Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution

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July 11, 2019

Abstract

We investigate the short- and long-term effects of a shale gas boom in an economy where energy can be produced with coal, natural gas, or clean energy sources. In the short run, cheaper natural gas has counteracting effects on CO2 emissions: on the one hand it allows substitution away from coal which reduces CO2 emissions, ceteris paribus; on the other hand the shale gas boom may increase pollution as it increases the scale of aggregate production. We then empirically document another potentially important effect, namely that the shale boom was associated with a decline in innovation in green relative to fossil fuels-based electricity generation technologies. Introducing directed technical change dynamics in our model, we derive conditions under which a shale gas boom reduces emissions in the short-run but increases emissions in the long-run by inducing firms to direct innovation away from clean towards fossil fuels innovation. We further show the possibility of an infinitely delayed switch from fossil fuels to clean energy as a result of the boom. Finally, we present a quantitative version of the model calibrated to the U.S. economy, and analyze the implications of the shale boom for optimal climate policy.

1 Introduction

Technological progress in shale gas extraction (specifically the combination of hydraulic fracturing and horizontal drilling) has led to a boom in the natural gas industry in the United States. As shown in Figure 1, shale gas production in the United States increased more than threefold between January 2005 and January 2010, and it has increased close to 5 times more from January 2010 to December 2018. This shale gas boom has revolutionized energy production in the United States. Figure 2.A shows that natural gas started displacing coal at a much faster rate from 2009 so that today natural gas is more important than coal in electricity production. Panel.B shows the effect of the shale gas boom on carbon dioxide (CO_2) emissions in the electricity sector. Natural gas emits 40-50 percent less carbon per British thermal unit (Btu) of energy than coal, the carbon emission intensity of the electricity sector has declined by around a quarter in a few years. In fact, CO_2 emissions from the electricity sector peaked in 2007 and have kept declining since.[1]

Interestingly, at the time of the shale gas boom, innovation in clean technologies in electricity has collapsed. Figure 3 shows that patenting in renewables or more generally in green energy (which includes renewables, biofuels and nuclear) has collapsed with the shale gas boom, both as a share of total patents and as a ratio relative to patents in fossil fuel electricity generation.[2] For instance, patenting in renewables in the US has gone from representing 0.4% of total patents in 2009 to close to 0.1% in 2015. If the shale gas boom reduced emissions in the short-run at the cost of displacing innovation toward truly green technologies, then its overall effect on emissions and climate change is much less clear.

This paper investigates the short- and long-term effects of a shale gas boom in an economy with directed technical change where energy can be produced with coal, natural gas, or a clean source. In the first part of the paper we develop a simple framework to highlight the key trade-offs involved in allowing for improvements in the intermediate source of energy (specifically, in the extraction technology of natural gas). The final good is produced with an intermediate input and with energy. Energy is itself produced using coal, and/or natural gas, and/or a green source of energy (such as wind and solar power). Fossil fuel energy is produced using a combination of a resource - coal or natural gas - and an energy input (think of a power plant). The green energy is produced using only the intermediate input (representing wind mills, solar panels, etc.). Fossil resource use in energy production in turn generates pollution, with a higher pollution intensity for coal than for gas.

The model delivers the following insights. In the short run - given the current state of technology - there are two effects of a shale gas boom: a substitution effect and a scale effect. First, the substitution effect: a shale gas boom helps substitute natural gas energy for both coal energy (this reduces emission) and green energy (this increases emission). The overall substitution effect leads to a reduction in aggregate pollution when coal use is sufficiently more polluting than natural gas use. Second, the scale effect: a shale gas boom makes overall energy production cheaper which leads to an increase in overall energy consumption and therefore to an increase in aggregate pollution, ceteris paribus. Overall, in the short-run a shale gas boom will reduce pollution when the substitution effect is sufficiently negative and large compared to the scale effect. This in turn occurs when coal is sufficiently more polluting than natural

[1] All data here are taken from the EIA. The pattern of Figure 2.B also applies for the entire economy. See Appendix A.

[2] Details on data construction are given in section 2.

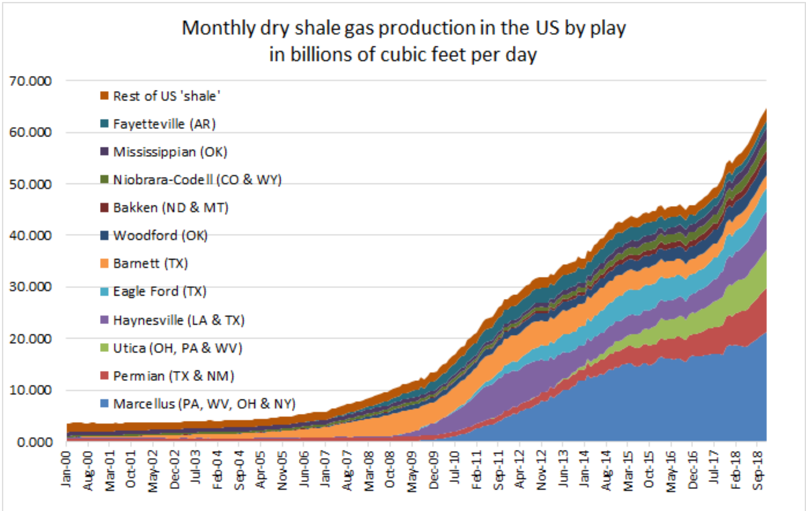

Figure 1: The shale gas boom

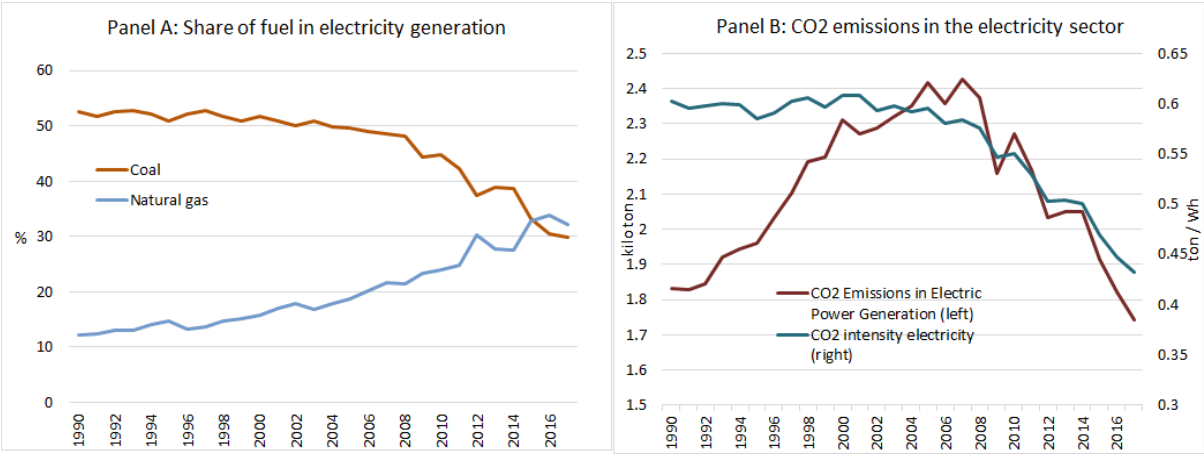

Figure 2: The shale gas boom in electricity

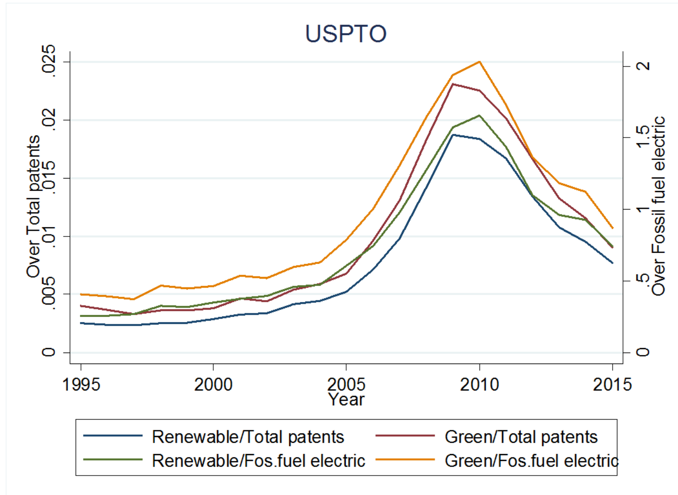

Figure 3: The collapse of clean innovation in electricity

gas at the margin.

In the long run, a shale gas boom tends to postpone the switch toward green innovation, i.e. towards innovating in the energy input production technology for clean energy. In fact, we provide sufficient conditions under which a shale gas boom results in the economy getting trapped in fossil fuels, which in turn results in a permanent increase in aggregate emissions whereas in the absence of the shale gas boom emissions would have converged to zero in the long run.

The theoretical analysis demonstrates the potential of an unmanaged shale boom to increase carbon dioxide emissions significantly in the long run. In order to gauge both the potential magnitudes and policy implications of these effects, the second part of the paper moves to a quantitative analysis.

To assess the short-run and long-run impacts of improving the shale extraction technology, we move to a quantitative analysis, which proceeds in three parts. We first calibrate an extended version of the static model to the electricity sector in the United States. We use data on electricity production and costs according to the energy source (coal, gas and the different green energies), aggregate data on output, capital, employment, etc., and estimates from the literature on the elasticity of substitution across fuels in order to estimate the initial technology levels. The benchmark results sugest that, for the United States, a reduction in the price of natural gas (akin to the "shale gas revolution") initially led to a decrease in CO_2 emissions (i.e. the intermediate technology has a positive environmental effect in the short-run). Second, we calibrate the dynamics of the model, setting innovation parameters to match our empirical estimates of the decline in the green innovation share observed in the aftermath of the shale boom. Simulating the economy forward, we find that an unmanaged shale gas revolution can lead to substantial increases in carbon emissions and climate damages in the medium- and long-run. Finally, we set up a social planner's problem to consider the implications of the shale boom for climate policy design. In the benchmark scenario, optimal management of the shale boom requires both increases in carbon taxes and higher subsidies for clean energy research.

This project belongs to the developing literature on macroeconomics and climate change. The first strand of that literature focuses on "integrated assessment models" (IAMs), which consist of dynamic models of the economy and the climate to evaluate the impact of climate change policies on welfare. This literature has been pioneered by Nordhaus (1980; 1991; 1994), whose seminal global DICE model is a benchmark in the literature and one of three models used by the U.S. government to value the social cost of carbon emissions (Interagency Working Group, 2011). The IAM literature now features a vast range of models and frameworks (Nordhaus, 2013) that consider rich details such as multi-regional and sectorally differentiated climate impacts (e.g., RICE, Nordhaus and Boyer, 2000; MERGE, Manne and Richels, 2005; PAGE, Hope, 2006; FUND, Anthoff and Tol, 2013; see also Stern, 2007, for another classic analysis). Within this broad literature, this project belongs to the sub-strand of IAMs that study interactions between climate policy and the macroeconomy in dynamic equilibrium. That is, while many IAMs take macroeconomic outcomes as given, a sub-strand of literature has emerged which focuses specifically on interactions between climate and the macroeconomy in general equilibrium (e.g., Golosov et al., 2014,; see review by Hassler, Krusell, and Smith, 2016). This literature building on DICE has analyzed issues ranging from climate tipping points (Lemoine and Traeger, 2014) and uncertainty (Cai, Judd, and Lontzek, 2018) to fiscal policy interactions (Barrage, 2019), and the role of intertemporal preferences (Gerlagh and Liski, 2018), among others.

A third strand focuses on endogenous technological change (ETC). Some early pioneers in this literature incorporated different representations of ETC into DICE (Goulder and Mathai, 2000; Nordhaus, 2002; Popp, 2004), often finding that the implications for optimal carbon pricing were modest. Gillingham et al. (2008) review competing modeling approaches, such as learning-by-doing, direct price-induced technical change, and research and development (R&D) knowledge stocks. Certain IAM groups now incorporate some of these mechanisms as baseline features (e.g., Bosetti et al., 2007).

Acemoglu et al. (2012, henceforth AABH) introduce a directed technical change model based on Acemoglu (2002) in order to study climate policy in this class of settings. In particular, AABH showed that, in a 2-sector model, market forces would naturally favor the sector which is already the more advanced. As a result, the social planner needs to redirect innovation from the dirty to the clean sector in order to reduce emissions in the long-run. Hémous (2016) pursued this type of analysis in a multi-country model. Both papers conduct numerical simulations but are essentially theoretical projects and do not carry out a comprehensive calibration exercise. Similarly to this paper, Lemoine (2018) extends AABH by modelling separately the resource used in energy production and the complementary inputs necessary to produce energy. He considers the implications of a more general (CES) specification for the technology used to combine resource and intermediate inputs, and studies both historical energy transitions and climate policy design in a calibrated model. Acemoglu et al. (2016) expands on these ideas and present a quantitative model of transition from dirty to clean technologies using firm-level data. Fried (2018) also introduces a carefully calibrated model featuring directed technical change using historical oil price shocks. None of these papers focus on the effects of technological breakthroughs in bridge technologies such as natural gas. Aghion et al. (2016) provide empirical evidence for directed technical change between clean and dirty technologies and path dependence in the car industry (see also Newell, Jaffe and Stavins, 1999, Popp, 2002, Calel and Dechezleprêtre, 2012 or Meng, 2019).

A fourth strand of related literature builds computational energy-economic or detailed electricity sector models which can be used to simulate the implications of changes in resource prices and policies. Leading examples include the U.S. Energy Information Administration's NEMS model, the MIT EPPA Model (Paltsev et al., 2005), and the RFF HAIKU Model, inter alia. Applications of these frameworks to study the impacts of the shale gas boom have found mixed results. Several studies project significant declines in short- and medium-run greenhouse gas emissions from the electricity sector due to fuel substitution away from coal (e.g., Burtraw et al., 2012; Venkatesh et al., 2012). Brown and Krupnick (2010) project higher overall CO_2 emissions in 2030 due to cheap and abundant natural gas (in the absence of climate policy), and that 2030 electricity generation will include higher natural gas consumption along with lower use of coal, nuclear, and renewables. A recent multi-model comparison study finds estimates of the CO_2 emissions impacts of the shale gas revolution ranging from -2% to +11% (McJeon et al., 2014). Our analysis seeks to add to this literature in two main dimensions. On the one hand, while these models are often extraordinarily detailed in their representations of the electricity sector, their complexity make them black-box and prevent from deriving general lessons. Our paper makes a step in that direction while retaining the ability to derive analytical results. On the other hand, though several models account for learning-by-doing effects (i.e., a reduction in capital costs of power plants of a new technology with increased past construction), they typically take progress in the technological frontier as exogenously given. Our analysis focuses on this channel and its implications for the greenhouse gas emissions impacts of shale gas in addition to the fuel switching and scale effects at play.

Finally, a recent empirical literature investigated the effects of the shale gas revolution. Most related to our analysis are studies which quantify the impacts of the shale boom on greenhouse gas emissions in electricity generation (e.g., Knittel, Metaxoglou, and Trinade, 2015; Holladay and LaRiviere, 2017; Cullen and Mansur, 2017; Fell and Kaffine, 2018; Linn and Muehlenbachs, 2018)), although these studies typically focus on short-run effects. For example, Cullen and Mansur's (2017) central estimates imply that a 67% decrease in gas prices as observed from 2008 to 2012 changes the CO_2 emission intensity of electricity generation by around -10% in the short-run. A broader empirical literature has investigated the impacts of hydraulic fracking on a range of outcomes including housing prices (Muehlenbachs et al., 2015), local economies (Alcott and Keniston, 2017; Feyer et al., 2017), and local/sectoral welfare (Hausman and Kellogg, 2015; Bartik et al., 2018). Our analysis relates to this rich literature by investigating a new channel through which the shale boom may affect long-run welfare, namely its effects on the direction of innovation and technological change.

2 Motivational evidence: the decline in green innovations

Our first contribution is to document a decline in green innovation in electricity generation since the shale gas boom. We use the World Patent Statistical Database (PATSTAT) from 2018 which contains the bibliographic information of patents from most patent offices in the world. A patent gives the right to use a technology exclusively in a given market and filing a patent in each country involves costs. Therefore, the location of the patent office at which a specific innovation is protected indicates how profitable a market is for the innovator. Patents are classified using different technological codes. We use the International Patent Classification (IPC) (and the Cooperative Patent Classification (CPC), which is a simple extension of the IPC). Each patent may contain several codes. To identify patents relevant to the generation of electricity using fossil fuels, we use Lanzi, Verdolini and Hascic (2011), who identified IPC codes corresponding to fossil fuel technologies for electricity generation.[3] To identify green innovations, we directly rely on the CPC classification, which contains a technological subclass Y02 for the reduction of greenhouse gases. Innovation in renewable electricity (geothermal, hydro, tidal, solar thermal, photovoltaic and wind) is contained in the main group Y02E10. To define green innovation, we add the main groups Y02E30 (which corresponds to nuclear energy) and Y02E50 (which corresponds to biofuels and fuel from waste).[4] We use patent applications from 1995 to 2015 to give a long time period before the shale gas boom and because the data for the most recent years are incomplete.

Figure 3 in the introduction plots patent applications at the USPTO (where the date of application is the date of first filing).[5] To give an idea of the order of magnitude, there are on average 3264 renewable electricity patents at the USPTO per year between 1995 and 2015. Figures 4 and 5 then plot the ratio of renewable to total or to fossil fuel electric patent applications, respectively, at USPTO, the Canadian, the French and the German patent offices. When inventors from multiple locations are listed, we count patents fractionally. The figures reveal that the pattern observed at the USPTO generalizes to other countries: while patents in renewables were quickly catching up and even overtook patents in fossil fuel electricity until around 2009-2010, the pattern has since sharply reversed. Moreover, the reversal occurred sooner for the United States and Canada, the two countries which first exploited shale gas.

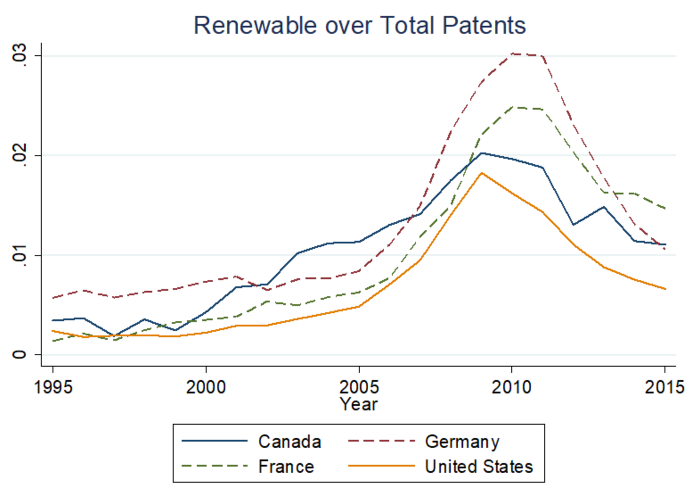

Figure 4: Cross-Country Comparison: Renewable over Total Patents

[3] We count as fossil fuel patents all patents with an IPC or CPC code in that list. The full list of codes is given in their Appendix A.1.

[4] Nuclear energy poses environmental and safety concern but is considered as "green" when it comes to greenhouse gases. Biofuels are used for transportation but also for electricity generations. We do not include innovation aiming at making fossil fuel electricity less polutting (Y02E20), at improving the efficiency of the grid (Y02E40), or at improving electricity storage (Y02E60), since those are not technologies which compete with fossil fuel technologies directly.

[5] We use patent applications instead of granted patents because we want to use recent years for which only few patents are already granted.

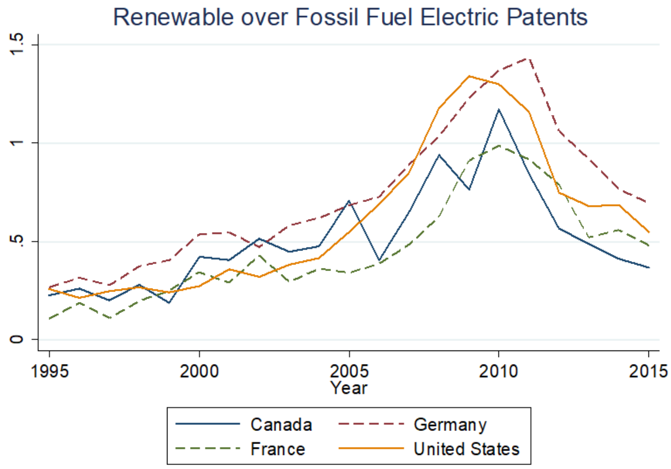

Figure 5: Cross-Country Comparison: Renewable over Fossil Fuel Electric Patents

One may wonder how the shale gas boom could have affected green patenting in Germany and France by local innovators though. There are at least three reasons: First, even domestic innovators may have their incentives shaped by foreign markets, so that German innovators may be less likely to undertake research in renewable energy because the US market is less profitable, leading to a decline in German patents. Second, if innovation in the US is redirected away from renewable energy, the relative amount of spillovers in renewables should decline. Third, innovation is forward looking and as the shale gas revolution unfolded in the US, there was an active political debate in Europe about the exploitation of shale gas. Indeed, natural gas prices in Europe have declined substantially in recent years, beginning to catch up to U.S. trends (see Appendix A).

Nevertheless, to further assess whether the United States and Canada have experienced a decline in green patenting relative to fossil fuel electric patents, we conduct a simple difference-in-difference exercise, where we compute the ratio of renewable or green patents to fossil fuel electric patents for the most important patent offices. We date the shale gas boom from 2009, following Holladay and LaRiviere (2017), and study its effects allowing a two-year lag. For a subset of countries, we are also able to assess whether and when the exploitation of shale gas is banned (see Appendix A for a list and data sources). As further controls, we include year- and country-fixed effects, and GDP per capita (from the OECD).

Table 1 reports the results both for all patents and when we restrict to patents by domestic inventors. The baseline regression in column (4) suggests that after the shale gas boom the ratio of renewable to fossil fuel electric patents declined by 0.4. The coefficient on the shale gas boom is always negative and of the same order of magnitude, but not always precisely estimated. The coefficient on the shale gas ban generally has the expected sign, but is not estimated with precision.

