Takes time $O(\mathrm{poly}(MN))$. $\leftarrow$ involving evaluating inner product $\langle \vec{u}, \vec{v}_m \rangle$
- The *swap test* for computing the inner product.



The controlled-swap gate

$$\Pr(0) = \frac{1}{2} + \frac{1}{2}|\langle\psi|\phi\rangle|^2$$

Computing in Amplitude Encoding (2/2)

- The clustering can be done in $O(\log MN)$ in a quantum computer.

Exponential speed-up!

- Caveats:
  - If wanted to read-out the value of any specific entry x_i of $|x\rangle = \sum_{i=1}^{N} x_i |i\rangle$, then in general would require repeating the algorithm roughly $O(N)$ times, this then kills the exponential speed-up.

    $\leftarrow$ measurement outcome x_i with $\Pr(x_i^2)$
  - Once again, if preparing the amplitude encoding in $|x\rangle$ requires super-logarithmic time, then the speed-up unfortunately vanishes.

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Quantum machine learning: a classical perspective

Carlo Ciliberto, Mark Herbster, Alessandro Davide Ialongo, Massimiliano Pontil, Andrea Rocchetto, Simone Severini, Leonard Wossnig

Published 17 January 2018. DOI: 10.1098/rspa.2017.0551

nature physics

Published: 02 April 2015

Read the fine print

Scott Aaronson

Nature Physics **11**, 291–293 (2015) | Cite this article

Support Vector Machine with Quantum RAM

- The Support Vector Machine (SVM) can be formulated as a quadratic programming problem, which can be solved in time $O\big(\mathrm{poly}(M,N)\big)$, i.e.
  $\max_{\vec{\alpha}} \left\{ L(\vec{\alpha}) = \langle \vec{\alpha}, \vec{y} \rangle - \frac{1}{2} \langle \vec{\alpha}, K\vec{\alpha} \rangle : \sum_{j=1}^{M} \alpha_j = 0 \,, y_j \alpha_j \geq 0 \right\}, [K]_{ij} := k\big(\vec{x}_i, \vec{x}_j\big).$
- Efficient inner product evaluation via quantum algorithm takes $O(\mathrm{poly}(M) \log N)$.
- Using a non-sparse matrix exponentiation technique for approximating the kernel matrix inverse requires $O(\log MN)$.

← Exponential speed-up!

- Caveats:
  - Works with non-sparse kernel matrix, which has a small *condition number* (as in the HHL).
  - Again, the quantum RAM is indispensable.


PHYSICAL REVIEW LETTERS

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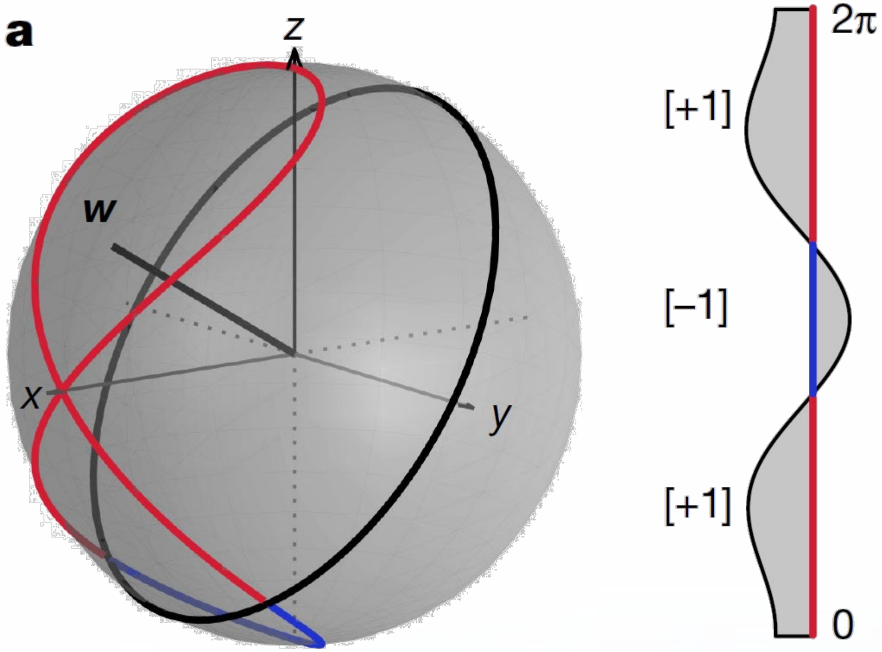
Quantum Support Vector Machine for Big Data Classification

