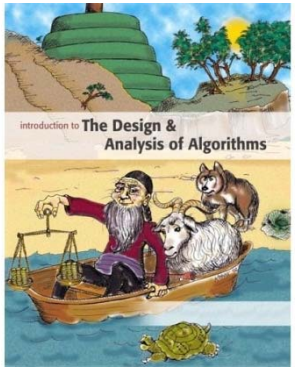
计算机科学与技术系
Department of Computer Science and Technology

南京大學
NANJING UNIVERSITY

Introduction to

Algorithm Design and Analysis

[1] Model of Computation



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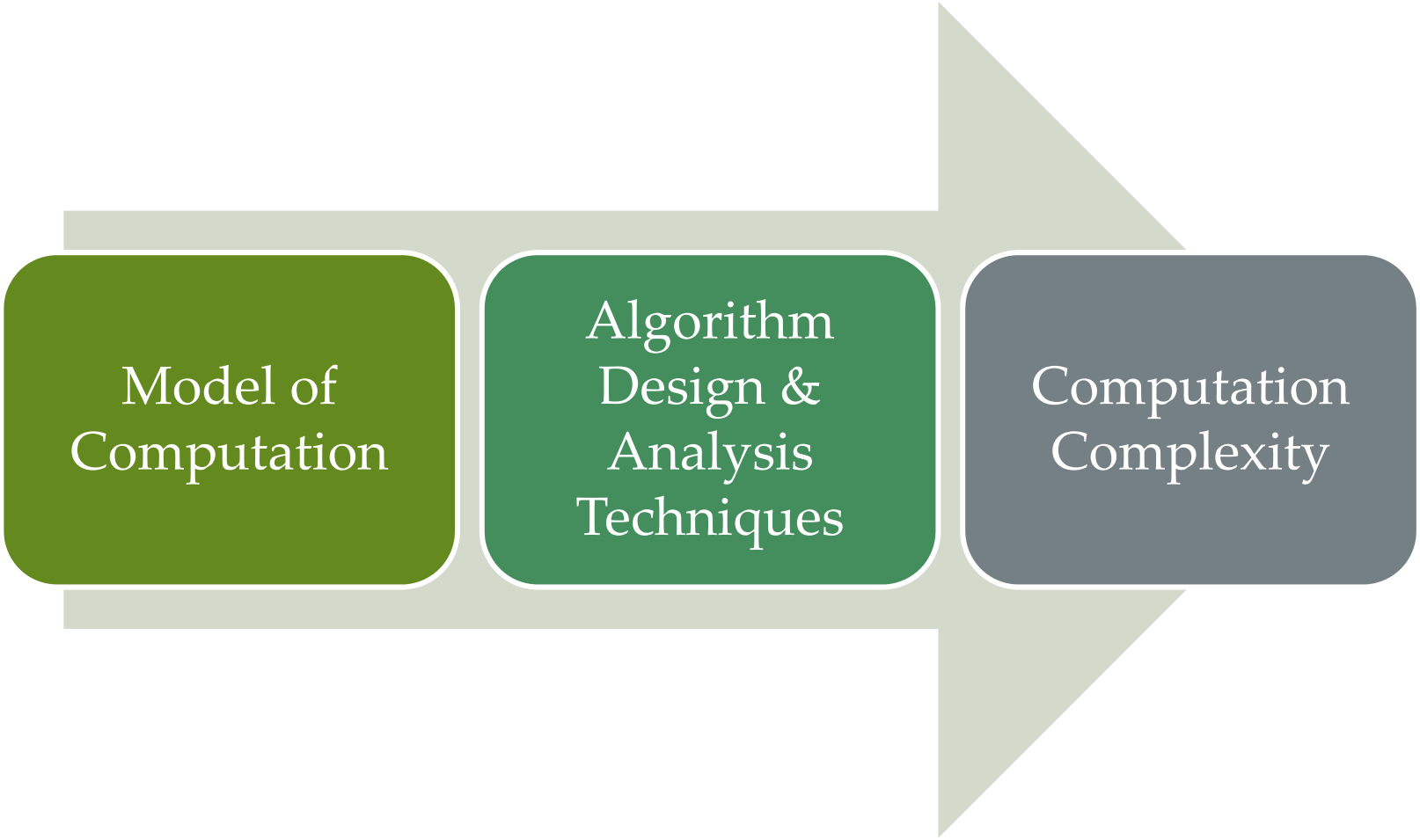
Institute of Computer Software
Nanjing University