One notable potential confounder in this analysis is that other relevant policies may have changed differentially across countries during the 2009 recession, as some countries may have adopted green technology stimulus funding, and others may have cut research support as a part of austerity. We therefore obtain data on countries' public R&D expenditures for different

	Patent Office: all				Patent Office: domestic			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel A: Renewable / Fossil fuel electric							
Shale Gas Boom	-0.228	-0.349*	-0.242	-0.400**	-0.448*	-0.482	-0.488*	-0.595
	(0.15)	(0.17)	(0.15)	(0.17)	(0.22)	(0.38)	(0.24)	(0.41)
Ban		0.536		0.468		0.248		0.084
		(0.51)		(0.48)		(0.62)		(0.53)
N	719	379	719	379	675	346	675	346
	Panel B: Green / Fossil fuel electric							
Shale Gas Boom	-0.172	-0.318*	-0.186	-0.367**	-0.445*	-0.473	-0.487*	-0.584
	(0.14)	(0.17)	(0.14)	(0.17)	(0.23)	(0.41)	(0.24)	(0.44)
Ban		0.441		0.375		0.016		-0.145
		(0.46)		(0.42)		(0.51)		(0.42)
N	719	379	719	379	675	346	675	346
FEs (C, T)	Y	Y	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)			Y	Y			Y	Y

Note: Difference-in-difference regressions. Independent variables lagged 2 periods. The shale gas boom is dated from 2009. Standard errors are clustered at the country-level. Column (2), (4), (6), (8) include AU, CA, CH, CL, CN, CZ, DE, DK, ES, FR, GB, HU, IE, IL, JP, PL, PT, NL, NZ, US, the other columns also include TW, AT, BE, IS, EE, FI, GR, IT, KR, LV, LT, LU, MX, NO, SK, SI, SE, TR.

Table 1: Association of shale boom with green versus fossil electricity innovation

types of technologies (energy efficiency, fossil fuels, renewables) from the International Energy Agency (IEA) and consider these as additional controls. Table 2 presents the results, which are broadly similar in magnitudes. Estimation precision improves for the "all patents" sample, but declines somewhat for the specification restricted to domestic inventors.

In the Appendix we show results for another specification looking at EPO or USPTO patents and allocating patents to countries depending on the nationality of their inventors. The coefficients on the shale boom are again consistently negative, larger in absolute magnitude, and significantly different from zero, suggesting that foreign inventors also patented less in the US and Canada after the shale gas boom.

Finally, we use data on natural gas prices indexed from the International Energy Agency (IEA) from a group of 12 countries to conduct a panel analysis. We regress the log ratio of renewable or green patents over fossil fuel electric patents at the patent offices of the different countries on the log price index, country- and year-fixed effects, and GDP per capita, again with a 2 year lag. Table 3 shows a positive correlation between these ratio and natural gas prices, with a significant coefficient when considering all patents. In the Appendix, we again present an analogous specification looking at EPO and USPTO patents allocated based on

	Patent Office: all				Patent Office: domestic			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Panel A: Renewable / Fossil fuel electric				
Shale Gas Boom	-0.284**	-0.320*	-0.291**	-0.357**	-0.366	-0.474	-0.389	-0.552
	(0.11)	(0.16)	(0.12)	(0.17)	(0.24)	(0.32)	(0.26)	(0.34)
Ban		0.554		0.485		0.220		0.045
		(0.51)		(0.47)		(0.63)		(0.53)
N	719	379	719	379	675	346	675	346
FEs (C, T)	Y	Y	Y	Y	Y	Y	Y	Y
Control Public Exp.	Y	Y	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)			Y	Y			Y	Y

Note: Difference-in-difference regressions. Independent variables lagged 2 periods. The shale gas boom is dated from 2009. Standard errors are clustered at the country-level. Column (2), (4), (6), (8) include AU, CA, CH, CL, CN, CZ, DE, DK, ES, FR, GB, HU, IE, IL, JP, PL, PT, NL, NZ, US, the other columns also include TW, AT, BE, IS, EE, FI, GR, IT, KR, LV, LT, LU, MX, NO, SK, SI, SE, TR. Controls separately for public RnD expenditures related to fossil fuels, renewables, and energy efficiency.

Table 2: Association of shale boom with green versus fossil electricity innovation, with public R&D controls

inventors' nationalities, which, in this case, are less precisely estimated.

Overall, this section shows that innovation in the electricity sector has been sharply redirected away from renewable and green electricity at the time of the shale gas revolution in the US. We provide suggestive evidence that the shale gas revolution may have been a factor behind this trend. A more thorough empirical exercise is beyond the scope of this paper.

3 Theory: Short-run and long-run effects of the shale gas boom

We now develop a simple tractable model which we will use to develop our main theoretical intuitions. We first describe the model, then solve for the static equilibrium and look at the short-term effects of the shale gas revolution, before analyzing the dynamic equilibrium and the long-run effects.

3.1 Model description

Production technology. Time is discrete and the economy comprises a continuum of researchers and a continuum of identical individuals whose utility depends positively on consumption and negatively on aggregate pollution.[6] The final (consumption) good is produced

[6] In the quantitative analysis, we also consider climate change impacts on production possibilities within the United States and the possibility of disutility over other countries' climate damages.

	Patent Office: all			Patent Office: domestic		
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A: log (Renewable / Fossil fuel electric)					
log(Price Index)	0.334**	0.317***	0.321***	0.304	0.289	0.283
	(0.14)	(0.10)	(0.10)	(0.29)	(0.27)	(0.27)
N	213	213	213	155	155	155
	Panel B: log(Green / Fossil fuel electric)					
log(Price Index)	0.344**	0.331**	0.338***	0.333	0.312	0.311
	(0.13)	(0.11)	(0.10)	(0.27)	(0.24)	(0.24)
N	213	213	213	159	159	159
FEs (C, T)	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)		Y	Y		Y	Y
Control ln(Energy Consumption)			Y			Y

Note: Independent variable lagged 2 periods, star levels: * 0.10, ** 0.05, *** 0.010. Standard errors are clustered at the country-level. Includes: AU, BE, CA, FR, GR, JP, KR, MX, NZ, CH, GB, US.

Table 3: Association of natural gas prices with green versus fossil electricity innovation

according to:

$$Y_t = \left((1-\nu)\, Y_{Pt}^{\frac{\lambda-1}{\lambda}} + \nu \left(\widetilde{A}_{Et} E_t \right)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}},$$

where E_t is an energy composite, Y_{Pt} is a production input produced according to $Y_{Pt} = A_{Pt} L_{Pt}$ and A_{Pt} and $\widetilde{A}_{Et}$ represent respectively productivity in goods production and energy efficiency.

The energy composite is produced according to

$$E_t = \left(\kappa_c E_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_s E_{s,t}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

where each $E_{i,t}$ denotes a specific electricity type: $E_{c,t}, E_{s,t}$, and $E_{g,t}$ denote coal, natural gas and green energy, respectively. The $\kappa's$ are share parameters and ε is the elasticity of substitution between electricity types.[7]

The production of energy $i \in \{c, s, g\}$ is given by

$$E_{i,t} = \min\left(Q_{it}, R_{it}\right), \tag{1}$$

where Q_{it} represents an energy input and R_{it} is a resource use corresponding to that particular source of energy (e.g., coal, natural gas, and wind etc.). We then immediately get: $E_{it} = Q_{it} = R_{it}$. Green resource inputs are free but the extraction of natural gas and coal is costly.

[7] We note that the quantitative model will allow for differentiated elasticities of substitution between green and fossil energies, and adds capital to the production technology for intermediate production inputs.

Each resource i at date t involves a pollution intensity ξ_{it} so that: $P_{i,t} = \xi_i R_{i,t}$ with $\xi_c > \xi_s > 0 = \xi_g$. In other words, using the green resource does not pollute the atmosphere, and the use of natural gas pollutes the atmosphere but less than that of the coal resource. Aggregate pollution is then given by

$$P_t = \xi_g R_{i,t} + \xi_s R_{s,t} + \xi_c R_{c,t} = \xi_s R_{s,t} + \xi_c R_{c,t}. \tag{2}$$

We take the ξ's to remain fixed over time.

There are two dimensions of technical change: the first one is in the energy input production (which represents technological progress in power plants) and the other one is in the extraction technology.

Let us first formalize technological progress in the energy input production. We assume that the energy input i is produced at time t according to

$$Q_{it} = \exp\left(\int_0^1 \ln q_{ijt} dj\right) \tag{3}$$

where q_{ijt} is the intermediate input produced by local monopolist j in energy sector i. The production of this intermediate input occurs according to the linear technology:

$$q_{ijt} = A_{ijt} l_{ijt}^q, \tag{4}$$

where l_{ijt}^q denotes labor hired for the production of the intermediate and A_{ijt} denotes productivity in the production of intermediate j for energy sector i. Since coal and natural gas power plants share certain technologies and inputs (for instance steam turbines), we will assume that a share of the intermediates are common to both sectors.

Next we model technological progress in the extraction technology as follows. To produce one unit of resource, one needs to spend one unit of extraction input. Without loss of generality we denote the extraction input by R_{it} as well. We model the extraction technology exactly as the power plant technology. That is we write the production function as

$$R_{it} = \exp\left(\int_0^1 \ln r_{ijt} dj\right).$$

Each extraction input is produced according to

$$r_{ijt} = B_{ijt} l_{ijt}^r.$$

where l_{ijt}^r denotes labor hired for extraction input j and B_{ijt} denotes productivity in the production of intermediate j for extraction sector i. Coal and natural gas are in infinite supply so that the cost of the resource is simply equal to the cost of extraction.

Innovation. There is vertical innovation in A_{ijt} and B_{ijt} over time. The current monopolist has access to the latest vintage of the technology while its competitors have access to the previous vintage, which is γ times less productive.

$$A_{ijt} = \gamma A_{ijt-1}$$

if innovation occurs at date t in energy intermediate input ij and similarly

$$B_{ijt} = \gamma B_{ijt=1}$$

if innovation occurs at date t in energy extraction input ij.

We define the average productivities in energy production and resource extraction in sector i as:

$$\ln A_{it} = \int_0^1 \ln A_{ijt} dj \text{ and } \ln B_{it} = \int_0^1 \ln B_{ijt} dj. \tag{5}$$

We assume that there is a mass 1 of scientists who can decide to allocate their research efforts between the three energy input sectors (improving A_{ct}, A_{st} or A_{gt}) and the two resource extraction sectors (B_{ct} and B_{st}). In this theory section and for simplicity we consider that innovation in the extraction sector is exogenous. Each scientist has a probability of success given by $\eta_i s_{it}^{-\psi} A_{it}^{-\zeta_i}$, where η_i represents research productivity in sector i, ψ denotes a stepping-on-the toe externality and ζ_i represents decreasing returns to innovation. Finally, to reflect the fact that several inputs in coal and natural gas power plants are similar, we will assume that a share of innovations in fossil fuel technologies apply to both A_{ct} and A_{st}. Energy efficiency $\widetilde{A}_{Et}$ and productivity in the rest of the economy A_{Pt} evolve exogenously.

3.2 The short-run effects of the shale gas revolution

Static equilibrium. We now solve for the static equilibrium given productivity vectors A_{ijt}. For simplicity, we drop the subscript t in this subsection. The Leontief technology imposes that the price of electricity of type i is given by

$$p_i = p_i^q + p_i^r, \tag{6}$$

where p_i^q is the price of the energy input and p_i^r is the price of the resource extraction input (with $p_g^r = 0$ since extraction is free in green technologies). Maximization by the producer of the energy input i implies that

$$p_{ij}^q y_{ij} = p_i^q Q_i,$$

where p_{ij}^q is the price of the energy intermediate input ij. Following Bertrand competition, we immediately obtain:

$$p_{ij}^q = \frac{\gamma w}{A_{ij}} \text{ so that } l_{ij}^q = \frac{p_i^q Q_i}{\gamma w},$$

where w is the wage. This leads to equilibrium profits:

$$\pi_{ij}^q = \left(1 - \frac{1}{\gamma}\right) p_i^q Q_i.$$

Aggregating across intermediates, the price of energy input i obeys:

$$p_i^q = \frac{\gamma w}{A_i}. \tag{7}$$

Following the same logic in the resource extraction sector, we obtain that

$$p_{it}^r = \frac{\gamma w}{B_{ij}},\ l_{it}^r = \frac{p_i^r R_i}{\gamma w} \text{ and } \pi_{ij}^r = \left(1 - \frac{1}{\gamma}\right) p_i^r R_i$$

and the resource price is

$$p_i^r = \frac{\gamma w}{B_i}. \tag{8}$$

We denote by C_i the harmonic mean of A_i and B_i, which is the overall productivity in the production of electricity of type i, so that the price of electricity of type i is simply given by

$$p_i = \frac{\gamma w}{C_i} \text{ where } \frac{1}{C_i} \equiv \frac{1}{A_i} + \frac{1}{B_i}. \tag{9}$$

Then, profits maximization for the energy composite producer implies that the quantity of energy i is given by:

$$E_i = \kappa_i^{\varepsilon} \left(\frac{C_{it}}{C_{Et}} \right)^{\varepsilon} E_t, \tag{10}$$

where C_{Et} is the overall productivity of the energy sector:

$$C_{Et} \equiv \left(\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon} C_{st}^{\varepsilon-1} + \kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1} \right)^{\frac{1}{\varepsilon-1}}. \tag{11}$$

The price of the energy composite is given by

$$p_E = \frac{\gamma w}{C_E}, \tag{12}$$

and we have that total energy production is given by

$$E = C_E L_E, \tag{13}$$

where L_E is total labor hired by the energy sector.

The relative sizes of the energy sectors (in revenues) are given by

$$\Theta_i = \frac{p_i E_i}{p_E E} = \kappa_i^{\varepsilon} \left(\frac{C_i}{C_E} \right)^{\varepsilon-1}. \tag{14}$$

To solve for labor allocation, we look at the maximization problem of the final good producer. We assume that the intermediate input Y_P is also sold at a mark-up γ.[8] Then, taking the ratio of the two first order conditions with respect to E and L_P we get

$$L_E = \frac{\nu^{\lambda} \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1}}{\nu^{\lambda} \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1} + (1-\nu)^{\lambda-1} A_P^{\lambda-1}} L. \tag{15}$$

Define

$$\xi_E \equiv \xi_c \kappa_c^{\varepsilon} \left(\frac{C_c}{C_E} \right)^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} \left(\frac{C_s}{C_E} \right)^{\varepsilon} \tag{16}$$

as the average emission intensity of energy production. Then the equilibrium level of pollution is given by:

$$P = \xi_E E \tag{17}$$

The shale gas revolution. We can now derive conditions under which an increase in natural gas extraction productivity B_s increases or decreases contemporaneous aggregate pollution P. The increase in natural gas extraction productivity is akin to the shale gas revolution. In the subsequent sections on dynamics we look at the long-run consequences of the shale gas

[8] Implicitly, the intermediate input Y_P is also an aggregate of intermediates which are sold by monopolists engaged in Bertrand competition.

revolution. The model allows us to decompose the overall effect of an improvement in shale gas technology on pollution into a substitution effect and a scale effect.

We can write the effect of an increase in natural gas extraction technology as

$$\frac{\partial \ln P}{\partial \ln B_s} = \underbrace{\frac{\partial \ln \xi_E}{\partial \ln B_s}}_{\text{substitution effects}} + \underbrace{\frac{\partial \ln E}{\partial \ln B_s}}_{\text{scale effect}}, \tag{18}$$

the first term corresponds to substitution effects in energy production (a change in extraction technology will affect the average pollution intensity), and the second effect is the Jevons scale effect (a change in extraction technology will increase the scale of the energy sector).

$$\begin{aligned} \frac{\partial \ln \xi_E}{\partial \ln B_s} &= \varepsilon \frac{C_s}{B_s} \frac{\kappa_s^\varepsilon \left(-\xi_c \kappa_c^\varepsilon C_c^\varepsilon C_s^{\varepsilon-1} + \xi_s C_s^\varepsilon \left(\kappa_c^\varepsilon C_c^{\varepsilon-1} + \kappa_g^\varepsilon C_g^{\varepsilon-1}\right)\right)}{C_E^{\varepsilon-1}\left(\xi_c \kappa_c^\varepsilon C_c^\varepsilon + \xi_s \kappa_s^\varepsilon C_s^\varepsilon\right)} \\ &= \varepsilon \frac{C_s}{B_s}\left(\frac{P_s}{P} - \Theta_s\right), \end{aligned}$$

where P_s represents pollution generated by natural gas. Therefore the substitution effect is negative when the revenue share of natural gas Θ_s in the energy sector is larger than its emission share P_s/P. This holds whenever:

$$\frac{\xi_c C_c}{\xi_s C_s} > 1 + \left(\frac{\kappa_g}{\kappa_c}\right)^\varepsilon \left(\frac{C_g}{C_c}\right)^{\varepsilon-1}. \tag{19}$$

The terms $\xi_i C_i$ correspond to the pollution intensity per unit of input. If $\xi_c C_c > \xi_s C_s$, then natural gas is effectively cleaner than coal, so that the substitution effect away from coal reduces average emissions. This is not enough to ensure that the overall substitution effect is negative because the substitution effect away from green is positive. To ensure that average emissions decrease following the shale gas boom, it must be that the coal technologies are sufficiently dirtier than natural gas compared to the backwardness of green technologies relative to coal (the term $\left(\frac{\kappa_g}{\kappa_c}\right)^\varepsilon \left(\frac{C_g}{C_c}\right)^{\varepsilon-1}$).

The scale effect is given by

$$\begin{aligned} \frac{\partial \ln E}{\partial \ln B_s} &= \frac{C_s}{B_s}\left(\lambda + (1-\lambda)\frac{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1}}{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1} + (1-\nu)^{\lambda-1} A_P^{\lambda-1}}\right)\frac{\kappa_s^\varepsilon C_s^{\varepsilon-1}}{C_E^{\varepsilon-1}} \\ &= \frac{C_s}{B_s}\Theta_s\left(\lambda + (1-\lambda)\frac{L_E}{L}\right), \end{aligned}$$

so that, given Θ_s and the labor share L_E/L, the scale effect is smaller when the energy input is more complement to production input (that is for λ low). The lower is λ, the more labor gets reallocated to the production input following an increase in extraction technology B_s.

Thus the overall effect of the shale gas boom on pollution is given by:

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{C_s}{B_s}\left(\underbrace{\varepsilon\left(\frac{P_s}{P} - \Theta_s\right)}_{\text{substitution effect}} + \underbrace{\Theta_s\left(\lambda + (1-\lambda)\frac{L_E}{L}\right)}_{\text{scale effect}}\right).$$

Since $\varepsilon > 1$ and $\lambda < 1$, the substitution effect may dominate the scale effect. In fact, we obtain that $\partial \ln P / \partial \ln B_s < 0$ if and only if

$$\frac{\xi_s}{\xi_c} < \frac{\kappa_c^\varepsilon C_c^\varepsilon \left[\varepsilon - \left(\lambda + (1-\lambda)\frac{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1}}{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1} + (1-\nu)^{\lambda-1} A_P^{\lambda-1}}\right)\right]}{\left[\kappa_s^\varepsilon C_s^\varepsilon \left(\lambda + (1-\lambda)\frac{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1}}{\nu^\lambda \widetilde{A}_{Et}^{\lambda-1} C_E^{\lambda-1} + (1-\nu)^{\lambda-1} A_P^{\lambda-1}}\right) + \varepsilon C_s \left(\kappa_c^\varepsilon C_c^{\varepsilon-1} + \kappa_g^\varepsilon C_g^{\varepsilon-1}\right)\right]}. \tag{20}$$

We summarize our discussion in the following Proposition:

Proposition 1 *A shale gas boom (that is a one time increase in B_s) leads to a decrease in emissions in the short-run provided that the natural gas is sufficiently clean compared to coal (for ξ_s/ξ_c small enough that (20) is satisfied).*

3.3 The innovation effect of a shale gas boom

We now solve for the allocation of innovation in laissez-faire, and look at how this allocation is affected by a shale gas boom. A first finding is that under suitable assumptions a shale gas boom induces firms to direct innovation away from clean innovation towards shale gas innovation. A second finding is that there exists a non empty set of parameter values such that a shale gas boom delays the switch towards clean innovation, with the possibility of an infinitely delayed switch.

For simplicity, we assume here that all energy inputs are common to the natural gas and the coal power plants (but the productivities of the intermediates may differ by a constant). Moreover, we assume that there are no decreasing returns to scale: $\zeta_i = 0$. Therefore innovators must decide whether they want to innovate in the green energy input or in the fossil fuel energy input. If an innovator innovates in the green energy input, she obtains expected profits

$$\Pi_{gt} = \eta_g s_g^{-\psi} \left(1 - \frac{1}{\gamma}\right) p_g E_g \tag{21}$$

If she innovates in fossil fuel energy inputs, she obtains expected profits

$$\begin{aligned} \Pi_{ft} &= \eta_f s_f^{-\psi} \left(1 - \frac{1}{\gamma}\right) \left(p_c^y Y_c + p_s^y Y_s\right) \\ &= \eta_f s_f^{-\psi} \left(1 - \frac{1}{\gamma}\right) \left(\frac{C_c}{A_c} p_c E_c + \frac{C_s}{A_s} p_s E_s\right). \end{aligned} \tag{22}$$

In equilibrium, expected profits in green and fossil fuel innovations must be the same. Therefore, using (14), we get:

$$\frac{\Pi_{gt}}{\Pi_{ft}} = \frac{\eta_g s_{gt}^{-\psi} \kappa_g^\varepsilon C_{gt}^{\varepsilon-1}}{\eta_f s_{ft}^{-\psi} \left(\kappa_c^\varepsilon \frac{C_{ct}^\varepsilon}{A_{ct}} + \kappa_s^\varepsilon \frac{C_{st}^\varepsilon}{A_{st}}\right)} = 1. \tag{23}$$

As shown in Appendix 8.1, the allocation of innovation is uniquely determined by this equation provided that the following Assumption, which we maintain for the rest of the section, holds:

Assumption 1 $(\ln \gamma) \max\left(\eta_g, \eta_f\right) < \psi / \left((\varepsilon - 1)(1 - \psi)\right)$.

Proposition 2 *Under Assumption 1 the equilibrium allocation of innovation is unique.*

For γ or η_g and η_f small enough, we get:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} \approx \frac{\eta_g \kappa_g^{\varepsilon} C_{gt-1}^{\varepsilon-1}}{\eta_f \left(\frac{1}{A_{ct-1}} \kappa_c^{\varepsilon} \left(\frac{1}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{-\varepsilon} + \frac{1}{A_{st-1}} \kappa_d^{\varepsilon} \left(\frac{1}{A_{st-1}} + \frac{1}{B_{st}}\right)^{-\varepsilon}\right)}. \tag{24}$$

This expression highlights that, as in AABH, the innovation allocation features some form of path dependence. A higher green productivity at time $t-1$ $A_{g(t-1)} = C_{g(t-1)}$ increases the relative size of the green energy sector and favors innovation in that sector at time t. Similarly higher productivity levels in the fossil fuel technologies A_{ct-1} and A_{st-1} tend to favor innovation in fossil fuel technologies. Yet, this is only the case as long as $B_{ct}/A_{c(t-1)}$ and $B_{st}/A_{s(t-1)}$ are not too low: otherwise the return to innovation in fossil fuel technologies A_{ct} or A_{st} decreases as such innovation would have little effect on the overall productivities of coal and natural gas technologies C_{ct} or C_{st}.