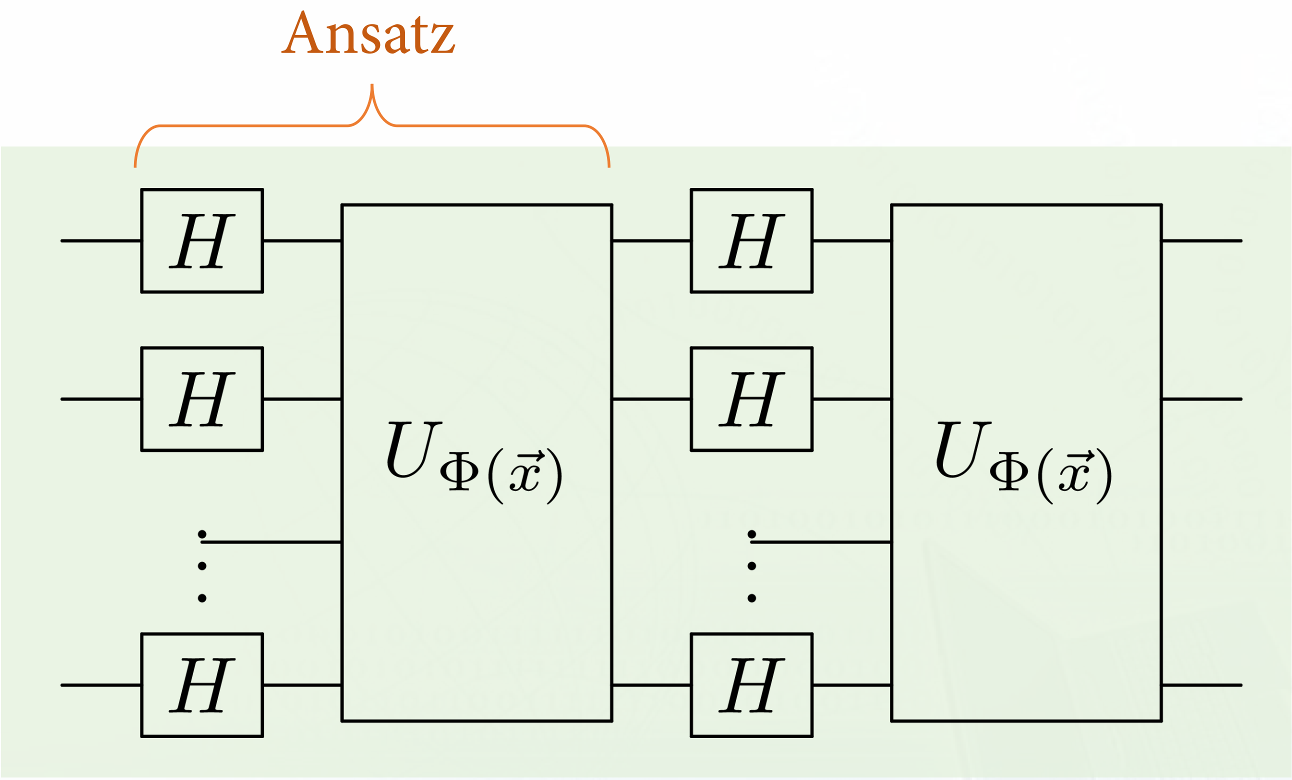
Patrick Rebentrost, Masoud Mohseni, and Seth Lloyd

Phys. Rev. Lett. **113**, 130503 – Published 25 September 2014

Support Vector Machine without Quantum RAM (1/3)

- *Rotation encoding*: $\Phi: \vec{x} \mapsto |\Phi(\vec{x})\rangle$ mapping the data to the *quantum feature space*.
- To implement the feature map, a rotation Z gate $U_{\Phi(\vec{x})}$ is used in the *ansatz* $U_{\Phi(\vec{x})}H$
  e.g. $U_{\Phi(x)} = \mathrm{e}^{-\mathrm{i}xZ}$ for 1-bit x; $U_{\Phi(x)} = \mathrm{e}^{\mathrm{i}(x_1 Z_1 + x_2 Z_2 + (\pi - x_1)(\pi - x_2) Z_1 Z_2)}$ for 2-bit $\vec{x}$.

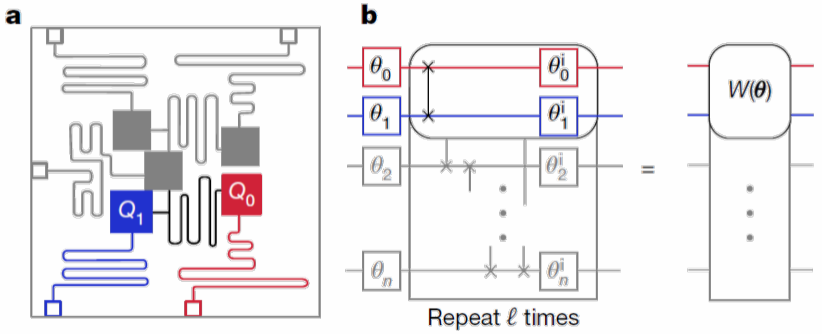


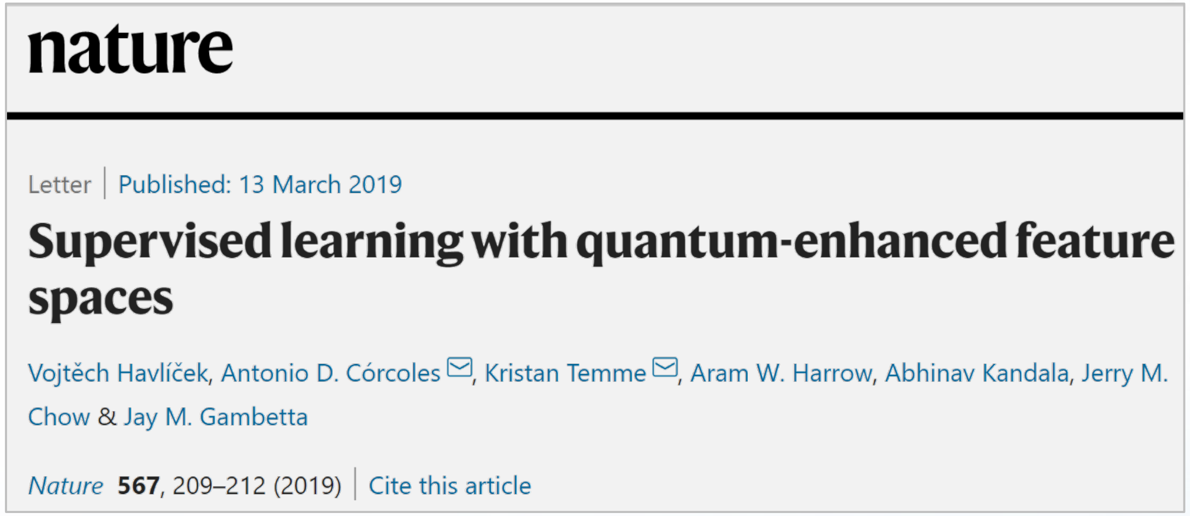


[Havlicek *et al.*, "Supervised learning with quantum enhanced feature spaces," *Nature*, 2018]

Support Vector Machine without Quantum RAM (2/3)

- With the quantum feature map, the classical kernel is $k(\vec{x}_i, \vec{x}_j) = \left|\langle \Phi(\vec{x}_i) | \Phi(\vec{x}_j) \rangle\right|^2$.
- Such a kernel (constructed from the above ansatz) is efficiently implementable via short-depth quantum circuits, but believed *classically hard*.