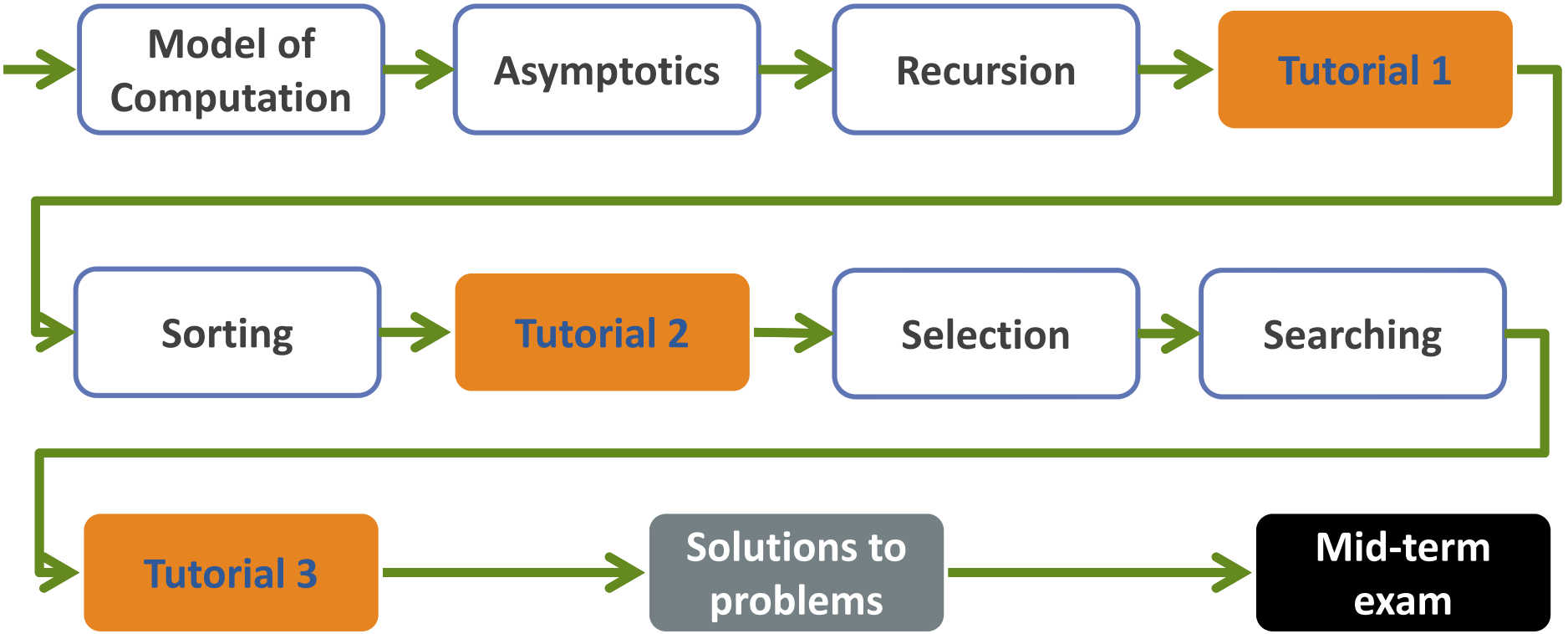
Course Information

- **Syllabus**
- **Textbook**
- **Website**

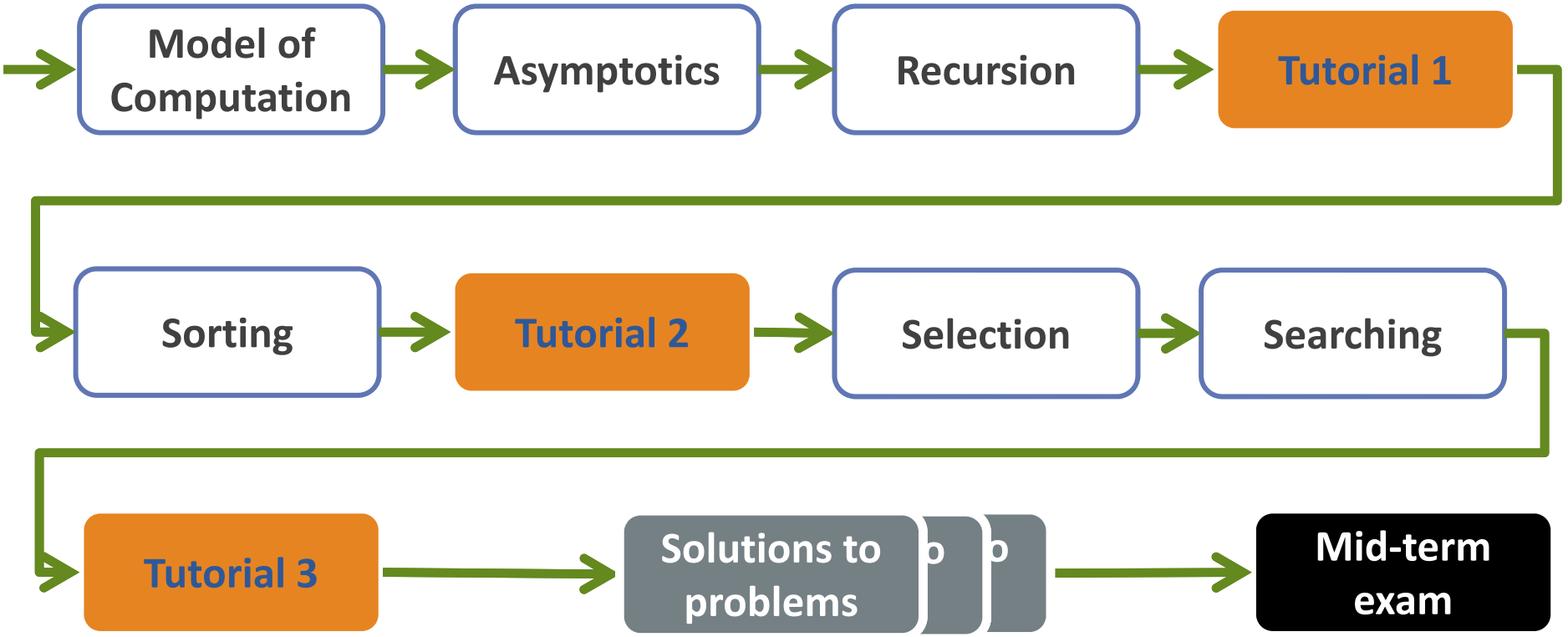
Syllabus


Model of Computation
Algorithm Design & Analysis Techniques
Computation Complexity

Syllabus


Model of Computation
Asymptotics
Recursion
Tutorial 1
Sorting
Tutorial 2
Selection
Searching
Tutorial 3
Solutions to problems
Mid-term exam