In addition, the right-hand of (24) is decreasing in B_{st}, so that an increase in B_{st} leads to a reallocation of scientists away from the green technology. Intuitively, this is due to two effects: first, progress in extraction technology is complementary with progress in the associated energy input because the two are linked in a Leontief way; second, progress in extraction technology makes fossil fuel overall more advanced than green technologies, which induces further innovation in fossil fuels (since the two are substitute).

Therefore, a shale gas boom at time $t = 1$ (an increase in B_{s1}) reduces innovation in green technologies contemporaneously (s_{g1} decreases). This leads to higher levels of A_{c1} and A_{s1} and a lower level for the green technology C_{g1}, which, under certain assumptions,[9] will then further reduce innovation in clean technologies at $t = 2$. More precisely, in Appendix 8.1, we prove the following Proposition:

Proposition 3 *Assume that Assumption 1 holds. Then, a shale gas boom (an increase in B_{s1}) leads to reduced innovation in green technologies at $t = 1$ (i.e., to a decrease in s_{g1}). Moreover, if $\min\left(B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta_f} / (\varepsilon - 1)$ for all $t > 1$, then green innovation declines for all $t \geq 1$.*

The Proposition states sufficient conditions under which a shale gas boom at time 1 (which increases the extraction technology B_{s1}) shifts innovation toward fossil fuel technologies for all $t \geq 1$. If the shale gas boom shifts extraction technology B_{st} up for all $t \geq 1$, then its negative effects on green innovation cumulate over time. That is, green innovation at time t, s_{gt}, will decrease not only because B_{st} moves up, but also because there is path dependence in the direction of innovation and the shale gas boom will have reduced green innovation in preceding periods.

To describe the overall dynamics of pollution following a shale gas boom, we need to make assumptions on the dynamic path followed by extraction technologies and by the production technology A_{Pt}. We proceed to do so in the next sections under two polar cases.

[9] As noted below equation (24), an increase in $A_{c(t-1)}$ or $A_{s(t-1)}$ may have a negative effect on fossil fuel innovation if the extraction technologies are too much behind the power plant technologies. The assumption that $\min\left(B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta_f} / (\varepsilon - 1)$ ensures that this is not the case.

3.4 Long-run equilibrium with fast progress in extraction technologies

We first consider the case where the extraction technologies grow exogenously at a fast rate. Specifically, we assume that $\eta_c = \eta_f = \eta$ and that B_{ct} and B_{st} grow both grow at factor rate γ^η. We also assume that A_{Pt} grow exogenously at the same factor. These assumptions ensure that in the long-run, the economy will grow at rate γ^η in all possible scenarii. We define a shale gas boom as a one time increase in B_{s1}, such that the entire path B_{st} is moved up by a constant factor.

In this case, productivity in extraction technologies must grow weakly faster than in power plant technologies, so that if $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right) > \gamma^\eta/(\varepsilon-1)$ for $t=1$, then this is also true for $t>1$. Using Proposition 3, we get that a shale gas boom will lead to a reallocation of innovation away from green technologies for all $t \geq 1$.

Since the extraction technologies in fossil fuel must grow at least as fast as the power plant technologies, then the innovation allocation problem looks asymptotically similar to that in AABH and features path dependence. That is, the innovation allocation is asymptotically "bang-bang" with either all researchers working on green innovation or all researchers working on fossil fuel innovation (except for a knife-edge case). More specifically, there exists a threshold value $\overline{A}_{g0}\left(A_{s0}, A_{c0}, B_{s1}, B_{c1}\right)$, which depends on the initial productivities in fossil fuel technologies, such that if the initial green productivity is below that threshold, i.e. if $A_{g0} < \overline{A}_{g0}$, then the economy is on a "fossil-fuel" path where eventually all innovation occurs in fossil fuel technologies. The opposite occurs if the initial green technology is above the threshold, i.e. if $A_{g0} > \overline{A}_{g0}$.[10]

By favoring innovation in fossil fuel technologies, a shale gas boom moves the threshold value $\overline{A}_{g0}$ upwards. For intermediate values of the initial green productivity A_{g0}, the economy will move from a "green" path to a "fossil fuel" path. On a fossil fuel path emissions grow asymptotically at factor γ^η while on a green path emissions decrease toward 0.[11] Hence, switching from green innovation to fossil fuel innovation has dramatic consequences on the emission path. In the Appendix we prove:

Proposition 4 *Assume that Assumption 1 holds, that B_{ct} and B_{st} grow exogenously at factor γ^η and that $\min\left(B_{c1}/A_{c0},\, B_{s1}/A_{s0}\right) > \gamma^\eta/(\varepsilon-1)$. Then a shale gas boom at $t=1$ leads to a decrease in green innovations for all $t \geq 1$. For small enough initial green productivity A_{g0}, emissions will grow forever regardless of a shale gas boom and for large enough initial A_{g0} emissions will converge to zero in either case, but for an intermediate range of A_{g0}, emissions will grow forever following a shale gas boom while they converge to zero over time absent a shale gas boom.*

[10] It is not possible to derive analytical expressions for the threshold $\overline{A}_{g0}$. Yet, a sufficient condition to be on the fossil fuel path is $\kappa_g^\varepsilon A_{g0}^{\varepsilon-1} \leq \frac{\kappa_c^\varepsilon}{A_{c0}}\left(\frac{1}{A_{c0}} + \frac{\gamma^{\eta/2^{1-\psi}}}{B_{c1}}\right)^{-\varepsilon} + \frac{\kappa_s^\varepsilon}{A_{s0}}\left(\frac{1}{A_{s0}} + \frac{\gamma^{\eta/2^{1-\psi}}}{B_{s1}}\right)^{-\varepsilon}$, which ensures that $s_{g1} \leq 1/2$. Similarly $\kappa_g^\varepsilon A_{g0}^{\varepsilon-1} > \kappa_c^\varepsilon A_{c0}^{\varepsilon-1} + \kappa_s^\varepsilon A_{s0}^{\varepsilon-1}$ is a sufficient condition to ensure that the economy is on a green path (regardless of the value of the extraction technology).

[11] On a fossil fuel path, C_{st} and C_{ct} grow asymptotically at factor γ^η, which ensures that C_{Et} grow at the same rate and that ξ_E tends toward a constant (16). Since A_{Pt} also grow at the same rate, L_E approaches a positive constant (see 15) and pollution grows asymptotically at factor γ^η. On a green path, $C_{Et} \to \kappa_g^{\varepsilon/(\varepsilon-1)} C_{gt}$ and both asymptotically grow at factor γ^η, L_E still approaches a positive constant but the emission rate ξ_E now tends toward 0. Using (13), (16) and (17), we then get $P_t \to \left(\xi_c \kappa_c^\varepsilon C_{ct}^\varepsilon + \xi_s \kappa_s^\varepsilon C_{st}^\varepsilon\right) \kappa_g^{-\varepsilon} C_{gt}^{1-\varepsilon} L_E \to 0$,since C_{ct} and C_{st} do not grow exponentially.

This proposition deals with the extreme case in which the shale gas boom may lead to (much) higher emissions in the long-run. It is interesting to note that this may occur even for parameters such that the initial effect of the shale gas boom is to reduce emissions. Indeed, the latter occurs whenever coal is sufficiently polluting compared to natural gas, but how polluting the two technologies are, has no bearing on the allocation of innovation, which is driving the result here.

3.5 Long-run equilibrium with no progress in extraction technologies

We now consider the polar case where B_{st} and B_{ct} remain constant over time (except for a possible shift of the B_{st} schedule following a shale gas boom). We maintain the assumptions that $\eta_c = \eta_f$ and that A_{Pt} grows at by factor γ^{η}.

When B_{st} and B_{ct} remain constant, it eventually becomes unprofitable for firms to innovate in energy input production technologies for coal or natural gas. In other words, in this case innovation will always end up occurring on green energy production. Intuitively, since extraction technologies do not improve and since extraction and power plant inputs are complements, the share of income within the fossil fuel sector going to power plant inputs goes to 0, which discourages innovation in fossil fuel power plant technologies. Emissions will then always asymptote 0.

Yet, by making extraction technologies more productive, a shale gas boom still favors innovation in fossil fuel technologies, which will have the effect of delaying the switch toward green innovations. Formally we establish (proof in Appendix 8.3):[12]

Proposition 5 *Assume that Assumption 1 holds and that B_{ct} and B_{st} are constant over time. i) Then there exists a time t_{switch} such that for all $t > t_{switch}$, $s_{gt} > 1/2$ and eventually all innovations occurs in green technologies. If $\varepsilon \geq 2$, a shale gas boom at $t = 1$ delays the time t_{switch} and reduces green innovation until then. ii) In addition for $\varepsilon \geq 2$ and for $\ln \gamma$ small, emissions are increased in the long-run.*

Overall, for suitable parameter values, a shale gas boom reduces emissions in the short-run, but it delays or (in the case of the previous subsection) prevents the switch towards clean innovation. As a result, the shale gas boom permanently lowers the clean technology and increases the fossil fuel technologies. In the long-run, clean technologies are still the most developed, so that coal's main competitor is clean energy and the negative effect on emissions coming from the substitution of coal with natural gas is dominated. As a result, emissions increase in the long-run following the shale gas boom.[13]

4 Quantitative Model

We now calibrate our model to the US economy. Subsection 4.1 extends the basic model studied so far to incorporate empirical nuances such as local pollution abatement. Subsection 4.2 presents the calibration. We then present numerical estimates of the effects of the shale

[12] The assumption $\varepsilon \geq 2$ is a sufficient condition and plays a role similar to the assumption $\min\left(B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right) > \gamma^{\eta_f}/(\varepsilon - 1)$ in Proposition 3.

[13] To establish the result formally, we require that the innovation step, $\ln \gamma$, is small. This assumption is made for analytical tractability, numerical simulations suggest that the result is robust to removing it.

boom in the short-run in Subsection 4.3, and for the long-run in Subsetction 4.4. Finally, we present results for optimal climate policy implications in Subsection 4.5.

4.1 From the basic to the calibrated model

To bring the model to the data, we introduce three extensions. First, we allow for a different elasticity of substitution between green electricity and fossil fuel electricity on one hand, and within the fossil fuel electricity nest on the other. That is, we assume that the energy composite is produced according to:

$$E_t = \left(\left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

with $1 < \varepsilon \leq \sigma$, so that natural gas and coal electricity may be more substitutable with each other than with green electricity—reflecting for instance the fact that some green resources are intermittent. Second, we relax the assumption that labor is the only factor of production and introduce capital. We assume that the production input Y_P is produced according to

$$Y_{Pt} = A_{Pt} L_{Pt}^{\varphi} K_{Pt}^{1-\varphi},$$

where K_P is the capital used and φ is the labor share in the production sector. The intermediate energy and extraction inputs are produced according to

$$q_{ijt} = A_{ijt} \left(l_{ijt}^q \right)^{\phi} \left(k_{ijt}^q \right)^{1-\phi} \text{ and } r_{ijt} = B_{ijt} \left(l_{ijt}^r \right)^{\phi} \left(k_{ijt}^r \right)^{1-\phi},$$

where k_{ijt}^q and k_{ijt}^r denote the capital used in the production of intermediate energy and extraction inputs and ϕ is the labor share in the energy sector.

Third, we now consider local pollutants and abatement expenditures that may be mandated to control them. That is, through regulations such as the Clean Air Act and the Clean Water Act, U.S. power plants are already subject to a range of mostly command-and-control regulations that enforce expenditures to control local pollutant emissions, such as sulfur dioxides, nitrogen oxides, fly ash (for coal plants), etc. Formally, let P^l denote a composite of local pollution, let ξ_i^l denote the baseline local pollution intensity of energy resource i, and let $\underline{\mu_i}$ denote the mandated minimum level of pollution abatement, where:

$$P^l = (1 - \mu_i) \xi_i^l R_i$$

We stipulate the following abatement technology:

$$\mu_i = \left(\frac{1}{\theta_{i1}} \left(\frac{Q_i}{R_i} - 1 \right) \right)^{\frac{1}{\theta_{i2}}}$$

where θ_{i1} and $\theta_{i2} > 1$ are parameters of the abatement cost function. It is straightforward to show that, if the abatement mandate is binding, the profit-maximizing input choices of energy producer of type i satisfy:

$$R_i = E_i \text{ and } Q_i = \left(1 + \Lambda \left(\underline{\mu_i} \right) \right) R_i.$$

where $\Lambda(\underline{\mu_i})$ denotes the fraction of the capital-labor input devoted to local pollution abatement. The equilibrium price of energy type j then moreover satisfies:

$$p_i = p_i^q\left(1+\Lambda\left(\underline{\mu_i}\right)\right)+p_i^r \tag{25}$$

Solving for the equilibrium follows the same steps as in section 3. In the price of the energy inputs or the resource, the wage is replaced by the input bundle price,

$$c_{Et} = \left(\frac{w_t}{\phi}\right)^{\phi}\left(\frac{\rho_t}{1-\phi}\right)^{1-\phi},$$

where ρ_t is the interest rate. Therefore (7), (8) and (9) are replaced with

$$p_{it}^y = \frac{\gamma c_{Et}}{A_{it}},\ p_{it}^r = \frac{\gamma c_{Et}}{B_{it}} \text{ and } p_{it} = \frac{\gamma c_{Et}}{C_{it}}. \tag{26}$$

One further difference is that the mean productivity term C_{it} is now defined via:

$$C_{it} = \left[\frac{1}{B_{it}}+\frac{1+\Lambda_i\left(\underline{\mu_i}\right)}{A_{it}}\right]^{-1} \tag{27}$$

The price of the aggregate production input Y_{Pt} is now given by:

$$p_{Pt} = \frac{\gamma c_{Pt}}{A_{Pt}} \text{ where } c_{Pt} = \left(\frac{w_t}{\varphi}\right)^{\varphi}\left(\frac{\rho_t}{1-\varphi}\right)^{1-\varphi}.$$

The effective productivity of energy C_{Et} is now given by

$$C_{Et} \equiv \left(C_{ft}^{\varepsilon-1}+\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} \text{ with } C_{ft} \equiv \left(\kappa_c^{\sigma}C_{ct}^{\sigma-1}+\kappa_s^{\sigma}C_{st}^{\sigma-1}\right)^{\frac{1}{\sigma-1}}. \tag{28}$$

where C_{ft} is the effective productivity of the fossil fuel bundle: $E_f \equiv \left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}}+\kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. We then obtain that the price of electricity in laissez-faire obeys

$$p_{Et} = \frac{\gamma c_{Et}}{C_{Et}}, \tag{29}$$

and that the quantity of energy composite produced is given by

$$E_t = C_{Et}L_{Et}^{\phi}K_{Et}^{1-\phi}, \tag{30}$$

where K_{Et} and L_{Et} are aggregate quantity of capital and labor involved in energy production.

Propositions 1 and Proposition 3 can be extended to this set-up under certain assumptions (see Appendix 8.4 for formal statements and proofs). In particular, we can still decompose the short-run effect of an increase in extraction technology between a substitution effect and a scale effect as in (18), and we can decompose the substitution effect between substitution away from green technologies and within fossil fuels:

$$\begin{aligned}\frac{\partial\ln\xi_{Et}}{\partial\ln(B_{st})} &= \frac{\theta_{sft}C_{st}}{B_{st}}\left[\underbrace{\varepsilon\Theta_{gt}}_{\text{substitution away from green}} - \underbrace{\sigma\frac{P_{ct}}{P_t}\left(1-\frac{\xi_s C_{st}}{\xi_c C_{ct}}\right)}_{\text{substitution within fossil fuels}}\right] \\ &= \frac{C_{st}}{B_{st}}\left[\sigma\frac{P_{st}}{P_t}-(\sigma-\varepsilon)\,\theta_{sft}-\varepsilon\Theta_{st}\right]\end{aligned} \tag{31}$$

where Θ_{gt} is the revenue share of the green industry in the energy sector, θ_{sft} is the revenue share of the gas industry within the fossil fuel energy subsector, and $P_{c,t}$ denotes emissions from coal energy. The substitution effect away from green is always positive if $\varepsilon > 0$ (substituting away from green always increases pollution). As before, the substitution effect within fossil fuels is negative as long as the pollution intensity in terms of input units is larger for coal electricity than for natural gas ($\xi_{s,t}C_{st} < \xi_{c,t}C_{ct}$). In this case, a shale gas boom is more likely to lead to a short-run reduction in pollution emissions when: the share of emissions caused by coal is large, the elasticity of substitution between fossil fuels is large relative to that with green electricity ($\sigma > \varepsilon$), green technologies are relatively less advanced (Θ_{gt} is low) and the scale effect is small.

4.2 Calibration

4.2.1 Base Year Model

We first describe the static calibration of the model to the pre-shale base year (2009). This calibration proceeds in three steps. First, we obtain measures of electricity generation costs (25) and other key moments from the data. Second, we select a number of parameters directly based on prior literature and data sources. Third, we simultaneously solve for the remaining parameters and initial equilibrium outcomes to match the data and other moments conditional on the model.

First, in order to quantify electricity generation costs and components by energy type (25), we collect plant- and generator-level data on electricity generation, fuel inputs and costs, operation and management (O&M) expenditures, plant capital, and abatement expenditures as outlined in Table 4.

Item	Data Source(s)
Plant O&M expenditures, capital, output $\rightarrow$ Annualized KL-costs/MWh $p^q_{it}(1+\overline{\Lambda_i})$	Federal Energy Regulatory Commission (FERC) "Form 1" Filings
Local abatement investment, O&M, output $\rightarrow$ Abatement costs/MWh $p^q_{it}\overline{\Lambda_i}$	Energy Information Administration (EIA) Form 767 (1985-2005), Form 923 (2007+)
Fuel resource costs/MWh p^r_{it}	FERC Form 423, EIA Forms 423, 923

Table 4: Cost Calibration Data Sources

One point to note is that we take advantage of the long history of available data on abatement capital investments (EIA Forms 767, 923) to construct abatement capital stock estimates using the perpetual inventory method. Before proceeding, we note that there are caveats associated with the use of these data in our analysis. The first is that the FERC data cover only investor-owned major utilities meeting certain generation thresholds. Consequently, thre 'green' energy generators represented in FERC tilt heavily towards nuclear plants. In order to improve our measure of green generation costs, we thus further consult levelized cost ("LCOE") estimates for renewable sources from NREL (Tidball et al., 2012) to compute the generation-weighted average capital-labor cost for green technologies in our base period.[14,15] More broadly, we have also performed a version of the calibration relying only on levelized cost estimates, including for coal and gas, and note that the results are qualitatively robust.

The second step of the calibration utilizes both the literature and matches selected moments in the data, as summarized in Table 5.

Parameter	Value(s)	Sources and Notes
ε	1.8561	Papageorgiou et al. (2013) avg. estimate of elasticity of subs. btw. clean, dirty inputs in electricity production
σ	2	Bosetti et al. (2007) calibration of fossil fuel electricity sub-nest elasticity based on empirical Ko and Dahl (2001), Sonderholm (1998)
κ_c, κ_s κ_g	0.3305, 0.3621 0.3074	Rationalize electricity demand equations (32),(33) at base year (2009) generation data (EIA) and costs (estimated from FERC, EIA data)
ϕ	0.403	Barrage (2019)
φ	0.67	Standard
λ	0.5	Literature (Chen et al., 2017; Van der Werf, 2008; Böringer and Rutherford, 2008; Bosetti et al., 2007; Hassler, Krusell, Olosvsson, 2012;); See also discussion in Appendix.
υ	0.5	Normalized (without loss of generality)
γ	1.07	Match 2004-2014 profits for Petroleum and Coal, Durable Manuf., Wholesale (U.S. Census)
$\widetilde{A_{E,0}}$	6.0818e+04	Rationalize final goods producer's electricity demand (37) in base year (2009) at observed GDP Y_0 (BEA)
ξ_c, ξ_s	1.001, 0.429	Billion metric tons of CO2 / trillion kWh (EIA, 2016)
$A_{g,0}, A_{c,0}, A_{s,0}, B_{c,0},$ $B_{s,0}, C_{f,0}, C_{E,0}, A_{P,0}$		Match 14 equilibrium conditions at observed employment $L_{E,0}$ and $L_{P,0}$ (BEA), capital K_0 (BEA), and generation cost estimates ($p^y_{i,t}$, $p^r_{i,t}$) (See Appendix C)

Table 5: Summary of Calibration Method

First, the benchmark substitution elasticities (ε, σ) are calibrated externally based on empirical estimates and other studies in the literature (Papageorgiou et al., 2013; Bosetti et al., 2007). Next, the $\kappa's$ are chosen to rationalize base year (2009) electricity generation data (from the EIA) at baseline costs.

Electricity	$E_{i,0}$ (tril. kWh)	$p_{i,0}$ (\$/MWh)	$p^r_{i,0}$ (\$/MWh)	$\overline{\Lambda_i}$ (%)
Coal	1.7411	41.0	23.0	8%
Gas	0.8410	64.6	47.6	0.2%
Green	0.8994	52.2	-	-

Table 6: Base Year Data

Given these moments in the data, we specifically solve for the $\kappa's$ jointly with the fossil composite's initial price $p_{f,0}$ and output level $E_{f,0}$ through five equations in five unknowns, namely (i) the profit-maximizing fossil electricity input demands,

[14] We consider several methods of combining FERC and NREL estimates, and present estimates here which simply average the capacity-weighted average cost measures derived from each information source.