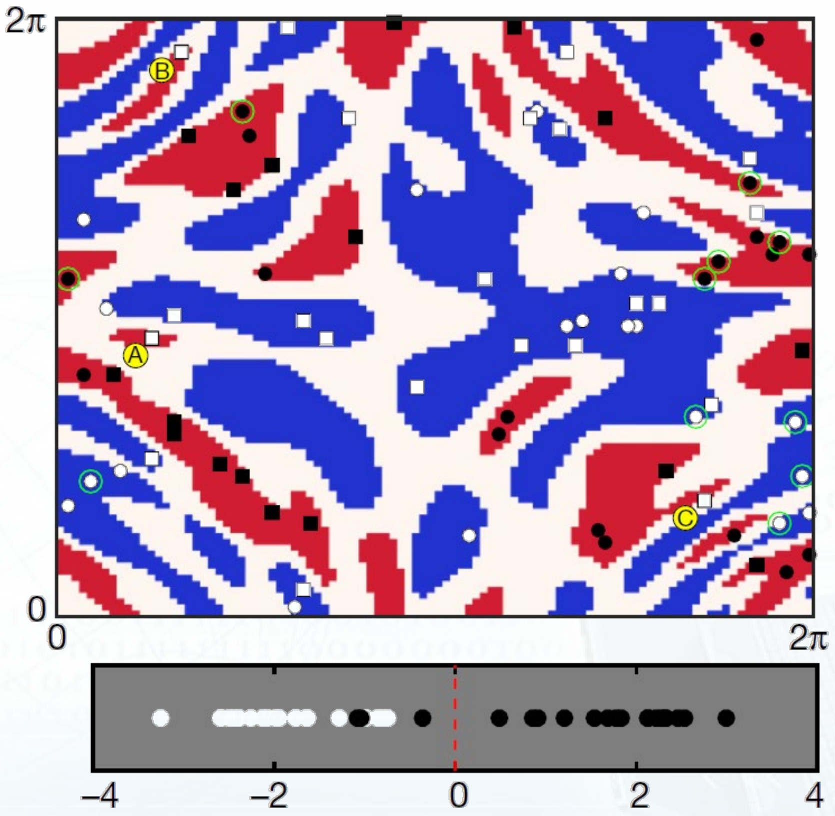
nature

Letter | Published: 13 March 2019

Supervised learning with quantum-enhanced feature spaces

Vojtěch Havlíček, Antonio D. Córcoles, Kristan Temme, Aram W. Harrow, Abhinav Kandala, Jerry M. Chow & Jay M. Gambetta

Nature **567**, 209–212 (2019) | Cite this article



Support Vector Machine without Quantum RAM (3/3)

qiskit.org/documentation/stubs/qiskit.aqua.algorithms.QSVM.html

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qiskit.aqua.algorithms.QSVM

CLASS `QSVM(feature_map, training_dataset=None, test_dataset=None, datapoints=None, multiclass_extension=None, lambda2=0.001, quantum_instance=None)` [SOURCE]

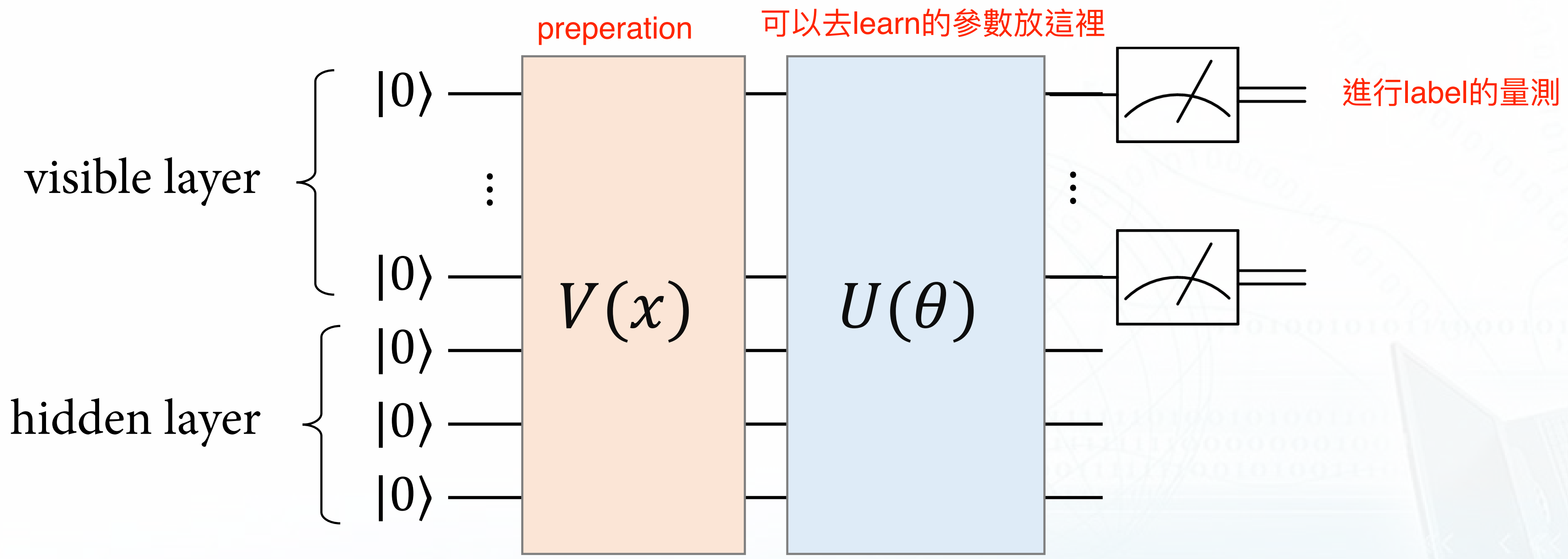
Quantum SVM algorithm.