Syllabus


Model of Computation
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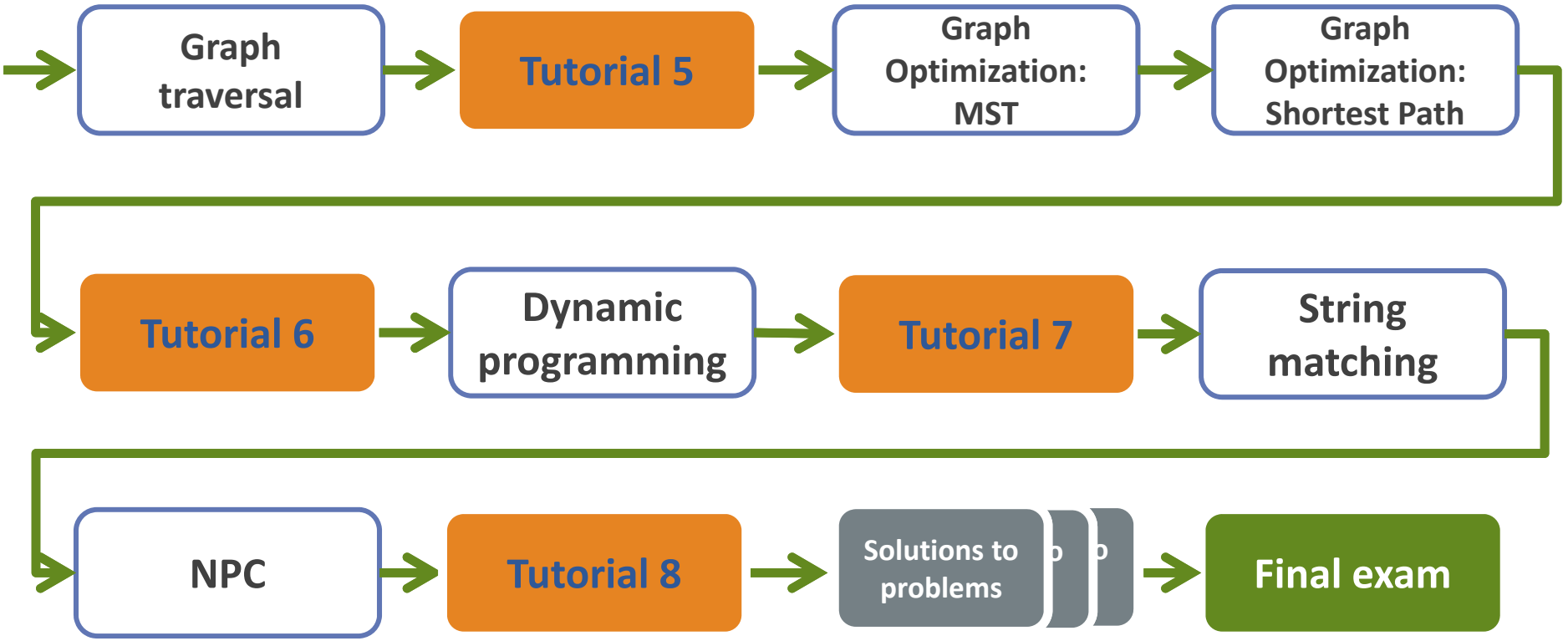
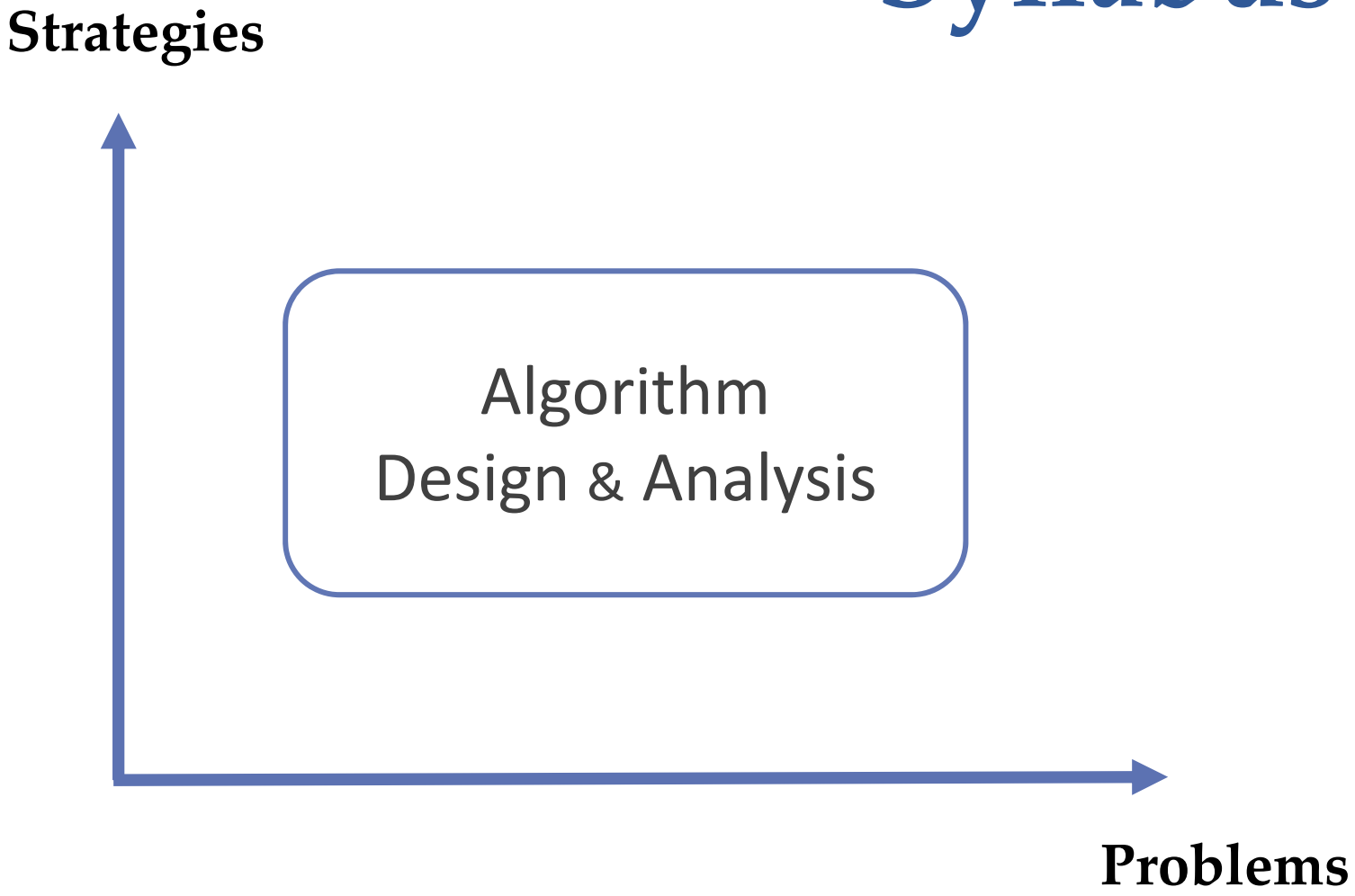
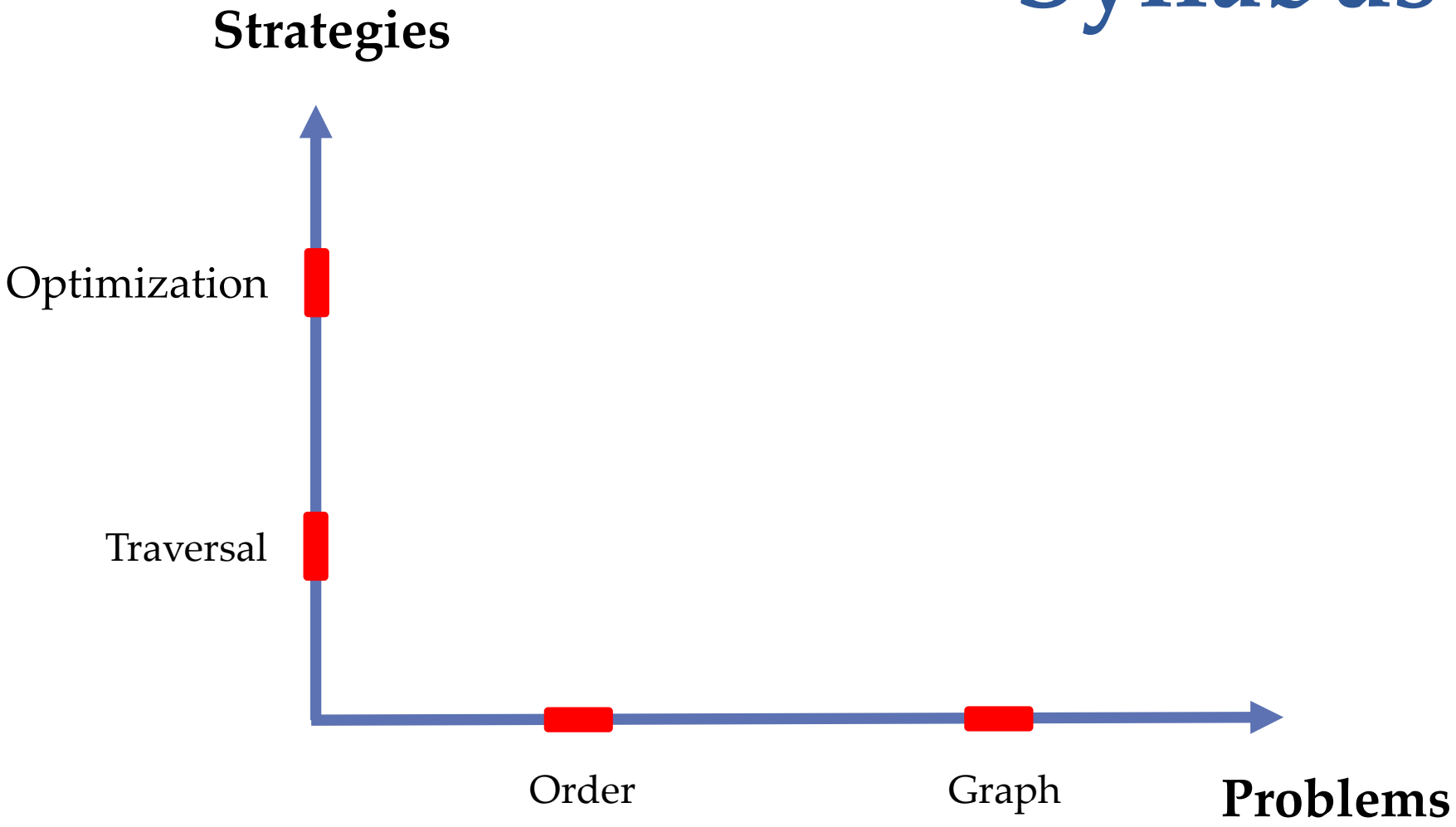
Syllabus