[15] We exclude hydroelectric generation from these calculations in light of limited projected expansion potential in hydroelectric generation (see, e.g., EIA *Annual Energy Outlook,* 2019).

$$\frac{E_{c,t}}{E_{s.t}} = \left(\frac{\kappa_c}{\kappa_s}\frac{p_{st}}{p_{ct}}\right)^{\sigma} \tag{32}$$

(ii) the profit-maximizing green versus fossil electricity input demands,

$$\frac{E_{g,t}}{E_{f.t}} = \left(\kappa_g \frac{p_{ft}}{p_{gt}}\right)^{\varepsilon} \tag{33}$$

(iii) the fossil composite's price index,

$$p_{ft} = \left(\kappa_c^{\sigma} p_{ct}^{1-\sigma} + \kappa_s^{\sigma} p_{st}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}; \tag{34}$$

(iv) the fossil composite's production technology,

$$E_f \equiv \left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}; \tag{35}$$

and the following basic restriction:

$$1 = \kappa_c + \kappa_g + \kappa_s \tag{36}$$

We can then back out the initial electricity composite quantity and price:

$$\begin{aligned} p_{E,0}(\$2010\ bil./tril.kWh - eq) &= \left(\kappa_g^{\varepsilon} p_{g,0}^{1-\varepsilon} + p_{f,0}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} = 147.0 \\ E_0(tril.kWh - eq) &= \left(E_{f,0}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{g,0}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1.17 \end{aligned}$$

Next, we solve for $\widetilde{A_{E,0}}$ based on the final goods producer's electricity first order condition:

$$p_{E,0} = \frac{\partial Y_0}{\partial E_0} = [Y_0]^{\frac{1}{\lambda}} v \widetilde{A_{E0}}^{\frac{\lambda-1}{\lambda}} E_0^{\frac{-1}{\lambda}} \tag{37}$$

where we bring in base year GDP Y_0 from the Bureau of Economic Analysis. In order to calibrate λ, we again refer to the literature with appropriate adjustments as our model focuses on electricity, whereas empirical estimates commonly measure elasticities of substitution between overall energy and a capital-labor composite. On the one hand, commonly utilized values for general energy-capital labor elasticities range around 0.4 to 0.5 (MIT EPPA Model, e.g., Chen et al., 2017; Böringer and Rutherford, 2008; Bosetti et al., 2007; Van der Werf, 2008), and electricity-other energy elasticities of 0.5 (e.g., Chen et al., 2017; Bosetti et al., 2007). We thus use $\lambda = 0.5$ as a benchmark. On the other hand, we also consider lower values as new empirical evidence from Hassler, Krusell, and Olovsson (2012) finds near-zero substitution elasticities at an annual frequency. We set $\nu = 0.5$ without loss of generality since different values of ν can be accommodated by adjusting the level of $\widetilde{A_{E0}}$. A final set of parameters set based on the literatureas are the labor shares. In the non-electricity sector, we set a standard value of $\phi = 0.67$. For electricity and resource production, we assume $\varphi = 0.403$ based on estimates from Barrage (2019).

In order to calibrate the remaining parameters, we obtain the following additional data. First, we collect employment shares for the extraction and electricity-related sectors from the Bureau of Labor Statistics for the calibration base year 2007 (see Appendix C). Normalizing the total labor force size to $L_0 = 1$ then yields values $L_{E,0} = .00797$ and $L_{P,0} = 0.99203$. Similarly, we obtain the aggregate initial capital stock $K_0 = \$50,567$ billion from the BEA ('Fixed Assets and Consumer Durables,' \$2010). Next, we calibrate $\gamma = 1.07$ based on profit data for 2004-2014 from the U.S. Census Bureau (Quarterly Financial Reports), specifically to match that profits are a share $1 - 1/\gamma$ of sectoral income in laissez-faire. Details are provided in Appendix C.

Finally, given these values, we then search for the remaining 13 parameters and unknown variables in initial equilibrium $(A_{g,0}, A_{c,0}, A_{s,0}, B_{c,0}, B_{s,0}, C_{f,0}, C_{E,0}, A_{P,0}, K_{E,0}, K_{P,0}, c_{E,0}, w_0, \rho_0)$ through a system of equilibrium conditions (given in Appendix 9.4). We then set pollution intensities ξ_c and ξ_s based on the benchmark pollution intensity of each type of electricity generation E_{it} (EIA, 2016).

4.2.2 Dynamic Calibration

We next describe our implementation of dynamics in the quantitative model. With regards to the innovation process, we assume that $\eta_f = \eta_g$ and choose η such that should innovation in energy occurs in green technology only, A_{gt} would grow with a factor 1.02^{ϕ} per year (specifically as $\eta = 5\phi \ln 1.02/\ln \gamma = 0.5898$). Importantly, we then select the exponent ψ parameter to match our empirical estimates of the decline in the green innovation share in the immediate aftermath of the shale gas boom from Section 2. We specifically match a short-run (within 5 years) decline of 0.367 (as in Table 1, Panel B, Column 4) by setting ψ equal to 0.125 in the benchmark. For the scenarios where extraction productivity is assumed to continue to grow, for numerical reasons, we also consider a value of $\psi = 0.18$, which is associated with a lower effect of the shale revolution on the green innovation share (-0.25 rather than -0.367 as in the benchmark), and thus likely a more conservative choice.

On the climate side, we adopt the carbon cycle and climate change damage parameters from Golosov et al. (2014). We specifically consider both their baseline and 'high' damage specifications. Since our model endogenizes only greenhouse gas emissions from the U.S. electricity sector, we must specify a path for emissions from other countries and sources. As a benchmark, we use the business-as-usual emissions projections from the 2010 RICE model (Nordhaus, 2010) for all but one-third of U.S. emissions for this purpose.[16] To the extent that changes in U.S. technology spill over to other countries in reality, our estimates thus likely represent a lower bound on the global climate impacts of the shale gas boom.

The remainder of the dynamic calibration is standard. We choose one period as corresponding to 5 years. For simplicity, we assume that the capital stock grows at 2% a year, that $\widetilde{A}_{Et}$ is constant, and that A_{Pt} grows with a factor 1.02^{φ} per year. These assumptions guarantee that the long-run growth rate of the economy is 2% a year. Finally, as in section 3.3, we assume that up to a constant productivity term the energy intermediates in fossil fuel power plant technologies q_{cj} and q_{sj} are shared.

[16] Specifically, we adopt all but 31.5% of U.S. business-as-usual emissions in RICE as exogenous rest-of-world-and-economy emissions, given that 31.5% is the average U.S. electricity greenhouse gas emissions share between 1990-2008 as per EPA data.

4.3 Short-Run Impacts

This subsection presents quantitative estimates for the static effects of improvements in shale gas extraction technology. We mainly focus on increases in $B_{s,0}$ of 50%.[17] The impact of changing B_s on the average effective emissions rate per unit of electricity $\xi_{E,t}$ can be directly computed from (48) using also (28). In order to compute the change in overall energy demand, we then solve for the new macroeconomic equilibrium (see Appendix 8.4 for details). Table 7 presents the results. As expected, the net effect of an improvement in shale extraction technology on contemporaneous carbon emissions is consistently negative, with a benchmark estimate of an around 13% decline in emissions. The sensitivity of this projection with respect to the model parameters is also as expected. First, a higher (lower) elasticity of substitution ε between fossil fuels and green technologies is associated with slightly lower (higher) declines in CO_2 emissions, as the substitution effect of natural gas away from clean technologies is stronger (weaker). Second, a higher (smaller) elasticity of substitution within fossil fuels σ is associated with larger (lower) declines in CO_2 emissions, as in that case natural gas is a better competitor to coal. Finally, a lower value for the elasticity of substitution between the production input and energy is associated with a larger decline in CO_2 emissions since it limits the scale effect (as C_{Et} increases, more workers get reallocated toward the production input).

Short-Run Effects of Improved Shale Extraction Technology			
	$\%\Delta\xi_E$	$\%\Delta E$	$\%\Delta CO_2$
Baseline Parameters			
+10% Increase in $B_{s,0}$	-16.7%	+5.5%	-12.1%
+50% Increase in $B_{s,0}$	-21.0%	+9.6%	-13.4%
Higher $\varepsilon = 2.2$			
+50% Increase in $B_{s,0}$	-20.9%	+9.7%	-13.2%
Lower $\varepsilon = 1.5$			
+50% Increase in $B_{s,0}$	-21.9%	+9.6%	-14.4%
Higher $\sigma = 2.2$			
+50% Increase in $B_{s,0}$	-23.1%	+9.7%	-15.6%
Lower $\sigma = 1.8$			
+50% Increase in $B_{s,0}$	-19.0%	+9.5%	-11.2%
Lower $\lambda = 0.3$			
+50% Increase in $B_{s,0}$	-21.0%	+5.8%	-16.5%

Table 7: Static Effects of Shale Technology Improvements

Ideally, we would like to compare these simulation results to real data in order to validate the model. A simple comparison to emissions data would not be informative as the shale gas revolution coincided with the Great Recession, among other confounders. Instead, we thus consult the empirical literature wherein a number of studies have produced micro-econometric estimates of the short-run effects of natural gas price changes on electricity producers, typi-

[17] While the observed natural gas price decline from 2008 to 2012 is around 67%, this decline also reflects changes in macroeconomic conditions and general equilibrium effects, so that we ultimately focus on a 50% improvement in B_s as our central effect estimate.

cally using power plant-level generation and emissions data and variation in natural gas prices over the mid-2000's through 2012 period. Of course these studies' findings are not strictly comparable to our model's predictions as they represent very short-run partial equilibrium estimates that hold various aggregate factors constant. They nonetheless represent the best available empirical evidence on the impacts of the shale gas revolution on electricity generation. Cullen and Mansur (2017) estimate that a 67% natural gas price decline from $6/mmBTU to $2/mmBTU (as observed from 2008 to 2012) would lead to a 10% decline in the CO2 emissions intensity of electricity generation. Our benchmark result of a 21% decline is broadly similar but certainly larger. This difference is perhaps not surprising given that Cullen and Mansur study daily variation in CO2 emissions, and include quarter fixed-effects in their specification so that their estimates represent very short-run impacts which control for changes in, e.g., generating capacity. In contrast, our model results seek to capture projected changes in generating capacity over a longer time horizon (one to five years). Another important study in this literature is by Linn and Muehlenbachs (2018), who estimate that a 10% decrease in shale prices in 2008 would decrease the emissions intensity of electricity generation by only -0.59%. There are again important structural reasons for this relatively lower estimate. For example, Linn and Muehlenbachs include power plant-by-year interaction terms in their analysis. Consequently, their estimates capture within-plant variation in emissions, identified from cross-sectional variation in gas prices, and would thus be expected to be smaller than our model simulations representing changes in aggregate electricity generation. Finally, we note that the literature has produced several other important empirical assessments of the shale boom's impacts on electricity generation which we cannot compare to our model results due to structural differences and limited comparability of outcome metrics.[18]

4.4 Dynamic Impacts

We first examine the dynamic effect of the shale boom in a laissez-faire world when there is no further innovation in extraction technologies. Figure 6 showcases the effects of a 50% increase in B_s in 2014 in the baseline case. In line with Proposition 5, Panel A shows that the shale gas boom increases the share of scientists in fossil fuel innovations. Since B_{st} and B_{ct} are constant (after the boom), this share eventually goes toward zero, but the shale boom delays this transition. Panel B plots the resulting change in output and in emissions. The initial effect on emissions is negative, but it turns positive by 2023 and increases thereafter. The effect on net output is initially positive, but turns negative over time as climate damages accrue.

Figure 7 reproduces the same analysis when extraction technologies grow over time at the same rate as the maximal rate achievable for the power plant technologies (i.e. with a factor γ^{η}). Figure 7 is similar to Figure 6 except that since A_{g2009} is not large enough in the benchmark calibration, innovation occurs increasingly more in the fossil fuel sector, whether the shale boom occurs or not. The boom does, however, hasten this transition.

[18] For example, Knittel, Metaxoglou, and Trinade (2015) compare shale gas share and CO2 emissions responses to natural gas price variation between investor-owned utilities and independent power producers in vertically integrated and restructured electricity markets, but focus only on entities with both coal- and gas-fired capacity, rather than the overall generating system as represented in our model. Holladay and Jacob LaRiviere (2017) study the effects of natural gas price declines on electricity generators but focus on changes in *marginal* emissions rates in the very short run due to changes in the dispatch of existing generation capacity. Similarly, Fell and Kaffne (2018) study the joint impacts of natural gas prices and wind generation on capacity utilization of coal electricity plants, using high-frequency data at the daily level.

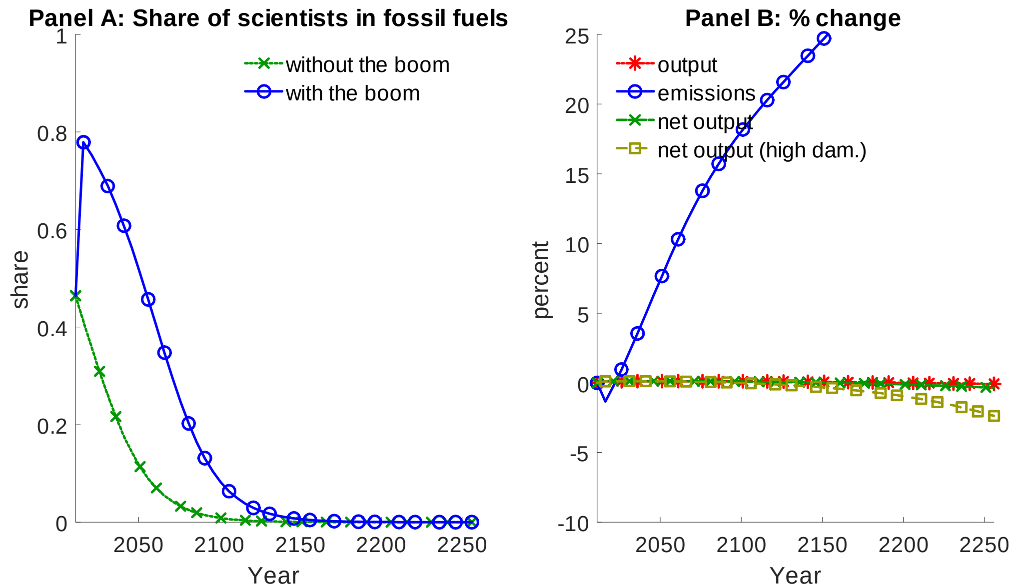

Figure 6: Shale Boom Impact on Laissez-Faire Outcomes (Constant Extraction Technology)

Proposition 4 tells us that the predicted impact of the shale boom depends qualitatively on the initial level of green technology, A_{g0}. Before proceding, we thus consider the quantitative sensitivity of the results to A_{g0}. Figure 8 repeats the analysis of Figure 7 assuming that initial green energy productivity was 70% higher than implied by our benchmark calibration. In this case, the model predicts that the shale boom pushes the economy out of a transition to a clean energy equilibrium back onto a fossil fuel path. That is, without the shale boom, the economy would transition to clean technology even with growing extraction productivities and absent policy interventions. The shale boom, however, shifts the balance of expected research rewards such that the post-boom economy is on a path towards a fossil fuel future, with correspondingly large impacts on greenhouse gas emissions and future climate damages.

One may ask how relevant it is to consider the implications of higher initial green productivity. We note that it could be relevant for at least two reasons. First, as noted above, green electricity generation costs are less well measured in standard data than for fossil fuels, implying additional uncertainty over our estimate of A_{g0}. Second, consideration of higher initial green productivity may be relevant for thinking about the potential future effects of another shale boom or equivalent event, such as improvements in Arctic extraction technologies which could open vast natural gas deposits to commercial exploration.

To conclude this discussion, Figure 9 presents results for even higher initial green productivity, double the benchmark level. As noted in Proposition 4, for high enough A_{g0}, the economy is expected to transition to a green path regardless of the shale boom. However, as shown in Figure 9, the shale boom delays this transition and increases emissions in the interim.

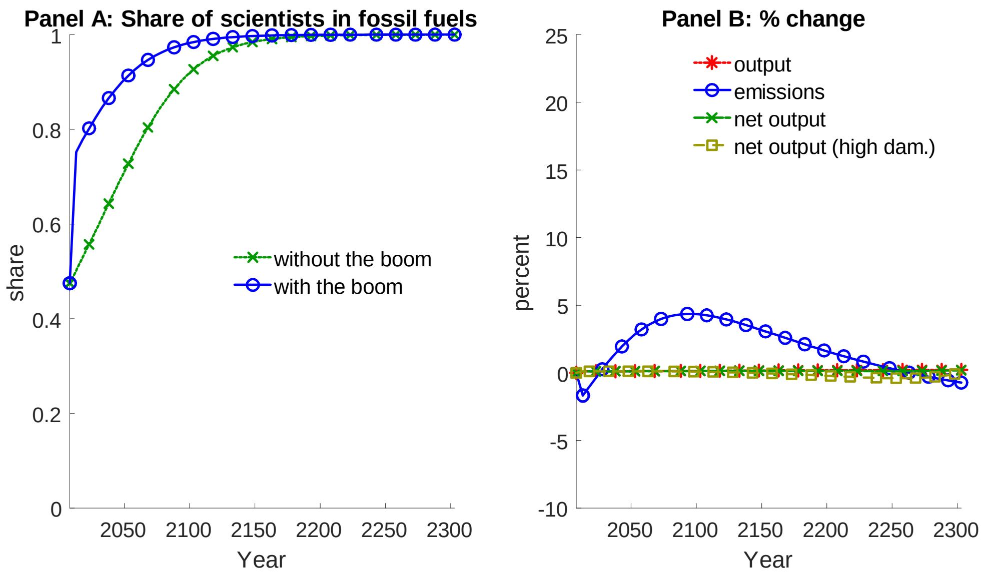

Figure 7: Shale Boom Impact on Laissez-Faire Outcomes (Growing Extraction Technology)

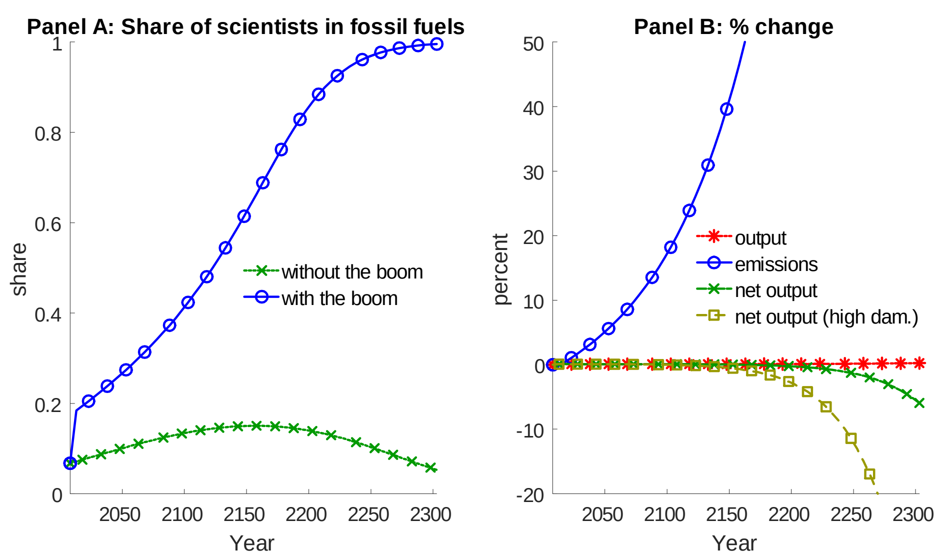

Figure 8: Shale Boom Impact on Laissez-Faire Outcomes (Growing Extraction Technology and 70% Higher A_{g0})

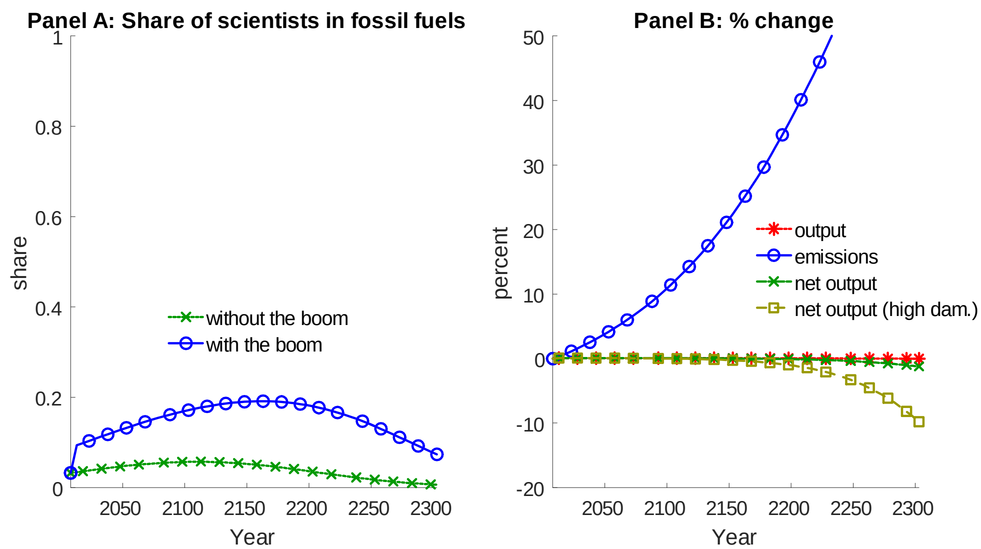

Figure 9: Shale Boom Impact on Laissez-Faire Outcomes (Growing Extraction Technology and 100% Higher A_{g0})

The results presented thus far consider the impacts of the shale boom in a laissez-faire world. We next consider policy design both with and without the shale boom.

4.5 Policy Implications

This section considers the implications of the shale boom for climate policy. We model a social planner who maximizes U.S. welfare while taking other countries' outcomes and non-electricity U.S. emissions as given. The planner has access to two policy instrument: a carbon tax (to correct the environmental externality), and a clean research subsidy (to take into account that the private value of innovation is too short-sighted).

We first consider optimal policy absent the shale boom. Figure 10 compares laissez-faire and policy (outcomes) in the setting with constant extraction technologies. Panels A and D showcase that, in the optimum, research efforts in fossil fuels and carbon emission should both be lower than in laissez-faire. Panel B shows that the optimal allocation can be decentralized by an increasing carbon tax, coupled with a clean research subsidy which is initially very high and gradually decreases over time.

Figure 11 presents analogous results to Figure 10 but for the case with exogenously growing extraction technologies. Here we see a more extreme difference between the optimal and laissez-faire research allocations (Panel A). Consequently, decentralizing the optimal allocation with growing extraction productivity requires an increasing research subsidy along with an increasing carbon tax (Panel B).

