## Shoelace formula

for a polygon with n vertices $(x_0, y_0)$ $(x_1, y_1)$ ...

The signed area is computed as:

$$A = \frac{1}{2}\left[\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)\right] \quad \text{where } (x_{n+1}, y_{n+1}) = (x_0, y_0)$$

For triangles with 3 vertices

$(x_0, y_0)$, $(x_1, y_1)$, $(x_2, y_2)$

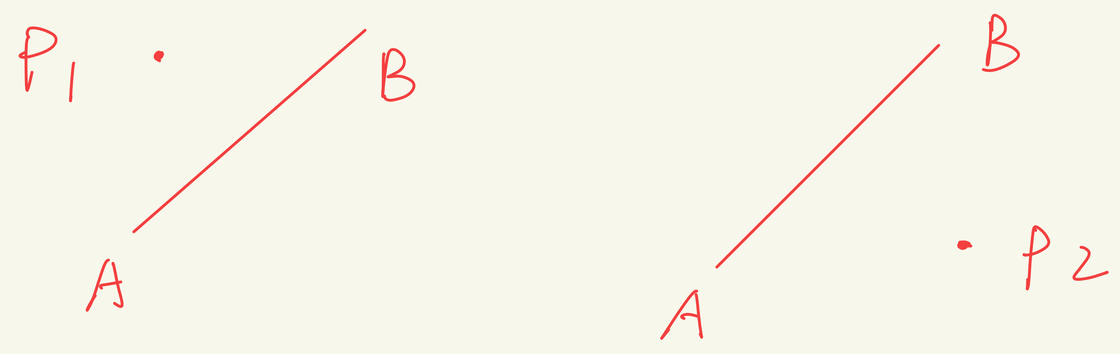
$$A = \frac{1}{2}(x_0 y_1 - x_1 y_0 + x_1 y_2 - x_2 y_1 + x_2 y_0 - x_0 y_2)$$

we can define an Edge function

$$E = x_0 y_1 - x_1 y_0 + x_1 y_2 - x_2 y_1 + x_2 y_0 - x_0 y_2$$

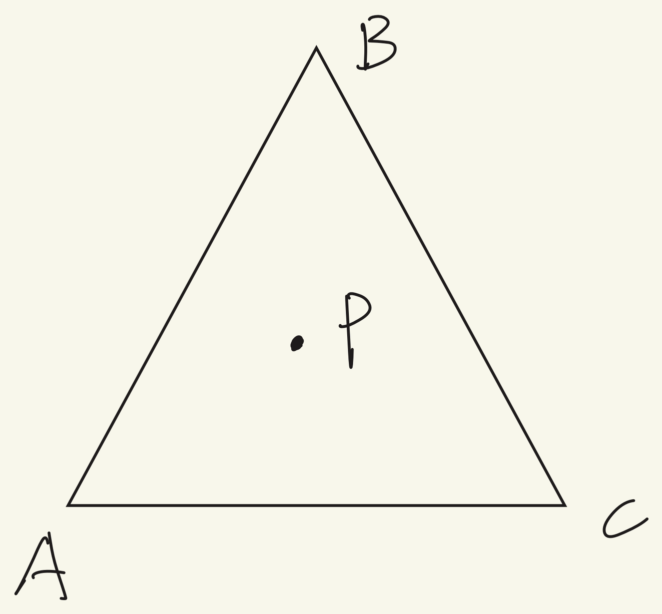
The nice thing about Edge function is:

Given A and B, its value (+/-) depends on which side P resides.




$$\begin{cases} E(A, B, P_1) < 0 \\ E(A, B, P_2) > 0 \end{cases}$$

Based on this nice property, we can easily use E to determine if a point falls inside a triangle.



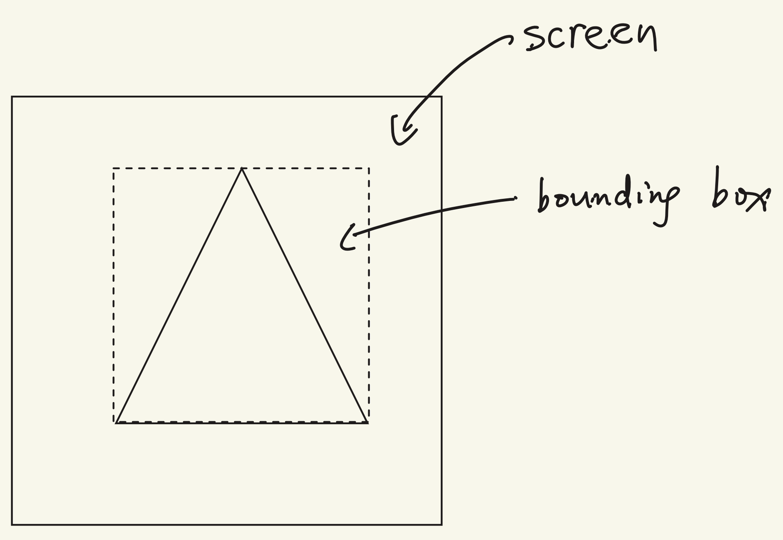

Calculate 3 edge functions

$$\begin{cases} E(A, B, P) = \lambda_1 \\ E(B, C, P) = \lambda_2 \\ E(C, A, P) = \lambda_3 \end{cases}$$

P is inside $\triangle ABC$ if $\lambda_1 \geq 0$ & $\lambda_2 \geq 0$ & $\lambda_3 \geq 0$

That's it!

To make the rasterization more efficient, we can check if a pixel is within the bounding box formed by the triangle.




one more thing, what is BaryCentric Coordinate?

$$\begin{cases} \lambda_1 = E(A, B, P) \\ \lambda_2 = E(B, C, P) \\ \lambda_3 = E(C, A, P) \\ \lambda_4 = E(A, B, C) \end{cases}$$

$$\frac{\lambda_1}{\lambda_4} + \frac{\lambda_2}{\lambda_4} + \frac{\lambda_3}{\lambda_4} = 1$$

$$\Downarrow$$

$$W_1 + W_2 + W_3 = 1$$

These weights can be used to do things like color interpolation.