Graph traversal
Tutorial 5
Graph Optimization: MST
Graph Optimization: Shortest Path
Tutorial 6
Dynamic programming
Tutorial 7
String matching
NPC
Tutorial 8
Solutions to problems
Final exam

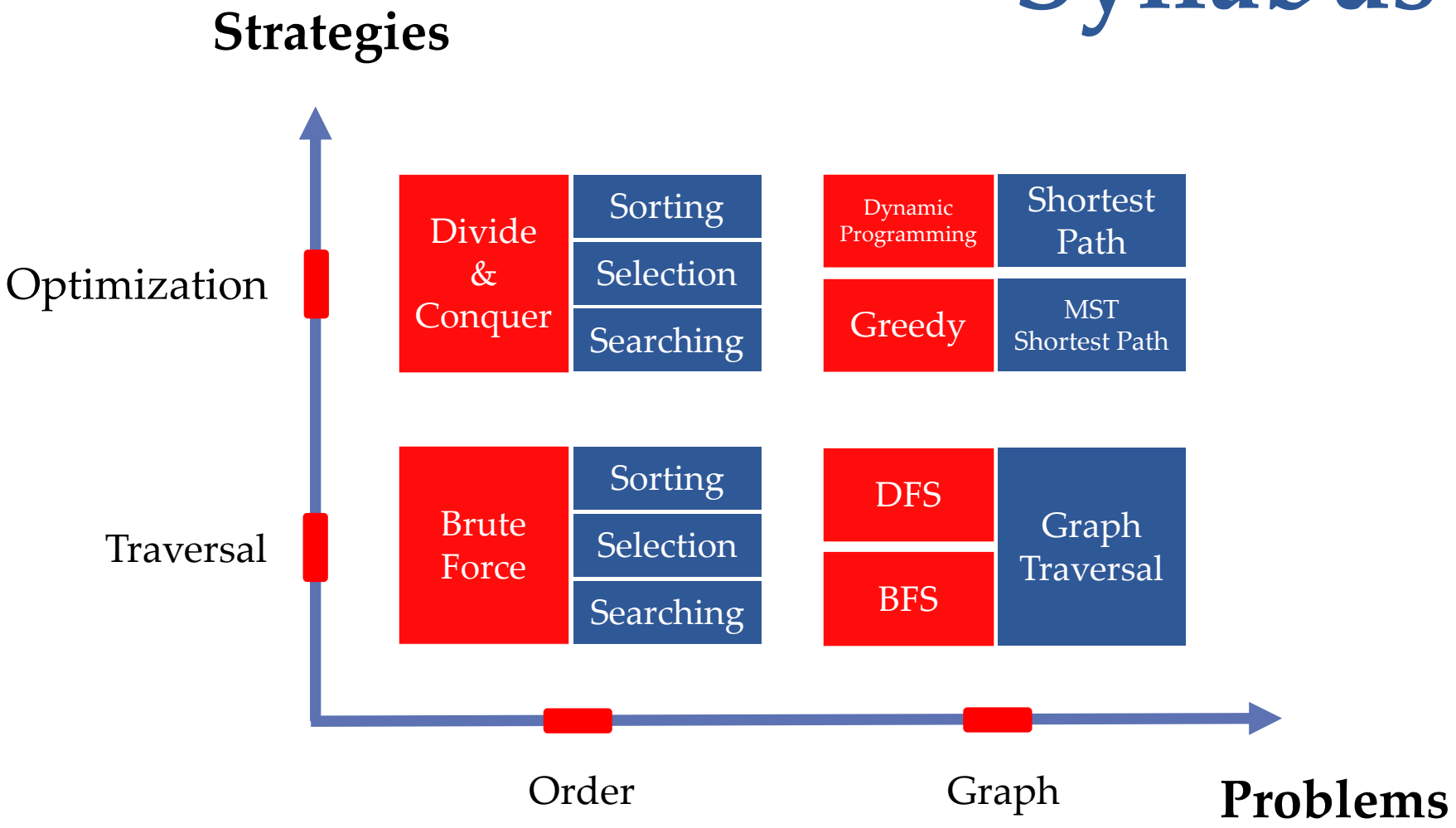
Syllabus



Syllabus

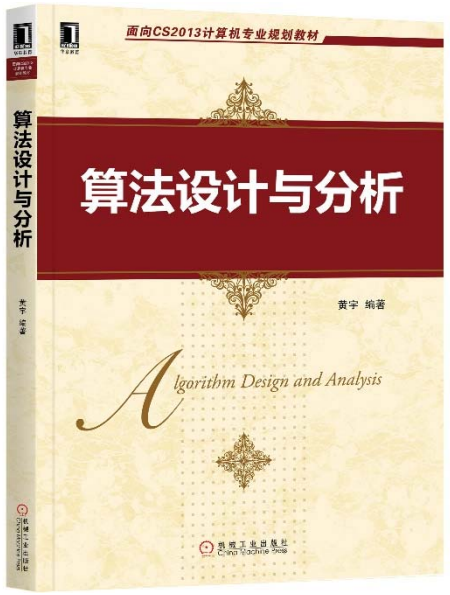


Syllabus


Strategies
Optimization
Traversal
Divide & Conquer
Sorting
Selection
Searching
Dynamic Programming
Shortest Path
Greedy
MST
Shortest Path
Brute Force
Sorting
Selection
Searching
DFS
BFS
Graph Traversal
Order
Graph
Problems

Textbooks

- **Course outline: LADA**
  - Lectures on Algorithm Design & Analysis (slides)
- **Course contents**
  - Algorithm Design and Analysis

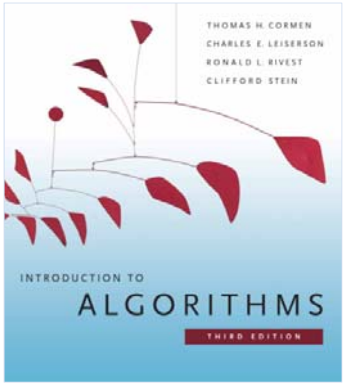


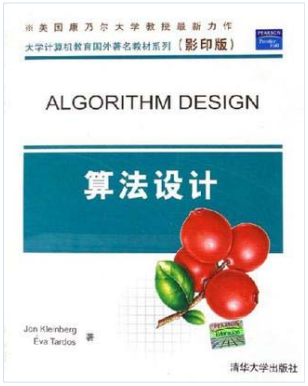
More info about the book:
https://zhuanlan.zhihu.com/p/24150569

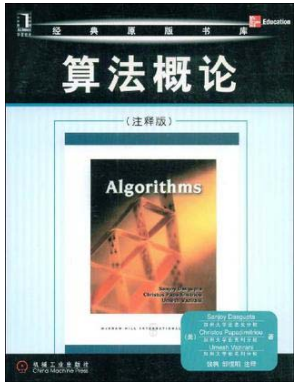
Textbooks

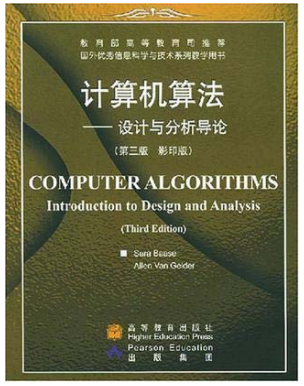
- **Further reading**
  - Introduction to Algorithms
  - Algorithm Design
  - Algorithms
  - Computer Algorithms*