We next compare the effects of these optimal baseline policies with the effects of the shale boom. Figure 12 presents results for the case with constant extraction technologies (after the boom). The main insight is that the shale boom pushes the economy further away from the

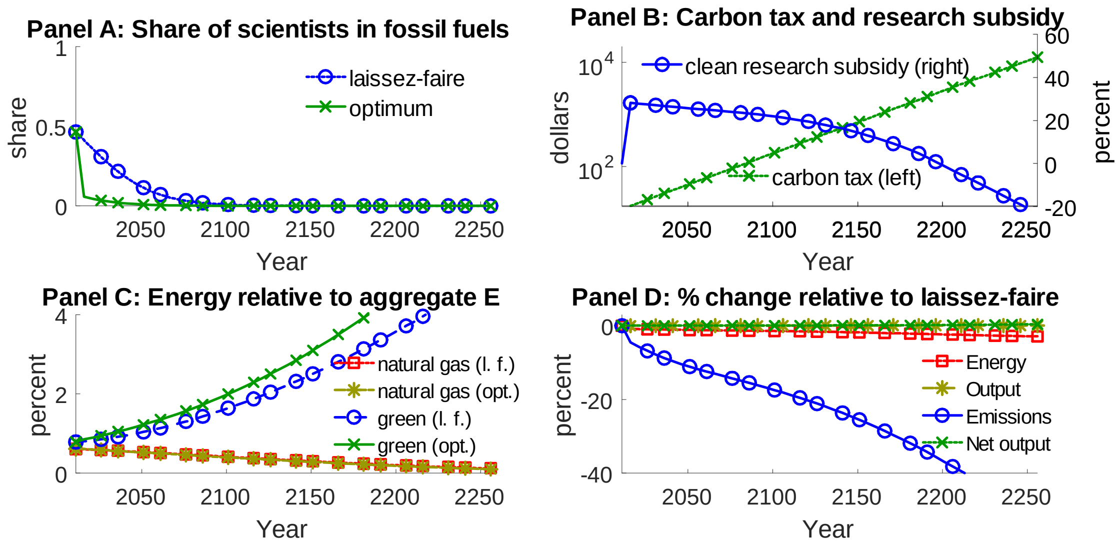

Figure 10: Optimal Outcomes, No Shale Boom (with Constant Extraction Technology)

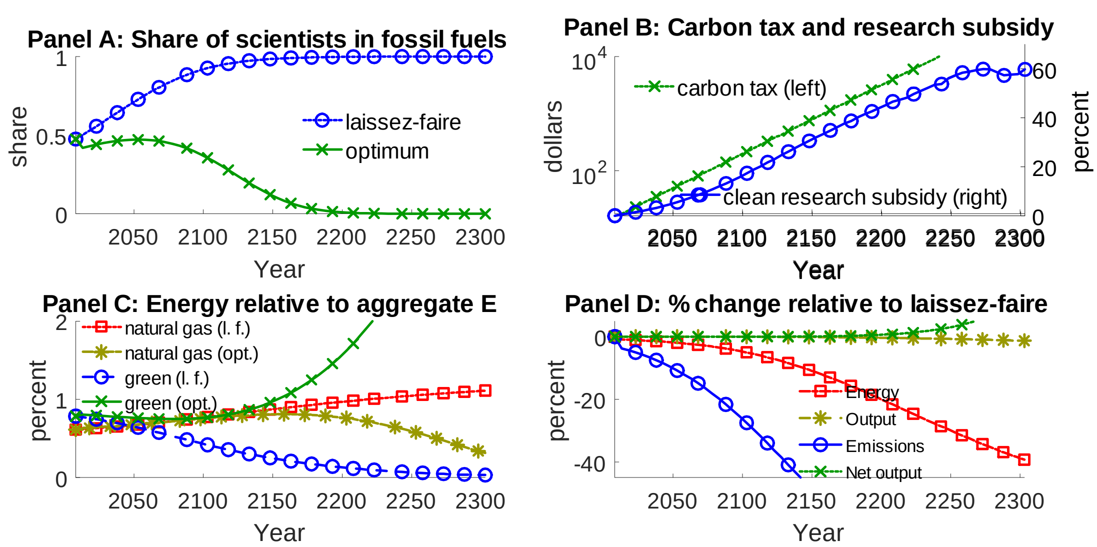

Figure 11: Optimal Outcomes, No Shale Boom (with Growing Extraction Technology)

baseline optimum than the initial laissez-faire allocation on several key dimensions. Panel A shows that the boom increases the fraction of research in fossil fuels technologies, whereas the baseline optimum would be to decrease it. Panel B shows that the boom leads to an increase in emissions (after an initial decline), whereas, again, the optimum would have been to decrease emissions. Finally, it should be noted that the shale boom does increase output for several periods relative to both the laissez faire and optimum outcomes. Eventually, however, the increase in climate damages in the laissez-faire boom outweighs the beneficial effects of cheaper energy, leading to a relative decline in long-run output.

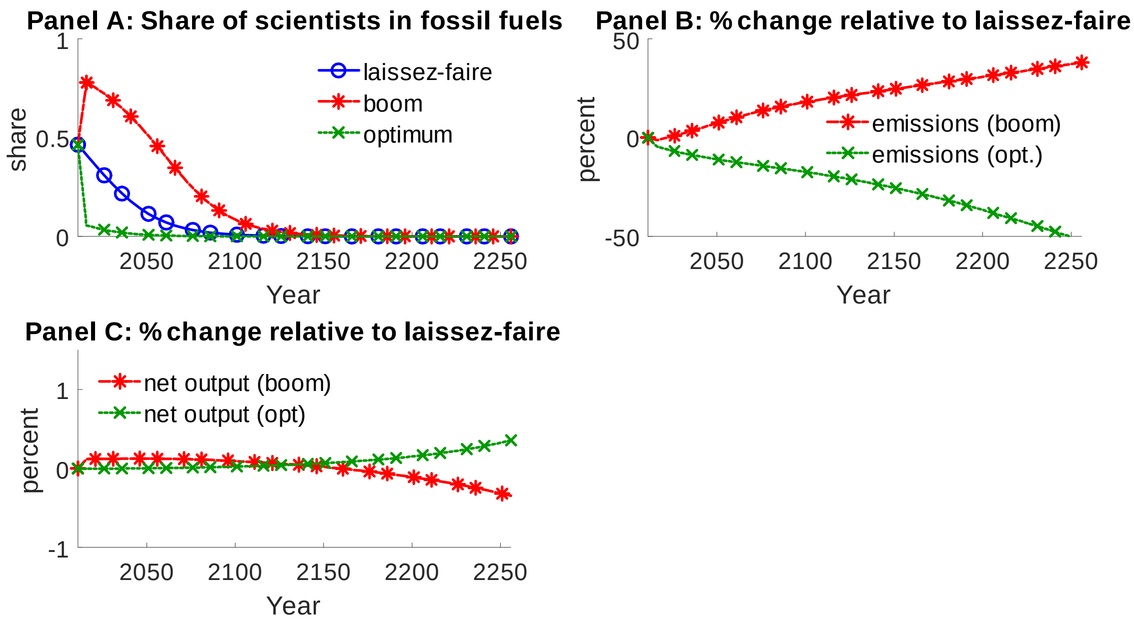

Figure 12: Comparison of Boom versus Policy Impacts Relative to Laissez-Faire (Constant Extraction Technology)

Finally, Figure 13 displays the estimated impacts of the shale boom on optimal outcomes and policy (with constant extraction technology), and compares these impacts under both standard and high climate damages (as per Golosov et al., 2014). Panel A shows that, with regular damages, the boom does lead to an *increase* in the optimal share of research devoted to fossil fuels in the benchmark case. Intuitively, this difference reflects the economic benefits of cheaper energy. At the same time, however, optimal management of the boom also requires (i) a higher carbon tax, and (ii) larger green research subsidies than in the baseline scenario. With this set of policies which balance the costs and benefits of shale gas, the boom has a sustained positive impact on net output (Panel D). The results are different if climate damages are very high. In this case, the optimal share of fossil fuel research quickly goes to zero regardless of the boom. Both carbon tax and green research subsidy levels are already high and not materially increased by the boom.[19]

Figure 14 displays analogous results to Figure 13 for the case with growing extraction technol-

[19] The spikes in late periods in Panel C reflect numerical issues associated with the corner solution and far-out time horizon in this scenario.

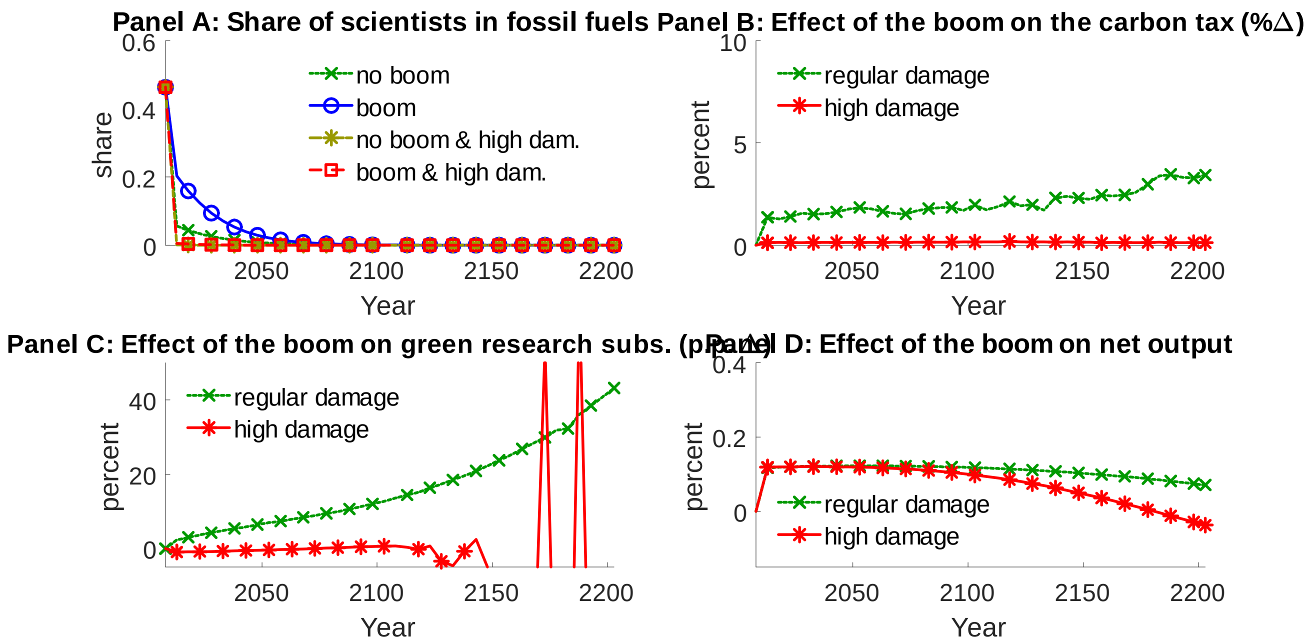

Figure 13: Shale Boom Impact on Optimal Policy (Constant Extraction Technology)

ogy. As before, the optimal allocation features a higher share of researchers working on fossil fuels technologies after the boom. In the benchmark calibration, this change initially implies a decline in the optimal green research subsidy, followed by a relative increase (Panel C). With high damages, the boom increases the green research subsidy throughout. The optimal carbon tax is again increased by the boom, although only to a very small degree.

5 Conclusion

This paper investigates the short- and long-term effects of a shale gas boom in an economy where energy can be produced with coal, natural gas, or a clean energy source. In the short run, a shale gas revolution has counteracting effects on CO2 emissions: on the one hand it allows countries to substitute away from coal which in turn reduces CO2 emissions everything else equal; on the other hand the shale gas boom may increase pollution as it increases the scale of aggregate production. Beyond this standard trade-off, however, we document empirically that the shale boom was associated with a decline in the share of green and renewable electricity generation technologies, relative to fossil fuels-based generation. Focusing on a model of directed technical change, we show that, in the long run a shale gas boom tends to increase CO2 emissions as it induces firms to direct innovation away from clean to fossil fuels technologies. A shale gas boom may even infinitely delay a switch from fossil fuel to clean energy.

To assess the short-run and long-run impacts of improving the shale extraction technology quantitatively, we calibrate our model to the U.S. economy using on electricity production and the costs of producing electricity using coal, gas and the different types of renewable energies. The results suggest that, in a laissez-faire world, the shale boom decreased CO2 emissions

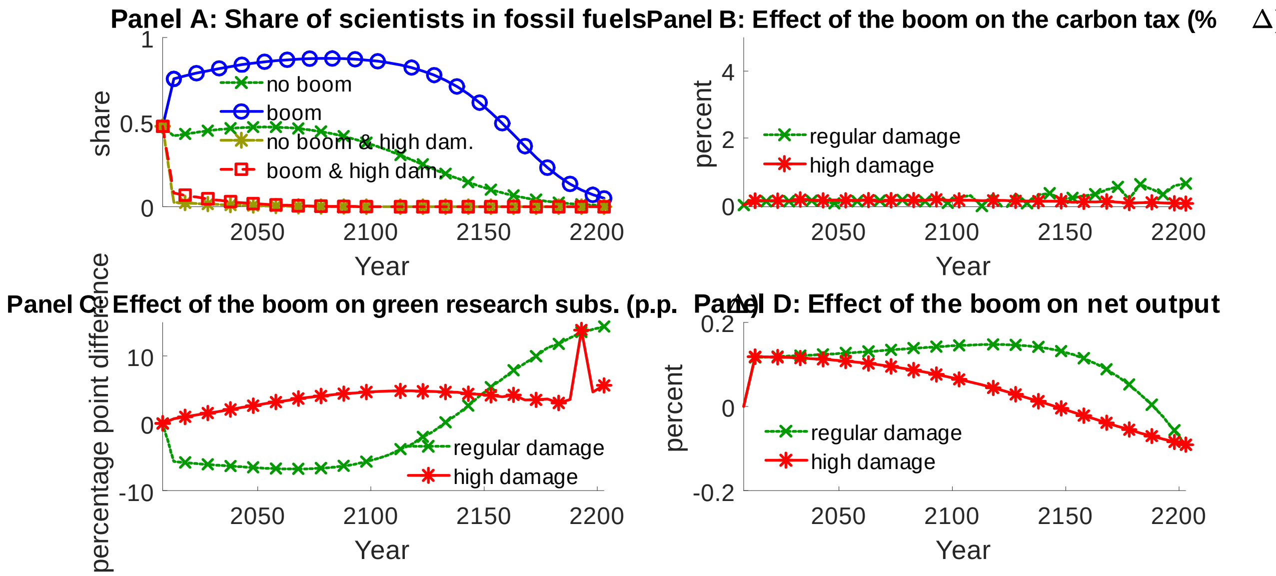

Figure 14: Shale Boom Impact on Optimal Policy (Growing Extraction Technology)

in the short-run, but may have significantly increased CO2 emissions in the longer run. One important point to highlight is that the quantitative predictions of our models depend on parameters and initial conditions, which suggests that a shale gas boom could have quite different effects across countries. A next step for this research agenda would be to calibrate the model to different countries.

To conclude, we characterize the optimal allocation and policies from the perspective of a U.S. social planner who takes other countries' emissions as given. In the benchmark case, optimal management of the shale boom does allow for an increase in the share of research efforts dedicated to fossil fuels generation, but also requires an increase in carbon taxes and higher subsidies for clean energy research. With an appropriate policy response that balances its economic and environmental costs and benefits, the shale boom can be harness to allow for sustained increases in net income and welfare.

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7 Appendix A: Details on the empirical analysis

Figure 15 reproduces Figure 2.B but for total emissions in the United States and for the CO2 intensity of primary energy consumption. The trends are similar.

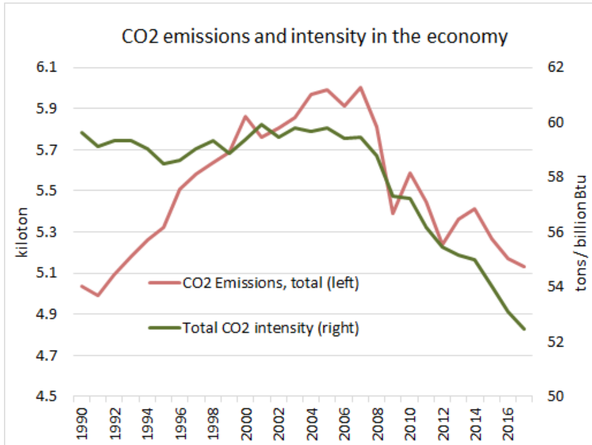

Figure 15: CO2 emissions and intensity for the whole economy

Figure 5 compares real natural gas prices over the past 25 years in Europe and the United States. In the United States, natural gas prices experienced a sustained decline since around 2009. In Europe, a decline is observed in later years. The data are from the World Bank Commodity Price Data (deflated based on the CPI from the U.S. Bureau of Economic Analysis).

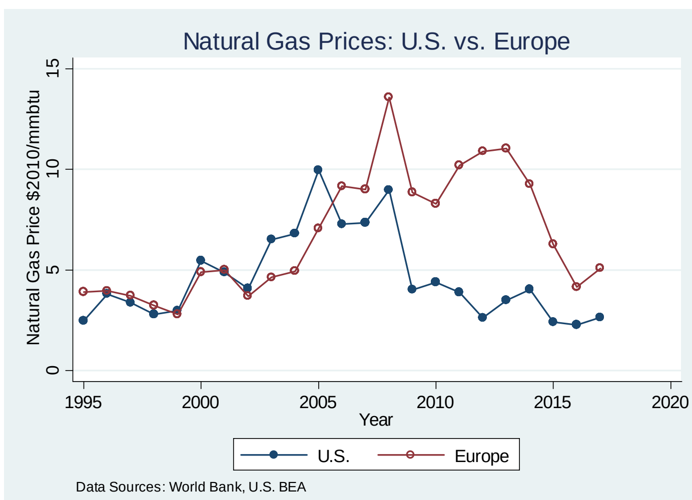

Figure 16: Real Natural Gas Prices

Figure 6 gives the list of shale gas bans per country and the source for our data.

Country	Year Ban	Source
AU (Australia)	0. No Ban until 2017. Minor regional moratoriums	Legal-Status-of-UOGE-across-the-world-31.03.18
CA (Canada)	0. No ban. Regional ban (New Brunswick) announced in 2015, but not in practice	Legal-Status-of-UOGE-across-the-world-31.03.18 https://www.cbc.ca/news/canada/new-brunswick/shale-gas-fracking-gallant-moratorium-1.4715225
CH (Switzerland)	0. No federal ban. Negligible production	https://www.eawag.ch/en/news-agenda/news-portal/news-detail/news/fracking-in-der-schweiz-im-zweifelsfall-nein/
CL (Chile)	0. No ban. Limited production. In 2018 senate begins regulation proposal	https://www.terram.cl/2018/06/proyecto-que-regula-la-practica-del-fracking-esta-en-primer-tramite-constitucional/
CZ (Czech Republic)	0. No ban. Proposals not passed until 2015	http://frackfreeworld.org/931
DE (Germany)	0. No ban. Ban introduced in 2017	Legal-Status-of-UOGE-across-the-world-31.03.18
DK (Denmark)	1. (2012)	Legal-Status-of-UOGE-across-the-world-31.03.18
ES (Spain)	0. No ban, little production	Legal-Status-of-UOGE-across-the-world-31.03.18
FR (France)	1. (2011)	Legal-Status-of-UOGE-across-the-world-31.03.18
GB (Great Britain)	1.(only during 2011). After 2015, ban introduced in some regions	Legal-Status-of-UOGE-across-the-world-31.03.18
HU (Hungary)	0. No ban	https://falconoilandgas.com/mako-hungary/
IE (Ireland)	0. No ban. Ban introduced in 2017	Legal-Status-of-UOGE-across-the-world-31.03.18
JP (Japan)	0. No ban	https://www.washingtonpost.com/blogs/post-partisan/wp/2016/11/27/japan-shows-why-the-world-needs-fracking-and-nuclear-power/?utm_term=.2d04e2a633f0 http://upstreampm.com/japan-slowly-becoming-model-fracking-nation-invaluable-ally-us-energy/
NL (Netherlands)	1. (2013)	Legal-Status-of-UOGE-across-the-world-31.03.18
US (United States)	0. No ban. Vermont banned in 2012, but was minor producer. 2015 high volume ban in NY	Legal-Status-of-UOGE-across-the-world-31.03.18
CN (China)	0. No Ban. However, regulations are very particular in China	In Italian: http://www.glawcal.org.uk/academic-articles/regulation-and-prospects-of-the-shale-gas-market-in-china-e
AT (Austria)	Not included	Negotiations in Parliament
BE (Belgium)	Not included	
IS (Iceland)	Not included	
IL (Israel)	0. No ban. After 2015 restrictions introduced	https://www.huffingtonpost.com/tom-szaky/the-israeli-public-says-n_b_11696222.html
EE (Estonia)	Not included	
FI (Finland)	Not included	
GR (Greece)	Not included	
IT (Italy)	Not included	
PL (Poland)	0. No ban. Very limited production	Legal-Status-of-UOGE-across-the-world-31.03.18
PT (Portugal)	0. No ban	Legal-Status-of-UOGE-across-the-world-31.03.18
NZ (New Zealand)	0. No ban. Except for one region in 2012, but limited production	https://keeptapwatersafe.org/global-bans-on-fracking/

Figure 17: Shale Gas Regulations

Table 7 presents results for the analysis at the level of the country of invention. We look in turn at EPO and USPTO patents and attribute patent applications to a country according to the nationality of its inventor.

	EPO				USPTO			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel A: Renewable / Fossil fuel electric							
Shale Gas Boom	-0.896***	-1.122***	-0.902***	-1.239**	-0.774**	-0.884**	-0.784*	-1.008**
	(0.26)	(0.38)	(0.31)	(0.47)	(0.33)	(0.36)	(0.41)	(0.48)
Ban		0.683		0.526		1.640		1.473
		(1.05)		(0.95)		(1.80)		(1.66)
N	718	379	718	379	718	379	718	379
	Panel B: Green / Fossil fuel electric							
Shale Gas Boom	-1.173***	-1.608***	-1.195***	-1.738***	-0.948**	-1.238**	-0.974**	-1.382**
	(0.30)	(0.49)	(0.36)	(0.59)	(0.38)	(0.53)	(0.46)	(0.65)
Ban		0.367		0.193		1.302		1.109
		(1.05)		(0.95)		(1.78)		(1.63)
N	718	379	718	379	718	379	718	379
FEs (C, T)	Y	Y	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)			Y	Y			Y	Y

Note: Difference-in-difference regressions. The shale gas boom is dated from 2009. Standard errors are clustered at the country-level. Column (2), (4), (6), (8) include AU, CA, CH, CL, CN, CZ, DE, DK, ES, FR, GB, HU, IE, IL, JP, PL, PT, NL, NZ, US, the other columns also include TW, AT, BE, IS,, EE, FI, GR, IT, KR, LV, LT, LU, MX, NO, SK, SI, SE, TR.

Table 7 presents results analogous to Table 7 but using variation in natural gas price indexes from IEA data and for a smaller sample of countries.