Part IV – Learning with Quantum Machines

Data & goal	Learning mechanism	Physical device
classical	quantum	classical/quantum

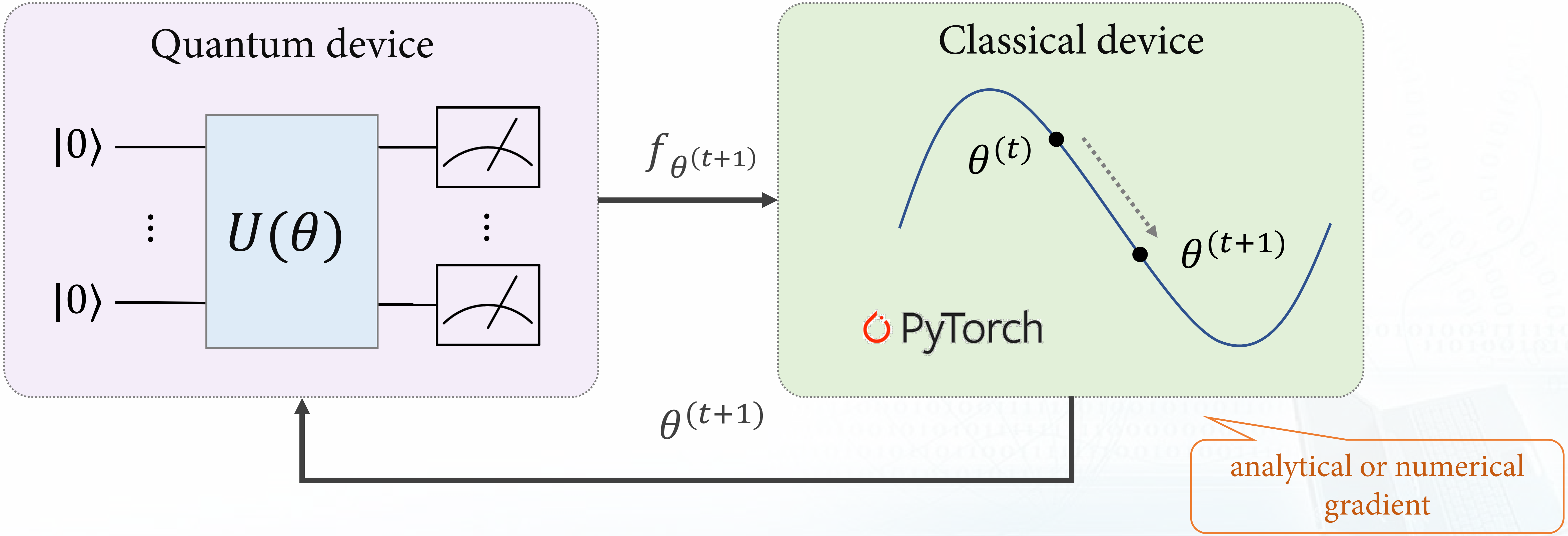
Variational (Parametric) Quantum Circuits

- The QNN is characterized by a unitary ansatz $U(\theta) \coloneqq \mathrm{e}^{-\mathrm{i}H_n\theta_n} \cdots \mathrm{e}^{-\mathrm{i}H_n\theta_n}$, where $\theta = \{\theta_1, \dots, \theta_n\}$ are the parameters we aim to learn and H_i are Hermitian operators.
- Goal: find a good assignment of parameters (e.g. via gradient descent) in terms of an objective function $f_\theta(x) \coloneqq \langle\psi_{\theta,x}|\Pi \otimes I|\psi_{\theta,x}\rangle$, where $|\psi_{\theta,x}\rangle \coloneqq U(\theta)V(x)|0\rangle$.



Hybrid Training for Variational Algorithms

- Idea: Use the quantum machine to compute the objective function $f\left(\theta^{(t)}\right)$, and then use a classical device to compute better circuit parameters $\theta^{(t+1)}$ with respect to the objective. Iterate the routine until the objective is optimized.



Optimization Methods (1/2)

- Quantum Approximate Optimization Algorithm (QAOA) for the MaxCut Problem.
  → heuristic, highly problem-specific, could be computationally expensive.
- Iterative derivative-free methods (e.g. the PSO).
- Numerical gradient (finite difference).
- Analytical gradient (**parameter shift rule**).
  - In general, $\partial U(\theta)$ might not be unitary.
  - By rotation gate, $\partial U(\theta)$ can be expressed as linear combinations of unitaries; hence computable by circuits.
  - Other gradient methods.

arXiv.org > quant-ph > arXiv:1411.4028

Quantum Physics

[Submitted on 14 Nov 2014]

A Quantum Approximate Optimization Algorithm

Edward Farhi, Jeffrey Goldstone, Sam Gutmann

arXiv.org > quant-ph > arXiv:1701.01450

Quantum Physics

[Submitted on 5 Jan 2017]

Practical optimization for hybrid quantum-classical algorithms

Gian Giacomo Guerreschi, Mikhail Smelyanskiy

Quantum
the open journal for quantum science
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Quantum Natural Gradient

James Stokes[1], Josh Izaac[2], Nathan Killoran[2], and Giuseppe Carleo[3]

[1]Center for Computational Quantum Physics and Center for Computational Mathematics, Flatiron Institute, New York, NY 10010 USA
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[3]Center for Computational Quantum Physics, Flatiron Institute, New York, NY 10010 USA

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Eprint: arXiv:1909.02108v3
Doi: https://doi.org/10.22331/q-2020-05-25-269
Citation: Quantum 4, 269 (2020).