See the "douban list" for more info:
http://book.douban.com/doulist/1155824/









Course Websites

http://www.bigoh.net/JudgeOnline/

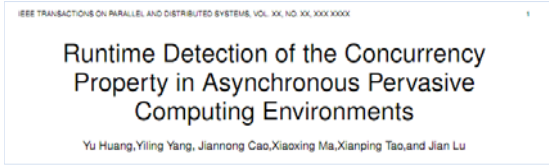
QQ group: 2105 15746

Algorithm – Design & Analysis

- **Algorithm - the spirit of computing**
  - Model of computation
- **Algorithm by example**
  - Greatest common divisor
  - Sequential search
- **Algorithm design & analysis**
  - Correctness
  - Worst-case / average-case cost analysis

Computer and Computing

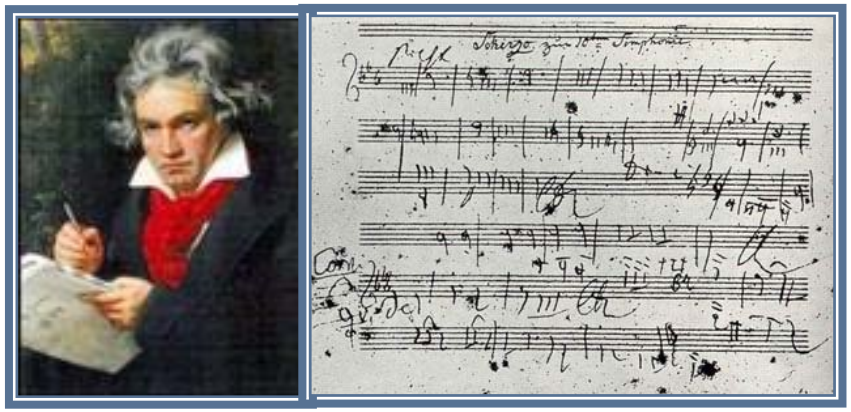
- **Problem 1**
  - Why the computer seems to be able to do anything?
    - Scientific computing, document processing, computer games, ebooks, movies, computer games, …



Computer and Computing

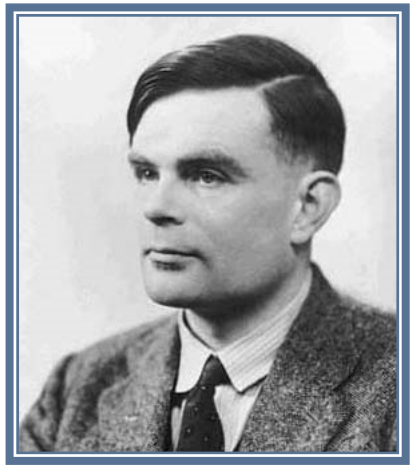
- **Problem 2**
  - What can / cannot be efficiently done by a computer?
    - Manage millions of songs vs. music composition

新歌TOP100			歌曲TOP500	
1	只是太爱你	张敬轩	没那么简单	黄小琥
2	绿旋风	凤凰传奇	走天涯	降央卓玛
3	好朋友只是...	郁可唯	漂亮的姑娘...	龙梅子
4	配角	sara	小三	冷漠
5	有时候	张靓颖	无法原谅	李佯璐
6	曾经太年轻	蓝又时	都要好好的	小沈阳
7	你和我的时...	张惠妹	老男孩	筷子兄弟
8	爱情掉在哪...	井柏然	我相信	杨培安
9	一瞬之光	a-lin	爱的供养	杨幂
10	狂想曲	萧敬腾	春天里	汪峰
更多>>		试听全部	更多>>	试听全部

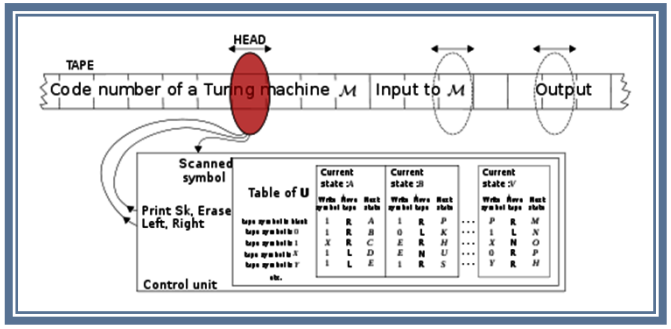


Computer and Computing

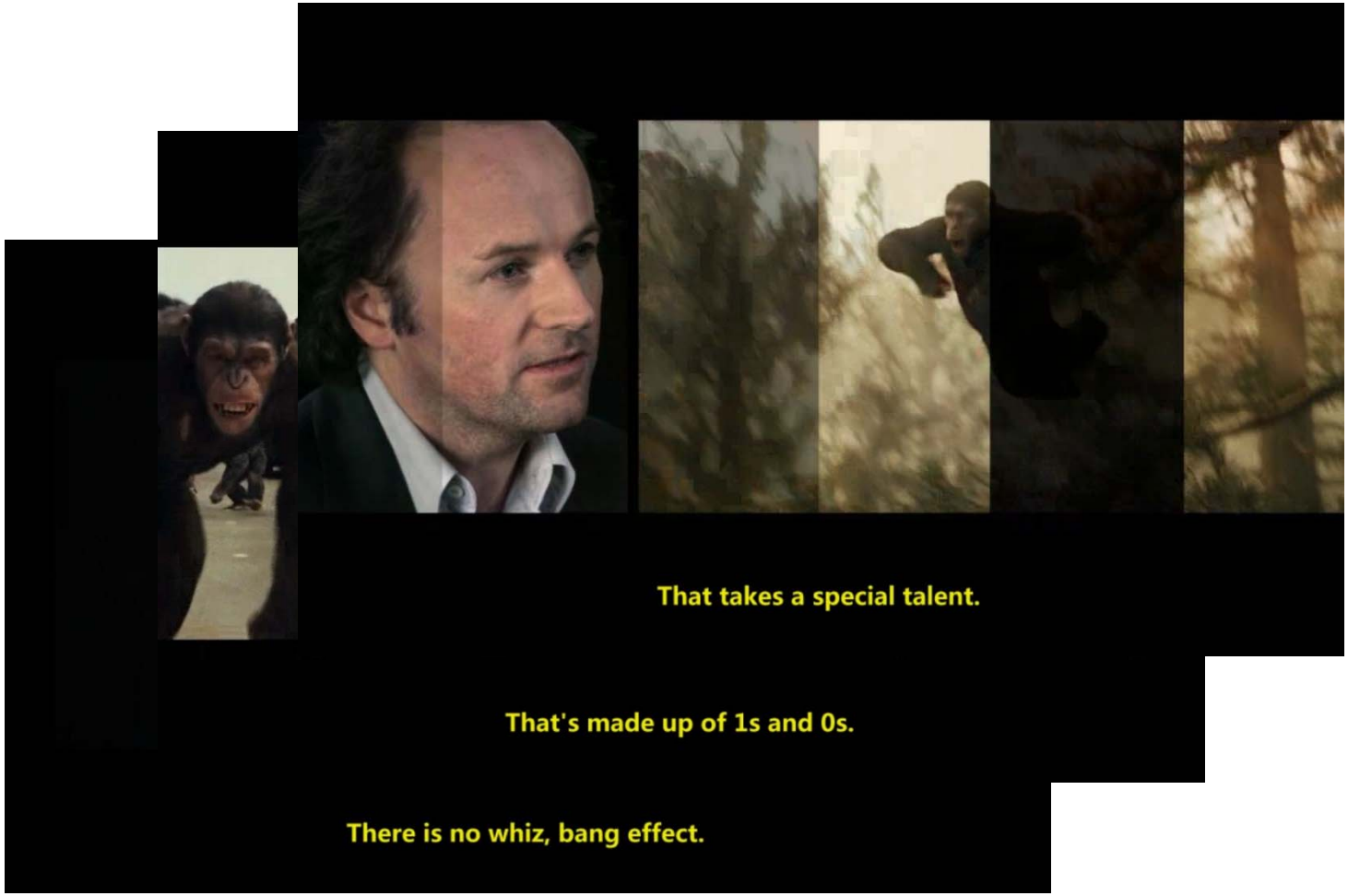
- **Computing**
  - Encoding everything into `0's and `1's
  - Operations over '1's and '0's
  - Decoding the '1's and '0's