	EPO			USPTO		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: log (Renewable / Fossil fuel electric)						
log(Price Index)	0.394	0.349	0.398	0.292	0.126	0.219
	(0.32)	(0.30)	(0.30)	(0.31)	(0.18)	(0.14)
Panel B: log(Green / Fossil fuel electric)						
log(Price Index)	0.148	0.107	0.148	0.108	-0.019	-0.006
	(0.39)	(0.37)	(0.40)	(0.23)	(0.13)	(0.14)
FEs (C, T)	Y	Y	Y	Y	Y	Y
Control ln(GDPCap)		Y	Y		Y	Y
Control ln(Energy Consumption)			Y			Y

Note: Independent variable lagged 2 periods, star levels: * 0.10, ** 0.05, *** 0.010. Standard errors are clustered at the country-level. Includes: AU, BE, CA, FR, GR, JP, KR, MX, NZ, CH, GB, US.

8 Appendix B: Theoretical results

8.1 Uniqueness of the equilibrium and proof of Proposition 3

We can rewrite (23) as:

$$f\left(s_{gt}, A_{ct-1}, B_{ct}, A_{st-1}, B_{st}, C_{gt-1}\right) = 1 \tag{38}$$

where the function f is defined as

$$f \equiv \frac{\eta_f \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} \kappa_c^{\varepsilon} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}} \right)^{-\varepsilon} + \frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} \kappa_s^{\varepsilon} \left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}} \right)^{-\varepsilon} \right) s_{gt}^{\psi}}{\eta_g \kappa_g^{\varepsilon} C_{gt-1}^{\varepsilon-1} s_{ft}^{\psi} \gamma^{\eta_g s_{gt}^{1-\psi}(\varepsilon-1)}}.$$

We then get that

$$\begin{aligned} & \frac{\partial \ln f}{\partial \ln s_{gt}} \\ = \quad & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln \gamma) s_{gt}^{1-\psi} + \psi \frac{s_{gt}}{s_{ft}} \\ & + \frac{\eta_f (1-\psi) \ln(\gamma) s_{ft}^{1-\psi} \frac{s_{gt}}{s_{ft}} \left(\kappa_c^{\varepsilon} \frac{C_{ct}^{\varepsilon}}{A_{ct}} \left(1 - \varepsilon \frac{B_{ct}}{B_{ct}+A_{ct}}\right) + \kappa_s^{\varepsilon} \frac{C_{st}^{\varepsilon}}{A_{st}} \left(1 - \varepsilon \frac{B_{st}}{B_{st}+A_{st}}\right)\right)}{\kappa_c^{\varepsilon} \frac{C_t^{\varepsilon}}{A_{ct}} + \kappa_s^{\varepsilon} \frac{C_{st}^{\varepsilon}}{A_{st}}} \\ \geq \quad & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln \gamma) s_{gt}^{1-\psi} + \left(\psi - \eta_f (\varepsilon - 1)(1-\psi)(\ln \gamma) s_{ft}^{1-\psi} \right) \frac{s_{gt}}{s_{ft}}. \end{aligned}$$

Therefore we get that $\frac{\partial \ln f}{\partial \ln s_{gt}} > 0$ if Assumption 1 holds. In that case since $f(0,.) = 0$ and $\lim_{s_g \to 1} f(s_g,.) = \infty$, we obtain that (23) defines a unique equilibrium innovation allocation.

We directly get that $\frac{\partial f}{\partial B_{st}} > 0$ which establishes that an increase in B_{s1} leads to a lower value for s_{g1}.

Further, we obtain that $\frac{\partial f}{\partial C_{gt-1}} < 0$, so that a higher value for C_{gt-1} leads to more clean innovation. Further, we get

$$\frac{\partial \ln f}{\partial \ln A_{ct-1}} = \frac{\frac{1}{A_{ct}}\kappa_c^\varepsilon C_{ct}^\varepsilon}{\frac{1}{A_{ct}}\kappa_c^\varepsilon C_{ct}^\varepsilon + \frac{1}{A_{st}}\kappa_s^\varepsilon C_{st}^\varepsilon}\left(\varepsilon \frac{B_{ct}}{B_{ct} + \gamma^{\eta_f s_{ft}^{1-\psi}} A_{ct-1}} - 1\right).$$

Therefore $\frac{\partial \ln f}{\partial \ln A_{ct-1}} \geq 0$ for all values of s_{ft} provided that $\frac{B_{ct}}{A_{ct-1}} > \frac{\gamma^{\eta_f}}{\varepsilon - 1}$. Similarly, $\frac{\partial \ln f}{\partial \ln A_{st-1}} \geq 0$ for all values of s_{ft} provided that $\frac{B_{st}}{A_{st-1}} > \frac{\gamma^{\eta_f}}{\varepsilon-1}$. If these conditions are satisfied, then an increase in B_{s1} leads to higher values of A_{s1}, A_{c1} and a lower value of C_{g1}, which imply a lower value of s_{g2}. This in turns leads to even higher values of A_{s2}, A_{c2} and a lower value for C_{g2}. By iteration, we then get that all s_{gt} decrease for $t \geq 1$.

8.2 Proof of Proposition 4

We first note that as argued in the text, if $\min\left(B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right) > \gamma^\eta/(\varepsilon - 1)$ at $t = 1$, then this holds for all $t > 1$, so that Lemma 3 applies.

We then prove the following lemma:

Lemma 1 *Assume that Assumption 1 holds, that B_{ct} and B_{st} grow exogenously at factor γ^η and that $\min(B_{c1}/A_{c0}, B_{s1}/A_{s0}) > \gamma^\eta/(\varepsilon - 1)$. Then the economy features $s_{gt} \to 1$ or $s_{gt} \to 0$ (except for a knife-edge case where $s_{gt} \to 1/2$).*

Proof. Assume that for some time period τ, $s_{g\tau} \leq 1/2$, we first establish that $s_{gt} < 1/2$ for all $t > \tau$. For ease of notations, define $f_\tau(s_{g\tau}) \equiv f\left(s_{g\tau}, A_{c(\tau-1)}, B_{c\tau}, A_{s(\tau-1)}, B_{s\tau}, C_{g(\tau-1)}\right)$. We then get that:

$$\begin{aligned}
& f_{\tau+1}(s_{g\tau}) \\
= \; & \frac{\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c\tau}}\kappa_c^\varepsilon\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c\tau}} + \frac{1}{B_{c(\tau+1)}}\right)^{-\varepsilon} + \frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s\tau}}\kappa_s^\varepsilon\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s\tau}} + \frac{1}{B_{s(\tau+1)}}\right)^{-\varepsilon}}{\kappa_g^\varepsilon C_{g\tau}^{\varepsilon-1}\gamma^{\eta s_{g\tau}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^\psi \\
= \; & \gamma^{(\varepsilon-1)\eta\left(s_{f\tau}^{1-\psi} - s_{g\tau}^{1-\psi}\right)} \frac{\frac{\kappa_c^\varepsilon \gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c(\tau-1)}}\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{c(\tau-1)}} + \frac{\gamma^{-\eta\left(1-s_{f\tau}^{1-\psi}\right)}}{B_{c\tau}}\right)^{-\varepsilon} + \frac{\kappa_s^\varepsilon \gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s(\tau-1)}}\left(\frac{\gamma^{-\eta s_{f\tau}^{1-\psi}}}{A_{s(\tau-1)}} + \frac{\gamma^{-\eta\left(1-s_{f\tau}^{1-\psi}\right)}}{B_{s\tau}}\right)^{-\varepsilon}}{\kappa_g^\varepsilon C_{g\tau-1}^{\varepsilon-1}\gamma^{\eta s_{g\tau}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^\psi \\
> \; & f_\tau(s_{g\tau}) = 1,
\end{aligned}$$

where we use that $s_{f\tau} < 1$ so that $\gamma^{-\eta\left(1-s_{f\tau}^{1-\psi}\right)} < 1$ and that $s_{f\tau} \geq s_{g\tau}$. Since $f_{\tau+1}$ is increasing then it must be that $s_{g(\tau+1)} < s_{g\tau}$, which implies that $s_{gt} > 1/2$ for all $t > \tau$. Since s_{gt} is increasing (from τ), it must tend toward a constant s_g^* smaller than $1/2$. As a result, $\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}}\kappa_c^\varepsilon\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}}\kappa_s^\varepsilon\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}}+\frac{1}{B_{st}}\right)^{-\varepsilon}$ will grow at factor $\gamma^{\eta(\varepsilon-1)\left(1-s_g^*\right)^{1-\psi}}$ while $C_{g(t-1)}^{\varepsilon-1}$ will grow less fast with a factor $\gamma^{\eta(\varepsilon-1)s_g^{*1-\psi}}$ if $s_g^*>0$ (or will not grow exponentially if $s_g^*=0$). As a result $f_t\left(s_t\right)\to\infty$ for s_t bounded above 0, therefore it must be that $s_g^*=0$. In other words, all innovation tend toward the fossil fuel sector.

Assume instead that for all $t's$, $s_{gt}>1/2$. We want to establish that $\lim_\infty s_{gt}=1/2$ is only possible for a knife-edge case. To do that consider A_{c0}, A_{s0} and C_{g0} such that $\lim_\infty s_{gt}=1/2$. Now consider an alternative set-up where the initial green productivity $\widetilde{C}_{g0}$ is higher. Since $\min\left(B_{ct}/A_{c(t-1)},\, B_{st}/A_{s(t-1)}\right)>\gamma^\eta/\left(\varepsilon-1\right)$ for all t, the reasoning of Appendix 8.1 applies and we get that under the alternative path (denoted with $\widetilde{\ }$), $\widetilde{s}_{gt}>s_{gt}$ so that $\widetilde{C}_{gt}>C_{gt}$, $\widetilde{A}_{ct}<A_{ct}$ and $\widetilde{A}_{st}<A_{st}$. In fact one gets:

$$\widetilde{A}_{st}<\gamma^{\eta\left(\widetilde{s}_{f1}^{1-\psi}-s_{f1}^{1-\psi}\right)}A_{st},\ \widetilde{A}_{ct}<\gamma^{\eta\left(\widetilde{s}_{f1}^{1-\psi}-s_{f1}^{1-\psi}\right)}A_{ct}\text{ and }\widetilde{C}_{gt}>\gamma^{\eta\left(\widetilde{s}_{g1}^{1-\psi}-s_{g1}^{1-\psi}\right)}C_{gt}.$$

Since B_{ct} and B_{st} grow faster than A_{ct} and A_{st}, we have that

$$f_t\left(s_{gt}\right)\sim\gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)}\frac{\left(\kappa_c^\varepsilon A_{c(t-1)}^{\varepsilon-1}+\kappa_s^\varepsilon A_{s(t-1)}^{\varepsilon-1}\right)s_{gt}^\psi}{\kappa_g^\varepsilon C_{g(t-1)}^{\varepsilon-1}s_{ft}^\psi},$$

since by assumption $\lim s_{gt}=1/2$, then $\lim\frac{\kappa_c^\varepsilon A_{c(t-1)}^{\varepsilon-1}+\kappa_s^\varepsilon A_{s(t-1)}^{\varepsilon-1}}{\kappa_g^\varepsilon C_{g(t-1)}^{\varepsilon-1}}=1$. Therefore (as B_{ct} and B_{st} still grow faster than $A_{c(t-1)}^{\varepsilon-1}$ and $A_{s(t-1)}^{\varepsilon-1}$).

$$\lim\widetilde{f}_t\left(\frac{1}{2}\right)=\lim\frac{\kappa_c^\varepsilon\widetilde{A}_{c(t-1)}^{\varepsilon-1}+\kappa_s^\varepsilon\widetilde{A}_{s(t-1)}^{\varepsilon-1}}{\kappa_g^\varepsilon\widetilde{C}_{g(t-1)}^{\varepsilon-1}}\leq\gamma^{\eta(\varepsilon-1)\left(\left(\widetilde{s}_{f1}^{1-\psi}-s_{f1}^{1-\psi}\right)-\left(\widetilde{s}_{g1}^{1-\psi}-s_{g1}^{1-\psi}\right)\right)}<1.$$

Therefore $\lim\widetilde{s}_{gt}\neq1/2$ as this would impose that $\lim\widetilde{f}_t\left(\frac{1}{2}\right)=1$ which is impossible. A similar reasoning can be applied for an alternative path with a lower C_{g0}, which then results in $\widetilde{s}_{gt}\to0$. In other words $s_{gt}\to1/2$ corresponds to a knife-edge case.

Then, consider a path such that $s_{gt}>1/2$ and $s_{gt}\not\to1/2$. For t large enough, we get that:

$$f_t\left(s_{gt}\right)\sim\gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)}\frac{\left(\kappa_c^\varepsilon A_{c(t-1)}^{\varepsilon-1}+\kappa_s^\varepsilon A_{s(t-1)}^{\varepsilon-1}\right)s_{gt}^\psi}{\kappa_g^\varepsilon C_{g(t-1)}^{\varepsilon-1}s_{ft}^\psi}.$$

As C_{gt-1} grows faster than $A_{c(t-1)}^{\varepsilon-1}$ and $A_{s(t-1)}^{\varepsilon-1}$, (38) can then only be satisfied if $s_{gt}\to1$. This achieves the proof of the lemma. ■

A sufficient condition to get that $s_{gt}\to0$ is obtained for $s_{g1}\leq1/2$ which corresponds to $f_1\left(1/2\right)\geq1$, which is equivalent to

$$\kappa_g^\varepsilon A_{g0}^{\varepsilon-1}\leq\frac{\kappa_c^\varepsilon}{A_{c0}}\left(\frac{1}{A_{c0}}+\frac{\gamma^{\eta/2^{1-\psi}}}{B_{c1}}\right)^{-\varepsilon}+\frac{\kappa_s^\varepsilon}{A_{s0}}\left(\frac{1}{A_{s0}}+\frac{\gamma^{\eta/2^{1-\psi}}}{B_{s1}}\right)^{-\varepsilon}.$$

In contrast assume now that $\kappa_g^\varepsilon A_{g0}^{\varepsilon-1} > \kappa_c^\varepsilon A_{c0}^{\varepsilon-1} + \kappa_s^\varepsilon A_{s0}^{\varepsilon-1}$, then since

$$f_1(s_{g1}) < \gamma^{\eta(\varepsilon-1)\left(s_{f1}^{1-\psi} - s_{g1}^{1-\psi}\right)} \frac{\left(A_{c0}^{\varepsilon-1} + A_{s0}^{\varepsilon-1}\right) s_{g1}^\psi}{C_{g0}^{\varepsilon-1} s_{f1}^\psi},$$

we must have $s_{g1} > 1/2$. This ensures that $s_{gt} > 1/2$ for all $t's$. For t large enough, we then have

$$f_t(s_{gt}) \sim \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi} - s_{gt}^{1-\psi}\right)} \frac{\left(A_{c(t-1)}^{\varepsilon-1} + A_{s(t-1)}^{\varepsilon-1}\right) s_{gt}^\psi}{C_{g(t-1)}^{\varepsilon-1} s_{ft}^\psi} < \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi} - s_{gt}^{1-\psi}\right)} \frac{\left(A_{c0}^{\varepsilon-1} + A_{s0}^{\varepsilon-1}\right) s_{gt}^\psi}{C_{g0}^{\varepsilon-1} s_{ft}^\psi},$$

so that $s_{gt} \nrightarrow 1/2$. Therefore for sufficiently low C_{g0} the economy will be on a path toward with $s_{gt} \to 0$ and for C_{g0} sufficiently high toward a path with $s_{gt} \to 1$. Since the only other possibility is that $s_{gt} \to 1/2$ and is obtained for a knife-edge case (where a higher C_{g0} leads to $s_{gt} \to 1$ and a lower one leads to $s_{gt} \to 0$), we get that there exists a C^* (which depends on the other parameters) so that for $C_{g0} > C^*$, $s_{gt} \to 1$, for $C_{g0} = C^*$, $s_{gt} \to 1/2$ and for $C_{g0} < C^*$, $s_{gt} \to 0$.

As already established, an increase in B_{s0} implies that s_{gt} decreases at all $t's$. Therefore, following the same reasoning that established that $s_{gt} \to 1/2$ is a knife-edge case for a given value of C_{g0}, if $s_{gt} \to 0$ prior to the increase it will still do so after the shale gas boom; if $s_{gt} \to 1/2$, it will tend toward 0; and if $s_{gt} \to 1$ it will either still do so, or tend toward 1/2 for a knife-edge case or tend toward 0 for a larger increase in B_{s0} (these latter two cases being only possible for intermediate values of C_{g0}).

8.3 Proof of Proposition 5

8.3.1 Proof of part i)

First, suppose that $s_{ft} \nrightarrow 0$, so that A_{ct} and A_{st} are unbounded. With B_{ct} and B_{st} constant, we get that

$$f_t(s_{gt}) \sim \left(\kappa_c^\varepsilon \frac{B_{c0}^\varepsilon}{A_{ct-1}} + \kappa_s^\varepsilon \frac{B_{s0}^\varepsilon}{A_{st-1}}\right) \frac{\gamma^{-\eta s_{ft}^{1-\psi}} s_{gt}^\psi}{\kappa_g^\varepsilon C_{gt-1}^{\varepsilon-1} \gamma^{\eta_g s_{gt}^{1-\psi}(\varepsilon-1)} s_{ft}^\psi},$$

which tends toward 0 unless s_{ft} is arbitrarily small. Therefore, it must be that $s_{ft} \to 0$.

We then establish the existence of a time t_{switch} by showing that if $s_{gt} \geq 1/2$ then $s_{g(t+1)} >$

1/2. Assume that $s_{gt} \geq 1/2$, then one gets

$$
\begin{aligned}
f_{t+1}\left(\frac{1}{2}\right) &= \frac{\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{\eta 2^{\psi-1}}\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{\eta 2^{\psi-1}}\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}\gamma^{\eta s_{gt}^{1-\psi}(\varepsilon-1)}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta(\varepsilon-1)2^{\psi-1}}} \\
&= \gamma^{\eta(\varepsilon-1)\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)}\frac{\frac{\kappa_c^{\varepsilon}}{A_{c(t-1)}}\left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{c(t-1)}}+\frac{\gamma^{\eta s_{ft}^{1-\psi}}}{B_{c0}}\right)^{-\varepsilon}+\frac{\kappa_s^{\varepsilon}}{A_{s(t-1)}}\left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{s(t-1)}}+\frac{\gamma^{\eta s_{ft}^{1-\psi}}}{B_{s0}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta(\varepsilon-1)2^{\psi-1}}} \\
&< \frac{\frac{\kappa_c^{\varepsilon}}{A_{c(t-1)}}\left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{c(t-1)}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}+\frac{\kappa_s^{\varepsilon}}{A_{s(t-1)}}\left(\frac{\gamma^{\eta 2^{\psi-1}}}{A_{s(t-1)}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta(\varepsilon-1)2^{\psi-1}}} = f_t\left(\frac{1}{2}\right) \leq 1.
\end{aligned}
$$

Therefore $s_{g(t+1)} > 1/2$, which establishes the existence of a time t_{switch} ($t_{switch} = 1$ if $s_{g1} \geq 1/2$).

We now show that an increase in B_{s0} increases t_{switch}, to do that we establish that an increase in B_{s0} leads to an increase in s_{gt} as long as $s_{gt} \leq 1/2$. We define

$$
\widehat{f}_t\left(s_{gt}, s_{g(t-1)}, ..., s_{g1}, B_{s0}\right) = \frac{s_{gt}^{\psi}}{\kappa_g^{\varepsilon}C_{g0}^{\varepsilon-1}s_{ft}^{\psi}\gamma^{\eta(\varepsilon-1)\sum\limits_{\tau=1}^{t}s_{g\tau}^{1-\psi}}}\left(\begin{array}{c}\frac{\kappa_c^{\varepsilon}\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{c0}}\left(\frac{\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{c0}}+\frac{1}{B_{c0}}\right)^{-\varepsilon} \\ +\frac{\kappa_s^{\varepsilon}\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{s0}}\left(\frac{\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{s0}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}\end{array}\right),
$$

so that the equilibrium innovation allocation is still defined through $\widehat{f}_t\left(s_{gt}, s_{g(t-1)}, ..., s_{g1}, B_{s0}\right) = 1$ with $\widehat{f}_t$ increasing in s_{gt} and in B_{s0}. We obtain for $\widetilde{\tau} \in [1, t-1)$

$$
\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} = \left[\frac{\begin{array}{c}\frac{\kappa_c^{\varepsilon}}{A_{ct}}\left(\frac{1}{A_{ct}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}\left(1-\varepsilon\frac{\frac{1}{A_{ct}}}{\frac{1}{A_{ct}}+\frac{1}{B_{c0}}}\right) \\ +\frac{\kappa_s^{\varepsilon}}{A_{st}}\left(\frac{1}{A_{st}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}\left(1-\varepsilon\frac{\frac{1}{A_{st}}}{\frac{1}{A_{st}}+\frac{1}{B_{s0}}}\right)\end{array}}{\frac{\kappa_c^{\varepsilon}}{A_{ct}}\left(\frac{1}{A_{ct}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}+\frac{\kappa_s^{\varepsilon}}{A_{st}}\left(\frac{1}{A_{st}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}}s_{f\widetilde{\tau}}^{-\psi}-(\varepsilon-1)s_{g\widetilde{\tau}}^{-\psi}\right]s_{g\widetilde{\tau}}\eta(1-\psi)\ln\gamma.
$$

Yet if $t < t_{switch}$, then $s_{f\widetilde{\tau}} > s_{g\widetilde{\tau}}$, so that

$$
\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} < -\left[\varepsilon - 2 + \varepsilon\frac{\frac{\kappa_c^{\varepsilon}}{A_{ct}^2}\left(\frac{1}{A_{ct}}+\frac{1}{B_{c0}}\right)^{-\varepsilon-1}+\frac{\kappa_s^{\varepsilon}}{A_{st}^2}\left(\frac{1}{A_{st}}+\frac{1}{B_{s0}}\right)^{-\varepsilon-1}}{\frac{\kappa_c^{\varepsilon}}{A_{ct}}\left(\frac{1}{A_{ct}}+\frac{1}{B_{c0}}\right)^{-\varepsilon}+\frac{\kappa_s^{\varepsilon}}{A_{st}}\left(\frac{1}{A_{st}}+\frac{1}{B_{s0}}\right)^{-\varepsilon}}\right]s_{f\widetilde{\tau}}^{-\psi}s_{g\widetilde{\tau}}\eta(1-\psi)\ln\gamma.
$$

Therefore if $\varepsilon \geq 2$, we have that $\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} < 0$.