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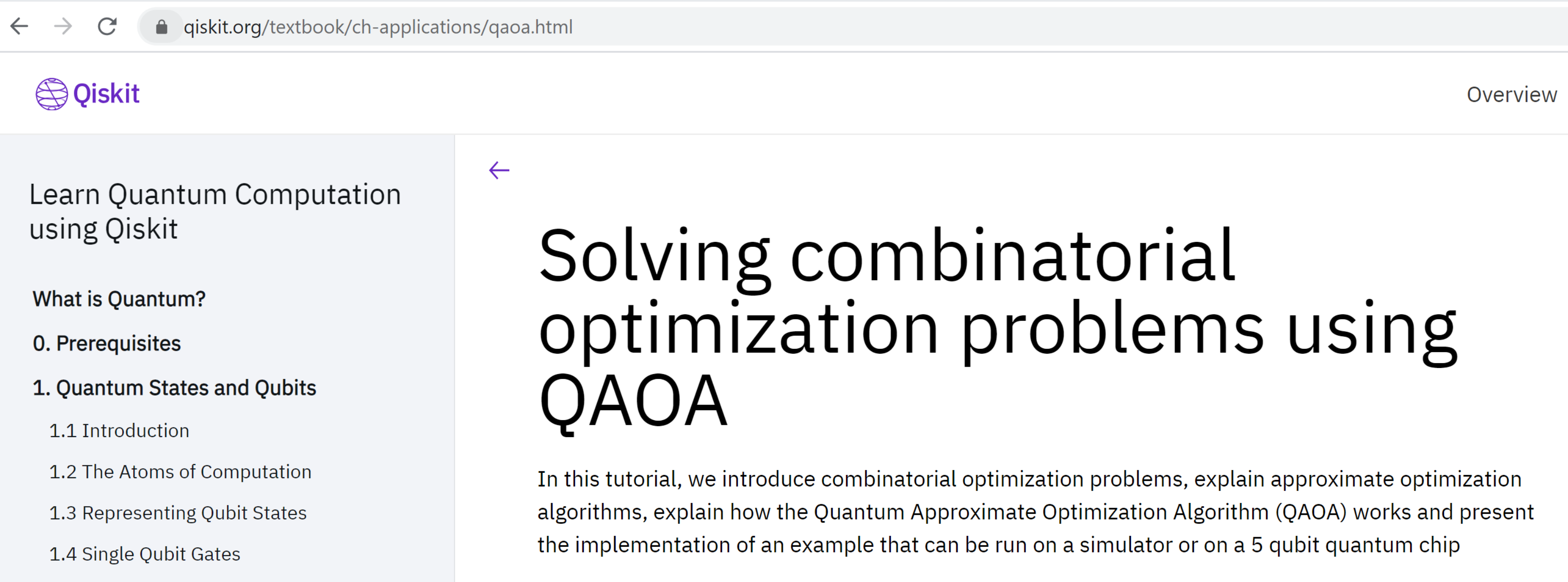
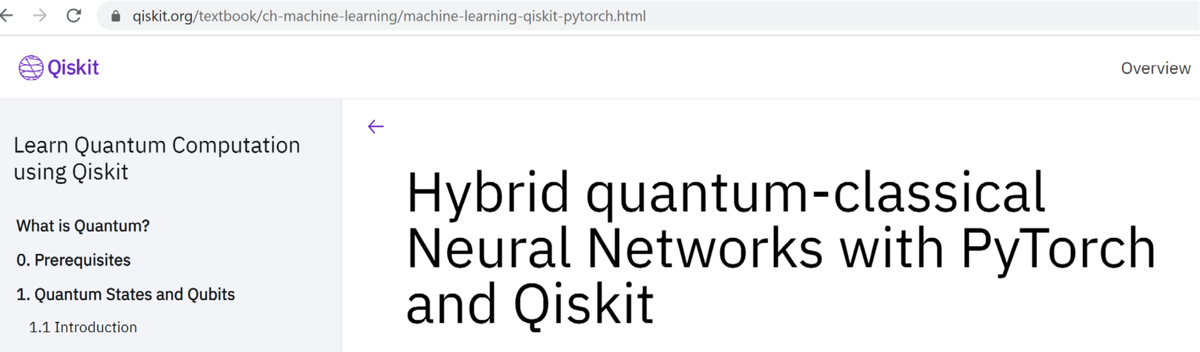
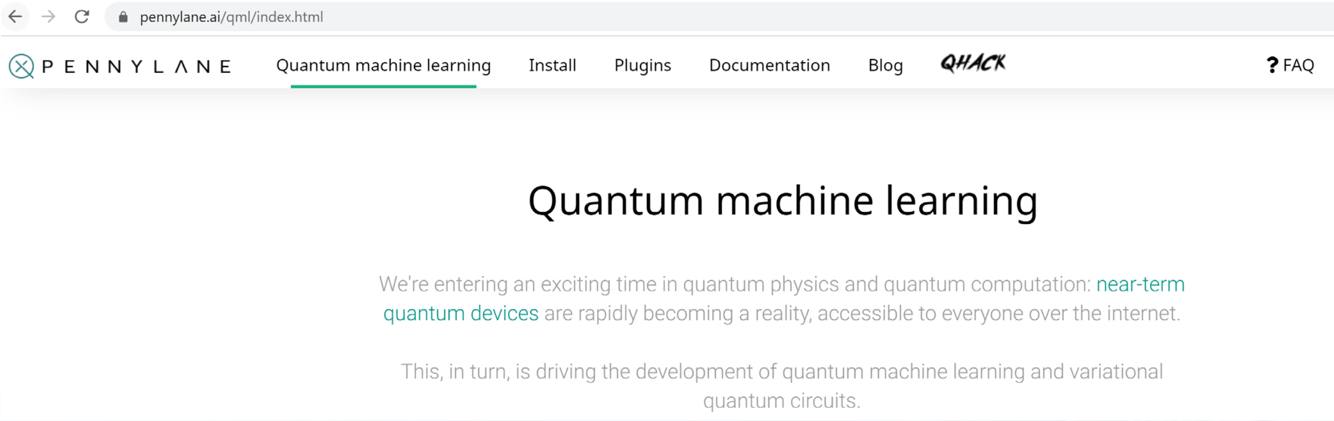
Quantum circuit learning

K. Mitarai, M. Negoro, M. Kitagawa, and K. Fujii
Phys. Rev. A **98**, 032309 – Published 10 September 2018

Optimization Methods (2/2)