- **Turing machine**
  - An abstract/logical computer



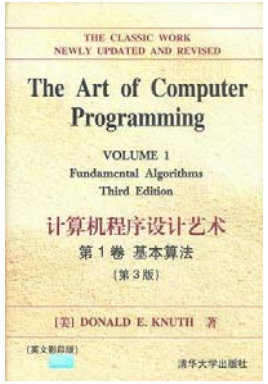
Computing in Everyday Life



Algorithm

Algorithm

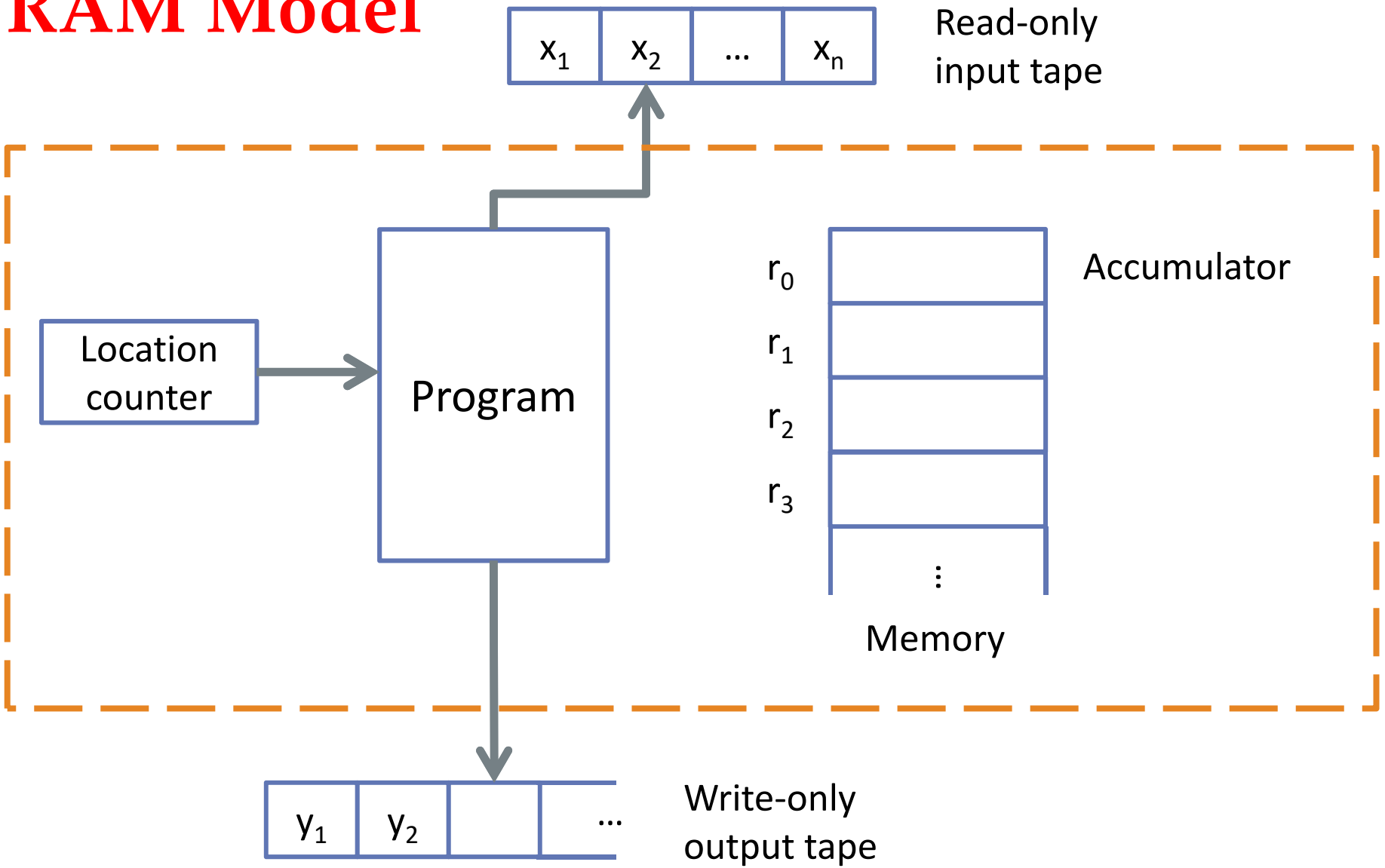
- **Algorithm is the spirit of computing**
  - To solve a specific problem (so called an *algorithmic problem*)
  - Combination of basic operations
    - in a precise and elegant way
- **Essential issues**
  - Model of computation
  - Algorithm design
  - Algorithm analysis



Model of Computation

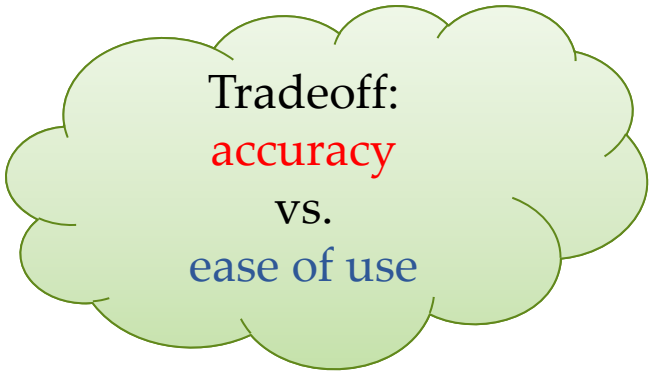
- **Problems**
  - Why the algorithms we learn can run almost everywhere?
  - Why the algorithms we learn can be implemented in any language?
- **Machine- and language- independent algorithms, running on an abstract machine**
  - Turing machine: over-qualify
  - RAM model: simple but powerful

RAM Model


x_1 x_2 ... x_n
Read-only input tape
Location counter
Program
r_0
r_1
r_2
r_3
Accumulator
Memory
y_1 y_2 ...
Write-only output tape

The RAM Model of Computation

- **Each *simple operation* takes one time step**
  - E.g., key comparison, +/-, memory access, …
- **Non-simple operations should be decomposed**
  - Loop
  - Subroutine
- **Memory**
  - Memory access is a simple operation
  - Unlimited memory