Therefore, the shale gas boom reduces $\widehat{f_1}$ leading to a lower value for s_{g1}. It then reduces $\widehat{f_2}$ both directly and because of its negative effect on s_{g1}, leading to a lower value for s_{g2}. By iteration, the shale gas boom will reduce all s_{gt} at least until the switch toward green innovation occurs.

8.3.2 Proof of Part ii)

We prove that emissions in the long-run must be decreasing following a shale gas boom for $\ln \gamma$ sufficiently small. To establish this result, we first show the following Lemma:

Lemma 2 *For $t > t_{switch}$, $s_{gt} > s_{g(t-1)}$.*

Proof. To establish the result, define:

$$f_{\gamma,t}\left(s_{g(t-1)},\gamma\right)=\frac{\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}}+\frac{1}{B_s}\right)^{-\varepsilon}\right)s_{g(t-1)}^{\psi}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta(\varepsilon-1)s_{g(t-1)}^{1-\psi}}s_{f(t-1)}^{\psi}}.$$

We then obtain:

$$\begin{aligned}
&\frac{\partial \ln f_{\gamma,t}}{\partial \ln \gamma}\\
=\;&\frac{\left(\begin{array}{c}\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}\right)^{-\varepsilon}\left(\varepsilon\frac{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}}{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}}-1\right)\\
+\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}+\frac{1}{B_s}\right)^{-\varepsilon}\left(\varepsilon\frac{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}}}{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-1}}+\frac{1}{B_c}}-1\right)\end{array}\right)}{\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{ct-1}}+\frac{1}{B_c}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{f(t-1)}^{1-\psi}}}{A_{st-2}}+\frac{1}{B_s}\right)^{-\varepsilon}}\eta s_{f(t-1)}^{1-\psi}-(\varepsilon-1)\,\eta s_{g(t-1)}^{1-\psi}\\
<\;&(\varepsilon-1)\,\eta\left(s_{f(t-1)}^{1-\psi}-s_{g(t-1)}^{1-\psi}\right)\leq 0,
\end{aligned}$$

since $t-1 \geq t_{swicth}$ so that $s_{f(t-1)} \geq s_{g(t-1)}$. Note that $f_{\gamma,t}\left(s_{g(t-1)},1\right) = f_{t-1}\left(s_{g(t-1)}\right) = 1$, therefore $f_t\left(s_{g(t-1)}\right) < f_{\gamma,t}\left(s_{g(t-1)},\gamma\right) < 1$, so that $s_{g(t-1)} < s_{gt}$. ■

We then establish the following Lemma:

Lemma 3 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d\ln A_{st_A} < 0$ and assume that $t_A < \infty$. Then $d\ln C_{gt_A} > d\ln A_{st_A}$.*

Proof. Since

$$\ln A_{ct} = \ln A_{c0} + \eta\left(\ln \gamma\right)\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi} \text{ and } \ln A_{st} = \ln A_{s0} + \eta\left(\ln \gamma\right)\sum_{\tau=1}^{t} s_{f\tau}^{1-\psi}$$

we have that

$$d\ln A_{ct} = d\ln A_{st} = \eta\left(1-\psi\right)\left(\ln\gamma\right)\sum_{\tau=1}^{t} s_{f\tau}^{-\psi} ds_{f\tau}.$$

By definition of t_A, $d\ln A_{c(t_A-1)} > 0$ and $d\ln A_{ct_A} < 0$, therefore we must have $ds_{ft_A} < 0$. Since $ds_{ft} > 0$ for $t \leq t_{switch}$, we must have $t_A > t_{switch}$.
In addition, we have:

$$d\ln C_{gt} = -\eta\left(1-\psi\right)\left(\ln\gamma\right)\sum_{\tau=1}^{t} s_{g\tau}^{-\psi} ds_{f\tau}.$$

Therefore, we can write

$$d\ln A_{st_A} - d\ln C_{gt_A} = \eta\left(1-\psi\right)\left(\ln\gamma\right)\left(\sum_{\tau=1}^{t_A}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau}\right).$$

We know that $ds_{ft} > 0$ for $t \leq t_{switch}$ and that $ds_{ft_A} < 0$, therefore ds_{ft} must change sign as t increases at least once. We indexes the times where ds_{ft} switches signs by t_{2p} and t_{2p+1}, such that ds_{ft} becomes negative at t_{2p+1} and positive at t_{2p} and p is a weakly positive integer in the integer set $[0, P-1]$ with $P \geq 1$. We denote by $t_0 = t_{switch}+1$ and $t_{2P} = t_A + 1$. We can then write

$$\begin{aligned}
& d\ln A_{st_A} - d\ln C_{gt_A} & (39)\\
= \;& \eta\left(1-\psi\right)\left(\ln\gamma\right)\left(\begin{array}{c}\sum_{\tau=1}^{t_{switch}}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ +\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau}\right)\end{array}\right) \\
= \;& \eta\left(1-\psi\right)\left(\ln\gamma\right)\left(\begin{array}{c}\sum_{\tau=1}^{t_{switch}}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ +\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(1-\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}\right) s_{f\tau}^{-\psi} ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(1-\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}\right) s_{f\tau}^{-\psi} ds_{f\tau}\right)\end{array}\right)
\end{aligned}$$

Using that $s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} < 0$ for $\tau \leq t_{switch}$, that $\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}$ is decreasing for $\tau > t_{switch}$ (following lemma 2), that $ds_{f\tau} > 0$ on intervals $[t_{2p}, t_{2p+1}-1]$ and negative otherwise, we get

$$d\ln A_{st_A} - d\ln C_{gt_A} < \eta\left(1-\psi\right)\left(\ln\gamma\right)\sum_{p=0}^{P-1}\left(1-\frac{s_{ft_{2p+1}}^{\psi}}{s_{gt_{2p+1}}^{\psi}}\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau}$$

By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$, therefore for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{f\tau}^{-\psi} ds_{f\tau} > 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$. Therefore, we get that

$$\begin{aligned}
& \sum_{p=P-2}^{P-1} \left(1 - \frac{s_{ft_{2p+1}}^{\psi}}{s_{gt_{2p+1}}^{\psi}}\right) \sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau} \\
= & \left(1 - \frac{s_{ft_{2P-3}}^{\psi}}{s_{gt_{2P-3}}^{\psi}}\right) \sum_{\tau=t_{2P-4}}^{t_{2P-2}-1} s_{f\tau}^{-\psi} ds_{f\tau} + \left(1 - \frac{s_{ft_{2P-1}}^{\psi}}{s_{gt_{2P-1}}^{\psi}}\right) \sum_{\tau=t_{2P-2}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \\
< & \left(1 - \frac{s_{ft_{2P-3}}^{\psi}}{s_{gt_{2P-3}}^{\psi}}\right) \sum_{\tau=t_{2P-4}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau}.
\end{aligned}$$

Iterating, we get

$$d \ln A_{st_A} - d \ln C_{gt_A} < \eta (1-\psi) (\ln \gamma) \left(1 - \frac{s_{ft_1}^{\psi}}{s_{gt_1}^{\psi}}\right) \sum_{\tau=t_{switch}+1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \leq 0.$$

Therefore $d \ln C_{gt_A} > d \ln A_{st_A}$, q.e.d. ■

We establish a symmetric lemma:

Lemma 4 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d \ln C_{gt_A} > 0$ and assume that $t_A < \infty$. Then $d \ln C_{gt_A} > d \ln A_{st_A}$.*

Proof. The beginning of the proof is the same as in the previous lemma: $d \ln C_{gt_A} > 0$ requires that $ds_{ft_A} < 0$, which implies $t_A > t_{switch}$ and that ds_{ft} switches sign an odd number of times. We use (39) to write:

$$\begin{aligned}
& d \ln A_{st_A} - d \ln C_{gt_A} \\
= & \eta (1-\psi) (\ln \gamma) \left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}} \left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ + \sum_{p=0}^{P-1} \left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1} \left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}} - 1\right) s_{g\tau}^{-\psi} ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1} \left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}} - 1\right) s_{g\tau}^{-\psi} ds_{f\tau} \right) \end{array} \right) \\
< & \eta (1-\psi) (\ln \gamma) \sum_{p=0}^{P-1} \left(\frac{s_{gt_{2p+1}}^{\psi}}{s_{ft_{2p+1}}^{\psi}} - 1\right) \sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{g\tau}^{-\psi} ds_{f\tau},
\end{aligned}$$

following the same logic as before. By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{g\tau}^{-\psi} ds_{g\tau} > 0$, therefore for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{g\tau}^{-\psi} ds_{g\tau} < 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi} ds_{g\tau} > 0$. Given that $ds_{g\tau} = -ds_{f\tau}$, then $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi} ds_{g\tau} < 0$. Using exactly the same reasoning as before, we obtain:

$$d \ln A_{st_A} - d \ln C_{gt_A} < 0.$$

■

We can now establish the result. Using (15), (16) and (17), we get.

$$P=\left(\xi_c\kappa_c^{\varepsilon}\left(\frac{C_c}{C_E}\right)^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}\left(\frac{C_s}{C_E}\right)^{\varepsilon}\right)C_E\frac{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda}A_P^{\lambda-1}}L.$$

Therefore, for a large t, as C_{gt} grows faster than C_{ct} or C_{st}, we get that:

$$P_t\rightarrow\frac{\xi_c\kappa_c^{\varepsilon}C_{ct}^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}{\kappa_g^{\varepsilon}}\frac{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon}{\varepsilon-1}(\lambda-1)}C_{gt}^{\lambda-\varepsilon}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon}{\varepsilon-1}(\lambda-1)}C_{gt}^{\lambda-1}+(1-\nu)^{\lambda}A_{Pt}^{\lambda-1}}L.$$

Using $d\ln A_{ct}=d\ln A_{st}$, this implies that

$$\begin{aligned}d\ln P_t\quad&\rightarrow\quad-\left(\varepsilon-1+\frac{(1-\lambda)(1-\nu)^{\lambda}A_P^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon(\lambda-1)}{\varepsilon-1}}C_{gt}^{\lambda-1}+(1-\nu)^{\lambda}A_P^{\lambda-1}}\right)d\ln C_{gt}\\&+\varepsilon\frac{\xi_c\kappa_c^{\varepsilon}C_c^{\varepsilon}\frac{C_c}{A_c}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}\frac{C_s}{A_s}}{\xi_c\kappa_c^{\varepsilon}C_c^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}d\ln A_{ct}+\varepsilon\frac{\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}{\xi_c\kappa_c^{\varepsilon}C_c^{\varepsilon}+\xi_s\kappa_s^{\varepsilon}C_s^{\varepsilon}}\frac{C_s}{B_s}d\ln B_s.\end{aligned}$$

Therefore emissions will increase asymptotically following the shale gas boom provided that C_{gt} decreases and A_{ct} and A_{st} increase. We prove that this is the case by contradiction.

Assume that C_{gt} does not decrease for all t. Denote by t_A the first time that $d\ln C_{gt}>0$, then if $\ln\gamma$ is small enough, it must be that $d\ln C_{gt_A}\approx d\ln C_{gt_A-1}\approx 0$, so that $d\ln A_{ct_A}<0$ according to Lemma 3 and A_{ct} must decline at some point.

Assume now that A_{ct} does not increase for all t. Denote by t_A the first time that $d\ln A_{ct_A}<0$, as argued before it must be that $ds_{ft_A}<0$. Log differentiate f_{t_A} to obtain:

$$\begin{aligned}d\ln f_{t_A}\quad&=\quad-(\varepsilon-1)\,d\ln C_{g(t_A-1)}+\frac{\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}\frac{C_{st_A}}{B_s}\varepsilon d\ln B_s\\&+\frac{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}\left(\varepsilon\frac{C_{ct_A}}{A_{ct_A}}-1\right)+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}\left(\varepsilon\frac{C_{st_A}}{A_{st_A}}-1\right)}{\frac{1}{A_{ct_A}}\kappa_c^{\varepsilon}C_{ct_A}^{\varepsilon}+\frac{1}{A_{st_A}}\kappa_s^{\varepsilon}C_{st_A}^{\varepsilon}}d\ln A_{c(t-1)}.\end{aligned}$$

Following a shale gas boom $d\ln B_s>0$. Since $d\ln A_{ct_{A-1}}>0>d\ln A_{ct_A}$, then for $\ln\gamma$ small, we have $d\ln A_{ct_{A-1}}\approx d\ln A_{ct_A}\approx 0$, using lemma 2 we have that $d\ln C_{g(t_A-1)}<0$, so that we must have $d\ln f_{t_A}>0$ but this implies that $ds_{ft_A}>0$ which is a contradiction. Therefore A_{ct} must increase for all t's.

This establishes that emissions must increase asymptotically.

8.4 Extending the theoretical results to the calibrated model

8.4.1 Equilibrium

Following similar steps as those used in the baseline model to derive (10) and using the definition of E_{ft}, we get that given technologies and the level of overall demand for energy E_t, the demand for the different type of electricities are given by:

$$E_{c,t}=\kappa_c^{\sigma}\left(\frac{C_{ct}}{C_{ft}}\right)^{\sigma}E_{ft}\text{ and }E_{s,t}=\kappa_s^{\sigma}\left(\frac{C_{st}}{C_{ft}}\right)^{\sigma}E_{ft},\qquad(40)$$

within fossil fuels and

$$E_{f,t}=\left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon}E_t \text{ and } E_{g,t}=\kappa_g^{\varepsilon}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon}E_t, \tag{41}$$

for fossil fuel and clean energy. The quantity of energy is itself given by (30).

To determine the level of E_t (that is to solve for the input allocation), note that cost minimization in energy production and the production of good Y_{Pt} leads directly:

$$\frac{K_{Et}}{L_{Et}}=\frac{1-\phi}{\phi}\frac{w_t}{\rho_t}, \tag{42}$$

$$\frac{K_{Pt}}{L_{Pt}}=\frac{1-\varphi}{\varphi}\frac{w_t}{\rho_t}. \tag{43}$$

Profit maximization in the final good sector leads to the relative demand:

$$\frac{\left(\frac{w_t}{\phi}\right)^{\phi}\left(\frac{\rho_t}{1-\phi}\right)^{1-\phi}}{\left(\frac{w_t}{\varphi}\right)^{\varphi}\left(\frac{\rho_t}{1-\varphi}\right)^{1-\varphi}}=\frac{\nu\widetilde{A}_{Et}^{\frac{\lambda-1}{\lambda}}C_{Et}^{\frac{\lambda-1}{\lambda}}\left(L_{Et}^{\phi}K_{Et}^{1-\phi}\right)_t^{\frac{-1}{\lambda}}}{(1-\nu)A_{Pt}^{\frac{\lambda-1}{\lambda}}\left(L_{Pt}^{\varphi}K_{Pt}^{1-\varphi}\right)^{\frac{-1}{\lambda}}}. \tag{44}$$

Normalizing the price of the final good to 1, we obtain:

$$1=(1-\nu)^{\lambda}\left(\frac{\gamma}{A_{Pt}}\left(\frac{w_t}{\varphi}\right)^{\varphi}\left(\frac{\rho_t}{1-\varphi}\right)^{1-\varphi}\right)^{1-\lambda}+\nu^{\lambda}\left(\frac{\gamma}{\widetilde{A}_{Et}C_{Et}}\left(\frac{w_t}{\phi}\right)^{\phi}\left(\frac{\rho_t}{1-\phi}\right)^{1-\phi}\right)^{1-\lambda}. \tag{45}$$

Together with the two factor market clearing equations, (42), (43), factor resource constraints ($\overline{L}=L_{Et}+L_{Pt}$; $K_t=K_{Et}+K_{Pt}$), (44) and (45) determine the equilibrium value of w_t, ρ_t, L_{Pt}, K_{Pt}, L_{Et}, K_{Et}. In particular, the calibration results are based on a re-computation of the macroeconomic equilibrium using these conditions.

From this, we obtain that $E_t=g\left(C_{Et}\right)$ with g increasing. This is intuitive but to derive it formally, note that the system simplifies in two equations which determine $\frac{w_t}{\rho_t}$ and E_t:

$$\left(\frac{1-\varphi}{\varphi}L\frac{w_t}{\rho_t}+\left(\frac{1}{\phi}-\frac{1}{\varphi}\right)\left(\frac{\phi}{1-\phi}\right)^{1-\phi}\frac{E_t}{C_{Et}}\left(\frac{w_t}{\rho_t}\right)^{\phi}\right)=K, \tag{46}$$

$$\frac{\varphi^{\varphi+(1-\varphi)\frac{1}{\lambda}}(1-\varphi)^{(1-\varphi)\left(1-\frac{1}{\lambda}\right)}}{\phi^{\phi}(1-\phi)^{1-\phi}}\left(\frac{w_t}{\rho_t}\right)^{(\phi-\varphi)-\frac{1}{\lambda}(1-\varphi)}=\frac{\nu\widetilde{A}_{Et}^{\frac{\lambda-1}{\lambda}}C_{Et}}{(1-\nu)A_{Pt}^{\frac{\lambda-1}{\lambda}}}\left(\frac{L}{E_t}-\frac{1}{C_{Et}\left(\frac{1-\phi}{\phi}\frac{w_t}{\rho_t}\right)^{1-\phi}}\right)^{\frac{1}{\lambda}}. \tag{47}$$

Assuming that $\varphi>\phi$, the first equation traces a negative relationship between $\frac{w_t}{\rho_t}$ and E_t while an increase in C_{Et} moves the relationship to the right in the $(E_t, w_t/\rho_t)$ space. The second equation leads to a positive relationship which also moves to the right as C_{Et} increases. Therefore E_t increases in C_{Et}. By symmetry this also holds when $\varphi\leq\phi$.

We can then write the equilibrium level of pollution as $P_t=\xi_{Et}E_t$, where the average effective emission rate per unit of electricity is now given by

$$\xi_{E,t}=\left(\xi_{c,t}\kappa_c^{\sigma}\left(\frac{C_{ct}}{C_{ft}}\right)^{\sigma}+\xi_{s,t}\kappa_s^{\sigma}\left(\frac{C_{st}}{C_{ft}}\right)^{\sigma}\right)\left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon}. \tag{48}$$

8.4.2 Comparative statics

We now derive the comparative statics results. We get that

$$\frac{\partial \ln \xi_E}{\partial \ln B_{st}} = \underbrace{\varepsilon \frac{\partial \ln (C_{ft}/C_{Et})}{\partial \ln B_{st}}}_{Sub_g\text{: substitution effect away from green}} + \underbrace{\frac{\partial \ln \left(\xi_c \kappa_c^\sigma \left(\frac{C_{ct}}{C_{ft}}\right)^\sigma + \xi_s \kappa_s^\sigma \left(\frac{C_{st}}{C_{ft}}\right)^\sigma \right)}{\partial \ln (B_{st})}}_{Sub_f\text{: substitution within fossil fuels}}$$

The substitution effect away from green electricity is naturally positive:

$$Sub_g = \varepsilon \frac{\kappa_g^\varepsilon A_{gt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}} \frac{\kappa_s^\sigma C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}} \frac{B_{st}}{C_{st}}. \tag{49}$$

We use (26), (40) and the fact that the price of the fossil fuel aggregate is given by

$$p_{ft} = \left(\kappa_c^\sigma p_{ct}^{1-\sigma} + \kappa_s^\sigma p_{st}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = \frac{\gamma c_{Et}}{C_{ft}},$$

to get that the expenditure share of gas electricity in fossil fuel electricity obeys:

$$\theta_{sft} = \frac{p_{st} E_{st}}{p_{ft} E_{ft}} = \frac{\kappa_s^\sigma C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}}.$$

The expenditure share on clean energy, using (26), (29) and (41), is given by:

$$\Theta_{gt} = \frac{p_{gt} E_{gt}}{p_{Et} E_t} = \frac{\kappa_g^\varepsilon A_{gt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}.$$

We then can rewrite (49) as

$$Sub_g = \varepsilon \Theta_{gt} \theta_{sft} \frac{B_{st}}{C_{st}}.$$

Further, we have

$$\begin{aligned} Sub_f &= -\sigma \frac{\kappa_c^\sigma C_{ct}^{\sigma-1} \kappa_s^\sigma C_{st}^{\sigma-1}}{\left(\xi_{c,t} \kappa_c^\sigma C_{ct}^\sigma + \xi_{s,t} \kappa_s^\sigma C_{st}^\sigma\right) C_{ft}^{\sigma-1}} \left(\xi_c C_{ct} - \xi_s C_{st}\right) \frac{B_{st}}{C_{st}} \\ &= -\sigma \theta_{sft} \frac{P_{c,t}}{P_t} \left(1 - \frac{\xi_s C_{st}}{\xi_c C_{ct}}\right) \frac{B_{st}}{C_{st}}, \end{aligned}$$

where

$$\frac{P_{ct}}{P_t} = \frac{\xi_c \kappa_c^\sigma C_{ct}^\sigma}{\xi_{c,t} \kappa_c^\sigma C_{ct}^\sigma + \xi_{s,t} \kappa_s^\sigma C_{st}^\sigma}$$

is the pollution share of coal based electricity. Therefore the substitution effect within fossil fuel is negative as long as $\xi_c C_{ct} > \xi_s C_{st}$ holds. Overall, we obtain equation (31).