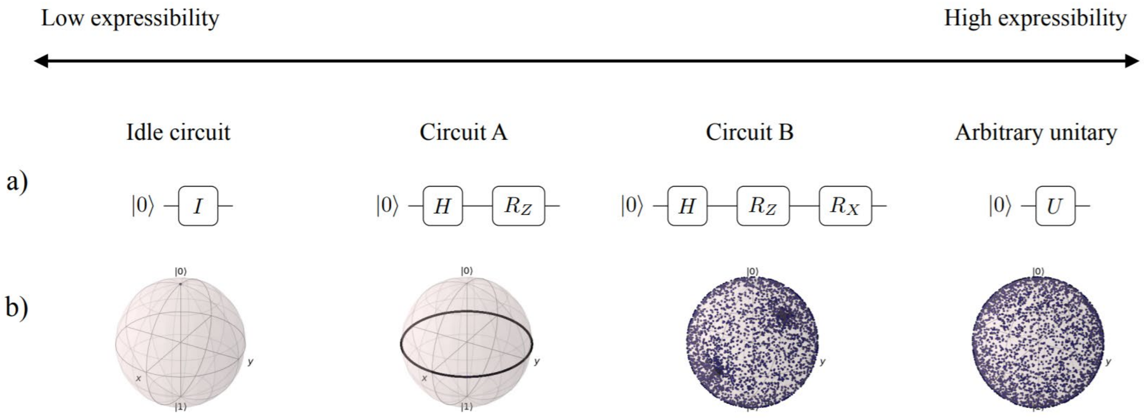


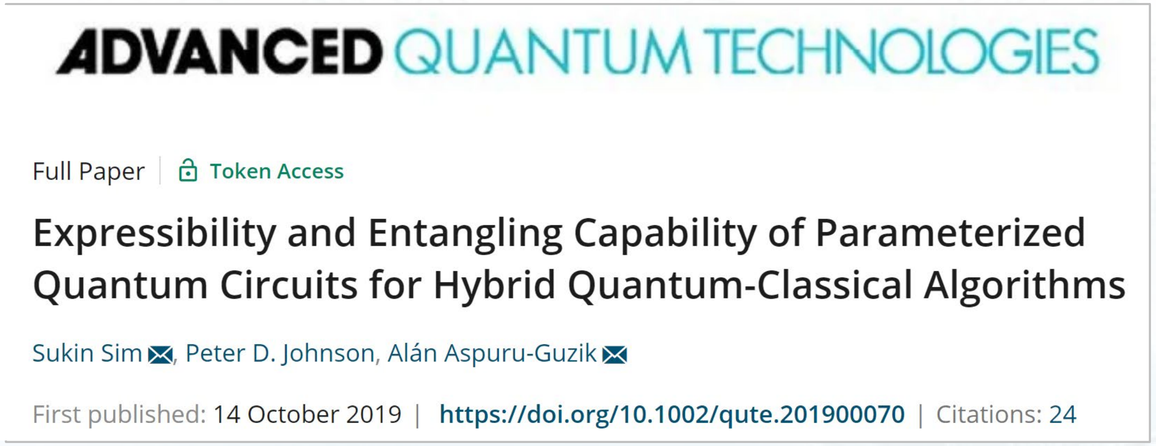




Various Ansatzs for QNNs (1/2)

- The dimension of the parameter space is typically mush smaller than the space of all unitary operators (e.g. $O(2^{2n})$ for n-qubits).
- *Goal*: designing an ansatz for the QNN such that it is rich enough to allow the parameterized circuit to approximate the solution with as few parameters as possible.
- It is desirable that the qubit-efficient circuits is hard for classical simulation – the objective function cannot be efficiently computed on a classical computer. (But it does not rule out the possibility that there are other efficient classical ways.)