Further Reading

“哼，你让他们成楔形攻击队形不就行了？”秦始皇轻蔑地看着冯·诺伊曼。牛顿不知从什么地方掏出六面小旗．三白三黑，冯·诺伊曼接过来分给三名士兵，每人一白一黑，说：“白色代表0，黑色代表1。好，现在听我说，出，你转身看着入1和入2，如果他们都举黑旗，你就举黑旗，其他的情况你都举白旗，这种情况有三种：入1白，入2黑；入1黑，入2白；入1、入2都是白。”

“不需要，我们组建一千万个这样的门部件，再将这些部件组合成一个系统，这个系统就能进行我们所需要的运算，解出那些预测太阳运行的微分方程。这个系统，我们把它叫做……嗯，叫做……”

“计算机。”汪淼说。

“啊——好！”冯·诺伊曼对汪淼竖起一根指头，“计算机，这个名字好，整个系统实际上就是一部庞大的机器，是有史以来最复杂的机器！”

刘慈欣，《三体、牛顿、冯·诺依曼、秦始皇、三日连珠》，《三体》第一部

To Create an Algorithm

- **Algorithm design**
  - Composition of simple operations, to solve an algorithmic problem
- **Algorithm analysis**
  - Amount of work done / memory used
    - In the worst/average case
  - Advanced issues
    - Optimality, approximation ratio, …

Algorithm by Example

- **Algorithmic Problem 1**
  - Find the greatest common divisor of two non-negative integers *m* and *n*

- **Algorithmic Problem 2**
  - Is a specific key *K* stored in array E[1..n]?

Probably the Oldest Algorithm

- **Euclid Algorithm**

Problem
- Find the greatest common divisor of two non-negative integers *m* and *n*

Specification

Input: non-negative integer m, n
Output: gcd(m, n)

Euclid algorithm

[E1] n divides m, the remainder -> r
[E2] if r = 0 then return n
[E3] n -> m; r-> n; goto E1

Euclid algorithm – recursive version

Euclid(m,n)
[E1] if n=0 then return m
[E2] else return Euclid(n, m mod n)

Sequential Search

Problem
- Search an array for a specific key

Specification

Input: K, E[1..n]
Output: Location of K (1,2,…,n; -1: K is not in E[])

Sequential searchEuclid algorithm

```
Int seqSearch(int[] E, int n, int K)
   int ans, index;
   ans=-1;
   for (index=1; index<=n; index++)
      if (K==E[index])
         ans=index;
         break;
   Return ans;
```

Algorithm Design

- **Criteria**
  - Defining correctness
- **Main challenge**
  - For proving correctness
- **Our strategy**
  - Mathematical induction
  - …

Specification

Input: non-negative integer m, n
Output: gcd(m, n)

Main challenge

- The output is **always** correct, for **any** legal input.
- Infinite possible inputs

Mathematical induction

- Weak principle
- Strong principle

For Your Reference

- **Mathematical induction**

The Weak Principle of Mathematical Induction

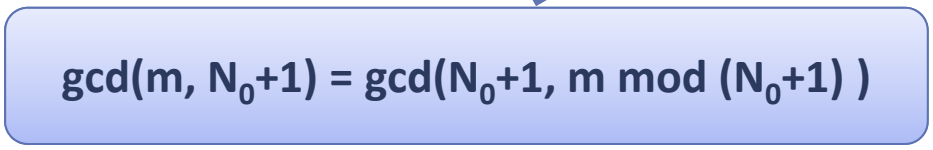
- If the statement p(b) is true and the statement p(n-1) => p(n) is true for all n>b, then p(n) is true for all integers n>=b.

The Strong Principle of Mathematical Induction

- If the statement p(b) is true, and the statement {p(b) and p(b+1) and … and p(n-1) => p(n)} is true, for all n>b, then p(n) is true for all integers n>=b.

Correctness of the Euclid Algorithm

- **Induction on n**
  - Base case
    - n = 0: for any m, Euclid(m, 0) = m;
    - n = 1: for any m, Euclid(m, 1) = 1;
    - n = 2: …
  - Assumption
    - For any $n \le N_0$, Euclid(m, n) is correct;
  - Induction
    - Euclid(m, N_0+1) = Euclid(N_0+1, m mod (N_0+1));



Notes on Induction

"Notes on Structured Programming", E.W. Dijkstra

I have mentioned **mathematical induction** explicitly, because it is the only pattern of reasoning that I am aware of, that eventually enables us to cope with loops and recursive procedures

Algorithm Analysis

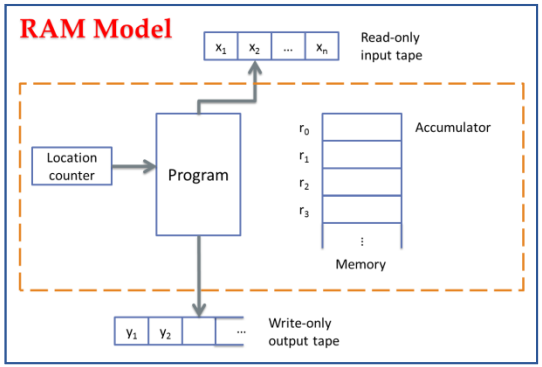
- **Criteria**
  - Performance metrics
- **Worst case**
  - Best case?
- **Average case**
  - Average cost?
- **Advanced topics**
  - Lower bound, optimality, …

Algorithm Analysis

- **How to measure**
  - Not too general
    - Giving essential indication in comparison of algorithms
  - Not too precise
    - Machine independent
    - Language independent
    - Programming paradigm independent
    - Implementation independent

Algorithm Analysis

- **Criteria**
  - Critical operation
  - How many critical operation are conducted
- **For example**



Algorithmic problem	Critical operation
Sorting, selection, searching String matching	Comparison (of keys)
Graph traversal	Processing a node/edge
Matrix multiplication	Multiplication

Algorithm Analysis

- **Amount of work done**
  - usually depends on size of the input
  - usually does not depend on size of the input only

Worst-case Complexity

- **W(n)**
  - Upper bound of cost
    - For any possible input
  - $W(n) = \max_{I \in D_n} f(I)$

Average-case Complexity

- **A(n)**
  - Weighted average
  - $A(n) = \sum_{I \in D(n)} \Pr(I)\, f(I)$
- **A special case**
  - Average cost
    - Total cost of all inputs, averaged over the input size
  - $Average(n) = \frac{1}{|D(n)|} \sum_{I \in D(n)} f(I)$

Average-case Cost of *SeqSearch*

- **Case 1: K is in E[]**
  - Assumptions:
    1. Assuming that K is in E[]
    2. Assuming no same entries in E[]
    3. Each possible input appears with equality (thus, K in the i^{th} location with probability $\frac{1}{n}$)

  - $A_{succ}(n) = \sum_{i=0}^{n-1} \Pr(I_i|succ)\, t(I_i)$

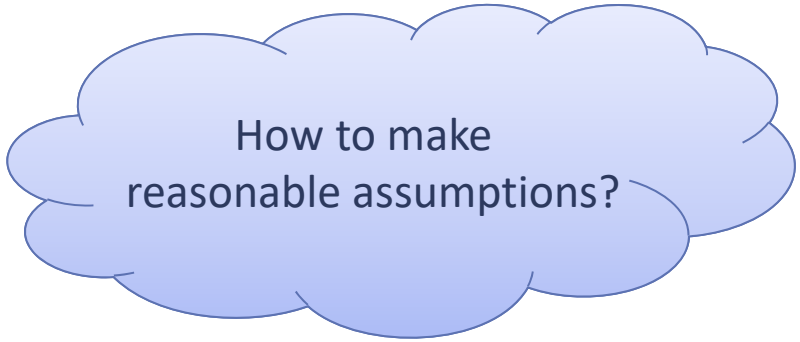
    $= \sum_{i=0}^{n-1} \frac{1}{n}(i+1)$

    $= \frac{n+1}{2}$

Average-case Cost of *SeqSearch*

- **Case 2: K may (or may not) be in E[]**
  - Assume that K is in E[] with probability q
  - $A(n) = \Pr(succ)\, A_{succ}(n) + \Pr(fail)\, A_{fail}(n)$

    $= q\frac{n+1}{2} + (1-q)n$



Algorithm Analysis

- **Advanced topics**
  - Lower bound (Selection)
  - Optimality (Greedy, DP)
  - Computation complexity
  - Approximate / online / randomized algorithms

Thank you!