To obtain $\frac{\partial \ln E_t}{\partial \ln C_{Et}}$, we log differentiate (46) and (47), from which we get:

$$\frac{1-\varphi}{\varphi} L \frac{\partial \ln \left(\frac{w_t}{\rho_t}\right)}{\partial \ln C_{Et}} + \left(\frac{1}{\phi} - \frac{1}{\varphi}\right) L_E \left(\frac{\partial \ln E_t}{\partial \ln C_{Et}} - 1 + \phi \frac{\partial \ln \left(\frac{w_t}{\rho_t}\right)}{\partial \ln C_{Et}}\right) = 0, \tag{50}$$

$$\left(\left(\phi-\varphi\right)-\frac{1}{\lambda}\left(1-\varphi\right)\right)\frac{\partial\ln\left(\frac{w_t}{\rho_t}\right)}{\partial\ln C_{Et}}=1+\frac{1}{\lambda}\left(-\frac{\partial\ln E_t}{\partial\ln C_{Et}}\frac{L}{L_P}+\frac{L_E}{L_P}\left(1+\left(1-\phi\right)\frac{\partial\ln\left(\frac{w_t}{\rho_t}\right)}{\partial\ln C_{Et}}\right)\right), \tag{51}$$

where we used that $L_E=\left(\frac{\phi}{1-\phi}\frac{\rho_t}{w_t}\right)^{1-\phi}\frac{E}{C_E}$. Re-arranging terms we then get that:

$$\frac{\partial\ln E_t}{\partial\ln C_{Et}}=1-\frac{\left(1-\lambda\right)\phi\left(\left(1-\varphi\right)L_P+\left(1-\phi\right)L_E\right)L_P}{\lambda\left(\varphi-\phi\right)^2L_PL_E+\left(\varphi L_E+\phi L_P\right)\left(\left(\left(1-\varphi\right)L_P+\left(1-\phi\right)L_E\right)\right)},$$

so that $\frac{\partial\ln E_t}{\partial\ln C_{Et}}\in(0,1)$: since energy and the production inputs are complement, $\lambda<1$, resources move toward the production input when the productivity of the energy sector goes up). The scale effect is given by

$$\frac{\partial\ln E_t}{\partial\ln B_{st}}=\frac{\partial\ln E_t}{\partial\ln C_{Et}}\frac{\partial\ln C_{Et}}{\partial\ln B_{st}}=\Theta_{st}\frac{C_{st}}{B_{st}}\frac{\partial\ln E_t}{\partial\ln C_{Et}}$$

We then obtain

$$\frac{\partial\ln P_t}{\partial\ln B_{st}}=\frac{\Theta_{st}C_{st}}{B_{st}}\left[\underbrace{\varepsilon\frac{\Theta_{gt}}{\Theta_{ft}}}_{\text{substitution away from green}}-\underbrace{\frac{1}{\Theta_{ft}}\sigma\frac{P_{ct}}{P_t}\left(1-\frac{\xi_sC_{st}}{\xi_cC_{ct}}\right)}_{\text{substitution within fossil fuels}}+\underbrace{\frac{\partial\ln E_t}{\partial\ln C_{Et}}}_{\text{scale effect}}\right].$$

For $\xi_c>>\xi_s$, we get that

$$\varepsilon\frac{\Theta_{gt}}{\Theta_{ft}}-\frac{1}{\Theta_{ft}}\sigma\frac{P_{ct}}{P_t}\left(1-\frac{\xi_sC_{st}}{\xi_cC_{ct}}\right)|_{\xi_c>>\xi_s}\approx-\varepsilon-\frac{1}{\Theta_{ft}}\left(\sigma-\varepsilon\right).$$

Therefore since $\sigma\geq\varepsilon>1$ and since $\frac{\partial\ln E_t}{\partial\ln C_{Et}}<1$, we get that $\frac{\partial\ln P_t}{\partial\ln B_{st}}<0$ for $\xi_c>>\xi_s$.

Furthermore, if C_{gt} grows while C_{st} and C_{ct} stay constant, then $\Theta_{gt}\to1$, so that

$$\varepsilon\frac{\Theta_{gt}}{\Theta_{ft}}-\frac{1}{\Theta_{ft}}\sigma\frac{P_{ct}}{P_t}\left(1-\frac{\xi_sC_{st}}{\xi_cC_{ct}}\right)|_{\Theta_{gt}\to1}\approx\frac{1}{\Theta_{ft}}\left(\varepsilon-\sigma\frac{P_{ct}}{P_t}\left(1-\frac{\xi_sC_{st}}{\xi_cC_{ct}}\right)\right).$$

As $\Theta_{ft}\to0$, the substitution effect dominates the scale effect for t large enough, so that a shale gas boom at $t=0$ will eventually lead to an increase in emissions only if

$$\varepsilon>\frac{P_{c0}}{P_0}\left(1-\frac{\xi_sC_{s0}}{\xi_cC_{c0}}\right)\sigma.$$

We then obtain the modified Proposition 1:

Proposition 6 *i) A shale gas boom (that is a one time increase in B_s at time $t=0$) leads to a decrease in emissions in the short-run provided that the natural gas is sufficiently clean compared to coal (for ξ_s/ξ_c small enough).*
ii) If all future innovations in the energy sector occur in clean technologies, then for t large enough, the shale gas boom will increase emissions if $\varepsilon>\frac{P_{c0}}{P_0}\left(1-\frac{\xi_sC_{s0}}{\xi_cC_{c0}}\right)\sigma$.

8.4.3 Innovation allocation

Assume that innovation occurs as in Section 3.3. Then the expected profits of an innovator in clean technologies are still given by (21) and those of an innovator in fossil fuel power plant technologies by (22). Therefore (23) is replaced by:

$$\frac{\Pi_{gt}}{\Pi_{ft}} = \frac{\eta_g s_{gt}^{-\psi} \kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1}}{\eta_f s_{ft}^{-\psi} \left(\frac{\kappa_c^{\sigma} C_{ct}^{\sigma}}{A_c} + \frac{\kappa_s^{\sigma} C_{st}^{\sigma}}{A_s}\right) C_{ft}^{\varepsilon-\sigma}} = 1.$$

We can rewrite this equation as:

$$f\left(s_{gt}, A_{ct-1}, B_{ct}, A_{st-1}, B_{st}, C_{gt-1}\right) = 1$$

where the function f is now defined as

$$\begin{aligned} f \quad \equiv \quad & \frac{\eta_f s_{gt}^{\psi}\left(\kappa_c^{\sigma} \frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{-\sigma} + \kappa_s^{\sigma} \frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}}\right)^{-\sigma}\right)}{\eta_g \kappa_g^{\varepsilon} s_{ft}^{\psi} C_{gt-1}^{\varepsilon-1} \gamma^{\eta_g s_{gt}^{1-\psi}(\varepsilon-1)}} \\ & \times \left(\kappa_c^{\sigma}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{1-\sigma} + \kappa_s^{\sigma}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}}\right)^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}} \end{aligned}$$

We then get that

$$\begin{aligned} & \frac{\partial \ln f}{\partial \ln s_{gt}} \\ = \quad & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln\gamma) s_{gt}^{1-\psi} + \psi \frac{s_{gt}}{s_{ft}} \\ & + \frac{\eta_f (1-\psi) \ln(\gamma) s_{ft}^{1-\psi} \frac{s_{gt}}{s_{ft}} \left(\kappa_c^{\sigma} \frac{C_{ct}^{\sigma}}{A_{ct}}\left(1 - \sigma \frac{B_{ct}}{B_{ct}+A_{ct}}\right) + \kappa_s^{\sigma} \frac{C_{st}^{\sigma}}{A_{st}}\left(1 - \sigma \frac{B_{st}}{B_{st}+A_{st}}\right)\right)}{\kappa_c^{\sigma} \frac{C_{ct}^{\sigma}}{A_{ct}} + \kappa_s^{\sigma} \frac{C_{st}^{\sigma}}{A_{st}}} \\ & + \frac{\varepsilon-\sigma}{\sigma-1} \frac{\partial \ln \left(\kappa_c^{\sigma}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{1-\sigma} + \kappa_s^{\sigma}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}}\right)^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}}}{\partial \ln s_{gt}} \\ \geq \quad & \psi - \eta_g (\varepsilon - 1)(1-\psi)(\ln\gamma) s_{gt}^{1-\psi} + \left(\psi - \eta_f (\sigma-1)(1-\psi)(\ln\gamma) s_{ft}^{1-\psi}\right) \frac{s_{gt}}{s_{ft}}, \end{aligned}$$

since $\sigma \geq \varepsilon$ and $\left(\kappa_c^{\sigma}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{ct-1}} + \frac{1}{B_{ct}}\right)^{1-\sigma} + \kappa_s^{\sigma}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{st-1}} + \frac{1}{B_{st}}\right)^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}}$ is decreasing in s_{gt}. Therefore f increases in s_g provided that $(\ln\gamma)\max\left(\eta_g(\varepsilon-1), \eta_f(\sigma-1)\right) < \psi/(1-\psi)$ in which case the equilibrium is uniquely defined.

We get that

$$\frac{\partial \ln f}{\partial \ln B_{st}} = \frac{\kappa_c^{\sigma} C_{ct}^{\sigma-1}\left(\sigma \frac{C_{st}}{A_{st}} - (\sigma-\varepsilon)\frac{C_{ct}}{A_{ct}}\right) + \varepsilon \kappa_s^{\sigma} \frac{C_{st}^{\sigma}}{A_{st}}}{\left(\kappa_c^{\sigma} \frac{C_{ct}^{\sigma}}{A_{ct}} + \kappa_s^{\sigma} \frac{C_{st}^{\sigma}}{A_{st}}\right)\left(\kappa_c^{\sigma} C_{ct}^{\sigma-1} + \kappa_s^{\sigma} C_{st}^{\sigma-1}\right)} \kappa_s^{\sigma} \frac{C_{st}^{\sigma}}{B_{st}},$$

so that $\frac{\partial \ln f}{\partial \ln B_{st}} > 0$ if and only if

$$\varepsilon\left(1+\frac{\kappa_s^\sigma C_{st}^{\sigma-1}}{\kappa_c^\sigma C_{ct}^{\sigma-1}}\right) > (\sigma-\varepsilon)\left(\frac{1+\frac{A_{st}}{B_{st}}}{1+\frac{A_{ct}}{B_{ct}}}-1\right),$$

that is provided that either σ is close enough to ε or $\frac{A_{st}}{B_{st}}$ is not too large relative to $\frac{A_{ct}}{B_{ct}}$—in fact, if $\sigma > \varepsilon$, the above inequality will be violated for B_{st} low enough. Intuitively, innovation toward the fossil fuel sector is higher when there is a large gap between productivity in coal and natural gas technology; an increase in B_{st} may not lead to more innovation in fossil fuel technologies when it is not very useful, that is when B_{st} is low.

Further,

$$\begin{aligned}
\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} &= \left(\frac{\sigma\frac{C_{ct}}{A_{ct}}-1}{\kappa_c^\sigma\frac{C_{ct}^\sigma}{A_c}+\kappa_s^\sigma\frac{C_{st}^\sigma}{A_s}}+\frac{\varepsilon-\sigma}{\kappa_c^\sigma C_{ct}^{\sigma-1}+\kappa_s^\sigma C_{st}^{\sigma-1}}\right)\kappa_c^\sigma\frac{C_{ct}^\sigma}{A_c} \\
&= \left(\frac{\left(\varepsilon\frac{C_{ct}}{A_{ct}}-1\right)\left(\kappa_c^\sigma C_{ct}^{\sigma-1}+\kappa_s^\sigma C_{st}^{\sigma-1}\right)-(\sigma-\varepsilon)\left(\frac{C_{st}}{A_s}-\frac{C_{ct}}{A_{ct}}\right)\kappa_s^\sigma C_{st}^{\sigma-1}}{\left(\kappa_c^\sigma\frac{C_{ct}^\sigma}{A_c}+\kappa_s^\sigma\frac{C_{st}^\sigma}{A_s}\right)\left(\kappa_c^\sigma C_{ct}^{\sigma-1}+\kappa_s^\sigma C_{st}^{\sigma-1}\right)}\right)\kappa_c^\sigma\frac{C_{ct}^\sigma}{A_c}
\end{aligned}$$

so that $\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} > 0$ if and only if

$$\left(\varepsilon-\frac{A_{ct}}{C_{ct}}\right)\left(1+\frac{\kappa_c^\sigma C_{ct}^{\sigma-1}}{\kappa_s^\sigma C_{st}^{\sigma-1}}\right) > (\sigma-\varepsilon)\left(\frac{1+\frac{A_{ct}}{B_{ct}}}{1+\frac{A_{st}}{B_{st}}}-1\right).$$

By symmetry we get that $\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} > 0$ if and only if

$$\left(\varepsilon-\frac{A_{st}}{C_{st}}\right)\left(1+\frac{\kappa_s^\sigma C_{st}^{\sigma-1}}{\kappa_c^\sigma C_{ct}^{\sigma-1}}\right) > (\sigma-\varepsilon)\left(\frac{1+\frac{A_{st}}{B_{st}}}{1+\frac{A_{ct}}{B_{ct}}}-1\right).$$

Therefore we obtain similar comparative statics as in the baseline model provided that either $\frac{A_{st}}{B_{st}}$ and $\frac{A_{ct}}{B_{ct}}$ are not too far away or σ is not too large relative to ε. Therefore Proposition 3 is modified and becomes:

Proposition 7 *Assume that* $(\ln\gamma)\max\left(\eta_g(\varepsilon-1),\eta_f(\sigma-1)\right) < \frac{\psi}{1-\psi}$. *Then a shale gas boom at* $t=0$ *(an increase in* B_{s0}*) leads to a decrease in innovation in green technology at* $t=0$ *(a decrease in* s_{g0}*) provided that* σ *is not too large relative to* ε *and* $\frac{A_{s0}}{B_{s0}}$ *is not too large relative to* $\frac{A_{c0}}{B_{c0}}$ *Furthermore if (i)* $\min\left(\frac{B_{ct}}{A_{ct-1}},\frac{B_{st}}{A_{st-1}}\right) > \frac{\gamma^{\eta_f}}{\varepsilon-1}$ *and (ii) either* σ *is close to* ε *or* $\frac{A_{st-1}}{B_{st}}$ *is close to* $\frac{A_{ct-1}}{B_{ct}}$ *for all* $t>0$*, then green innovation declines for all* $t>0$.

9 Appendix C: Calibration details

9.1 Calibration of electricity substitution parameter λ

The elasticity of substitution λ is calibrated based on the literature with appropriate modification since we are focused on electricity. The literature, when differentiating electricity, typically estimates or calibrates parameters for nested final goods production functions with electricity and non-electric energy. In the 'background' of our framework we might thus imagine a production function:

$$F_t = \{\gamma_Y (A_{Yt} Y_t)^{\frac{\sigma_1 - 1}{\sigma_1}} + (1-\gamma_Y) \left[\gamma_{Elec} (E_{Elec})^{\frac{v_1 - 1}{v_1}} + (1-\gamma_{Elec})(E_{NonElec})^{\frac{v_1 - 1}{v_1}}\right]^{\left(\frac{v_1 - 1}{v_1}\right)\left(\frac{\sigma_1 - 1}{\sigma_1}\right)} \}^{\frac{\sigma_1 - 1}{\sigma_1}} \tag{52}$$

For our calibration, we need $\sigma_{Y_P,Elec}$ and/or σ_{Elec,Y_P}. The literature provides some examples or estimates of the parameters in (52). The Morishima elasticities are then (Anderson and Moroney, 1993):[20]

$$\begin{aligned} \sigma_{Elec,Y_P} &= \gamma_{Elec} \cdot \sigma_1 + (1-\gamma_{Elec}) \cdot v_1 \\ \sigma_{Y_P,Elec} &= \sigma_1 \end{aligned}$$

Standard values for $\sigma_1 \sim \sigma_{KL,E}$ from the literature are $0.4 - 0.5$ (e.g., Chen et al., 2017; Van der Werf, 2008; Böringer and Rutherford, 2008; Bosetti et al., 2007). As several major models moreover assume $v_1 = 0.5$ (e.g., Chen et al., 2017; Bosetti et al., 2007), for our purposes, we would have $\sigma_{Elec,Y_P} = \sigma_{Y_P,Elec} = 0.5$ for any value of γ_{Elec}. We thus keep $\lambda = 0.5$ as a benchmark, but consider considerably lower values given the empirical evidence of very low capital-labor and energy substitution elasticities presented by Hassler, Krusell, and Olovsson (2012).

9.2 Benchmark employment in electricity (and resource) sectors

BLS Industry Employment Statistics		
	2009 ('000s)	Share
Mining - Oil and gas extraction	162.1	
Mining - Coal mining	82.0	
Mining - Support activities for mining	287.6	
Utilities - Electric power generation,	404.1	
transmission and distribution		
Utilities - Natural gas distribution	108.7	
Manufacturing - Engine, turbine, and	95.3	
power transmission equipment manufacturing		
Total Energy:	1,139.8	
Total:	143,053.1	0.7968%

9.3 Profit Margins and γ Calibration

The following tables present after-tax profits per dollar of sales for corporations in three relevant industries ("Petroleum and coal products," "All Durable Manufacturing," and "All Wholesale Trade") for 2004-2014 (Source: U.S. Census Bureau Quarterly Financial Report for Manufacturing, Mining, and Trade Corporations, 2004-2014).

[20] Intuitively, they are not symmetric (whereas the Allen-Uzawa elasticities would be) because a change in the price of electricity also changes the relative prices of electric and non-electric energy, whereas a change in the price of Y_P does not (Frieling and Madner, 2016).

Petroleum and Coal Products: Profits per Dollar of Sales (cents)					
After-tax (cents/dollar)	Q1	Q2	Q3	Q4	Avg.
2004	8.2	9.5	8.9	10.6	9.3
2005	10.1	9.2	8.4	9.6	9.325
2006	9.8	11.5	11.3	9.6	10.55
2007	10.9	10.6	8.7	8.2	9.6
2008	8.4	8.0	10.2	-8.5	4.525
2009	6.5	4.8	5.9	3.9	5.275
2010	6.8	0.7	6.4	6.2	5.025
2011	8.4	7.9	7.8	6.7	7.700
2012	6.8	8.6	6.8	6.6	7.200
2013	7.5	3.8	4.8	5.7	5.450
2014	6.3	5.7	6.3	4.8	5.775
Average					**7.25**

All Durable Manufacturing: Profits per Dollar of Sales (cents)					
After-tax (cents/dollar)	Q1	Q2	Q3	Q4	Avg.
2004	5.7	7.0	5.8	5.7	6.05
2005	10.1	9.2	8.4	9.6	9.325
2006	7.4	6.8	6.9	6.2	6.825
2007	6.6	7.9	2.4	5.8	5.675
2008	6.0	4.2	5.1	-7.1	2.05
2009	-1.9	0.6	4.6	5.0	2.075
2010	7.2	9.6	8.7	8.6	8.525
2011	9.5	10.3	9.6	9.3	9.675
2012	9.1	9.5	8.0	7.3	8.475
2013	9.0	9.2	9.5	9.2	9.225
2014	8.4	10.0	10.1	9.2	9.425
Average					**7.03**

All Wholesale Trade: Profits per Dollar of Sales (cents)					
After-tax (cents/dollar)	Q1	Q2	Q3	Q4	Avg.
2004	2.0	2.3	2.4	2.0	2.175
2005	1.9	2.3	2.1	2.4	2.175
2006	2.1	2.1	2.4	1.8	2.1
2007	2.1	2.3	2.0	1.7	2.025
2008	1.3	1.8	1.8	0.1	1.25
2009	-0.1	0.9	1.1	1.4	0.825
2010	1.2	1.7	1.7	1.4	1.5
2011	1.8	1.8	1.7	1.1	1.6
2012	1.3	2.0	1.8	1.3	1.6
2013	2.0	1.6	1.8	1.5	1.725
2014	1.7	1.8	2.2	1.2	1.725
Average					**1.7**

Weighted Average After-Tax Profits per Dollar of Sales (cents)		
Industry	Avg.	Income Share (2011)
Petroleum and coal products	7.25	.23
All durable manufacturing	7.03	.70
All wholesale trade	1.7	.06
Weighted Average:		6.69
$\Rightarrow$ Implied γ :		1.07

9.4 Equilibrium conditions matched by calibration

After the calibration of the parameters as described in sub-section 4.2, we solve for the remaining unknowns to satisfy the following set of equations at the initial observed labor shares L_{E0}, L_{P0}, GDP Y_0, aggregate capital K_0, energy prices, etc.:

$$
\begin{aligned}
\text{Unknowns} \quad &: \quad A_{g,0}, A_{c,0}, A_{s,0}, B_{c,0}, B_{s,0}, C_{f,0}, C_{E,0}, A_{P,0}, K_{E,0}, K_{P,0}, c_{E,0}, w_0, \rho_0 \\
Y_0 &= \left((1-\nu) \left(A_{P,0} L_{P,0}^{\varphi} K_{P,0}^{1-\varphi} \right)^{\frac{\lambda-1}{\lambda}} + \nu \left(\widetilde{A}_{E,0} E_0 \right)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}} \\
K_0 &= K_{E,0} + K_{P,0} \\
c_{E,0} &= \left(\frac{w_0}{\phi} \right)^{\phi} \left(\frac{\rho_0}{1-\phi} \right)^{1-\phi} \\
E_0 &= C_{E,0} L_{E,0}^{\phi} K_{E,0}^{1-\phi} \\
\frac{K_{E,0}}{L_{E,0}} &= \frac{1-\phi}{\phi} \frac{w_0}{\rho_0} \text{ and } \frac{K_{P,0}}{L_{P,0}} = \frac{1-\varphi}{\varphi} \frac{w_0}{\rho_0} \text{ from (42) and (43).} \\
A_{g,0} &= \frac{\gamma c_{E,0}}{p_{g,0}^{q}}, \; A_{c,0} = \frac{\gamma c_{E,0}}{p_{c,0}^{q}} \text{ and } A_{s,0} = \frac{\gamma c_{E,0}}{p_{s,0}^{q}} \\
B_{c,0} &= \frac{\gamma c_{E,0}}{p_{c,0}^{r}} \text{ and } B_{s,0} = \frac{\gamma c_{E,0}}{p_{s,0}^{r}} \\
C_{f,0} &= \left(\kappa_c^{\sigma} \left(\frac{1 + \Lambda_c \left(\underline{\mu_c} \right)}{A_{c,0}} + \frac{1}{B_{c,0}} \right)^{1-\sigma} + \kappa_s^{\sigma} \left(\frac{1 + \Lambda_s \left(\underline{\mu_s} \right)}{A_{s,0}} + \frac{1}{B_{s,0}} \right)^{1-\sigma} \right)^{\frac{1}{\sigma-1}} \\
C_{E,0} &= \left(\kappa_g^{\varepsilon} A_{g,0}^{\varepsilon-1} + C_{f,0}^{\varepsilon-1} \right)^{\frac{1}{\varepsilon-1}} \\
\frac{\gamma}{A_{P0}} \left(\frac{w_0}{\varphi} \right)^{\varphi} \left(\frac{\rho_0}{1-\varphi} \right)^{1-\varphi} &= (1-\nu) Y_0^{\frac{1}{\lambda}} \left(A_{P,0} L_{P,0}^{\varphi} K_{P,0}^{1-\varphi} \right)^{\frac{-1}{\lambda}}
\end{aligned}
\tag{53}
$$

As there are 14 equations but only 13 unknowns, we select parameters that minimize the sum of squared deviations between the model and the data.

9.4.1 Extended parameter value table

Finally, the parameters and initial endogenous unknowns whose values are not listed in Table (5) already are as follows in the benchmark:

Parameter	Value
$A_{g,0}$	0.1471
$A_{c,0}$	0.4264
$A_{s,0}$	0.4518
$B_{c,0}$	0.3346
$B_{s,0}$	0.1613
$C_{f,0}$	0.0333
$C_{E,0}$	0.0474
$A_{P,0}$	44.3196
$K_{E,0}$	1.1927e+03
$K_{P,0}$	4.9375e+04
$c_{E,0}$	7.1771
w_0	899.19
ρ_0	0.0890