ADVANCED QUANTUM TECHNOLOGIES

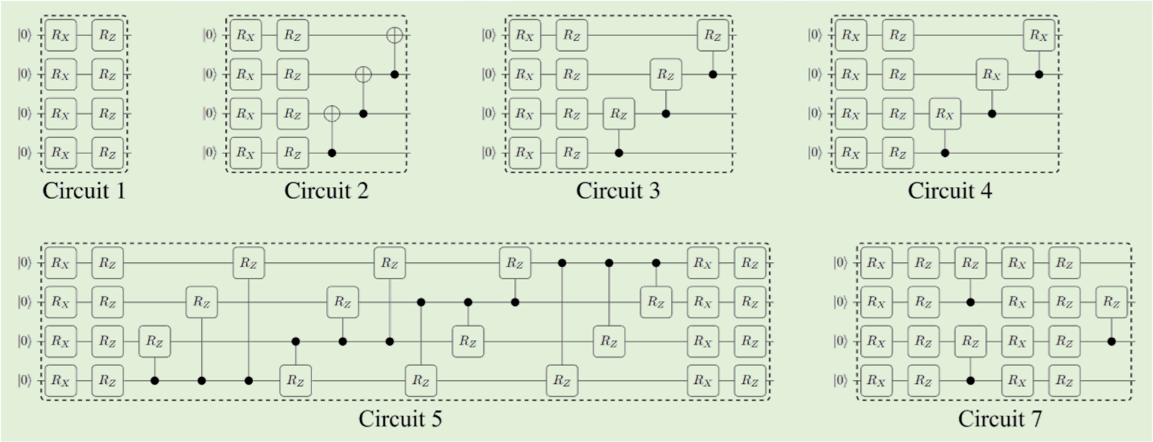
Full Paper | Token Access

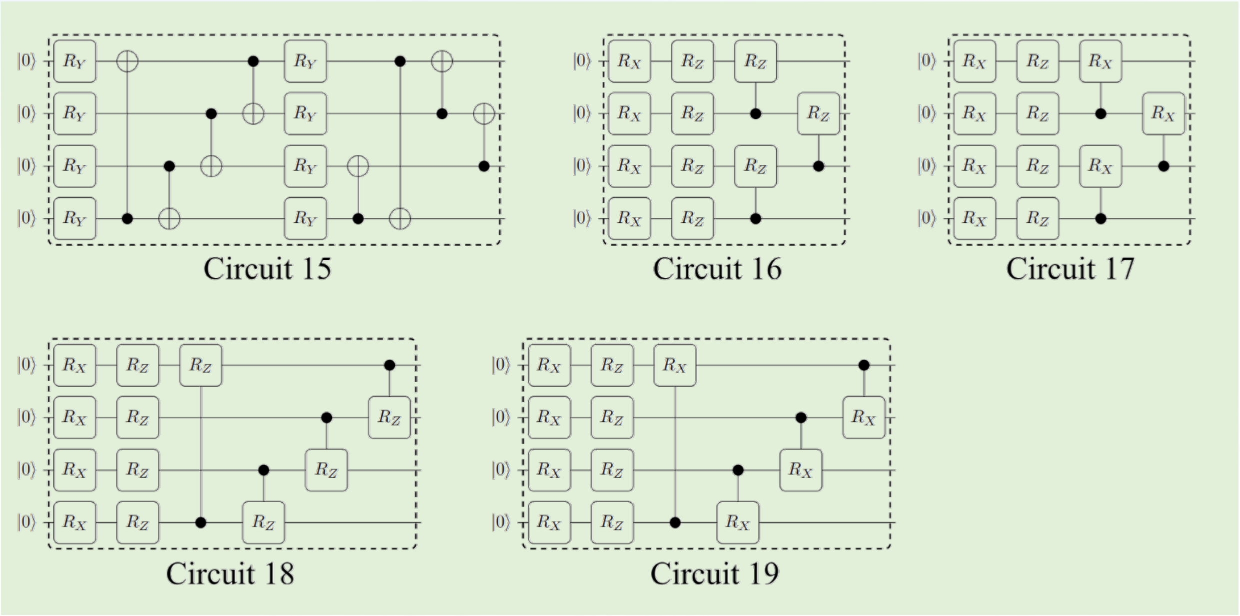
Expressibility and Entangling Capability of Parameterized Quantum Circuits for Hybrid Quantum-Classical Algorithms

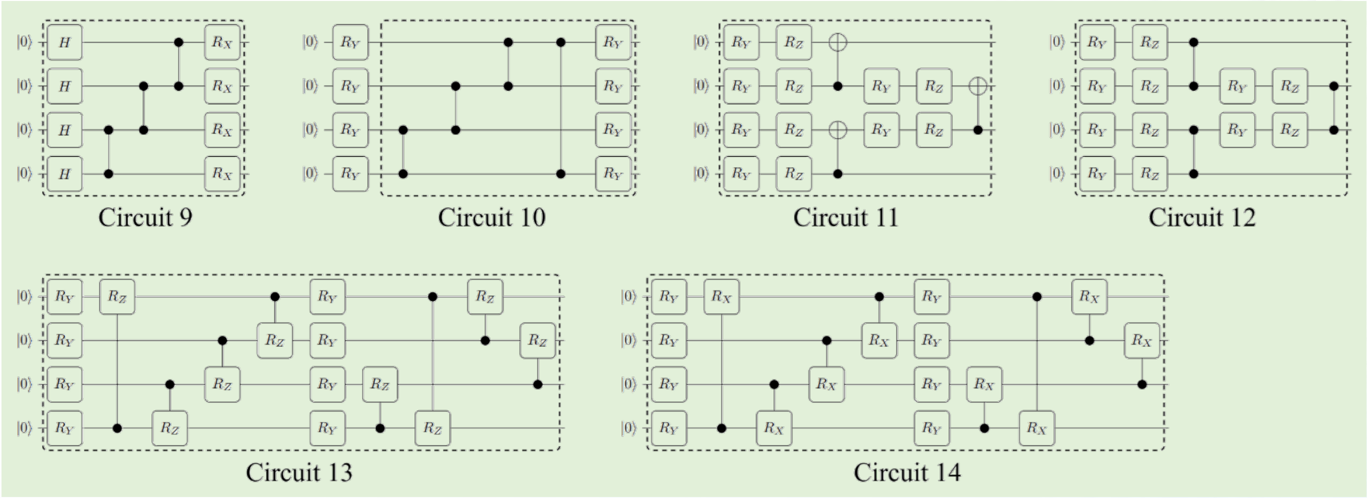
Sukin Sim, Peter D. Johnson, Alán Aspuru-Guzik

First published: 14 October 2019 | https://doi.org/10.1002/qute.201900070 | Citations: 24

Various Ansatzs for QNNs (2/2)