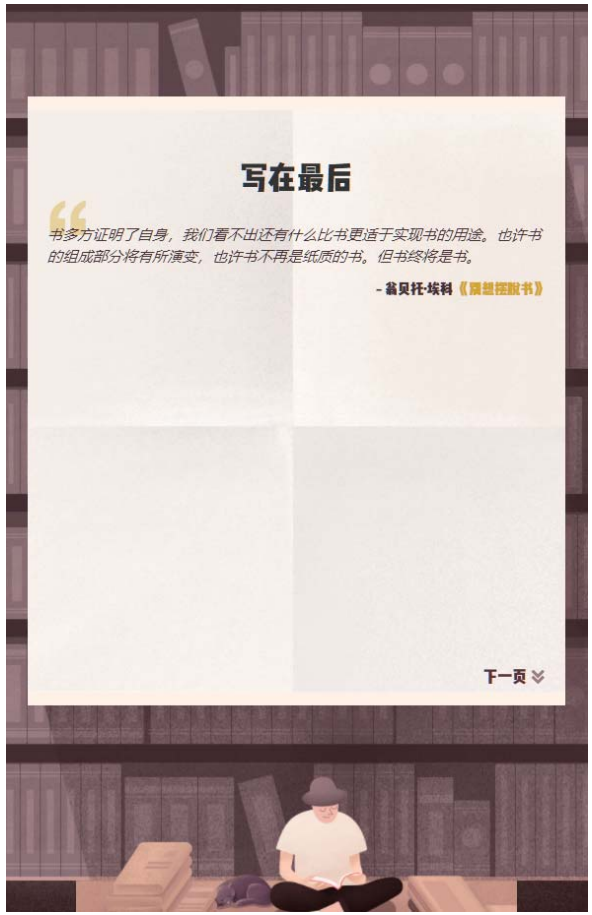
Q & A

Yu Huang
http://cs.nju.edu.cn/yuhuang

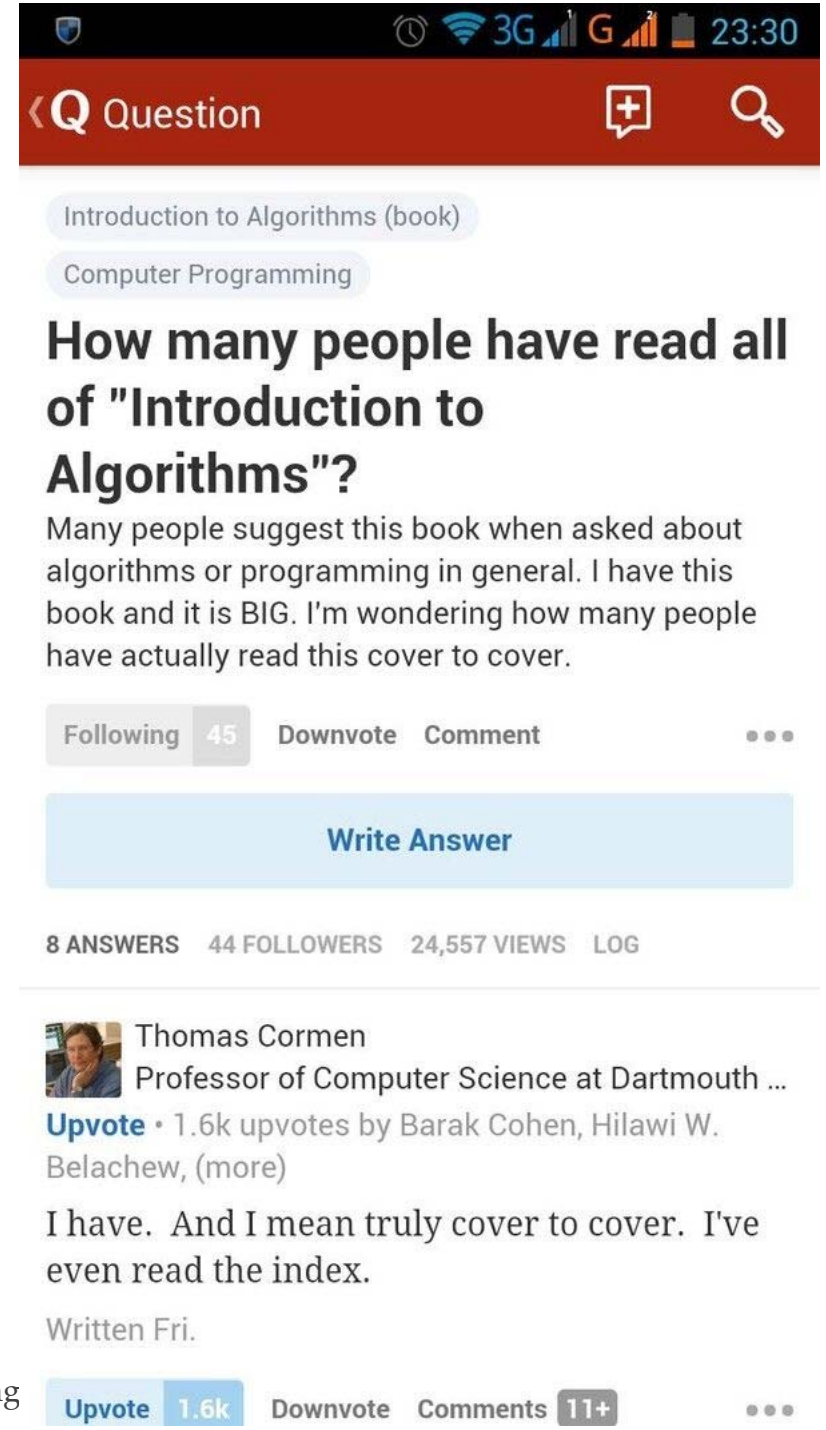
Appendix

也许计算的设备会变得天翻地覆，但是算法终将是算法

– 黄宇 《算法设计与分析讲义》



Appendix


23:30
Question
Introduction to Algorithms (book)
Computer Programming
How many people have read all of "Introduction to Algorithms"?
Many people suggest this book when asked about algorithms or programming in general. I have this book and it is BIG. I'm wondering how many people have actually read this cover to cover.
Following 45
Downvote
Comment
Write Answer
8 ANSWERS
44 FOLLOWERS
24,557 VIEWS
LOG
Thomas Cormen
Professor of Computer Science at Dartmouth ...
Upvote • 1.6k upvotes by Barak Cohen, Hilawi W. Belachew, (more)
I have. And I mean truly cover to cover. I've even read the index.
Written Fri.
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Comments 11+