

概率论第四次作业

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习题三

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3.2

$$(1) P(X < 0, Y \leq 0) = P(X = -1, Y = -1) + P(X = -1, Y = 0) = a = 0.2.$$

由所有概率的和为1,

$$\Rightarrow b = 0.2;$$

(2) Z的可能取值为-2, -1, 0, 1

$$P(Z = -2) = P(X = -1, Y = -1) = 0.2;$$

$$P(Z = -1) = P(X = 0, Y = -1) = 0.1;$$

$$P(Z = 0) = P(X = -1, Y = 1) + P(X = 0, Y = 0) = 0.4;$$

$$P(Z = 1) = P(X = 0, Y = 1) + P(X = 1, Y = 0) = 0.3;$$

$$(3) P(X = Z) = P(X = X + Y) = P(Y = 0) = 0.3;$$

3.3

$$P(X + Y = 2) = P(X = 1, Y = 1) + P(X = 2, Y = 0) + P(X = 3, Y = -1).$$

由于随机变量X和Y相互独立, 上式可化为

$$P(X = 1)P(Y = 1) + P(X = 2)P(Y = 0) + P(X = 3)P(Y = -1) = \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{8} \cdot \frac{1}{2} + \frac{1}{8} \cdot \frac{1}{4} = \frac{5}{32}.$$

3.4

$$(1) P(X > 2Y) = \int \int_{x > 2y} p(x, y) dx dy = \int_0^1 dx \int_0^{\frac{x}{2}} (2 - x - y) dy = \int_0^1 (x - \frac{5}{8}x^2) dx = \frac{7}{24}.$$

$$(2) p_Z(z) = \int_{-\infty}^{+\infty} p(x, z - x) dx;$$

$$\text{当 } 0 < z \leq 1 \text{ 时, } p_Z(z) = \int_0^z (2 - x - (z - x)) dx = \int_0^z (2 - z) dx = (2 - z)x|_0^z = z(2 - z);$$

当 $1 < z \leq 2$ 时,

$$p_Z(z) = \int_{z-1}^1 (2 - x - (z - x)) dx = \int_{z-1}^1 (2 - z) dx = (2 - z)x|_{z-1}^1 = 2 - z - (2 - z)(z - 1) = (2 - z)^2;$$

;

对于其他情况, $p_Z(z) = 0$.