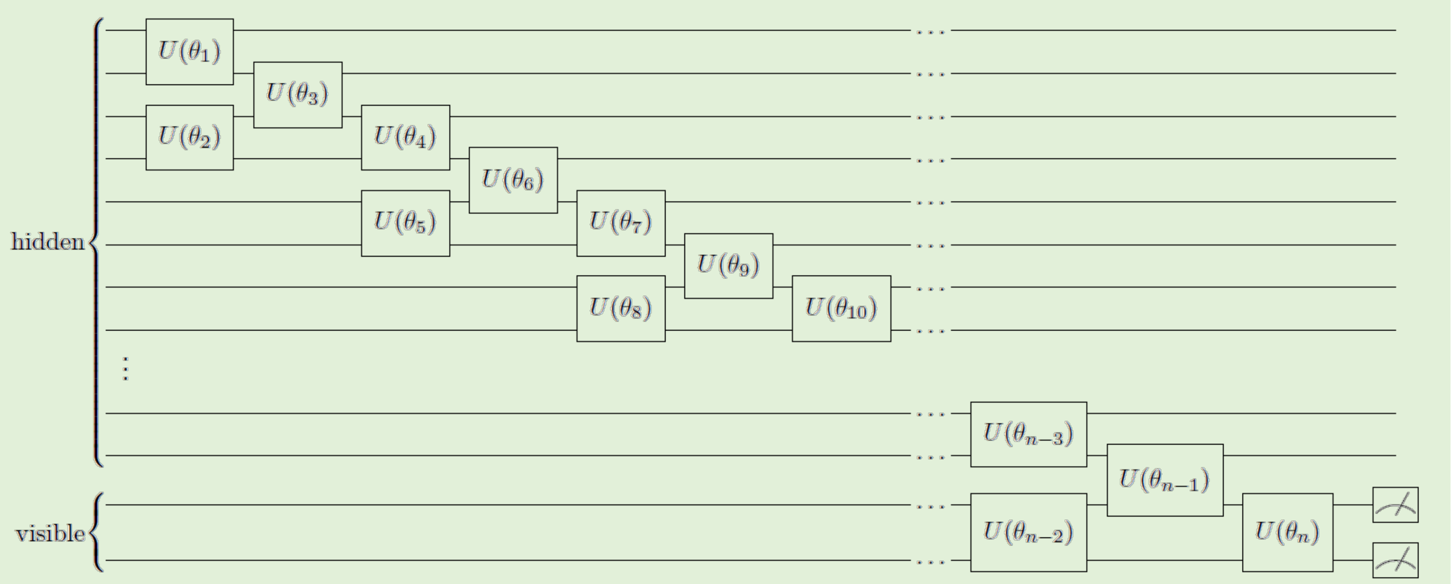






Caveat: Barren Plateaus

- On one hand, we desire *entanglement* between qubits to exploit full *quantumness*.
- On the other hand, excess of entanglement between visible and hidden layers may cause barren plateaus; hence both gradient descent and gradient methods fail.
- Any trade-off? How to quantify/characterize them?



arXiv.org > quant-ph > arXiv:2010.15968

Quantum Physics

[Submitted on 29 Oct 2020 (v1), last revised 10 Mar 2021 (this version, v2)]

Entanglement Induced Barren Plateaus

Carlos Ortiz Marrero, Mária Kieferová, Nathan Wiebe

Various Quantum Software Packages

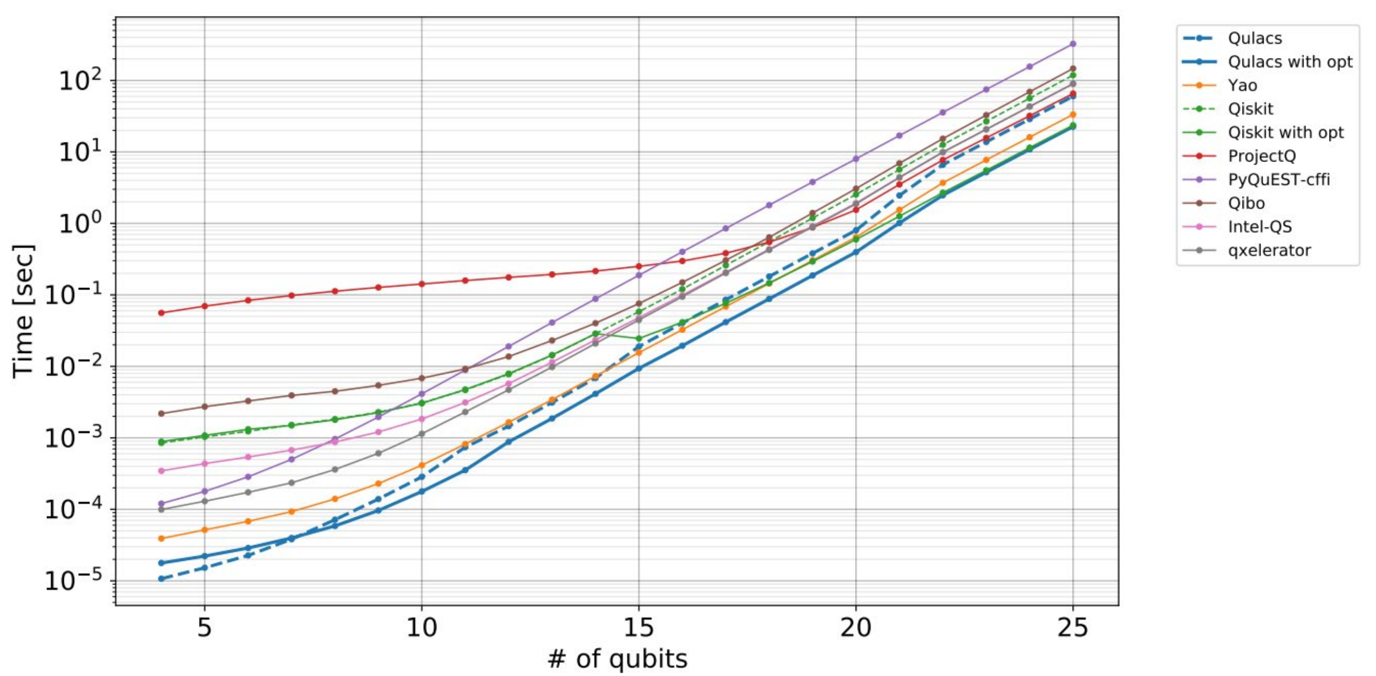
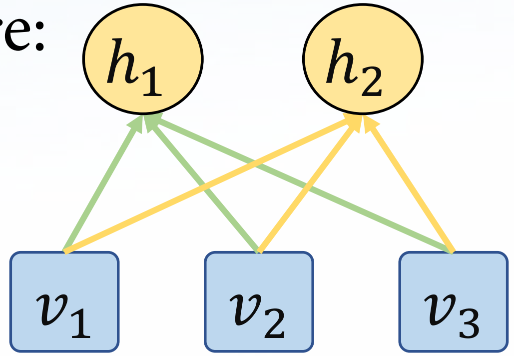


FIG. 8. Times for simulating random quantum circuits with a single thread using several libraries.

Epilogue

Conclusions and Outlooks

- There are other QML models that we haven't discussed in the lecture:
  - Quantum Boltzmann machines $\mathrm{e}^{-\sum_i \theta_i H_i}/Z(\theta)$.
  - Quantum example oracle $|0^n\rangle \mapsto \sum_x \sqrt{p(x)}|x\rangle|f(x)\rangle$.
  - Quantum PAC-learning model.



- QML is still at its very early stage of research. How to demonstrate quantum advantages, to provide theoretical evidence, and to identify the field it can apply? On the other hand, can one prove that the quantum circuit learning is just a lure?
- Conversely, existing ML techniques could help quantum information development.
- Even if QML does not help classical problems, it might help quantum problems.
- My ultimate goal: to invent a truly quantumly-meaning learning paradigm.

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